Math 4242 Sec 40 Quiz 4

1. (2 points) Does there exist an invertible diagonal matrix A such that $A_{ii} = 0$ for some i? If so, give an example of such a matrix A. If not, show that such a matrix cannot exist.

Answer: There does not exist such a matrix: If A is a diagonal matrix with $A_{ii} = 0$ then in fact $A_{*i} = \vec{0}$. If A is invertible then by the element-wise definition of matrix multiplication, $(A^{-1}A)_{ii} = A^{-1}_{i*}A_{*i} = A^{-1}_{i*}\vec{0} = 0$. But by the definition of an inverse, we must have $A^{-1}A = I$, and in particular $(A^{-1}A)_{ii} = 1$, contradicting the above. We conclude that an inverse of A cannot exist.

2. (3 points) For

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix},$$

Find a matrix E such that

$$EA = \begin{pmatrix} 3 & 3 & 3 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}.$$

Answer: We obtain the second matrix from A by first switching rows 1 and 2 in A, and then switching rows 3 and 1 in the resulting matrix. Therefore $E = E_2E_1$, where

$$E_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Thus,

$$E = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

3. (2 points) For E as in problem 2, what is E^{-1}? [Hint: Recall from class that for P an elementary matrix of type one, $P^{-1} = P$.]

Answer: $E^{-1} = (E_2E_1)^{-1} = E_1^{-1}E_2^{-1}$. From the hint, $E_1^{-1} = E_1$ and $E_2^{-1} = E_2$, so

$$E^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

4. (3 points) By performing Gaussian Elimination on the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 6 & 2 \\ -2 & 2 \end{pmatrix}.$$

Find a matrix E such that EA is in row echelon form.

[Hint: What elementary row operations do I need to do in order to put A into row echelon form? What are the matrix representatives of these row operations?]

Answer: To put A in row echelon form via Gaussian Elimination, we perform the following 3 elementary matrix operations:

(1) add -3 times the first row to the second row,
(2) add the first row to the third row,
(3) add 3 times the second row to the third row.

Thus we may take

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -8 & 3 & 1 \end{pmatrix}.$$