MATH 4242 SEC 40 Quiz 9 Solutions

Name: ______________________

Question 1. Let $B = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$. Note that B is an (ordered) basis for $\mathbb{R}^2$. Find $\left[\begin{pmatrix} 2 \\ 4 \end{pmatrix} \right]_B$, the coordinate representation of $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ with respect to B.

Answer: $\left[\begin{pmatrix} 2 \\ 4 \end{pmatrix} \right]_B = \begin{pmatrix} a \\ b \end{pmatrix}$, where

$$a \begin{pmatrix} 1 \\ 1 \end{pmatrix} + b \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

Solving the system via the usual method we find $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

Question 2. Let $M^{2\times 2}$ denote the usual vector space of 2×2 matrices with coefficients in R. Let

$$\mathcal{S} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \right\}$$

denote the standard (ordered) basis for $M^{2\times 2}$. Let

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

For $T : M^{2\times 2} \to M^{2\times 2}$ the linear map given by $T(X) = AX$, find $[T]_{\mathcal{S}}$, the coordinate representation of T with respect to $\mathcal{S}$.

Answer: Denote the elements of $\mathcal{S}$ as b_1, b_2, b_3, b_4, respectively. Then

$$[T]_{\mathcal{S}} = \left([T(b_1)]_{\mathcal{S}} \,|\, [T(b_2)]_{\mathcal{S}} \,|\, [T(b_3)]_{\mathcal{S}} \,|\, [T(b_4)]_{\mathcal{S}} \right).$$

$$T(b_1) = b_1, \quad T(b_2) = b_2, \quad T(b_3) = b_1 + b_3, \quad T(b_4) = b_2 + b_4.$$

Thus

$$[T]_{\mathcal{S}} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$