TMAT/AMAT 118 Diagnostic Quiz 1 (not for credit)

Name: ____________________________

1. Evaluate each expression:

   (a) -4^3 **Answer**: -64
   (b) 4^{-3} **Answer**: $\frac{1}{64}$
   (c) $8^{-4/3}$ **Answer**: $\frac{1}{16}$

2. Simplify the expression $\left(\frac{x^{3/2}y^3}{x^{-1/2}y^{-1}}\right)^{-2}$. **Answer**: $\frac{1}{x^4y^8}$

3. Factor the polynomial $x^4 - 5x^3 + 6x^2$. **Answer**: $x^2(x-3)(x-2)$

4. Simplify the expression $\frac{\frac{x}{y}-\frac{y}{x}}{\frac{1}{y}-\frac{1}{x}}$. **Answer**: $x+y$

5. Solve the following equations:

   (a) $3|x-4| = -9$, **Answer**: No solution exists: The product of two non-negative numbers is never negative. A common incorrect solution was $x = 1, 7$. This is what you get if you apply the usual solution method, but forget an important step.
   (b) $-2x(4-x)^{-1/2} + 3\sqrt{4-x} = 0$. **Answer**: $\frac{5}{12}$

6. Solve the inequality $\frac{2x-3}{x+1} \leq 1$. Write your answer using interval notation. **Answer**: $(-1, 4]$. Many students put $(-\infty, 4]$, which is wrong. The key to this problem is to remember that if you multiply both sides of an inequality by a negative number, then the direction of the inequality sign flips. So you need to break up the solution process into two cases. First, assume that $x+1$ is positive, i.e. $x > -1$, and solve. Then assume $x+1$ is negative, i.e. $x < 1$, and solve.

7. State whether each equation is true for all possible vales of x and y. (Write true or false).

(a) $(x+y)^2 = x^2 + y^2$ **Answer**: False.

(b) $(xy)^{1/3} = x^{1/3}y^{1/3}$ **Answer**: True. It's true for any x, y, $a > 0$ that $(xy)^a = x^a y^a$.

(c) $\sqrt{x^2 + y^2} = |x| + |y|$ **Answer**: False

(d) $\frac{1+xy}{y} = \frac{1}{y} + x$. (Assume $y \neq 0$.) **Answer**: True

(e) $\frac{1}{x-y} = \frac{1}{x} - \frac{1}{y}$. (Assume $x \neq 0$, $y \neq 0$, and $x - y \neq 0$.) **Answer**: False.

8. Find the equation for the line that:

(a) passes through the points $(1, 2)$ and $(0, 1)$, **Answer**: $y = x + 1$

(b) passes through (1,2) and is vertical, **Answer**: $x = 1$

(c) passes through (1,2) and is parallel to the line $y = x$. **Answer**: $y = x + 1$

9. Find the equation for the circle which has the line segment from $(1, 1)$ to $(-1, -1)$ as a diameter. **Answer**: $x^2 + y^2 = 2$. The center of the circle is the midpoint of the line segment, which is $(0, 0)$. By the pathagorian theorem the radius of the circle is $\sqrt{2}$. The result now follows from the standard formula for the equation of a circle.

10. Sketch the region in the xy-plane defined by the inequalities $x^2 \leq y \leq 1$.