Math 4242 Sec 40 Quiz 3

Name: ______________________

1. (2 points) Which of the following maps $F : \mathbb{R}^2 \to \mathbb{R}^2$ is linear?
(Select all.)

$$F(x,y) = \begin{pmatrix} x \\ y^2 \end{pmatrix}, \qquad F(x,y) = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad F(x,y) = \begin{pmatrix} x \\ 1 \end{pmatrix}, \qquad F(x,y) = \begin{pmatrix} x \\ 0 \end{pmatrix}.$$

Answer: Simple examples show that the first and third functions do not satisfy either property 1) or property 2) in the definition of linearity. On the other hand, it can be verified directly from the definition of a linear function that the second and forth functions are linear.

2. (2 points) Let $F : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear function given by

$$F(x,y,z) = \begin{pmatrix} x+z \\ y+z \end{pmatrix}.$$

Find a 2×3 matrix A such that for any $\vec{v} \in \mathbb{R}^3$, $F(\vec{v}) = A\vec{v}$.

Answer: The quick way to do this is to observe

$$\begin{pmatrix} x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 1x & +0y & +1z \\ 0x & +1y & +1z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

Thus $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$.

For a more conceptual solution, we may observe that we are being asked for the matrix A such that $F = T_A$, for T_A as defined in class. Now $[F] = [T_A] = A$, where the last equality was shown in class. Thus to obtain A, we just need to compute

$$[F] = (F(\vec{e}_1) \mid F(\vec{e}_2) \mid F(\vec{e}_3)) = \left(F\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \middle| F\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \middle| F\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

3. (2 points) Let

$$A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 0 & 3 \\ 1 & 1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 1 \\ 1 & 3 & 0 \end{pmatrix}.$$

Compute the matrix product AB.

4. (2 points) For A, B as in problem 3, find $c_1, c_2, c_3 \in \mathbb{R}$ such that $(AB)_{*2} = c_1 A_{*1} + c_2 A_{*2} + c_3 A_{*3}$.

Answer: Although you can set up a linear system to solve for c_1, c_2, c_3, the quickest way to solve this problem is to consider the column-wise interpretation of matrix multiplication. From this, it is immediately clear that

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = B_{*2} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

is a solution.

5. (2 points) Compute the matrix product

$$\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$$

Answer:

$$\begin{pmatrix} 2 & 4 & 6 \\ 0 & 0 & 0 \\ 2 & 4 & 6 \end{pmatrix}.$$