Math 4242 Sec 40 — Quiz 10: Last Quiz!!!

Name: ______________________

Question 1. (3 points) Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$. Compute the determinant of A.

Answer: By the formula from class, the answer is 1*-1+0*1+1*2=1.

Question 2. (1 point) Let $B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$. Building on your work from Question 1, give the determinant of B.

Answer: B is obtained from A by flipping the first and second rows, so by the properties of the determinant mentioned in class or in Meyer, det(B)=-det(A)=-1.

Question 3. (3 points) Let $\mathrm{Id}_{\mathbb{R}^3} : \mathbb{R}^3 \to \mathbb{R}^3$ denote the identity map. What are the eigenvalues and eigenvectors of $\mathrm{Id}_{\mathbb{R}^3}$? [Hint: if you understand the definitions, you shouldn't have to do a big computation to answer this.]

Answer: Every $\vec{v} \in \mathbb{R}^3$ satisfies $\mathrm{Id}_{\mathbb{R}^3}(\vec{v}) = \vec{v}$. Hence 1 is an eigenvalue for $\mathrm{Id}_{\mathbb{R}^3}$, and it is the only eigenvalue. Further, every non-zero vector in $\mathbb{R}^3$ is a 1-eigenvector of $\mathrm{Id}_{\mathbb{R}^3}$. $\vec{0}$ is not an eigenvector, by definition, so the eigenvectors of $\mathrm{Id}_{\mathbb{R}^3}$ are the non-zero vectors in $\mathbb{R}^3$.

Question 4. (3 points) By writing down and solving the characteristic polynomial, find all of the eigenvalues of $\begin{pmatrix} 3 & 0 \\ 1 & 4 \end{pmatrix}$.

Answer: $c_A(\lambda) = det \begin{pmatrix} 3-\lambda & 0 \\ 1 & 4-\lambda \end{pmatrix} = (3-\lambda)(4-\lambda)$. Setting $c_A(\lambda) = 0$, we find that the eigenvalues of A are 3 and 4.