Endogenous Liquidity and the Business Cycle*

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Abstract

I present a model in which asymmetric information in the quality of capital endogenously determines the degree of liquidity in an economy. Liquidity is used to relax financial constraints that affect investment and employment decisions. I explain how liquidity is determined by the wedges induced by financial frictions and, in turn, how these wedges depend on liquidity.

Unlike real business cycle theory, aggregate fluctuations can be attributed to both mean preserving spreads in the quality of capital and real shocks. Quantitatively, the model generates sizeable recessions similar in magnitude to the financial crisis of 2008-2009.

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1 Introduction

The recent financial crisis began with an abrupt collapse in liquidity. A common view is that the crisis arose when lenders found it difficult to distinguish the value of collateral assets. This shortfall in liquidity would have spread to the real economy as firms were restrained from accessing funds required to hire workers and invest. The overall outcome of the crisis was the deepest recession of the post-war era.

This paper develops a theory to formalize and quantify this chain of events. The theory builds on the interaction of two frictions: *limited enforcement* in contractual agreements and *asymmetric information* about the quality of capital. On one hand, limited enforcement prevents agents from carrying out transactions solely with a promise of repayment. This market imperfection imposes constraints on the possible contractual agreements and necessitates the use of assets as collateral. Asymmetric information, on the other hand, induces a shadow cost on collateralizing assets that increases as the quality of assets becomes more disperse. Theoretically, the paper characterizes, in general equilibrium, the firm's strategic decision to use assets of privately known quality as collateral to relax enforcement constraints. Quantitatively, a calibration exercise assesses the effects of a mean preserving spread in collateral quality. This shock is tailored to induce a fall in aggregate liquidity of similar magnitude as the observed reduction in issued syndicated loans or asset-backed commercial paper during 2008. The model explains a considerable fraction of the reduction in output, employment and hours observed in the data, although the shock has no effect on the economy's production possibility frontier or the wealth distribution.

The model makes two contributions to the literature. First, limited enforcement is introduced to both labor contracts (or, more broadly interpreted as variable inputs) and claims to investment goods in the spirit of Hart and Moore [1994]. Limited enforcement in labor contracts prevents firms from hiring workers without securing some portion of their payroll with liquid funds. In reality, payroll and inventories are partly financed by short-term instruments.[1] In fact, a key policy concern during the crisis was that the lack of liquid funds would cause firms to demand less employment. This feature distinguishes the model from the bulk of the literature on financial frictions, which focuses on financial frictions that affect investment decisions. Investment frictions alone cannot generate strong output responses to fundamental shocks. Consequently, other non-financial frictions that indirectly affect employment decisions are typically introduced.[2] By incorporating limited enforcement on labor

[1]This fact is very well documented in the corporate finance literature. For references, see Berk and DeMarzo [2007] or Anderson and Gascon [2009].

[2]Shocks that exacerbate financial friction on investment resemble investment shocks. Barro and King [1984] found that investments shocks are not an important source of business cycle fluctuations. Since then,

contracts, the model provides a mechanism by which liquidity affects employment directly and leads to a strong output response without resorting to nominal rigidities.

Second, the paper explains how asymmetric information affects the strategic decision to use assets as collateral to relax financial frictions. In symmetric information environments, asset values are correctly assessed by every party so there are no strategic decisions when issuing collateral: no value is lost in the process. This is not the case with asymmetric information, however. When agents sell assets in a pooling market and repurchase them on a spot market, they must forgo part of the asset's value. A similar insight holds in a more complex environment where assets are used to relax financial constraints. In this setting, firms sell assets in a pooling market because they need liquid funds to operate. Financial institutions provide the service of pooling assets together and in doing so, they dilute trading risk. At the end of the production cycle, firms can accumulate capital by (re)purchasing at a spot price. Thus, an asset is (endogenously) liquid only if the additional profits obtained by relaxing financial constraints compensate for the loss caused by trading it under asymmetric information. The outcome of these transactions resemble sell and repurchase (Repo), or commercial paper issues.

The collapse of asset-backed security markets was at the epicenter of the financial crisis. Anderson and Gascon [2009] argue that, as in the model, commercial paper is a major funding source in the U.S. and its main uses include funding payroll. Economists such as Brunnermeier [2009] and Gorton and Metrick [2009] in part attribute this collapse to problems of asymmetric information similar to the ones described here. The focus on endogenous liquidity contrasts with most of the literature on financial frictions that studies how shocks are amplified by affecting the relative wealth of constrained agents or by directly worsening enforcement problems.

The paper shows that the decision to use assets as collateral resembles the solution to Akerlof [1970] lemons problem but differs in a crucial way. The solution to the lemons problem equates the seller's valuation of a threshold asset quality to the buyer's valuation of the average quality under that threshold. These valuations are given exogenously. When assets are used to relax financial constraints, however, these valuations are replaced by endogenous wedges that reflect binding financial constraints.

This characterization uncovers an inefficiency result: when efficient allocations require liquidity, efficient employment and investment allocations never occur in equilibrium. This

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there has been substantial research pointing out that investment specific shocks carry significant effects on output only in combination with some mechanism that causes variation in hours. In Greenwood et al. [1988] or in Greenwood et al. [2000] this mechanism is variable capital utilization. Bernanke et al. [1999], Christiano et al. [2009], del Negro et al. [2010] or Justiniano et al. [2010a], obtain this through New Keynesian features (nominal rigidities and monopolistic competition).

occurs because substantial liquidity is required to relax financial constraints to the point where employment and investment are efficient. However, if employment and investment are chosen efficiently, on the margin, there are no additional benefits from incrementing liquidity. Since there is a marginal cost to trading assets under asymmetric information, rational individuals never choose the efficient amount of liquidity. This implies that equilibria are characterized by insufficient liquidity in combination with always binding financial constraints. This is a constrained inefficiency result that is not a consequence of the environment but rather an outcome consistent with rational decisions.[3]

The quantitative analysis shows that mild increases in the dispersion of asset quality can generate strong responses in aggregate liquidity that cause considerable fluctuations in hours and investment. Fluctuations generated by these shocks are consistent with several business cycle features. [1] The model explains sizeable liquidity-driven recessions which operate primarily through movements in the labor wedge. This is a salient feature of the business cycle decomposition of Chari et al. [2007]. [2] Liquidity driven recessions are characterized by increases in labor productivity. This feature cannot be generated through total factor productivity (TFP) shocks but was characteristic of the 2008-2009 crisis (see Ohanian [2010]). [3] The model accounts for a negative correlation between investment and labor wedges. This supports the view in Justiniano et al. [2010b] that financial factors are responsible for this co-movement. [4] It is also in line with evidence on counter-cyclical capital reallocation documented by Eisfeldt and Rampini [2006]. [5] The features of this model produce two forces that counterbalance the relation between Tobin's Q and investment: TFP shocks induce a positive correlation (as in standard Q-theory) but dispersion shocks reverse the correlation. This second force explains how capital reallocation slows down in periods where its benefits seem greater as described in Eisfeldt and Rampini [2006].

In the model, firms hold a portfolio of assets of different quality. Dispersion shocks affect the quality distribution of those assets. These shocks have empirical and theoretical motivations. Bloom [2009] and Bloom et al. [2009] provide evidence that the dispersion of profits and revenues increase during recessions, both at industry and firm levels. Fundamental shocks that affect the distribution of firm level profits should also reflect changes in the distribution of the value of assets within a firm. After all, medium-size firms are typically involved in multiple production lines. Moreover, firms operate in complex production and financial networks. A firm's assets typically include risky receivable accounts from other firms and portfolio investments in other firms. Thus, the dispersion in the value of assets

[3] A common feature in models with financial frictions is that constrained agents tend to "grow away" from their financial constraints. For this reason, many models introduce ingredients such as heterogeneity in discount factors or sudden deaths.

within a firm can be tied down to the dispersion of more fundamental microeconomic shocks, a natural feature of any economy.

On theoretical grounds, we know from Long and Plosser [1987] that small sectoral productivity shocks can propagate to the entire economy. Among others, the recent works of Acemoglu et al. [2010] and Carvalho [2010] explain that small idiosyncratic shocks can lead to a non-stationary cross-sectional distribution of output depending on the structure of production networks.[4] Although these papers interpret production units as firms, there is no reason why one shouldn't interpret a firm as a collection of such units. Considering the extreme complexity of inter-firm production and financial links, dispersion shocks are a natural and parsimonious way to model the entangled effects.

Moreover, dispersion shocks may be interpreted as also stemming from beliefs. The model cast in such a way that these shocks have no effect on the production possibility frontier or the cross-sectional distribution of wealth. Consequently, in principle, one can think of these perturbations as shocks to beliefs that affect the amount of liquidity without a physical counterpart.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 provides a characterization of equilibria. Section 4 presents a calibration exercise and Section 5 concludes. A detailed discussion about the relationship between the present paper and the literature on financial frictions is found in the online Appendix C. The rest of the appendices include proofs and extensions.

2 The Model

2.1 Environment

The model is formulated in discrete time with an infinite horizon. There are two goods: a perishable consumption good (the *numeraire*) and capital. Every period there are two aggregate shocks: a TFP shock $A_t \in \mathbb{A}$, and a shock $\phi_t \in \{\phi_1, \phi_2, ..., \phi_N\}$ to the distribution of capital quality. ϕ is an index on a family of distributions, $\{f_\phi\}$, of capital quality. The nature of this shock is explained later. (A_t, ϕ_t) form a joint Markov process that evolves according to a transition probability $\Pi : (\mathbb{A}\times\Phi)\times(\mathbb{A}\times\Phi) \to [0, 1]$ with standard assumptions.

[4] Caballero and Simsek [2010] construct an environment in which firms are involved in a network of financial transactions. Shocks in one sector of the system may have different propagation properties when firms are unaware of the exact network structure. Reality is further more complex because firms face different degrees of complementarities (as in Jones [2010]) and the size distribution is such that large firms have a non-vanishing impact on the system (as in Gabaix [2010]).

2.2 Demography and Preferences

There are three classes of agents in this economy: workers, entrepreneurs and financial firms. The measure of entrepreneurs is normalized to a unit and the mass of workers to ϖ. The measure of financial firms is irrelevant.

Entrepreneurs. Each entrepreneur is identified by a number $z \in [0,1]$. Every period, entrepreneurs are randomly assigned one of two possible types: investors and producers. I will also refer to these types as i-entrepreneurs and p-entrepreneurs. At the beginning of each period, entrepreneurs draw a type where the probability of becoming an i-entrepreneur is always equal to π. Thus, every period there is a mass π of i-entrepreneurs and $1-\pi$ of p-entrepreneurs.[5]

Entrepreneurs carry a capital stock across periods. Investors have access to a technology that transforms consumption goods into investment goods but cannot use their capital for the production of consumption goods during that period. In contrast, producers can use their capital for the production of consumption goods but cannot produce investment goods. This segmentation generates the double coincidence of needs that sustains trade: investors have access to investment projects but lack consumption goods to carry them out. In contrast, producers have these funds but cannot increase their capital stock unless they trade with investors. Entrepreneurs manage their firms without earning labor income from this activity.

The entrepreneur's preferences over consumption streams are evaluated according to an expected utility criterion:

$$\mathbb{E}\left[\sum_{t\geq 0}\beta^t U\left(c_t\right)\right]$$

where $U(c) \equiv \frac{c^{1-\gamma}}{1-\gamma}$, and c_t is the entrepreneur's consumption at date t.

Financial Firms. Financial firms are intermediaries that purchase capital under asymmetric information and resell it in a market with full disclosure. These firms are competitive profit maximizers. As in Prescott and Townsend [1984] and Bisin and Gottardi [1999], financial firms simplify the definition of equilibria.

Workers. Workers choose consumption and labor but don't save. Their period utility is given by:

$$U^w(c,l) \equiv \max_{c\geq 0, l\geq 0} c - \frac{l^{1+\nu}}{(1+\nu)}$$

where l is the labor supply and c consumption. ν is the inverse of the Frisch-elasticity. Workers satisfy a static budget constraint in every period: $c = w_t l$ where w_t is their wage.

[5] Randomization is convenient to avoid keeping track of wealth distributions across groups. This feature reduces the dimension of the state space.

The only role for workers in the model is to provide an elastic labor supply schedule.[6]

2.3 Technology

Production of consumption goods. A p-entrepreneur, z, carries out production using his capital stock, $k_t(z)$. By assumption, $k_t(z)$ is fixed at the time of production. Capital is combined with labor inputs, l_t, according to a Cobb-Douglas technology, $F(k,l) \equiv k^{\alpha} l^{(1-\alpha)}$, to produce output. The entrepreneur's profits are $y_t(z) = A_t F(k_t(z), l_t) - w_t l_t$.

Limited enforcement in labor contracts. There is limited enforcement in contractual relations between workers and entrepreneurs. Before carrying out production, z, hires an amount of labor promising to pay $w_t l$ per unit of labor. The entire wage bill cannot be credibly pledged to be paid post-production because the entrepreneur may choose to default on his payroll. In that case, workers are capable to seize a fraction θ^L of output. In other words, z has a technology to divert the fraction $(1-\theta^L)$ of output for his own benefit without any consequence.

The possibility of defaulting on labor contracts has two implications. First, this possibility imposes a constraint on the entrepreneur's employment decision. This contrasts with an underlying assumption in RBC models where the wage bill is credibly paid after production. Second, it induces p-entrepreneurs to engage in financial transactions with the purpose of relaxing the constraints imposed by this friction.[7] This constraint is a milder version than the working capital constraints that require the entire wage bill to be paid up-front. Working capital constraints correspond to the case where θ^L is 0, and are imposed in Christiano et al. [2005] and Jermann and Quadrini [2009], to name some examples. The idea that workers are lenders of hours is also found in Michelacci and Quadrini [2005] and Michelacci and Quadrini [2009], who study long-term relations between workers and employers. An alternative way to interpret these constraints is to conceive of labor as a complement to other variable inputs. If variable inputs are supplied through short-term and easy-to-default contracts, their purchase would require an up-front payment that would lead to a similar set of constraints as the ones found here.

Production of investment goods. The i-entrepreneurs have access to a constant returns to scale technology that transforms a single consumption good into a unit of capital. Thus, the economy operates like a two sector economy model where capital goods are produced with a linear technology and consumption goods are produced using a Cobb-Douglass technology.

[6] The model could be modified to allow workers to save. The effect of this change would be to make the labor supply more elastic to temporal changes in wages without altering the results considerably.

[7] Implicitly, it is assumed that workers are not willing to take capital as collateral directly because they are small. Financial firms are bigger and, therefore, are capable of diluting that risk.

Limited enforcement in investment. i-entrepreneurs sell claims on capital goods in exchange for consumption goods that are used as inputs for investment projects. Following KM, an i-entrepreneur has access to a technology that allows him to divert a fraction $(1-\theta^I)$ of his investment projects at the expense of the rest of the project. This enforcement problem imposes a constraint on the issues of claims and provides a motive for trade with financial firms. Similar restrictions are obtained in problems of hidden-effort (e.g., Holmstrom and Tirole [1997] and Holmstrom and Tirole [1997]).

Evolution of capital. At the beginning of every period, capital held by each entrepreneur becomes divisible into a continuum of pieces. Each piece is identified by a quality $\omega \in [0,1]$. There is an increasing and differentiable function $\lambda(\omega) : [0,1] \to R_+$ that determines the corresponding future efficiency units remaining from a piece of quality ω by the end of the period. Efficiency units can also be interpreted as random depreciation shocks.[8]

The distribution of qualities assigned to each piece is randomly changing over time. In particular, at a given point in time the distribution of capital qualities is determined by a density function f_ϕ, which, in turn, depends on the current realization of the aggregate state ϕ_t. To simplify the analysis, I assume that the distribution is the same for every entrepreneur but differs across periods. Therefore, the measure of units of quality ω is $k(\omega) = kf_\phi(\omega)$. Between periods, each piece is transformed into future efficiency units by scaling the qualities by $\lambda(\omega)$. Thus, $\lambda(\omega)\,k(\omega)$ efficiency units remain from the $\omega-qualities$. Once capital units are scaled by depreciation, they are merged back into a homogeneous capital stock. Thus, by the end of the period, the capital stock that remains from k is,

$$\tilde{k} = \int \lambda(\omega)\,k(\omega)\,d\omega = k \int \lambda(\omega)\,f_\phi(\omega)\,d\omega.$$

In the following period, capital is again divided in the same way and the process is repeated indefinitely.

This does not mean that the entrepreneur will necessarily hold all of $\tilde{k}$ in subsequent periods. On the contrary, entrepreneurs may choose whether to sell particular qualities. $\iota^s(\omega) : [0,1] \to \{0,1\}$ represents the indicator for the decision on selling the capital units of quality ω. Because each quality has 0 measure, the restriction to all-or-nothing sales is without loss of generality.

In equilibrium, financial firms purchase all the units sold by entrepreneurs. This means that an entrepreneur transfers $k\int \lambda(\omega)\,\iota^s(\omega)\,f_\phi(\omega)\,d\omega$ units of capital to the financial sector. Accounting shows that the efficiency units that remain with the entrepreneur are

[8] Shocks to the efficiency units are commonly used in continuous time settings as Brunnermeier and Sannikov [2009], for example.

$k\int \lambda(\omega)[1-\iota^s(\omega)] f_\phi(\omega)\, d\omega$. Including investments and purchases of capital, the entrepreneur's capital stock evolves according to:

$$k' = i - i^s + k^b + k\int \lambda(\omega)[1-\iota^s(\omega)] f_\phi(\omega)\, d\omega, \tag{1}$$

where $i - i^s$, is the total investment i carried out by the entrepreneur net issued claims, i^s. Finally, this stock is augmented by capital purchases, k^b.

I impose some structure on the quality distributions $\{f_\phi\}$:

Assumption 1. *The set* $\{f_\phi\}$ *satisfies the following:*

1. *For any* $\phi \in \Phi$, $\int \lambda(\omega) f_\phi(\omega)\, d\omega = \bar{\lambda}$.
2. $\mathbb{E}_\phi[\lambda|\lambda < \lambda^*] \equiv \int I_{[\lambda(\omega)<\lambda^*]}\lambda(\omega) f_\phi(\omega)\, d\omega$ *is weakly decreasing in* ϕ *for any* λ^*.

The first condition states that for any ϕ, the mean of the distribution of capital quality is always $\bar{\lambda}$. That is, ϕ-shocks are mean preserving shocks (MPS). The implication of this condition is that the production possibility of the economy is invariant to this shock. In fact, any permanent shock that also affects the mean of the quality distribution can be decomposed into a permanent TFP shock and mean preserving shock to the distribution. Both representations yield the same allocations so the environment accounts for mean and variance shocks. Isolating MPS from TFP shocks has the connotation that if ϕ has any effect on allocations, it is because these shocks affect equilibrium but not the feasible set of allocations.

2.4 Timing, Information and Markets

Information. Aggregate capital, $K_t \in \mathbb{K} \equiv [0, \bar{K}]$, is the only endogenous aggregate state variable. The aggregate state of the economy is therefore summarized by the vector $X_t = \{A_t, \phi_t, K_t\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$. At the beginning of each period, X_t and the entrepreneurs' type become common knowledge. This means that financial firms can discriminate between an entrepreneur's activity.

On the other hand, the $\omega-$qualities are only known to the entrepreneur that owns them. This means that financial firms can observe the amount of capital being transferred to them, $k\int \iota^s(\omega) f_\phi(\omega) d\omega$, but ignore how much capital will remain from those units, $k\int \lambda(\omega)\,\iota^s(\omega) f_\phi(\omega) d\omega$.

Timing. The sequence of actions taken by the agents in this economy is as follows. At the beginning of each period all the relevant information is revealed. Then, p-entrepreneurs choose which qualities to transfer to financial firms in exchange for claims to consumption goods (liquid funds) denoted by x. Financial firms credibly guarantee to deliver these goods to its owner by the end of the period. Entrepreneurs transfer consumption claims to workers as an upfront payment of the fraction $1-\sigma$ of salaries.

Workers then provide labor and production is carried out. The consumption goods created by this activity are then used by p-entrepreneurs to: (i) pay for the remaining fraction of the wage bill $(1-\sigma)w_t$, (ii) to consume, and (iii) to purchase (or repurchase) capital to be delivered by financial firms.

When selling capital to p-entrepreneurs, financial firms obtain consumption goods which partially settle the claims on x and, in addition, are used to transact with i-entrepreneurs. In exchange for consumption goods, i-entrepreneurs sell capital qualities and claims to new investment projects, i^s, to financial firms. After the production of capital, all capital claims are settled.

This sequence of events is consistent with the physical requirement that consumption goods must be created before capital goods. For the rest of the paper, I treat these actions as if they occur simultaneously without further reference to the timing of transactions.

Markets. Labor markets are perfectly competitive. I impose the following assumptions on the capital market:

Assumption 2. *Financial firms are **competitive** and capital markets are **anonymous** and **non-exclusive**.*

The condition that financial firms are competitive ensures they earn 0 profits every period. Anonymity and non-exclusiveness guarantees that the market for used capital is a pooling market. Without anonymity and exclusivity, financial firms pay a different price depending on the capital traded by the entrepreneur. This would recover the full information outcome: firms could offer a price proportional to the CDF of f_ϕ.

Notation: For the rest of the paper, I will append terms like $^j(k, X)$ to indicate the policy function of an entrepreneur of type j in state (k, X). I will also use $\iota^j(\omega, k, X)$ to refer to an entrepreneur of type j's decision to sell a quality ω when his capital stock is k and the current state is X. Finally, I denote by $\mathbb{E}_\phi$ the expectations operator with respect to the quality distribution f_ϕ, and use $\mathbb{E}$ to denote expectations about future states.

2.5 Entrepreneur Problems and Equilibria

Entrepreneurs solve different problems according to their types. Both problems can be written recursively so from now on I drop time subscripts. I begin with the description of the p-entrepreneur's problem:

Problem 1 (Producer's Problem)

$$V^p(k, X) = \max_{c \geq 0, k^b \geq 0, \iota^s(\omega), l, \sigma \in [0,1]} U(c) + \beta \mathbb{E}\left[V^j(k', X')|X\right], \; j \in \{i, p\}$$

subject to

$$\text{(Budget constraint)} \qquad c + q(X) k^b = AF(k, l) - wl \tag{2}$$

$$\text{(Capital accumulation)} \qquad k' = k^b + k \int \lambda(\omega)(1 - \iota^s(\omega)) f_\phi(\omega) \, d\omega \tag{3}$$

$$\text{(Incentive compatibility)} \qquad AF(k, l) - \sigma wl \geq \left(1 - \theta^L\right) AF(k, l) \tag{4}$$

$$\text{(Liquid funds)} \qquad x = p^p(X) \int \iota^s(\omega) f_\phi(\omega) \, d\omega \tag{5}$$

$$\text{(Working capital constraint)} \qquad (1 - \sigma) wl \leq xk \tag{6}$$

The first two constraints are standard. The right hand side of the budget constraint corresponds to the entrepreneur's profits. The entrepreneur uses these funds to consume c, and to purchase k^b at a full information price $q(X)$. The evolution of the entrepreneur's capital stock is (1) with the restriction that p-entrepreneurs cannot produce capital or issue claims. The last 3 constraints are novel to this environment. σ represents the fraction of the wage bill that is not paid up-front. The incentive compatibility constraint, (4), states that the amount of income after the entrepreneur pays the remaining part of the wage bill must exceed the amount of funds that can be diverted. Rational workers require this incentive compatibility because they can provide work to other entrepreneurs at the same wage. In (5), x represents the liquid funds per unit of capital that are obtained by transferring $k \int \iota^s(\omega) f_\phi(\omega) \, d\omega$ to financial firms at a pooling price $p^p(X)$. Thus, the entrepreneur has xk liquid funds that he uses to pay a fraction of the wage bill. Finally, (6) states that the fraction of the wage bill guaranteed in advance, $(1 - \sigma) wl$, may not exceed the liquid funds in the hands of the entrepreneur.

An i-entrepreneur's problem is similar except that he chooses an optimal financial structure for investment projects subject to different enforcement constraints:

Problem 2 (Investor's Problem)

$$V^i(k, X) = \max_{c \geq 0, i, i^s \geq 0, k^b \geq 0, \iota^s(\omega) \geq 0} U(c) + \beta \mathbb{E}\left[V^j(k', X')|X\right], j \in \{i, p\}$$

subject to

$$\text{(Budget constraint)} \qquad c + i + q(X)k^b = xk + q(X)i^s \tag{7}$$

$$\text{(Capital accumulation)} \qquad k' = k^b + i - i^s + k\int \lambda(\omega)(1-\iota^s(\omega))f_\phi(\omega)d\omega \tag{8}$$

$$\text{(Liquid funds)} \qquad x = p^i(X)\int \iota^s(\omega)f_\phi(\omega)d\omega \tag{9}$$

$$\text{(Incentive compatibility)} \qquad i - i^s \geq (1-\theta^I)i \tag{10}$$

The right hand side of the i-entrepreneur's budget constraint corresponds to the entrepreneur's funds. The liquid funds available to the entrepreneur, xk, are also obtained by transferring capital to financial firms $\int \iota^s(\omega) f_\phi(\omega) d\omega$, at a pooling price $p^i(X)$. In addition, he also obtains funding by issuing i^s claims to investment units at a market price $q(X)$ (because new units are of known quality). Funds are used to consume c, to purchase k^b, or to fund investment projects i. Finally, (10), is an incentive compatibility condition that prevents the entrepreneur from defaulting on his issued claims. The constraint states that investment net of issued claims must be larger than the capital kept upon default $\left(1-\theta^I\right)i$. This constraint is introduced in KM.

Financial firms. Financial firms purchase capital units of different qualities from both entrepreneur types at pooling prices p^p and p^i. In addition, financial firms also purchase and claims to investment projects at the full information price q. These units are merged and resold as homogeneous capital units. Competition in financial markets ensures zero profits from trading with either entrepreneur type. I assume and later verify that the decision to sell a unit of quality, ω, is only a function of the entrepreneur's type and the aggregate state X and independent of the size of his capital stock. Zero expected profits are equivalent to,

$$p^p(X) = q(X)\,\mathbb{E}_\phi\left[\lambda(\omega)\,|\text{quality } \omega \text{ is sold by a p-entrepreneur}\right] \tag{11}$$

and

$$p^i(X) = q(X)\,\mathbb{E}_\phi\left[\lambda(\omega)\,|\text{quality } \omega \text{ is sold by a i-entrepreneur}\right] \tag{12}$$

(11) is a pooling market equilibrium condition that requires that the price $p^p(X)$ paid to purchase a used unit from a producing entrepreneur equals the market value of the efficiency units bought. Condition (12) is the analog for i-entrepreneurs.

In every period there is a measure over capital holdings and entrepreneur types. I denote this measure by $\Gamma(k,j)$ for $j \in \{i,p\}$. By independence, this distribution satisfies:

$$\int \Gamma(dk,i) = \pi K \quad \text{and} \int \Gamma(dk,p) = (1-\pi) K \,. \tag{13}$$

The total aggregate demand for efficiency units and the supply of investment claims are respectively:

$$D(X) \equiv \underbrace{\int k^{b,p}(k,X)\,\Gamma(dk,p)}_{\text{Capital demand of p-types}} + \underbrace{\int k^{b,i}(k,X)\,\Gamma(dk,i)}_{\text{Capital demand of i-types}} \text{ and } I^s(X) \equiv \underbrace{\int i^s(k,X)\,\Gamma(dk,i)}_{\text{Supply of new units by i-types}} \,.$$

Finally, transfers of efficiency units from both groups to the financial sector are obtained by integrating over the corresponding qualities and capital stocks.

$$S(X) \equiv \underbrace{\int k\left[\int \iota^s(k,X,\omega)\lambda(\omega)\,f_\phi(\omega)\,d\omega\right]\Gamma(dk,i)}_{\text{Effective units supplied by i-types}} + \underbrace{\int k\left[\int \iota^s(k,X,\omega)\lambda(\omega)\,f_\phi(\omega)\,d\omega\right]\Gamma(dk,p)}_{\text{Effective units supplied by p-types}}$$

Capital market clearing is given by $D(X) = I^s(X) + S(X)$. Finally, the labor market clearing requires: $\int l(k,X)\,\Gamma(dk,p) = \varpi l^w(X)\,.$

The definition of equilibria does not depend on any particular measure for the individual capital stocks because, as shown later on, this economy admits aggregation:

Definition (Recursive Competitive Equilibrium)**.** *A recursive competitive equilibrium is (1) a set of price functions, $\{q(X), p^i(X), p^p(X), w(X)\}$, (2) a set of policy functions $\left\{c^j(k,X), k^{b,j}(k,X), \iota^{s,j}(\omega,k,X)\right\}_{j=p,i}, c^w(X), l^w(X), i(k,X), i^s(k;X), l(k,X), \sigma(k,X)$, (3) a pair of value functions, $\{V^j(k,X)_{j=p,i}\}$, and (4) a law of motion for the aggregate state X such that for distribution of over capital holdings Γ satisfying (13), the following hold:*

a set of policy functions (1) Optimality: taking price functions as given, the policy functions solve the entrepreneurs' and worker's problem and V^j is the value of the j-entrepreneur's problem. (2) $p^p(X)$ and $p^i(X)$satisfy the zero profit conditions (11) and (12). (3) The labor market clears. (4) The capital market clears. (5) Capital evolves according to $K' = \int i(k,X)\,\Gamma(dk,i) + \bar{\lambda}K$. (6) The law of motion for the aggregate state is consistent with the individual's policy functions and the transition function Π.

3 Characterization

Producer's employment and liquidity. I begin the characterization of equilibria by solving the p-entrepreneur's problem. The strategy consists on breaking up the problem into two sub-problems. The first sub-problem solves the entrepreneur's labor choice subject to the enforcement and working capital constraints given an amount of liquidity. The value of this problem yields an indirect profit function of liquidity. The second sub-problem determines the qualities using the indirect profit function obtained from the first sub-problem. Once these solutions are found, the original problem is collapsed into a standard consumption-savings decision with stochastic returns.

Let me begin describing the optimal employment given an amount of liquid funds x subject to the enforcement constraint (4) and the working capital constraints (6). I first solve this problem for $k = 1$ and then show that since the constraints and the objective of the problem are linear in k, the value function is also linear in k.

Problem 3 (Profit Maximization) *The profit maximization problem is*

$$r(x, X) = \max_{l,\sigma} \left[Al^{1-\alpha} - wl \right]$$

subject to $Al^{1-\alpha} - \sigma w(X) l \geq \left(1 - \theta^L\right) Al^{1-\alpha}$ *and* $(1-\sigma) w(X) l \leq x$.

The solution to this problem is:

Proposition 1 (Optimal Labor) *The solution to the profit maximization problem 3 is* $l^*(x, X) = \min\{l^{cons}(x, X), l^{unc}(X)\}$ *where* $l^{cons}(x, X) = \max\left\{l : \theta^L Al^{1-\alpha} + x = w(X) l\right\}$ *and* $l^{unc}(X)$ *is the unconstrained labor choice. Constraints are always slack if* $\theta^L \geq (1-\alpha)$.

Proposition 1 states that an entrepreneur may be constrained to hire less than the efficient amount of labor if he lacks sufficient liquid funds. When liquidity is insufficient, both constraints will bind because there is no point in leaving liquid funds without use. If constraints bind, the entrepreneur is must choose employment so that his wage bill equals his liquid funds plus the pledgeable fraction of income. The max simply takes care of not choosing $l = 0$ when $x = 0$ because the entrepreneur can still hire workers even if $x = 0$. If, $\theta^L < (1-\alpha)$, efficient employment requires some amount of liquidity because the labor share of output cannot be credibly pledged to workers.

The following Lemma shows that the entrepreneur's problem can be simplified by isolating labor input decisions once liquidity is chosen.

Lemma 1 (Producer's Problem II) *The problem of the p-entrepreneur is equivalent to:*

$$V^p(k, X) = \max_{c \geq 0, k^b \geq 0, \iota^s(\omega) \geq 0} U(c) + \beta \mathbb{E}\left[V^j(k', X') | X\right], \; j \in \{i, p\}$$

$$\text{subject to } c + q(X) k' = r(x, X) k + q(X) k \int \lambda(\omega)(1 - \iota^s(\omega)) f_\phi(\omega) d\omega \tag{14}$$

$$\textit{and } x = p^p(X) \int \iota^s(\omega) f_\phi(\omega)\, d\omega$$

where $r(x, X)$ is the value of Problem 3.

This lemma exploits the fact that employment does not directly affect any intertemporal decision. The profits the entrepreneur obtains when he chooses labor optimally subject to constraints (4) and (6) for a given value of x are $r(x, X)\, k$. Re-writing the entrepreneur's problem this way allows me to pin down the sold qualities, $\iota^{s,p}(k, X, \omega)$:

Proposition 2 (Producer's Equilibrium Liquidity) *Any recursive competitive equilibrium is characterized by a threshold quality function $\omega^p(X)$ such that in state X, all qualities under $\omega^p(X)$ are sold by all p-entrepreneurs. The equilibrium liquidity and price for p-entrepreneurs solves:*

$$x^p(X) = p^p(X)\, F(\omega^p(X)) \;\; \textit{and } p^p(X) = q(X)\, \mathbb{E}_\phi\left[\lambda(\omega)\, | \omega < \omega^p(X), X\right].$$

In addition $\omega^p(X)$ is either: [1] Interior solution: $\omega^p(X) \in (0, 1)$ and solves,

$$r_x(x, X)\, \mathbb{E}_\phi\left[\lambda(\omega)\, | \omega < \omega^p(X), X\right] = \lambda(\omega^p(X)), \tag{15}$$

[2] Fully liquid: $\omega^p(X) = 1$, [3] Market Shutdown: $\omega^p(X) = \emptyset$.

Proposition 2 establishes that all equilibria are characterized by a threshold quality, $\omega^p(X)$, such that all qualities below it are sold. The interesting cases correspond to interior solutions. Equation (15) resembles the equilibrium condition in Akerlof [1970]'s classical lemon's problem where a marginal quality object valued by a seller equals the buyers valuation of the expected quality under that threshold. There is a key distinction, however. Whereas in Akerlof [1970], buyer and seller valuations are given exogenously, here those valuations depend on the shadow costs and benefits of selling a marginal asset. This shadow benefit is the marginal profit obtained by an extra unit of liquid funds which equals $r_x(x, X)$. Liquidity has the property of relaxing the entrepreneur's constraint on employment thereby incrementing his profits. In equilibrium, this shadow value will equal a shadow cost of selling a marginal unit. This cost is reflected in the difference between the buyer's valuation of that threshold and the expected qualities under that threshold.

The amount of liquid funds available to the p-entrepreneur , $x^p(X)$ is determined by the threshold quality, $\omega^p(X)$. One can substitute the optimal policy for $\iota^{s,p}(k, X, \omega)$ into the p-entrepreneur's problem and collapse it into a standard consumption-savings problem where the return and price of capital depend on equilibrium liquidity:

Problem 4 (Producer's Reduced Problem)

$$V^{p}(k,X)=\max_{c\geq 0,k'\geq 0}U(c)+\beta\mathbb{E}\left[V^{j}(k',X')|X\right],\ j\in\{i,p\} \tag{16}$$

$$\textit{subject to } c+q\left(X\right)k'=W^{p}\left(X\right)k \tag{17}$$

$$\textit{where } W^{p}\left(X\right)\equiv\left[r\left(x^{p}\left(X\right),X\right)+q\left(X\right)\int_{\omega>\omega^{p}(X)}\lambda\left(\omega\right)f_{\phi}\left(\omega\right)d\omega\right] \tag{18}$$

$W^{p}\left(X\right)$ is the entrepreneur's virtual wealth per unit of capital. $W^{p}\left(X\right)$ is the sum of profits per unit of capital given $x^{p}\left(X\right)$ and the value of the fraction of the capital stock that remains with the entrepreneur. I return to this problem after reducing the i-entrepreneur's problem in the same way.

Investor's financing and liquidity decisions. Let $i^{d}\equiv i-q\left(X\right)i^{s}$ be the portion of an i-entrepreneur's investment that is financed internally (the down payment). Given i^{d}, the entrepreneur must choose i^{s} optimally. His value function is increasing in the capital stock, so this decision is equivalent to maximizing future capital holdings:

Problem 5 (Optimal Investment Financing) *The optimal financing problem is* $\max_{i^{s}\geq 0} i-i^{s}$ *subject to* $i^{d}=i-q\left(X\right)i^{s}$ *and* $i^{s}\leq\theta^{I}i$ *taking* $i_{t}^{d}>0$ *as given.*

Substituting the definition of i^{d} into the objective leads to $i^{d}+\left(q\left(X\right)-1\right)i^{s}$. This quantity is the sum of the entrepreneur's down payment and his arbitrage profits: the entrepreneur issues claims to 1 investment unit at $q\left(X\right)$, but generating that unit costs him only one unit of consumption. Therefore, in states where $q(X)>1$, the entrepreneur will want to issue as many claims as possible. The entrepreneur's enforcement constraint can be written in terms of i^{d}: $\left(1-\theta^{I}q\left(X\right)\right)i^{s}\leq\theta^{I}i^{d}$. This constraint binds whenever $q\left(X\right)>1$, so the choice of claims is $i^{s}=\theta^{I}i^{d}/\left(1-\theta^{I}q\left(X\right)\right)$. Substituting this to the objective function renders an expression for the increment in the entrepreneur's capital stock per unit of down payment $\left(1-\theta\right)/\left(1-\theta q\left(X\right)\right)$. The inverse of this term defines the cost of generating a unit of capital in terms of consumption goods by exploiting the optimal external financing structure:

$$q^{R}\left(X\right)\equiv\frac{\left(1-\theta q\left(X\right)\right)}{\left(1-\theta\right)}\leq 1 \text{ for } q\left(X\right)\in\left[1,1/\theta\right)$$

Therefore, $q^{R}\left(X\right)$ is the internal cost of generating a unit of capital. Thus, $q(X)/q^{R}\left(X\right)$ represents the marginal Tobin's Q. In states where $q\left(X\right)=1$, $q^{R}\left(X\right)$ is also 1 and i^{s} is indeterminate from the individual's point of view. A simple comparison of internal costs and the price of capital establishes:

Proposition 3 (Optimal Financing) *When* $q\left(X\right)>1$, *any solution to Problem 5 will be* $k^{b}=0$ *and* $i^{s}=\theta^{I}i$. *When* $q\left(X\right)=1$, $i^{s}\leq\theta^{I}i$ *and* $i^{s}=0$ *if* $q\left(X\right)<1$.

I use this proposition to write down a simplified version of the investor's problem:

Lemma 2 (Investor's Problem II) *The problem of the i-entrepreneur is equivalent to:*

$$V^i(k, X) = \max_{c \geq 0, k^b \geq 0, \iota^s(\omega) \geq 0} U(c) + \beta \mathbb{E}\left[V^j(k', X')|X\right], j \in \{i, p\}$$

subject to

$$c + q^R(X) k' = xk + q^R(X) \int \lambda(\omega)(1 - \iota^s(\omega)) f_\phi(\omega) dz \text{ and } x = p^i(X) \int \iota^s(\omega) f_\phi(\omega) d\omega.$$

The analysis of the investor's problem is similar to the one studied by KM except that here liquidity is determined endogenously. Proposition 4 is the analog of Proposition 2 which describes the equilibrium liquidity chosen by the i-entrepreneur:

Proposition 4 (Investors Equilibrium Liquidity) *Any recursive competitive equilibrium is characterized by a threshold quality function $\omega^i(X)$ such that in state X, all qualities below $\omega^i(X)$ are sold by all i-entrepreneurs. The equilibrium liquidity and price for i-entrepreneurs are given by:*

$$x^i(X) = p^i(X) F\left(\omega^i(X)\right) \text{ and } p^i(X) = q(X) \mathbb{E}_\phi\left[\lambda(\omega)|\omega < \omega^i(X), X\right].$$

In addition $\omega^i(X)$ is either: [1] Interior solution: $\omega^i(X) \in (0, 1)$ and solves,

$$\frac{q(X)}{q^R(X)} \mathbb{E}_\phi\left[\lambda(\omega)|\omega < \omega^i(X), X\right] = \lambda\left(\omega^i(X)\right), \tag{19}$$

[2] Fully liquid: $\omega^i(X) = 1$, [3] Market Shutdown: $\omega^i(X) = \emptyset$.

As with producers, Proposition 4 states that the solution to the threshold quality sold by investors also resembles the solution to the standard lemons problem but in this case exogenous valuations are replaced precisely by Tobin's Q. Substituting in for $\iota^s(k, X, \omega)$, the i-entrepreneur's problem simplifies to:

Problem 6 (Investor's Reduced Problem)

$$V^i(k, X) = \max_{c \geq 0, k' \geq 0} U(c) + \beta \mathbb{E}\left[V^j(k', X')|X\right], \ j \in \{i, p\}$$

$$\text{subject to } c + q(X) k' = W^i(X) k$$

$$\text{where } W^i(X) \equiv \left[q(X) \int_{\omega \leq \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega + q^R(X) \int_{\omega > \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega\right] \tag{20}$$

$W^i(X)$ is the investor's virtual wealth per unit of capital. This quantity is a weighted sum over the entrepreneur's capital qualities. The first term is the value of liquid funds: liquid funds correspond to the price of capital sold by i-entrepreneurs, $p^i(X)$, times the volume of capital sold, $F_\phi(\omega^p(X))$. In equilibrium, $p^i(X) = q(X)\,\mathbb{E}[\lambda(\omega)\,|\omega < \omega^p(X)]$. Substituting this condition, we obtain the first term in $W^i(X)$. The second term corresponds to the illiquid units are valued at their replacement cost, $q^R(X)$,

Optimal consumption-savings decisions. The entrepreneurs' problems can be summarized by two standard consumption-savings problems. For p-entrepreneurs, this is done by finding the indirect profit of liquidity, $r(x, X)$. The optimal amount of liquidity x is proportional to the entrepreneur's capital stock and it is determined by a threshold quality $\omega^p(X)$. The threshold quality $\omega^p(X)$ solves an indifference condition that equates the marginal profit per unit of liquidity to the loss incurred by selling a marginal unit under private information. The same strategy is used to characterize the i-entrepreneur's problem. The value of a unit of liquidity for the i-entrepreneur is $q(X)/q^R(X)$. Once the equilibrium amounts of liquidity are characterized, one can express either problem without further reference to liquidity, labor or claims to capital decisions.

Thus, Problems 4 and 6 determine the entrepreneurs' consumption-savings decisions. These problems are isomorphic to standard consumption-savings problems with homogeneous preferences and constant returns to scale. It is immediate to show that the policy functions are linear functions of the capital stocks and therefore, invoking Gorman's aggregation result, we have the necessary conditions for the existence of a representative agent. This result guarantees the internal consistency of the definition of competitive recursive equilibrium without any reference to the distribution of capital holdings.

The optimal consumption-savings decisions are:

Proposition 5 (Optimal Policies) *The policy functions for p-entrepreneurs are* $c^p(k, X) = (1 - \varsigma^p(X))\,W^p(X)\,k$ *and* $k'(k, X) = \frac{\varsigma^p(X)W^p(X)}{q(X)}k$*. For i-entrepreneurs these are:* $c^i(k, X) = (1 - \varsigma^i(X))\,W^i(X)\,k$ *and* $k'(k, X) = \frac{\varsigma^i(X)W^i(X)}{q^R(X)}k$*.*

The functions $\varsigma^p(X)$ and $\varsigma^i(X)$ are marginal propensities to save for p-entrepreneurs and i-entrepreneurs. These functions solve a pair of functional equations. Proposition 8 presented in the Appendix shows that marginal propensities to save of both types solve a non linear functional equation which can be easily solved by repeated iteration.[9] When $\gamma = 1$, one can verify that $\varsigma^s = \varsigma^i = \beta$.

[9]A similar operator is found in Angeletos [2007]. I cannot provide a direct proof for a theorem that guarantees that the repeated iteration of this operator converges for $\gamma > 1$. Nevertheless, Alvarez and Stokey [1998] show that the standard dynamic programming properties of this problem are guaranteed. Thus, if the operator converges, it converges to its unique fixed point.

Full information price of capital. The last object to be characterized is the full information price of capital, q. One can re arrange the i-entrepreneur's capital accumulation equation, substitute in the capital policy functions obtained from Proposition 5, and integrate across individuals to obtain their aggregate demand for investment net of issued claims:

$$I(X) - I^{s}(X) = \left[\frac{\varsigma^{i}(X)\, W^{i}(X)}{q^{R}(X)} - \int_{\omega > \omega^{i}(X)} \lambda(\omega)\, f_{\phi}(\omega)\, d\omega\right] \pi K. \tag{21}$$

Considering that in equilibrium only producers purchase capital, similar steps lead to an expression for the aggregate demand for capital purchases:

$$D(X) = \left[\frac{\varsigma^{p}(X)\, W^{p}(X)}{q(X)} - \int_{\omega > \omega^{p}(X)} \lambda(\omega)\, f_{\phi}(\omega)\, d\omega\right] (1-\pi)\, K. \tag{22}$$

The total sales of used capital under asymmetric information is obtained by aggregating over the capital sales of both types:

$$S(X) = \underbrace{\left[\int_{\omega \leq \omega^{p}(X)} \lambda(\omega)\, f_{\phi}(\omega)\, d\omega\right] (1-\pi)\, K}_{\text{Capital sales by p-types}} + \underbrace{\left[\int_{\omega \leq \omega^{i}(X)} \lambda(\omega)\, f_{\phi}(\omega)\, d\omega\right] \pi K}_{\text{Capital sales by i-types}} \tag{23}$$

Market clearing requires $D(X) = S(X) + I^{s}(X)$. To satisfy this expression, producers must repurchase all the units they sold in the period. In that sense, the equilibrium allocations resemble Repo markets. In addition, it must be the case that investors satisfy their constraints on the issuance of investment claims (10). Since these constraints are linear in the entrepreneur's capital stock, an aggregate version of this condition, $I(X) - I^{s}(X) \leq (1-\theta)\, I(X)$, holds if and only if there exists an allocation such that all the individual constraints are satisfied. Thus, any equilibrium must be characterized by a price function $q(X)$ such that $D(X) = S(X) + I^{s}(X)$ and $\theta I(X) \leq I^{s}(X)$.

Solving for $q(X)$ is not immediate from market clearing since enforcement constraints must also be met. A thought experiment can clarify how market clearing and enforcement constraints are satisfied together. First, observe that if $q(X) < 1$, there would not be any supply of investment claims since i-entrepreneurs would find it cheaper to purchase capital than to invest. Thus, $q(X) < 1$ cannot occur in equilibrium. Assume then that $q(X) = 1$. Given prices and policy functions, $I(X) - I^{s}(X)$ can be solved for from (21) whereas $I^{s}(X)$ equals $D(X) - S(X)$. Given that $I^{s}(X)$ and $I(X) - I^{s}(X)$ are known quantities, one can check whether they satisfy $\theta I(X) \leq I^{s}(X)$. When this condition is guaranteed, then $q(X) = 1$ is an equilibrium. When it is violated, $q(X)$ must be 1 to satisfy incentive compatibility.

Proposition 3 ensures that when $q(X) > 1$, enforcement constraints bind so $I^s(X) = \theta I(X)$. Substituting this equality into (21) yields a supply schedule for claims that is increasing in $q(X)$. In addition, the supply of capital $S(X)$ is increasing and demand $D(X)$ decreasing in $q(X)$. Thus, $q(X)$ is found by solving for the market clearing condition when enforcement constraints are binding. Proposition 7 in the Appendix provides an expression for $q(X)$ based on this analysis. The rest of the equilibrium conditions that close the model are described in Appendix.

Economic properties. A distinguishing feature of this environment is that, in equilibrium, despite the ability to do so, entrepreneurs will not choose to acquire the amount of liquidity that would allow them to entirely relax their enforcement constraints. This result follows from an existing tension between the enforcement constraints and the incentives to sell capital under asymmetric information. On one hand, selling a marginal unit of capital under asymmetric information is costly to the entrepreneur because he receives a pooling price for an object that he values above that price. On the other hand, when financial frictions are active, they provide the incentives that support trade under asymmetric information because relaxing these constraints is valued by the entrepreneur. When constraints are entirely relaxed by acquiring sufficient funds, liquidity has no value on the margin because there is no point in having additional funds. Nevertheless, to obtain this amount, the entrepreneur must incur a loss from selling a marginal asset in a pooling market.

Take for example a p-entrepreneur. If employment in his firm is efficient, the marginal loss of reducing liquidity is negligible because the marginal profit from labor is 0. Because selling the marginal quality asset is costly, the entrepreneur is better-off if he reduces part of his capital sales. A similar consideration is true for i-entrepreneurs. [10]

Thus, the takeaway is that financial frictions must be active in order to support trade under asymmetric information. In other words, when liquidity is needed to enforce efficient employment or investment, the economy will feature under-employment and under-investment. The conditions on parameters that generate these inefficiencies are summarized by:

Proposition 6 (Inefficiency) *Employment is always sub-efficient if $\theta^L < (1-\alpha)$. Investment ($I > 0$) is always sub-efficient iff $\theta^I \leq (1-\pi)$.*

When $\theta^L < (1-\alpha)$, producers cannot credibly pledge workers the labor share of output unless they use liquid funds. Thus, in order to attain efficient employment, liquidity is needed. Yet, if employment is efficient, liquidity has no marginal value. By Proposition 2, this implies $x^p = 0$. A contradiction. Similarly, efficient investment requires the price of

[10] If investment is efficient, then the physical cost of creating capital units equal its price $q(X) = 1$. Nevertheless, if $q(X) = 1$, trade under asymmetric information cannot be supported because liquidity has no value for the entrepreneur.

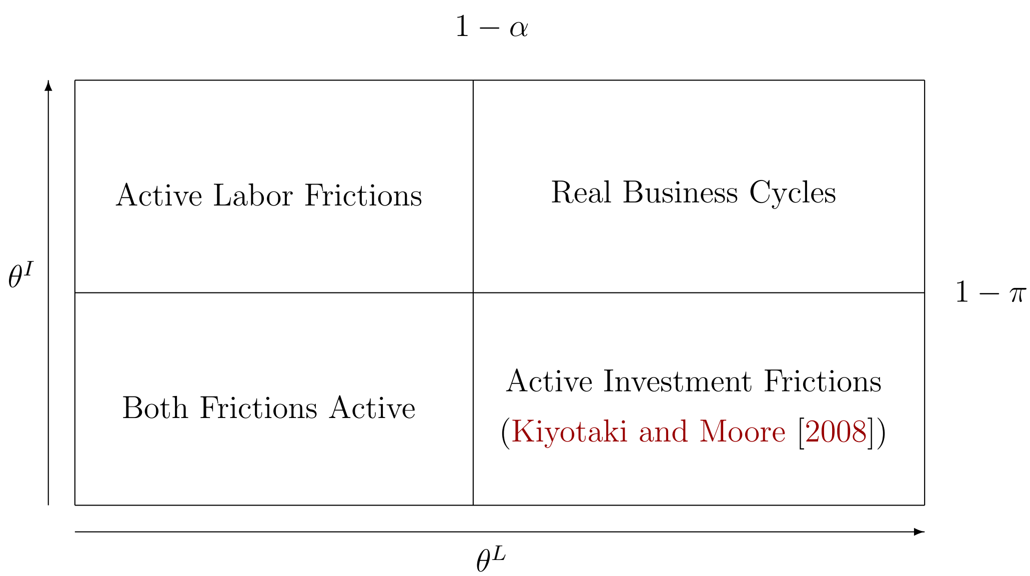

Figure 1: Activation of Constraints.

capital to equal its physical cost, $q(X) = 1$. When $q(X) = 1$, Proposition 8 implies that $\varsigma^p(X) = \varsigma^i(X)$ and that $W^p(X) \geq W^i(X)$. Market clearing conditions then imply that $(I(X) - I^s(X))/I^s(X) \geq \pi/(1-\pi)$. Hence, if $\theta^I \leq (1-\pi)$ holds, efficiency cannot be attained and investment frictions must be active.

Figure 1 provides a summary of the conditions on parameters that activate either friction. When $\theta^L < (1-\alpha)$, the financial frictions that affect labor markets are active so dispersion shocks impact the labor wedge. $\theta^L = 0$ corresponds to the working capital constraints in Christiano et al. [2005] or Jermann and Quadrini [2009]. When $\theta^I < (1-\pi)$, investment frictions are active, so dispersion shocks cause fluctuations that resemble fluctuations caused by investment specific shocks as in KM. The case where $(\theta^I, \theta^L) = (0, 1)$ is the specification in Kurlat [2009].

4 Results

4.1 Calibration

The model period is a quarter. The calibration of technology parameters is standard to business cycle theory. Technology shocks are modeled so that their log follows an AR(1) process where the autoregressive coefficient, ρ_A, is set to 0.95 and the standard deviation of the innovations, σ_A, is set to 0.008. The capital share, α, is set to 1/3.[11] λ is set to 0.9873

[11] Acemoglu and Guerrieri [2008] estimate the capital share of output to be roughly constant over the last 60 years. This accounting exercise is based on a frictionless labor market benchmark. Nevertheless, this

so that the annualized depreciation rate is 5%. π is set to 0.1 to match the plant investment frequencies in Cooper et al. [1999].[12]

I use log-utility for the calibration. Numerically, for any choice of (γ, β) one can find a corresponding alternative value for β such that the marginal propensities to consume under log-preferences are roughly the same.[13] Therefore, I set $\beta = 0.97$ and $\gamma = 1$ to approximate policy functions corresponding with a risk aversion of 2 and a discount factor of 0.991. Log-utility is convenient because the stochastic process that determines the quality distributions does not affect intertemporal decisions. This allows me to calibrate the dispersion shock to target a particular reduction in liquidity and guarantees that responses are independent of the process of ϕ.

The Frisch-elasticity is set to 2 ($\nu = 1/2$). This elasticity is within the range of calibrations used in macro models. The magnitude of the response of output is sensitive to this parameter so I present results for several values. θ^L and θ^I govern the extent of the limited enforcement problem. I do not have any microeconomic support to calibrate these parameters directly so I take an indirect approach and pick the parameters to match aggregate moments. θ^L is set to 0.375 in order to obtain an average labor wedge of 0.35. This number is close to the estimates of Shimer [2009] and Chari et al. [2007]. As an outcome of this calibration, the fraction of the wage bill secured in advance is on average close to 60%.[14] I follow Lorenzoni and Walentin [2009] and set θ^I to 0.4 in order to match the regression coefficients obtained by running I/K against the return to capital and Tobin's Q (as in the investment regressions of Gilchrist and Himmelberg [1998]). I use the invariant distribution of the model to obtain analytical estimators for these coefficients. The resulting coefficients are close to those obtained by Lorenzoni and Walentin [2009]. This parameter implies that the average leverage rate of investment projects is 1/2, which is close to the fraction of investment internally funded in the U.S..

I choose the family $\{f_\phi\}$ to be a set of exponentials. ϕ indexes a variance and a support that keeps the average depreciation rate constant. The particular choice of exponentials is immaterial for the results but is convenient because this family has a closed form solution for conditional expectations. I calibrate ϕ to be an i.i.d process to avoid built-in persistence. The set of standard deviations of f_ϕ ranges from 0.4 to 0.9. The calibration requires a 35%

value is used in other models that induce labor frictions (Chari et al. [2007], for example).

[12] The data suggests that around 20% to 40% plants augment a considerable part of their physical capital stock in a given year. These figures vary depending on plant age. By, setting π to 0.1, the arrival of investment opportunities is such that about 30% of firms invest in a given year.

[13] I perform these numerical experiments in Bigio [2009]. This result is also related to findings in Tallarini [2000] that show that under CRRA preferences, risk aversion does not affect allocations in a standard growth model.

[14] This figure is consistent with a production cycle of a quarter and wages paid at a monthly basis.

Parameter	Value	Notes
Preferences		
γ	1	2.5% risk free rate and risk aversion of 2.
β	0.97	2.5% risk free rate and risk aversion of 2.
ν	1/2	Frisch-elasticity of 1/2.
Technology		
α	1/3	Standard.
π	0.1	To match investment frequencies in Cooper et al. [1999].
λ	0.9781	10% annual depreciation.
Enforcement		
θ^L	0.375	Labor wedge in Shimer [2009] and Chari et al. [2007].
θ^I	0.4	Investment regressions in Gilchrist and Himmelberg [1998].
Aggregate Shocks		
μ_A	0	Standard.
ρ_A	0.95	Standard.
σ_A	0.016	Standard.
f_ϕ		Exponential family with adjusted support.
Φ	[0.4 - 0.9]	To match 60% fall in liquidity after 35% increase in MPS.

Table 1: Calibration and References.

increase in capital quality dispersion to generate a reduction of 60% in liquidity. This shock is more moderate than the twofold dispersion increase studied by Bloom [2009]. In addition, the calibration is such that output over the real value of liquidity is on average close to 2.5. This number is close to the historical mean velocity of M2 in the U.S..

Robustness in the choice of $\{f_\phi\}$: All of the exercises were corroborated using mean preserving families of log-Normal, Beta and Gamma distributions for the distribution of capital quality. Only minor changes in the quantitative results are found upon different choices. Figure 9 reports a graphical robustness test that explains this consistency. The figure shows several moments for each family of distributions. The last column of the figure reports the equilibrium liquidity for i-entrepreneurs corresponding to a distribution with given dispersion. For a given q, a particular value of equilibrium liquidity can be targeted by finding the right dispersion.

A summary of the calibration is reported in Table 1. I use global methods in the computation of equilibria and impulse responses. All the exercises use a grid of 6 elements for both $\mathbb{A}$ and Φ and 120 for the aggregate capital state. Increasing the grid size does not affect results.

4.2 A Glance at Equilibria

This section describes the calibrated recursive competitive equilibrium throughout the state-space.

Endogenous liquidity. Figure 2 presents several equilibrium objects reported in different panels. Within each panel, there are four curves corresponding to a combination of aggregate TFP (high and low) and a dispersion shock (high and low). The x-axis of each panel is the aggregate capital stock.

The top panels describe the equilibrium liquid funds per unit of capital, x, for both entrepreneur types. I begin by describing the liquid funds of p-entrepreneurs. For a given combination of TFP and dispersion shocks, liquidity per unit of capital decreases with the aggregate capital stock (although the total value is increasing). This negative relation follows from decreasing marginal profits in the aggregate capital stock.[15] With lower marginal benefits from increasing liquidity, p-entrepreneurs sell less capital. Comparing the curves that correspond to low and high dispersion shocks, we observe that liquidity shrinks with dispersion. This happens because quality dispersion increases the shadow cost of selling capital under asymmetric information as adverse selection becomes more severe. In contrast, TFP has the opposite effect. It improves liquidity due to a positive effect on marginal profits, which explains the increased benefit from selling capital. An analog pattern is found for the i-entrepreneur's liquidity. The shadow cost of selling capital increases with dispersion, is decreasing in the capital stock and increasing in TFP.

One way to understand the increase in the shadow cost of selling capital with dispersion is through the loss of value incurred by selling a threshold quality. This cost is reflected in the percent difference between the replacement cost of the threshold quality and its pooling price. These terms are the ratios $\left[\frac{q^R(X)\lambda^i(X)-p^i(X)}{q^R\lambda^i(X)}\right]\%$ and $\left[\frac{q(X)\lambda^p(X)-p^p(X)}{q(X)\lambda^p(X)}\right]\%$ for i and p-entrepreneurs respectively. The figure reveals that dispersion increases these shadow costs substantially. TFP, on the other hand, increases the loss entrepreneurs are willing to bear because the benefits of additional liquid funds are greater.

Hours, output, and investment. As dispersion reduces the liquidity of producers, their effective demand for hours leads to a reduction in output. However,when either TFP or the capital stock are large, employed hours and output increase . The figure also shows the perverse effects of dispersion shocks on investment. With less liquidity at disposal, the supply of investment claims shrinks in response to more severe enforcement problems. The reduction in the liquidity of p-entrepreneurs has ambiguous effects on their profits because this reduces the amount of labor hired but wages also fall. The ambiguous effect on profits

[15] With a larger capital stock, labor demand is higher pressing wages upwards.

implies that the demand for capital may increase even after liquidity shortages. For the calibration, the overall effect involves a strong reduction in investment and an increase in q.

The analysis shows how the correlation between Tobin's Q and investment is determined by two counterbalancing forces. The first force is TFP which produces a positive correlation between Q and investment. The second force is dispersion which causes an increase in Tobin's Q along with a reduction in investment. Similar countervailing forces are also obtained in other models with financial frictions as, for example, Lorenzoni and Walentin [2009].

Robustness. It is important to note that the monotonic relation between dispersion and liquidity shown in this exercise crucially depends on Assumption 1. More general results cannot be established between ϕ and the equilibrium objects because disposing of this assumption breaks the ordering of conditional expectations. Without this ordering, the direction of adverse selection effects can become state-dependent.

4.3 Impulse Response Analysis

Baseline Exercise. The first experiment quantifies the effects of a once-and-for-all mean preserving spread in the dispersion of capital quality. Figure 3 reports responses to a 35% increase in dispersion tailored to cause a 60% reduction in aggregate liquidity. This magnitude is close to the fall in the issuance of outstanding asset-backed commercial paper or syndicated loans during the financial crises of 2008-2009 (from peak to trough) as documented by Anderson and Gascon [2009] and Ivashina and Scharfstein [2010].[16]

In response to the shock, the value of liquid funds available to both entrepreneurs contracts immediately. This reaction is triggered by adverse selection effects that explain larger premia between pooling and spot prices. Larger premia imply a greater implicit cost of selling capital. As a consequence, entrepreneurs opt to scale down liquidity. The collapse in liquid funds has real effects on output. Aggregate output falls by 2.5% in the impact quarter due to the reduction in hours. In response to the lack of working capital funds, hours and wages fall by 4% and 2% respectively

The liquidity crisis also affects investors who face tighter limits in their capacity to self-finance investment projects which Consequently, issued claims fall to meet incentive constraints which leads aggregate investment to drop by 25%. With fewer investment projects carried out, the post-impact capital stock shrinks. In the subsequent periods after the shock, the capital stock's dynamics drive the response of the entire system.

The plots at the bottom present the responses of different measures of the inefficiencies

[16]The reduction in issued asset-backed commercial paper is smoother in the data than in the model. The model generates a smoother and more persistent responses in liquidity by fitting a particular sequence of dispersion shocks at the expense of making the analysis less transparent.

Variable	Model on Impact	U.S. Data - 2008I-2009I
Output	-2.5%	-4.1%
Hours	-4%	-7.3%
Wage	-2%	2.86%
MPL	1.3%	$\Uparrow$
Investment	-25%	-20.3%
Consumption	-0.5%	-2.1%-2.2%

Table 2: Model Response and Financial Crisis Data. Source: The data is obtained from the Federal Reserve Bank of St. Louis. Figures correspond to the percent change between the levels in the first quarter of 2008 and the levels the first quarter of 2009. The increase in the marginal product of labor is documented in Ohanian [2010].

caused by the financial frictions in the model. There are important responses in the investment wedges, q and q^R (10% and 20% respectively). This contrasts with the negligible response in the investment wedge computed from an aggregate consumption-euler equation (henceforth, CKM-wedge following from Chari et al. [2007]). The labor wedge, however, increases 5%. A higher labor wedge follows from the increase in the marginal product of labor (MPL) and the decrease in wages.

The experiment shows that contrary to TFP shocks, output contractions can be explained by liquidity crisis resulting from dispersion shocks that reduce labor productivity along with increases in MPL. Labor wedge increases are common to many of the modern recessions documented by Chari et al. [2007] and Shimer [2009]. Moreover, as shown by Ohanian [2010] or Hall [2010], the 2008-2009 crisis was an episode in which hours fell in parallel with increases in MPL. Note also that this happens without a response in the CKM-wedge. This last feature is incorrectly interpreted as evidence against the presence of financial frictions when, in fact, investment is being distorted dramatically.

The first column in Table 2 reports the response of several key variables during the quarter of impact. The second column reports the counterparts of the U.S. economy during the crisis. The table shows that the model delivers quantitative responses on par with the data. The model only fails to account for the increase in the hourly wage which can be explained by numerous reasons that are left out of the analysis such as sectoral differences in unemployment rates.

Which frictions matter? What is the relevant importance of the enforcement constraints on investment and labor for the results? Figure 4 presents responses to dispersion shocks when (a) only the investment friction is active, (b) only the labor friction is active, and (c) both frictions are active. The rest of the parameters keep the target moments constant.

This exercise shows that the enforcement constraint on labor is key to generate a strong output response. The contemporaneous output response is virtually the same with and without the investment friction but vanishes once the labor friction is turned off. The reason for this discrepancy is that output fluctuations can only be explained by hours since the capital stock is fixed in the short run. In contrast, the investment friction distorts the accumulation of capital so the investment friction is responsible for the output dynamics after the shock is realized. Without the labor friction, however, the response of output is mild for the simple reason that investment is a small portion of the total capital stock. In fact, without investment, the capital stock can drop at most by the depreciation rate. This limits the output response unless there is a change in hours or TFP.

The key lesson from this exercise is that investment frictions are responsible for the propagation of the shock but, without the labor market friction, they cannot generate a strong recession. Nevertheless, introducing the investment friction is important to explain the dynamics of aggregate investment. Indeed, in the absence of investment frictions, investment can react positively to a dispersion shock because profits can increase from the reduction in wages. Finally, it is important to note that with limited enforcement in the labor market there is no compromise between magnitude and persistence of shocks as in Cordoba and Ripoll [2004].

Labor supply elasticity. The key parameter for the magnitude of the output response is the labor supply elasticity. Figure 5 presents the response to a $\phi-$shock for different values of the Frisch-elasticity. As explained before, reductions in producers' liquidity affect the labor demand. This is met, in equilibrium, via a reduction of hours and wages. The relative response of either margin depends on the labor supply elasticity. Not surprisingly, the response of hours and, consequently, output is stronger as the Frisch-elasticity is increased. The overall magnitude in the responses of output varies from -2.5% (when the Frisch-elasticity is 2) to -0.5% (when it is equal to 1/2). One can argue in favor of a large Frisch-elasticity since the model abstracts from features such as wage rigidities or worker savings that would magnify the responses.

TFP shocks. Figure 11 presents the response to a positive TFP shock. The figure shows that the producers' liquidity increases immediately in the period after the shock is realized. With additional liquid funds, p-entrepreneurs relax their constraints on employment. This effect magnifies the response of hours. In addition, TFP strengthens the demand for capital, causing an increase in q. This provides i-entrepreneurs with incentives to obtain more liquid funds. The increase relaxes the investor's enforcement problem, which magnifies the response of investment. Figure 10 contrasts the response to technology shocks in the frictionless version of the model with the response activating the two frictions. The response of aggregate quantities is very similar in both models. Although there is an amplification mechanism within the model with active frictions, this does not mean that TFP shocks are magnified in relation to the RBC counterpart as the occupation times in states are also affected. This result is consistent with prior findings by Kocherlakota [2000] and Cordoba and Ripoll [2004] that suggest that financial frictions are weak amplification mechanisms of aggregate TFP shocks.

5 Conclusions

Summary. This paper describes how asymmetric information about capital quality endogenously determines the degree of liquidity in capital markets. In a world with non-enforceable contracts, liquidity is key to relax financial frictions. The dispersion of capital quality increases the cost of using it as collateral and has real effects by exacerbating financial frictions.

The main lessons are: [1] In equilibrium, liquidity is always insufficient to relax enforcement constraints completely. [2] A dispersion shock to capital quality can cause a collapse in liquidity. For example, a shock that generates a 60% reduction in liquidity may provoke a 2.5% output contraction. [3] The key friction to explain this large impact is limited enforcement in labor markets.

Research directions. A first direction for future research is to introduce other assets that are not subject to asymmetric information into the model. This may explain the relevant tensions that affect the use of assets as collateral when these differ in their information properties. The present analysis suggests that an increase in the supply of cash or bonds that targets liquidity could "crowd-out" the liquidity of other assets that feature asymmetric information. This could be caused if the incentives to use capital as collateral are reduced. This line of research is followed by Rocheteau [2009] in the context of money-search theory.

A richer model could also let financial firms play a more active role. The 2008-2009 crisis began with events that caught the financial system by surprise and impacted on its balance-sheets. This extension is important because balance-sheets are key in explaining the provision of liquidity by financial institutions. Moreover, adverse selection can be exacerbated by weaker balance sheets which, in turn, can slow the recovery of bank profits. This aspect could explain the persistence of shocks and the slow recovery of economies after a liquidity crises. I pursue this extension in Bigio [2010].

This paper explains a collapse in liquidity through perturbations that exacerbate adverse selection effects. Strategic delays and learning can lead to dynamic effects left out from the model (see Kurlat [2009] or Daley and Green [2010]). In Caballero and Krishnamurthy [2008], liquidity crises are explained by Knightian uncertainty. Yet, other "non-fundamental" shocks could explain financial collapses in liquidity. Lorenzoni [2009] or Angeletos and La'O

[2010] provide frameworks in which shocks that affect information structures have real effects by leading agents to take decisions away from full information responses. This phenomena could also operate through a liquidity channel.

Finally, the results of this paper hinge on tensions between two market imperfections: (a) asymmetric information and (b) financial frictions. In the aftermath of the financial crisis, there has been a fair amount of anecdotal evidence in both dimensions. Nevertheless, our profession lacks conclusive evidence that corroborates that these frictions are not a myth in a way that disentangles them from other phenomena interpreted as demand shocks.

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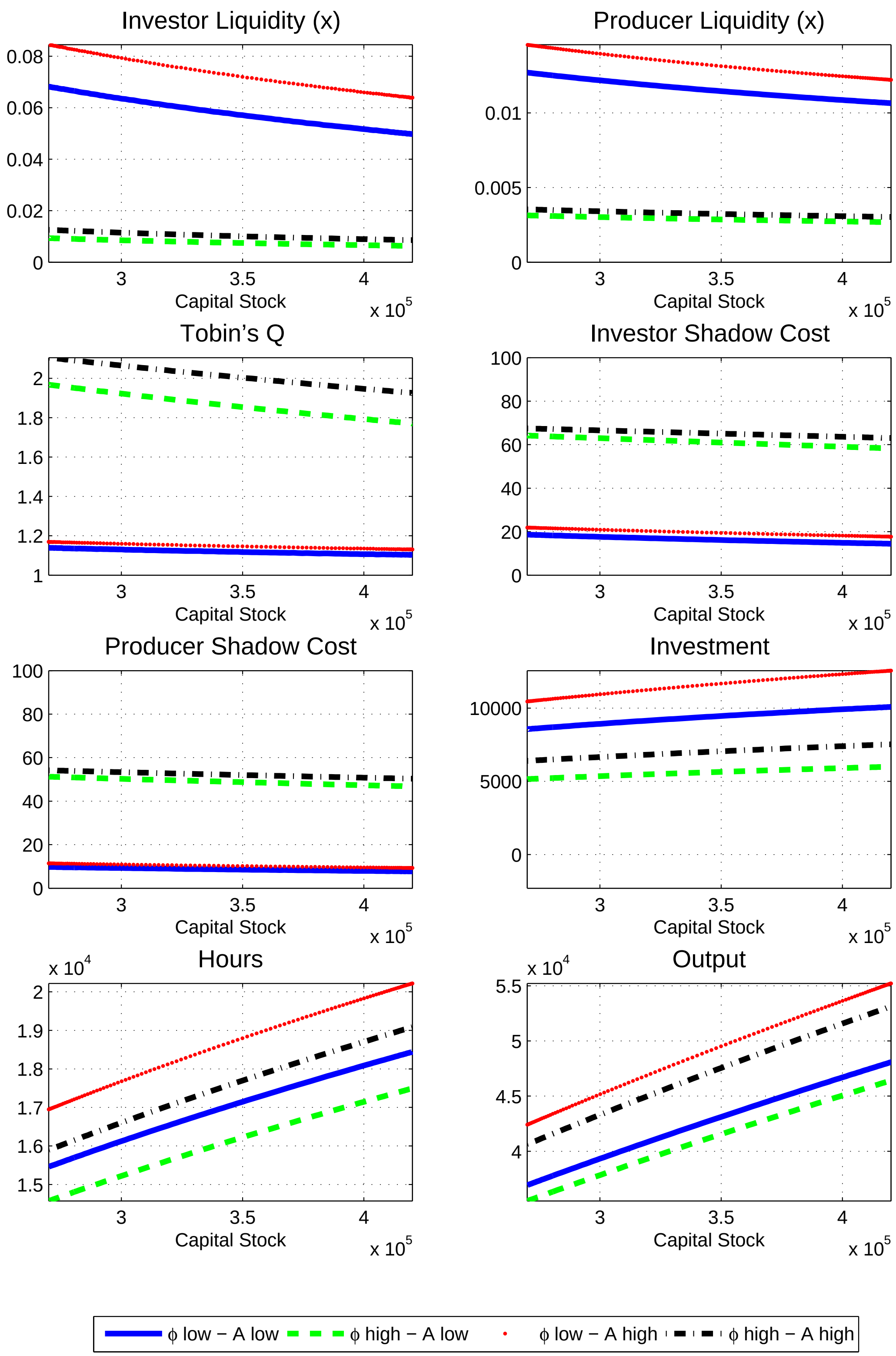

Figure 2: **Equilibrium Variables across State-Space**. The figure plots equilibrium objects as functions of aggregate capital stock for 4 possible combinations of TFP (high and low) and dispersion shocks (high and low).

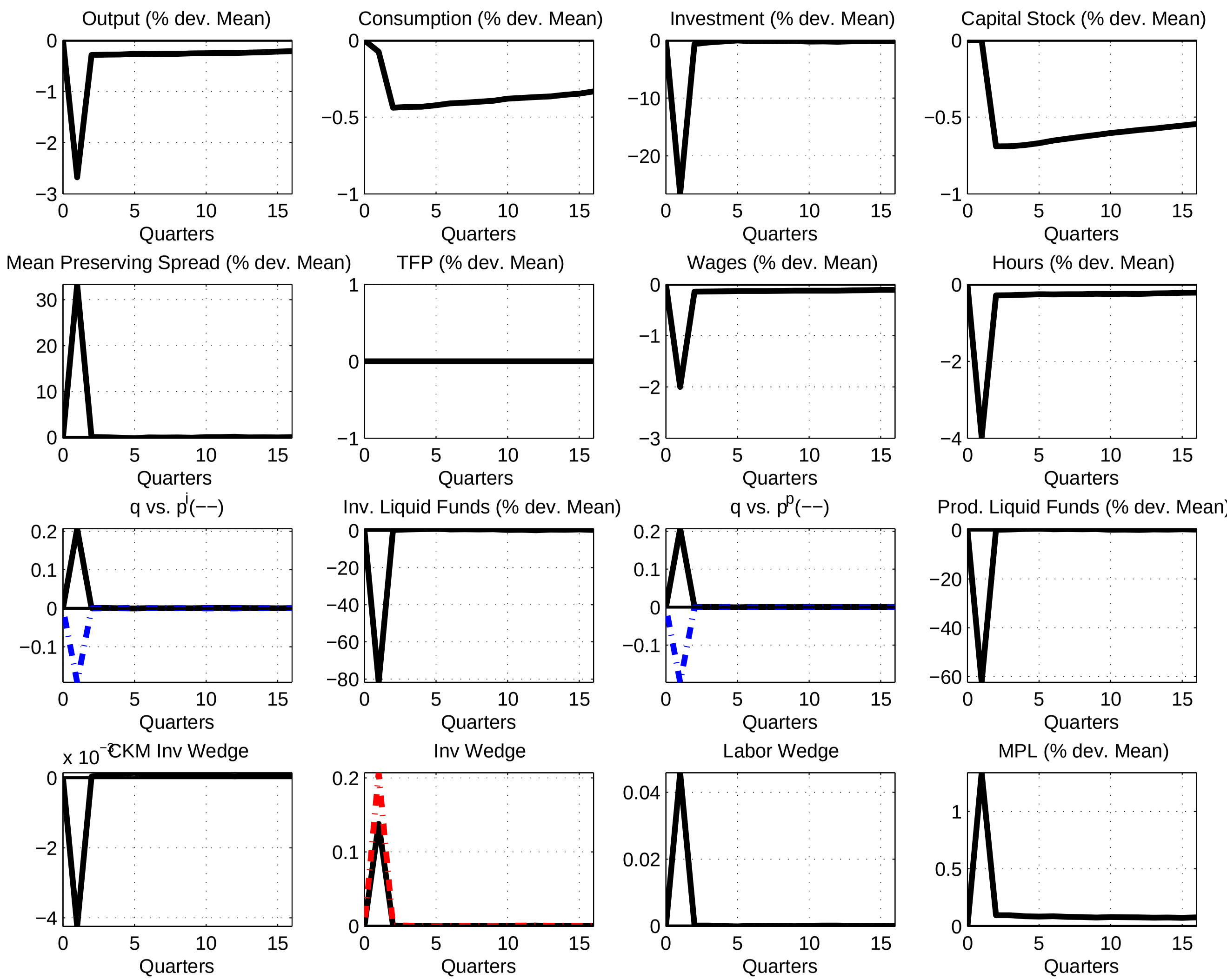

Figure 3: **Response to a MPS shock increase in the quality distribution of capital.**

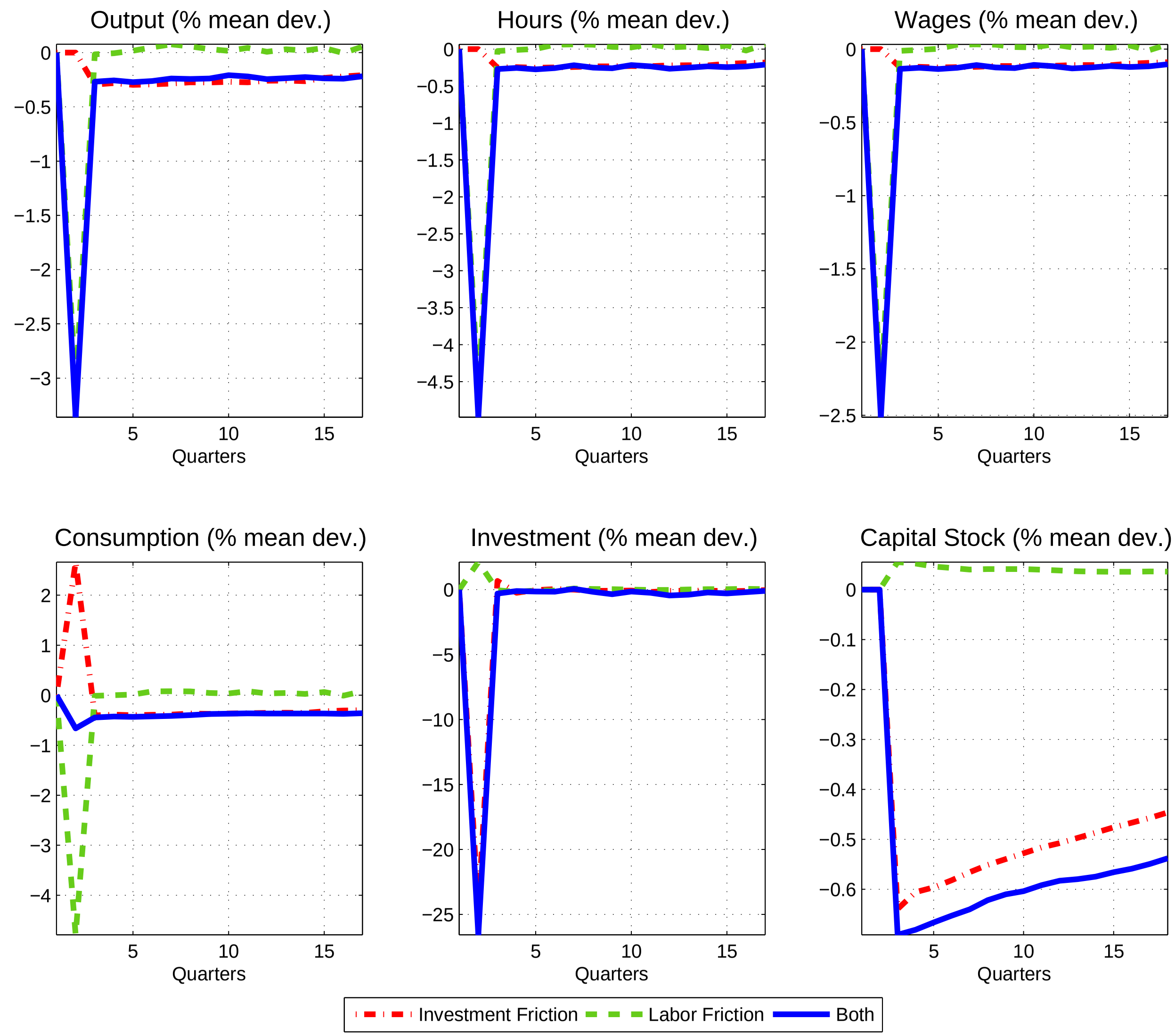

Figure 4: **Response with alternative frictions**. The figure shows the response of aggregate variables to a dispersion shock for the model with the friction on labor, on investment and both frictions combined.

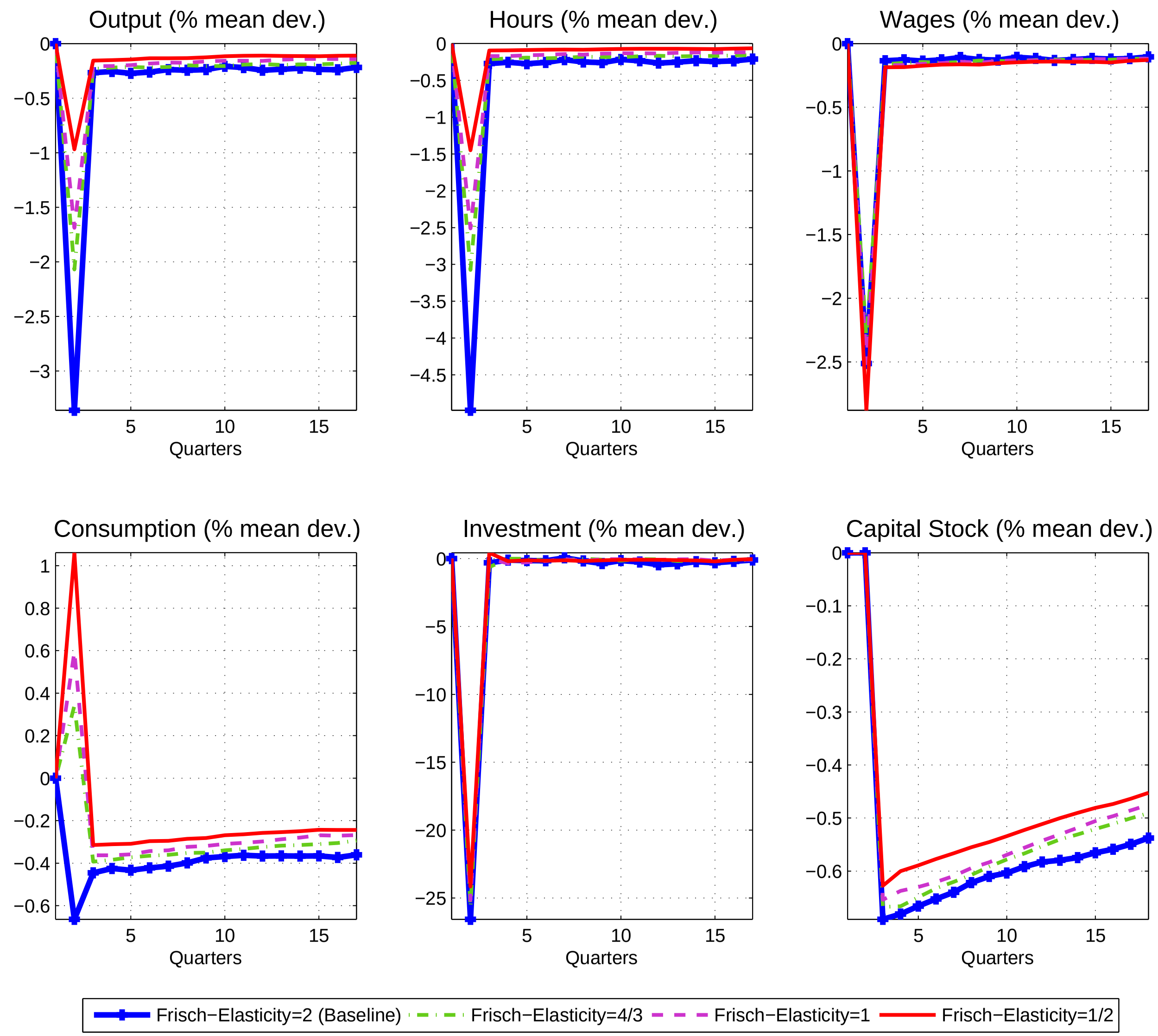

Figure 5: **Response with alternative Frisch elasticities**. The figure shows the response of aggregate variables to a dispersion shock for different values of the labor supply elasticity.

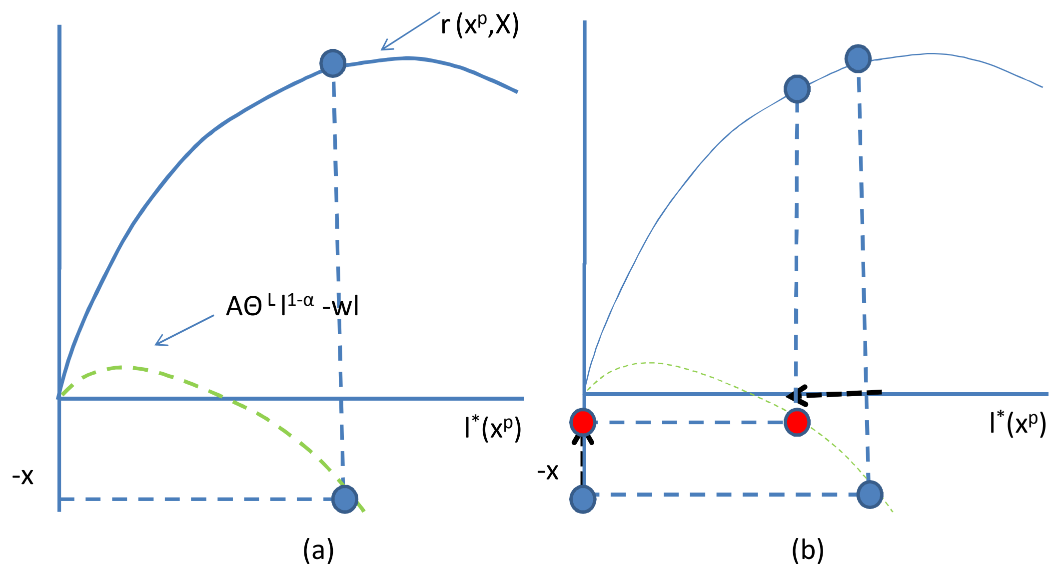

Figure 6: **Effect on Employment**.

A Understanding the Mechanics (Appendices not for publication)

A brief digression on the effects of a contraction in liquidity is useful to gain intuition on how dispersion shocks translate into the real economy. As discussed before, dispersion shocks have the effect of incrementing the costs of selling capital to obtain liquid funds. Here I take a reduction in liquidity as the starting point and focus on its partial equilibrium effects.

Effects on employment. For producers, lower liquidity means fewer funds to finance the fraction of payroll to be paid up-front. Thus, after the shock, the producer's labor demand must contract to the point where employment is consistent with this reduction. As shown earlier, the equilibrium employment is found by solving an implicit equation $\theta^L A l^{1-\alpha} - wl = -x$. The dashed curve in Panel (a) of Figure 6 is the left hand side of this equation and the solid line depicts the entrepreneur's profit as a function of employment size. Panel (b) shows how the reduction in liquidity restrains the equilibrium amount of labor as the enforcement constraints become tighter. It also shows how marginal profits increase. In general equilibrium, wages also respond to weaker labor demand conditions and partially mitigate the effect.

Effects on investment. To understand the effects on investment, assume the i-entrepreneur uses all his liquid funds for the internal funding of investment setting $i^d = x$. External financing is obtained by selling i^s claims to investment units at a price q but enforcement constraints impose a restriction on the amount of claims that can be issued, $i^s \leq \theta i$. Hence, total external funding is qi^s and the investment scale is $i = qi^s + i^d$. Whenever $q > 1$, i-entrepreneurs issue as many investment claims as possible. Because they own the fraction $(1-\theta)$ of total investment but only finance the $(1-q\theta)$ fraction, they obtain arbitrage rents

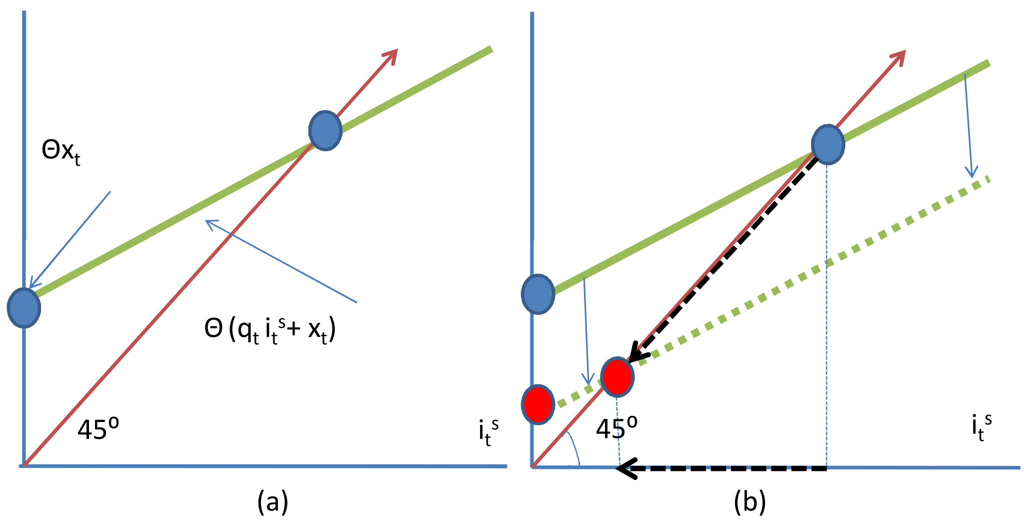

Figure 7: **Effect on Investment.**

by resorting to external financing.

Figure 7 describes the effects of a reduction in liquidity. Panel (a) plots the right and left hand sides of the limited enforcement constraints on claims: $i^s \leq \theta\left(qi^s + x\right)$. Thus, i^s must lie in the region where the affine function is above the 45-degree line. Panel (b) shows the effects of a decrease in x. The fall in the down payment reduces the intercept of the function defined by the right hand side of Figure 7. External funding falls together with the scale of the project. In general equilibrium, q also increases, something that mitigates the original effect.

B Isomorphic Environments

There are several isomorphic environments to the one presented in this paper.

Heterogeneity in investment productivity. Consider an environment with heterogeneity in the rate of investment. Let $a(z)$ be an investing entrepreneur z's technical rate of transformation: z can transform i consumption goods into $a\left(z\right)i$ capital goods. When $a(z)$ is observable, the corresponding incentive compatibility condition is $i^s \leq a\left(z\right)\theta^I i$. The entrepreneur's investment financing problem is, in this case:

Problem 7 (Investment financing with heterogeneity.) *The optimal financing problem is* $\max_{i^s \geq 0} a\left(z\right)i - i^s$ *subject to* $i^d + q\left(X\right)i^s = i$ *and* $i^s \leq \theta^I a\left(z\right)i$ *taking* $i_t^d > 0$ *as given.*

If $q\left(X\right) \geq a\left(z\right)$, the entrepreneur would find it optimal to invest. If this inequality is strict, a similar analysis to the one carried in the main text shows that the incentive constraint would also bind. The internal cost of capital for the investment entrepreneur is a z-dependent version for q^R:

$$q^R(z, X) = \frac{\left(1 - \theta^I a(z)\right)}{\left(1 - q(X)\,\theta^I a(z)\right)}$$

The threshold qualities are determined by equation (19) except that $q^R(X)$ is replaced by $q^R(z, X)$. Thus, cutoffs are also z-specific, $\omega^i(z, X)$ and consequently, i-entrepreneurs face different pooling prices that account for their incentives. When $a(z)$ is observable only after trading old capital units, financial firms reverse-engineer the qualities sold by each entrepreneur and $p^i(X) = \int \mathbb{E}_\phi[\lambda(\omega)\,|\omega < \omega^i(z, X), X]\, G(dz)$ where $G(z)$ is the distribution over technical rates of transformation. If $a(z)$ is unobservable throughout, an equilibrium can also be constructed, but equilibria would involve some entrepreneurs defaulting on claims. Kurlat [2009] avoids this by setting θ to 0. Indeed, this extension is a general version of that and the present paper. In any case, results are not substantially altered.

Heterogeneity in TPF. The case in which p-entrepreneurs are heterogeneous in individual TFP is similar to heterogeneity in investment productivity. In this case, $r(x, X)$ is also adjusted to be z-specific and the remaining conditions can be adapted in the same way.

Equity as collateral. If $\theta^L > (1 - \alpha)$, that is, when there are no enforcement problems in labor relations, the model is also isomorphic to a setup in which firms sell equity backed by capital units. In such an environment, entrepreneurs hold portfolios of equity issued by other entrepreneurs and depreciation shocks are allowed to be firm specific rather than capital unit specific. With enforcement constraints on employment, the model is more complex as entrepreneurs manage firms for other entrepreneurs. This version of the model is available upon request.

Repo contracts. There is a sense in which the model is identical to an environment in which agents write down Repo contracts with financial contracts and can independently default of capital units. It is not hard to show that the only equilibria are ones with trivial Repo contracts that replicate the allocations in the present model. In that case, the purchase price equals the pooling in this model and all units are defaulted upon.

C Relation with the literature

This paper relates to a number of prior studies on financial frictions. The seminal contributions of Bernanke and Gertler [1989], Carlstrom and Fuerst [1997] or Bernanke et al. [1999] explain how shocks that affect the relative wealth of borrowers have effects on lending because they exacerbate agency costs. The present paper is distinguished from this literature because dispersion shocks do not affect the relative wealth of constrained agents but affect the fraction of collateralized wealth.[17]

This paper also relates to Kiyotaki and Moore [1997] because there is a similar feedback effect from prices to constraints.[18] That paper shows how shocks that impact the price of collateral tighten enforcement constraints and, moreover, induce a multiplier effect. This multiplier follows from the reduction in future returns to capital which, in turn, further depress current prices magnifying the initial effect. A similar feedback occurs here: dispersion shocks tighten enforcement constraints having effects on future returns. In anticipation, the price of capital adjusts affecting the incentives to use capital as collateral.[19]

The insight of combining asymmetric information with limited enforcement follows from prior work by Kiyotaki and Moore [2008] (henceforth, KM). KM introduced a model with limited enforcement on claims to investment, but consider the fraction of capital that can be pledged to be exogenous and random. KM motivate their liquidity shocks by arguing that they possibly follow from a problem of asymmetric information that is not formalized. Indeed, asymmetric information is the natural market imperfection that motivates liquidity constraints because, as known at least since the work of Stiglitz and Weiss [1981], it can induce credit rationing outcomes. This paper formalizes the ideas of KM and extends them. On the one hand, it introduces private information in the quality of collateral as in Myers and Majluf [1984] and incorporates this feature into a general equilibrium environment. On the other hand, it extends the analysis beyond investment frictions and adds enforcement constraints on labor relations.

In fact, only until recently, there were but a few pieces that incorporated asymmetric information problems into general equilibrium. A notable exception is Eisfeldt [2004] who studies endogenous liquidity where consumption-smoothing motives provide the incentives to trade under asymmetric information. The closest to the present paper is Kurlat [2009], which independently develops similar ideas. Both papers take KM as starting point and extend their environment modeling liquidity through asymmetric information. The key distinction that distinguishes our work is that Kurlat's paper focuses on frictions that affect only investment (in the particular case where of $\theta^L = 0$ and $\theta^P = 1$). This distinction is important because, as argued throughout, imperfections in the labor market are necessary to obtain a quantitatively relevant theory. Current studies like Arellano et al. [2010] and

[17]In these papers, agency costs follow from a problem of costly state verification rather than from limited enforcement. Nonetheless, this distinction is immaterial.

[18]In both papers, capital is a form of collateral. In Kiyotaki and Moore [1997] entrepreneurs use all their capital stock as collateral. Here, entrepreneurs sell a fraction of capital to obtain liquid funds that are used as collateral.

[19]This feedback effect is only present for investing entrepreneurs and disappears completely under log-preferences.

Shourideh and Zetlin-Jones [2010] introduce asymmetric information a l*à* Stiglitz and Weiss [1981] that affect the aggregate effective TFP rather than the labor demand. Because these papers depart from frictions that only focus on investment, they can also deliver important quantitative responses to dispersion shocks.

A recent number of papers in the money-search literature use private information in the quality of capital to explain the use of money and equity as mediums of exchange. Rocheteau [2009] shows how money dominates equity when the quality of the latter is private information. As in the present model, the dispersion of asset quality reduces liquidity. Lester et al. [2009] provide a similar environment in which agents are also allowed to acquire information about assets. A lesson from these studies is that liquidity is not invariant to policy.

There are to reasons for why formalizing liquidity shocks through asymmetric information is important. First, a theory of endogenous liquidity provides a set of testable implications on the stochastic process behind liquidity. For example, Proposition 6 in the main text shows that liquidity is always inefficient. The implication of this result is that the liquidity shocks studied by KM should be restricted to regions were they are always or never binding. Second, and most important, policy experiments that treat liquidity as exogenous may be subject to the Lucas critique, as argued by Rocheteau [2009] in the context of money-search models. Several recent studies, for example Gertler and Karadi [2009], Curdia and Woodford [2009], Gertler and Kiyotaki [2010] or del Negro et al. [2010], analyze the effects of alternative government policies directed at replenishing liquidity. In those models, the disturbances that interrupt financial markets originate exogenously. The experiments studied in these papers are extremely valuable to asses the implications of the many policies undertaken during the financial crises of 2008-2009. Yet, it would be desirable to test the robustness of results in the context of endogenous liquidity. There are several reasons why assessments can depend on how the private sector reacts to policy. For example, policies that transfer liquid funds to constrained agents will, in parallel, reduce the benefit of selling capital under asymmetric information. Thus, a policy that targets liquidity may, in principle, be ineffective to reactivate the economy because it can be partially offset by inducing a reduction in the liquidity of assets that are sold under private information.

D Returns and State Equations

Labor Demand: Aggregate labor demand is obtained by aggregating across p-entrepreneurs. Since labor/capital ratios are constant, aggregate labor demand is,

$$L^d(X) = l^*(x^p(X), X)(1-\pi)K \tag{24}$$

Equilibrium employment. Worker's consumption is $c = w(X) L^s(X)$ where $L^s(X)$ is the labor supply. In equilibrium, the leisure-consumption defines the aggregate labor supply: $w(X)^{\frac{1}{\nu}} = L^s(X)$. Individual labor supply is given by: $w(X) = (l^*(x^p(X), X)(1-\pi)K/\bar{\omega})^{\nu}$. Recall that $l^*(x^p(X), X)$ is also a function of $w(X)$. Thus, $x^p(X), w(X)$ and $l^*(x^p(X), X)$ solve a non linear system of equations.

*Aggregate output.*Aggregate output, $Y(X)$, is the sum of the return to labor and capital:

$$Y(X) = r(x, X)(1-\pi)K + (l^*(x^p(X), X)(1-\pi)K)^{\nu+1}. \tag{25}$$

Aggregate variables. Using the results from Proposition 5, one can aggregate to obtain aggregate consumption and capital holdings:

$$\begin{aligned} C^p(X) &= (1-\varsigma^p(X)) W^p(X)(1-\pi)K \text{ and } C^i(X) = \left(1-\varsigma^i(X)\right) W^i(X)\pi K \\ K'^{,p}(X) &= \varsigma^p(X) W^p(X)(1-\pi)K/q(X) \text{ and } K'^{,i}(X) = \varsigma^i(X) W^i(X)\pi K/q^R(X) \end{aligned}$$

Aggregate capital evolves according to $K'(X) = K'^{,i}(X) + K'^{,p}(X)$. Finally, the following proposition describes the $q(x)$,

Proposition 7 (Market Clearing) *The equilibrium full information price of capital is given by:*

$$q(X) = \begin{cases} q^o(X) & \text{if } q^o(X) > 1 \\ 1 & \text{if otherwise} \end{cases} \tag{26}$$

where $q^o(X)$ is a function of $(W^p(X), W^i(X), \varsigma^s(X), \varsigma^i(X))$.

The proof is presented in Appendix E.

D.1 Optimal Policies and Aggregation

Finally, $\varsigma^s(X)$ and $\varsigma^i(X)$ remain to be characterized. For that purpose, define the virtual returns to capital conditional on the entrepreneur's type:

$$\begin{aligned} R^{pp}(X', X) &\equiv \frac{W^p(X')}{q(X)} \text{ and } R^{pi}(X', X) \equiv \frac{W^p(X')}{q^R(X)} \\ R^{ii}(X', X) &\equiv \frac{W^i(X')}{q^R(X)} \text{ and } R^{ip}(X', X) \equiv \frac{W^i(X')}{q(X)} \end{aligned}$$

$R^{jm}(X, X')$ is the X' contingent return of equity from to an entrepreneur m who becomes j at the current state X. These virtual returns are used to obtain the marginal propensities to save, $\varsigma^i(X)$ and $\varsigma^s(X)$ according to:

Proposition 8 (Recursion) *Marginal propensities to save, ς^i and ς^s satisfy:*

$$\left(1-\varsigma^i(X)\right)^{-1} = 1+\beta^{1/\gamma}\Omega^i\left(\left(1-\varsigma^p(X')\right), \left(1-\varsigma^i(X')\right)\right) \tag{27}$$

$$\left(1-\varsigma^p(X)\right)^{-1} = 1+\beta^{1/\gamma}\Omega^p\left(\left(1-\varsigma^p(X')\right), \left(1-\varsigma^i(X')\right)\right) \tag{28}$$

where

$$\Omega^i\left(a(X'), b(X')\right) \equiv \mathbb{E}\left[(1-\pi)\left(a(X')\right)^{\gamma} R^{pi}(X')^{1-\gamma} + \pi\left(b(X')\right)^{\gamma} R^{ii}(X')^{1-\gamma}\right]^{1/\gamma}$$

$$\Omega^s\left(a(X'), b(X')\right) \equiv \mathbb{E}\left[(1-\pi)\left(a(X')\right)^{\gamma} R^{pp}(X')^{1-\gamma} + \pi\left(b(X')\right)^{\gamma} R^{ip}(X')^{1-\gamma}\right]^{1/\gamma}$$

In addition, $\varsigma^i, \varsigma^i \in (0,1)$ and equal (β, β) if $\gamma = 1$.

E Proofs

E.1 Proof of Proposition 1

For the proof, $w(X)$ is referred simply as w. Rearranging the incentive compatibility constraints this problem consists of solving:

$$\begin{aligned} r(x, X) &= \max_{l\geq 0, \sigma\in(0,1)} Al^{1-\alpha} - wl \text{ subject to} \\ \sigma wl &\leq \theta^L Al^{1-\alpha} \text{ and } (1-\sigma) wl \leq x. \end{aligned}$$

Denote the solutions to this problem by (l^*, σ^*). The unconstrained labor demand is $l^{unc} \equiv \left[\frac{A(1-\alpha)}{w}\right]^{\frac{1}{\alpha}}$. A simple manipulation of the constraints yields a pair of equations that characterize the constraint set:

$$l \leq \left[A\frac{\theta^L}{\sigma w}\right]^{\frac{1}{\alpha}} \equiv l^1(\sigma) \tag{29}$$

$$l \leq \frac{x}{(1-\sigma) w} \equiv l^2(\sigma) \tag{30}$$

$$\sigma \in (0,1)$$

As long as l^{unc} is not a part of the constraint set, at least one of the constraints will be active because the objective is increasing in l for $l \leq l^{unc}$. In particular, the tighter constraint will bind as long as $l \leq l^{unc}$. Thus, $l^* = \min\{l^1(\sigma^*), l^2(\sigma^*)\}$ if $\min\{l^1(\sigma^*), l^2(\sigma^*)\} \leq l^{unc}$ and $l^* = l^{unc}$ otherwise. Therefore, note that (29) and (30) impose a cap on l depending on the choice of σ. Hence, in order to solve for l^*, we need to know σ^*. First observe that (29) is a decreasing function of σ. The following properties can be verified immediately:

$$\lim_{\sigma\to 0} l^1(\sigma) = \infty \text{ and } l^1(1) = \left(\frac{\theta^L}{(1-\alpha)}\right)^{\frac{1}{\alpha}} \left[\frac{A}{w}(1-\alpha)\right]^{\frac{1}{\alpha}} = \left(\frac{\theta^L}{(1-\alpha)}\right)^{\frac{1}{\alpha}} l^{unc} \tag{31}$$

The second constraint curve (30) presents the opposite behavior. It is increasing and has the following limits,

$$l^2(0) = \frac{x}{\omega} \text{ and } \lim_{\sigma\to 1} l^2(\sigma) = \infty$$

These properties imply that $l^1(\sigma)$ and $l^2(\sigma)$ will cross at most once if $x > 0$. Because the objective is independent of σ, the entrepreneur is free to choose σ in a to relax the constraint on l as much as possible. Since $l^1(\sigma)$ is decreasing and $l^2(\sigma)$ increasing, the optimal choice of σ^* solves $l^1(\sigma^*) = l^2(\sigma^*)$ so that l is as large as possible. This implies that both constraints will bind if one of them must bind. Adding them up, we find that $l^{cons}(x)$ is the largest solution to

$$\theta^L A l^{1-\alpha} - wl = -x \tag{32}$$

This equation defines $l^{cons}(x)$ as the largest solution of this implicit function: when $x = 0$, this function has two zeros so the restriction to the largest root prevents us from picking $l = 0$. Indeed, if $x = 0$, then $\sigma = 1$, does not imply that l should be 0.

Thus, we have that,

$$l^*(x) = \min\{l^{cons}(x), l^{unc}\}$$

Since $l^1(\sigma)$ is monotone decreasing, if $\theta^L \geq (1-\alpha)$, then, $l^1(1) \geq l^{unc}$, by (31). Because for $x > 0$, $l^1(\sigma)$ and $l^2(\sigma)$ cross at some $\sigma < 1$, then, $l^{cons} > l^{unc}$ and $l^* = l^{unc}$. Moreover, if $x = 0$, then the only possibility implied by the constraints of the problem is to set $\sigma = 1$. But since, $l^1(1) \geq l^{unc}$, then $l^* = l^{unc}$. Thus, we have shown that $\theta^L \geq (1-\alpha)$ is sufficient to guarantee that labor is optimally chosen regardless of the value of x. This proves the second claim in the proposition.

Assume now that $l^{unc} \leq \frac{x}{\omega}$. Then, the wage bill corresponding to the efficient employment can be guaranteed upfront by the entrepreneur. Obviously, $x \geq wl^{unc}$ is sufficient for optimal employment.

To pin down the necessary condition for the constraint to bind, observe that the profit function in (32) is concave with an positive interior maximum. Thus, at $l^{cons}(x)$, the left hand side of (32) is decreasing. Therefore, if $l^{cons}(x) < l^{unc}$, then it should be the case that

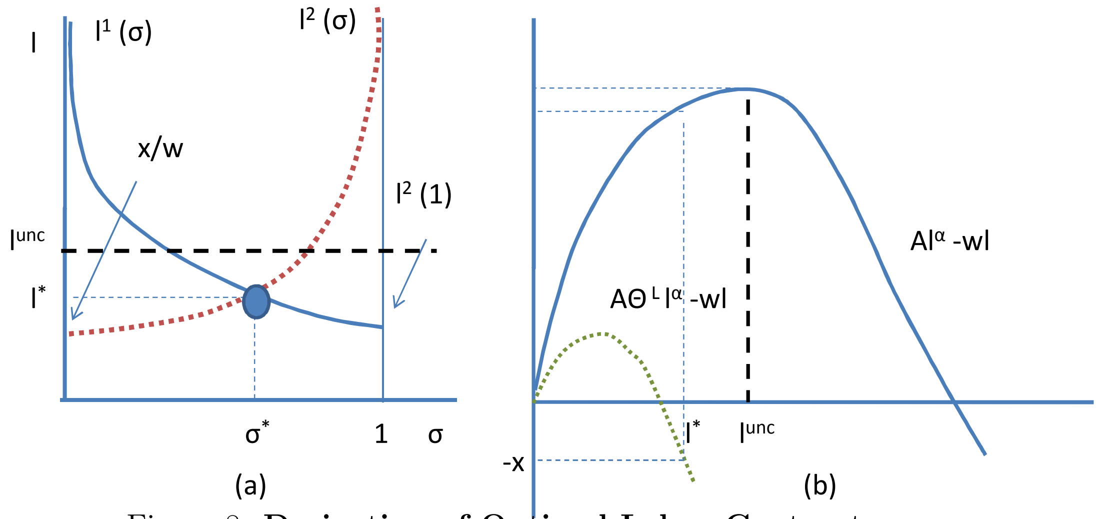

Figure 8: **Derivation of Optimal Labor Contract.**

$\theta^L A (l^{unc})^{1-\alpha} - w l^{unc} < -x$. Substituting the formula for l^{unc} yields the necessary condition for the constraints to be binding:

$$x < w^{1-\frac{1}{\alpha}} \left[A(1-\alpha)\right]^{\frac{1}{\alpha}} \left(1 - \frac{\theta^L}{(1-\alpha)}\right)$$

This shows that if $\frac{\theta^L}{(1-\alpha)} < 1$, at least some liquidity is needed to achieve efficient employment.

Figure 8 provides a graphical description of the arguments in the proof. The left panel plots l^1 and l^2 as functions of σ. It is clear from the figure that the constraint set is largest at the point where both curves meet. If l^{unc} is larger than the point where both curves meet, then, the optima is constrained. A necessary condition for constraints to be binding is that l^{unc} is above $1^2(1)$, otherwise l^{unc} will lie above. A sufficient condition for constraints to be binding is described in the right panel. The dashed line represents the left hand side of (32) as a function of labor. The figure shows that when the function is evaluated at l^{unc}, and the result is below $-x$, then the constraints are binding.

E.2 Proof of Lemma 1

This Lemma is an application of the principle of optimality. By homogeneity, given a labor-capital ratio l/k, p-entrepreneur profits are linear in capital stock:

$$\left[A(l/k)^{1-\alpha} - w(l/k)\right] k \tag{33}$$

Observe that once x is taken into account, the incentive compatibility constraint (4) and the working capital constraint (6) can be expressed in terms of the labor-capital ratio only:

$$A\left(l/k\right)^{1-\alpha}-\sigma w\left(X\right)\left(l/k\right)\geq\left(1-\theta^{L}\right)A\left(l/k\right)^{1-\alpha} \tag{34}$$

and

$$\left(1-\sigma\right)w\left(X\right)\left(l/k\right)\leq x. \tag{35}$$

l and σ don't enter the entrepreneur's problem anywhere else. Thus, optimally, the entrepreneur will maximize expected profits per unit of capital in (33) subject to (34) and (35). This problem is identical to the to Problem 3. Thus, the value of profits for the entrepreneur considering the optimal labor to capital ratio is $r\left(x,X\right)k$. Finally, substituting (3) into (2) and using $r\left(x,X\right)k$ as an indirect profit function yields the equivalent problem.

E.3 Proof of Proposition 2

The proof requires some preliminary computations. Note that given ι^{s}, x is independent of the entrepreneur's capital stock. In addition, Lemma 1 shows that the entrepreneur's profits are linear in the entrepreneur's capital stock. Thus, the following computations are normalized to the case when $k=1$.

Marginal labor of liquidity. For any x such that $l^{*}\left(x\right)=l^{unc}$, the constraints (4) and (6) are not binding. Therefore, when x is sufficiently large to guarantee efficient employment, an additional unit of liquidity has no value for increasing $r\left(x,X\right)$. For x low enough, constraints are binding. Applying the Implicit Function Theorem to the pseudo-profit function (32) and obtain an expression for the marginal increase in the labor with a marginal increase in liquidity,

$$\frac{\partial l^{cons}}{\partial x}=-\frac{1}{\left(1-\alpha\right)\theta^{L}Al\left(x\right)^{-\alpha}-w}$$

Note that the denominator satisfies,

$$\left(1-\alpha\right)\theta^{L}Al^{-\alpha}-w\leq\frac{\left[\theta^{L}Al^{1-\alpha}-wl\right]}{l}=\frac{-x}{l}<0$$

which verifies that $\frac{\partial l^{cons}}{\partial x}>0$.

Marginal profit of labor. Let $\Pi\left(l,X\right)=Al^{1-\alpha}-wl$. The marginal product of labor is,

$$\Pi_{l}\left(l,X\right)=A\left(1-\alpha\right)l^{-\alpha}-w>0\text{ for any }l<l^{unc}$$

Marginal profit of liquidity. Using the chain rule, we have an expression for the marginal profit obtained from an additional unit of liquidity.

$$\begin{aligned} r_x(x,X) &= \Pi_l(l^*(x))\, l^{*\prime}(x) = -\frac{A(1-\alpha)\, l^*(x)^{-\alpha} - w}{(1-\alpha)\,\theta^L A l^*(x)^{-\alpha} - w},\ l^*(x) \in (l^{cons}(0), l^{unc}) \\ &\quad \text{and 0 otherwise.} \end{aligned}$$

Thus, liquidity has a marginal value for the entrepreneur whenever constraints are binding. Since $l^*(x)$ is the optimal labor choice, $\Pi(l^*(x)) = r(x,X)$, which explains the first equality $r_x(x,X) = \Pi_l(l^*(x))\, l^{*\prime}(x)$. Since $A(1-\alpha)\, l(x)^{-\alpha} - w$ approaches 0 as $l(x) \to l^{unc}$, $r_x(x,X) \to 0$, as x approaches its efficient level. Hence, $r_x(x,X)$ is continuous and $r(x,X)$ everywhere differentiable. The marginal value of liquidity, $r_x(x,X)$, is decreasing in x ($r_{xx}(x,X) < 0$) since the numerator is decreasing and the denominator is increasing in x.

Equilibrium liquidity. To establish the result in Proposition 2, observe that as in the standard lemons problem of Akerlof [1970], if any capital unit of quality ω is sold in equilibrium, all the units of lower quality must be sold. Otherwise, the entrepreneur would be better-off by selling the lower qualities instead. This cutoff rule will define a threshold quality ω^* for which all qualities below ω will be sold. Choosing the qualities to be sold is equivalent to choosing a threshold quality ω^* to sell. The entrepreneur chooses that threshold to maximize the right hand side of the 14. Thus, $\omega^p(X)$ solves:

$$\omega^p(X) = \arg\max_{\omega^*} r(x,X)\, k + q(X)\, k \int_{\omega^*}^{1} \lambda(\omega)\, f_\phi(\omega)\, d\omega$$

where

$$x = p^p(X) \int_0^{\omega^*} \iota^s(\omega) f_\phi(\omega)\, d\omega$$

The objective function is continuous and differentiable, as long as $f_\phi(\omega)$ is absolutely continuous. Thus, interior solutions are characterized by first order conditions. Substituting x, in $r(x,X)$ and taking derivatives yields the following first order condition:

$$r_x(x,X)\, p^p(X) f_\phi(\omega^*) - q(X)\, \lambda(\omega^*)\, f_\phi(\omega^*) \geq 0 \text{ with equality if } \omega^* \in (0,1)$$

Points where $f_\phi(\omega^*) = 0$ are saddle points of the objective function, so without loss of generality $f_\phi(\omega^*)$ is canceled from both sides. There are three possibilities for equilibria: $\omega^* = 1, \omega^* \in (0,1)$, or $\omega^* \neq \varnothing$, where the latter case is interpreted as no qualities are sold.

Full liquidity. If $\omega^* = 1$, then it must be the case that

$$-r_x(x, X)\frac{p^p(X)}{q(X)} \geq \lambda(1) \tag{36}$$

If this condition is violated, by continuity of $r_x(x, X)$, the entrepreneur could find a lower threshold w^* that maximizes his budget.

Interior solutions. For an interior solution $\omega^* \in [0, 1)$, it must be the case that

$$-r_x(x, X)\frac{p^p(X)}{q(X)} = \lambda(\hat{\omega}) \tag{37}$$

for $x = p^p(X)\int_0^{\omega^*} \iota^s(\omega)d\omega$. Since $r_x(x, X)$ is continuous and decreasing, if the condition does not hold, the entrepreneur can be better of with a different cutoff: lowering the cutoff in the case where the equality is replaced by $<$.

Market Shutdowns. Finally, as in any lemons problem, there exists a market shutdown equilibrium where $\omega^* = \emptyset$, and $p^p(X) = 0$. This accounts for the case in which no ω^* satisfies the necessary first order condition. Moreover, a market shutdown equilibrium is unique iff

$$w(X)\left[\frac{(1-\alpha)}{\theta^L} - 1\right] < 1. \tag{38}$$

To see why, evaluate the market equilibrium price given by (11), when $\omega^* = 0$,that is $\lambda_L(X) = \lambda(0)$. Then, the (38) is equivalent to

$$q(X)\mathbb{E}_\phi[\lambda(\omega)|\lambda(\omega) < \lambda_L]w(X)\left[\frac{(1-\alpha)}{\theta^L} - 1\right] = q(X)\mathbb{E}_\phi[\lambda(\omega)|\lambda(\omega) < \lambda_L]r_x(0, X) < q(X)\lambda_L$$

$r_x(x, X)$ is decreasing and any conditional expectation also satisfies $\mathbb{E}_\phi[\lambda(\omega)|\lambda(\omega) < \lambda(\omega^*)] \leq \lambda(\omega^*)$, for any ω^*. Thus, if this condition holds, then it is also true that,

$$q(X)\mathbb{E}_\phi[\lambda(\omega)|\lambda(\omega) < \lambda_L]r_x(x, X) < q(X)\lambda(\omega^*)$$

contradicting the existence of another equilibrium pair $(\omega^*, p(X))$. The exact expression in the proposition is obtained by substituting the market equilibrium condition, (11), into (36).

Labor Wedge representation. The marginal value of liquidity admits a representation in terms of labor wedges commonly used in the literature. To express these benefit in terms of labor wedges, note that an entrepreneur with profit function $Al^{(1-\alpha)} - (1+\tau)wl$ would optimally choose labor to satisfy: $\tau = \frac{z(1-\alpha)l^{-\alpha} - w}{w}$. Let $\tau_l(z, x, X)$ be the expression to the right when $l = l^*(x)$ and $w = w(X)$. Substituting A and $A\theta^L$ for z in $\tau_l(z)$, leads to the following expression for the marginal value of a unit of liquidity in terms of labor wedges:

$$r_x(x,X) = -\frac{\tau_l(A,x,X)}{\tau_l(\theta^L A,x,X)}$$

E.4 Proofs of Lemma 2 and Proposition 4

The proof of Lemma 2 follows the same steps as the proof of Lemma 1. The proof of Proposition 4 is identical as to the proof of Proposition 2. Due to the linearity in the production of capital and the constraints, in this case the marginal value of an additional unit of liquidity is constant and equal to $\frac{q(x,X)}{q^R(x,X)}$.

E.5 Proof of Proposition 8

This section provides a proof of the optimal policies described in section 3. The strategy is guess and verify. For the proof, I omit the arguments of the functions. For *producing* entrepreneurs I guess: $V^p(W) = U(a^p W)$, $c^p(W) = (1-\varsigma^p)W$, $k^{',p}(W,q) = \frac{\varsigma^p W}{q}$ and for *investing* entrepreneurs, the guess is : $V^i(W) = U(a^i W)$, $c^i(W) = (1-\varsigma^i)W, k^{',i}(W) = \frac{\varsigma^i W^i}{q^R}$.

Guess. Using this guess, the first order condition for $k^{\prime,p}$ is:

$$(k^{\prime,p}) : U^{'}(c) = \beta\mathbb{E}\left((1-\pi)U'(a^p W^{\prime,p})a^p W^{\prime,p} + \pi U'\left(a^i W^{\prime,i}\right)a^i W^{\prime,i}\right) \tag{39}$$

where I use the fact that investment opportunities are independent random variables. There are four types of returns that map current wealth in both states to wealth in both states. These maps are given by $W^{\prime,p} = R^{pp}\varsigma^p W^{\prime,p}$, $W^{\prime,i} = R^{ip}\varsigma^p W^p$, $W^{\prime,i} = R^{ii}\varsigma^i W^i$, and $W^{\prime,p} = R^{pi}\varsigma^i W^i$. Using these definitions, W^p can be factored out from (39) to obtain:

$$(k^{\prime,p}) : U^{'}(c) = \beta(\varsigma^p W^p)^{-\gamma}\mathbb{E}\left[(1-\pi)U'(a^p R^{pp})a^p R^{pp} + \pi U'\left(a^i R^{ip}\right)a^i R^{ip}\right]$$

By replacing the guess for consumption and clearing out wealth, an Euler equation in terms of the marginal propensities to save is obtained:

$$(1-\varsigma^p)^{-\gamma} = \beta(\varsigma^p)^{-\gamma}\mathbb{E}\left[(1-\pi)U'(a^p R^{pp})a^p R^{pp} + \pi U'\left(a^i R^{ip}\right)a^i R^{ip}\right]$$

Proceeding in the same way the first order condition for the investment state is given by:

$$\left(1-\varsigma^i\right)^{-\gamma} = \beta\left(\varsigma^i\right)^{-\gamma}\mathbb{E}\left[(1-\pi)U'\left(a^p R^{pi}\right)a^p R^{pi} + \pi U'\left(a^i R^{ii}\right)a^i R^{ii}\right]$$

Verification. Replacing the guessed policies and using the Envelope Theorem, the following relation is obtained:

$$U^{'}\left(\left(1-\varsigma^{p}\right)W^{p}\right)=V_{w}^{\prime}\left(a^{p}W^{p}\right)a^{p}\rightarrow\left(a^{p}\right)^{1-\gamma}=\left(1-\varsigma^{p}\right)^{-\gamma}$$

Similarly, for the investment state: $\left(a^{i}\right)^{1-\gamma}=\left(1-\varsigma^{i}\right)^{-\gamma}$. Rewriting both relations yields $a_{t}^{j}=\left(1-\varsigma^{j}\right)^{\frac{-\gamma}{1-\gamma}}$, $j=i,s$. The certainty equivalent of a unit of future wealth is:

$$\Omega^{p}\left(a^{\prime,p},a^{\prime,i}\right)=\mathbb{E}\left[\left(1-\pi\right)\left(a^{\prime,p},R^{pp}\right)^{1-\gamma}+\pi\left(a^{\prime,i},R^{ip}\right)^{1-\gamma}\right]$$

which is homogeneous of degree 1. The same is true about:

$$\Omega^{i}\left(a^{\prime,p},a^{\prime,i}\right)=\mathbb{E}\left[\left(1-\pi\right)\left(a^{\prime,p},R^{pi}\right)^{1-\gamma}+\pi\left(a^{\prime,i},R^{ii}\right)^{1-\gamma}\right]$$

Rearranging terms delivers: $\left(1-\varsigma^{p}\right)^{-\gamma}=\beta\left(\varsigma^{p}\right)^{-\gamma}\Omega^{s}\left(a^{\prime,p},a^{\prime,i}\right)$ and $\left(1-\varsigma_{t}^{i}\right)^{-\gamma}=\beta\left(\varsigma^{i}\right)^{-\gamma}\Omega^{i}\left(a^{\prime,p},a^{\prime,i}\right)$. Clearing out ς^{p} from the right hand side, and adding 1 to both sides yields:

$$\left(1-\varsigma^{p}\right)^{-1}=1+\beta^{1/\gamma}\Omega^{s}\left(a^{\prime,p},a^{\prime,i}\right)^{1/\gamma} \tag{40}$$

and

$$\left(1-\varsigma^{i}\right)^{-1}=1+\beta^{1/\gamma}\Omega^{i}\left(a^{\prime,p},a^{\prime,i}\right)^{1/\gamma} \tag{41}$$

Note that any value function satisfies the envelope condition so any pair of functions $\varsigma^{p},\varsigma^{i}$ satisfying this recursion guarantee that the functional form guessed for V^{p} and V^{i} satisfies the Bellman equation. Since policies are independent of wealth, we have shown that the economy satisfies the conditions for Gorman's aggregation result.

Corollary 1 (Aggregation) *The economy admits a aggregation.*

Remark 1. *Any solution to the functional equations 40 and 41 solves the policy functions of the entrepreneurs' problem. As long as the value function is well defined, the solutions to these function are unique. The operator defined by the right hand side of 40 and 41 is a contraction for a sufficiently low returns and $\gamma\in(0,1)$. The policy functions may be obtained by iteration of this operator. In contrast, the contraction property is not guaranteed to work for $\gamma\geq1$, in which case convergence must be checked.*

Remark 2. *For the case in which $\gamma=1$, (40) and (41) are solved by constant functions equal to β.*

A formal statement of the last two remarks is provided in Bigio [2009].

E.6 Proof of Proposition 7

Substitute the optimal policies described in Proposition 5 into the expression for $D(X)$ and $S(X)$ to obtain $I^s(X) = D(X) - S(X)$. Then uses (21) and to clear out expressions for $I^s(X)$ and $I(X)$. In the proof the state X is fixed so I drop the arguments from the functions. Performing these substitutions, the aggregate version of the incentive compatibility condition becomes:

$$\frac{(1-\pi)\left(\varsigma^p\left(r+q\psi^p\right)/q-\psi^p\right)K-(1-\pi)\,\varphi^pK-\pi\varphi^iK}{\theta} \leq \frac{\pi\left[\varsigma^i\left(W^i/q^R\right)K-\psi^iK\right]}{(1-\theta)}$$

I have introduced the following variables:

$$\begin{array}{ll} \varphi^p=\int_{\omega\leq\omega^p}\lambda(\omega)\,f_\phi(\omega)\,d\omega & \varphi^i=\int_{\omega\leq\omega^i}\lambda(\omega)\,f_\phi(\omega)\,d\omega \\ \psi^p=\int_{\omega>\omega^p}\lambda(\omega)\,f_\phi(\omega)\,d\omega & \psi^i=\int_{\omega>\omega^i}\lambda(\omega)\,f_\phi(\omega)\,d\omega \end{array}$$

that correspond to the expectations over the sold and unsold qualities of both groups. K clears out from both sides. I then use the definition of q^i and rearrange the expression to obtain:

$$\begin{aligned} \frac{(1-\pi)\varsigma^pr-\left((1-\pi)\left(1-\varsigma^p\right)\psi^p+(1-\pi)\,\varphi^p+\pi\varphi^i\right)q}{\theta q} &\leq \frac{\pi\left[\varsigma^iq\varphi^i-\left(1-\varsigma^i\right)\psi^iq^R\right]}{(1-\theta)\,q^R} \\ &\leq \frac{q\pi\varsigma^i\varphi^i}{(1-\theta q)}-\frac{\pi\left(1-\varsigma^i\right)\psi^i}{(1-\theta)} \end{aligned}$$

I get rid of q from the denominators, rearrange terms and obtain,

$$\begin{aligned} & (1-\pi)\varsigma^pr\,(1-\theta q)-\left((1-\pi)\left(\left(1-\varsigma^p\right)\psi^p+\varphi^p\right)+\pi\varphi^i\right)q\,(1-\theta q) \\ \leq\; & \theta q^2\pi\varsigma^i\varphi^i-\theta q\,(1-\theta q)\,\pi\frac{\left(1-\varsigma^i\right)\psi^i}{(1-\theta)} \end{aligned}$$

By arranging terms, the inequality includes linear and quadratic terms for q. This expression takes the form:

$$\left(q^*\right)^2A+q^*B+C\geq 0 \tag{42}$$

where the coefficients are:

$$A=-\theta\left((1-\pi)\left(\left(1-\varsigma^p\right)\psi^p+\varphi^p\right)+\pi\left(1-\varsigma^i\right)\varphi^i-\pi\theta\frac{\left(1-\varsigma^i\right)}{(1-\theta)}\psi^i\right)$$

$$B = \theta(1-\pi)\varsigma^p r + \left((1-\pi)\left(\left(1-\varsigma^p\right)\psi^p + \varphi^p\right) + \pi\varphi^i - \pi\theta\frac{(1-\varsigma^i)}{(1-\theta)}\psi^i\right)$$

$$C = -(1-\pi)\varsigma^p r$$

C is negative. Observe that

$$\begin{aligned}
& (1-\pi)\left(\left(1-\varsigma^p\right)\psi^p + \varphi^p\right) + \pi\varphi^i - \pi\frac{(1-\varsigma^i)}{(1-\theta)}\psi^i\theta \\
\geq\ & (1-\pi)\left(\left(1-\varsigma^p\right)\psi^p + \varphi^p\right) + \pi\left(1-\varsigma^i\right)\varphi^i - \pi\frac{(1-\varsigma^i)}{(1-\theta)}\psi^i\theta \\
\geq\ & (1-\pi)\left(\left(1-\varsigma^p\right)\psi^p + \varphi^p\right) + \pi\left(1-\varsigma^i\right)\varphi^i - (1-\pi)\left(1-\varsigma^i\right)\psi^i \\
\geq\ & (1-\pi)\bar{\lambda} - (1-\pi)\varsigma^p\psi^p + \pi\left(1-\varsigma^i\right)\bar{\lambda} - \pi\left(1-\varsigma^i\right)\psi^i - (1-\pi)\left(1-\varsigma^i\right)\psi^i \\
\geq\ & \bar{\lambda} - (1-\pi)\varsigma^p\psi^p - \pi\psi^i \\
\geq\ & 0
\end{aligned}$$

where the second line follows from the assumption that $(1-\theta) \geq \pi$. The third linen uses the identity $\bar{\lambda} = \psi^p + \varphi^p = \psi^i + \varphi^i$. The fourth line uses the fact that $(1-\varsigma^i) < 1$ and the last line uses the fact that ψ^p and ψ^i are less than $\bar{\lambda}$. This shows that A is negative and B is positive. Evaluated at 0, (42) is negative. It reaches a maximum at $-\frac{B}{2A} > 0$. Thus, both roots of (42) are positive. Let the roots be (q_1, q_2) where q_2 is the largest. There are three possible cases: *Case 1:* If $1 \in (q_1, q_2)$, then $q = 1$ satisfies the constraint.

Case 2: If $1 < q_1$, then $q = q_1$, since it is the lowest price such that the constraints bind with equality.

Case 3: If $q_2 < 1$, then there exists no incentive compatible price. Thus, $I = 0$ and i-entrepreneurs consume part of their capital stock.

F Equilibrium Equations

An equilibrium can be entirely characterized by a fixed point problem in the functions $q(X), \omega^p(X), \omega^i(X), \varsigma^s(X)$ and $\varsigma^i(X)$. Once this fixed point is found, the rest of the equilibrium objects is obtained by equilibrium conditions. The following set of functional equations summarizes the equilibrium conditions. For presentation purposes, I present these in three blocks:

Capital Market Clearing Block:

$$q^{R}(X)=\frac{1-\theta q(X)}{1-\theta}$$

$$I(X)-I^{s}(X)=\left[\frac{\varsigma^{i}(X)W^{i}(X)}{q^{R}(X)}-\int_{\omega>\omega^{i}(X)}\lambda(\omega)f_{\phi}(\omega)d\omega\right]\pi K.$$

$$D(X)=\left[\frac{\varsigma^{p}(X)W^{p}(X)}{q(X)}-\int_{\omega>\omega^{p}(X)}\lambda(\omega)f_{\phi}(\omega)d\omega\right](1-\pi)K.$$

$$S(X)=\underbrace{\left[\int_{\omega\leq\omega^{p}(X)}\lambda(\omega)f_{\phi}(\omega)d\omega\right](1-\pi)K}_{\text{Capital sales by p-types}}+\underbrace{\left[\int_{\omega\leq\omega^{i}(X)}\lambda(\omega)f_{\phi}(\omega)d\omega\right]\pi K}_{\text{Capital sales by i-types}}$$

$$D(X)=S(X)+I(X)$$

$$I^{s}(X)(1-\theta)\leq\theta\left(I(X)-I^{s}(X)\right)$$

Marginal Propensities Block:

$$\begin{aligned}R^{pp}(X',X)&\equiv\frac{W^{p}(X')}{q(X)}\text{ and }R^{ip}(X',X)\equiv\frac{W^{p}(X')}{q^{R}(X)}\\R^{ii}(X',X)&\equiv\frac{W^{i}(X')}{q^{R}(X)}\text{ and }R^{ip}(X',X)\equiv\frac{W^{i}(X')}{q(X)}\end{aligned}$$

$$\begin{aligned}\left(1-\varsigma^{i}(X)\right)^{-1}&=1+\beta^{1/\gamma}\Omega^{i}\left((1-\varsigma^{p}(X')),\left(1-\varsigma^{i}(X')\right)\right)\\(1-\varsigma^{p}(X))^{-1}&=1+\beta^{1/\gamma}\Omega^{p}\left((1-\varsigma^{p}(X')),\left(1-\varsigma^{i}(X')\right)\right)\end{aligned}$$

$$\begin{aligned}\Omega^{i}(a(X'),b(X'))&\equiv\mathbb{E}\left[(1-\pi)(a(X'))^{\gamma}R^{pi}(X')^{1-\gamma}+\pi(b(X'))^{\gamma}R^{ii}(X')^{1-\gamma}\right]^{1/\gamma}\\\Omega^{p}(a(X'),b(X'))&\equiv\mathbb{E}\left[(1-\pi)(a(X'))^{\gamma}R^{pp}(X')^{1-\gamma}+\pi(b(X'))^{\gamma}R^{ip}(X')^{1-\gamma}\right]^{1/\gamma}\end{aligned}$$

$$W^{i}(X)\equiv\left[q(X)\int_{\omega\leq\omega^{i}(X)}\lambda(\omega)f_{\phi}(\omega)d\omega+q^{R}(X)\int_{\omega>\omega^{i}(X)}\lambda(\omega)f_{\phi}(\omega)d\omega\right]$$

$$W^{p}(X)\equiv\left[r(x^{p}(X),X)+q(X)\int_{\omega>\omega^{p}(X)}\lambda(\omega)f_{\phi}(\omega)d\omega\right]$$

Liquidity Block:

$$\frac{q(X)}{q^R(X)}\mathbb{E}_\phi\left[\lambda(\omega)|\omega<\omega^i(X),X\right]=\lambda\left(\omega^i(X)\right)$$

$$r_x(x^p,X)\,\mathbb{E}_\phi\left[\lambda(\omega)|\omega<\omega^p(X),X\right]=\lambda\left(\omega^p(X)\right)$$

$$x^p=q(X)\,\mathbb{E}_\phi\left[\lambda(\omega)|\omega<\omega^p(X),X\right]$$

$$w(X)=\left(l^*(x^p,X)\,K\right)^\nu$$

$$l^*(x^p,X)=\min\left\{\arg\max_l \theta^L A l^{1-\alpha}-wl=x^p,l^{unc}\right\}$$

$$r(x^p,X)=Al^{*1-\alpha}-\left(l^*(x,X)\,K\right)^{\nu+1}$$

G Additional Graphs

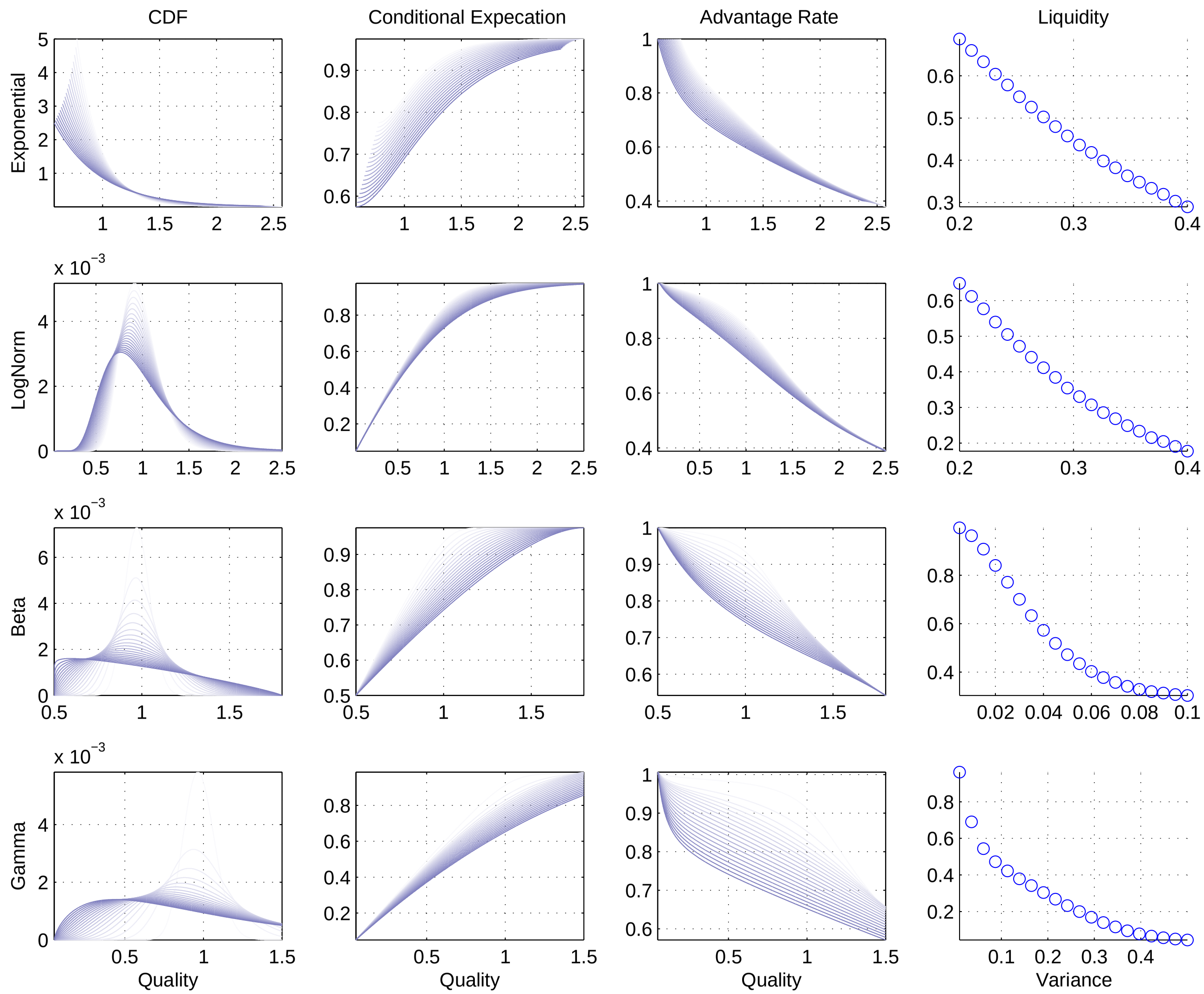

Figure 9: **Family of Distributions satisfying Assumption 1**. The figure plots several moments for families corresponding to exponential, log-Normal, Beta and Gamma distributions. From left to right, the figures plot their pdf, conditional expectations, advantage rates and the equilibrium liquidity for i-entrepreneurs holding q fixed.

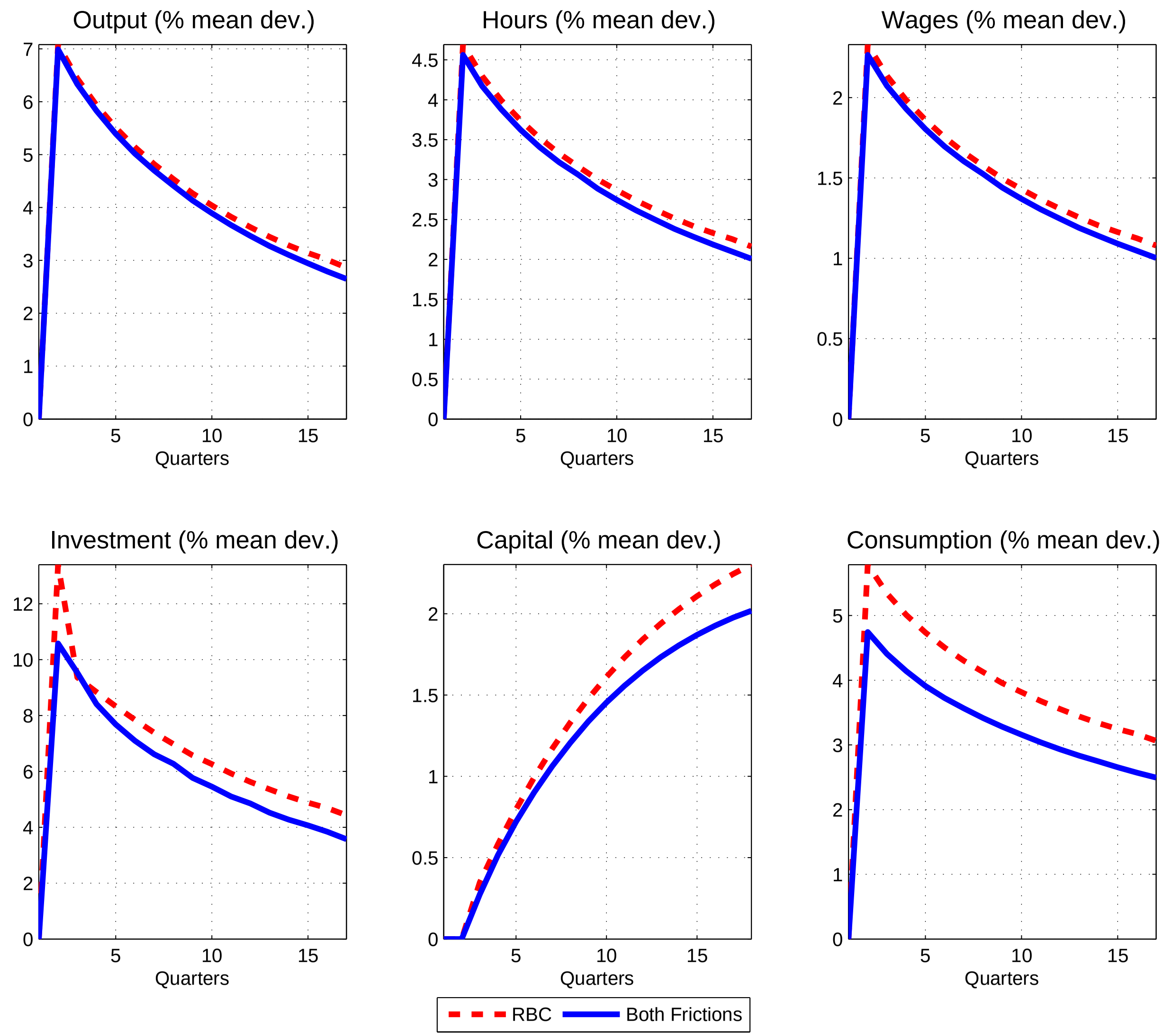

Figure 10: **Amplification of TFP shocks**. The figure contrasts the response of aggregate variables to a TFP shock in the model with active friction and an RBC benchmark.

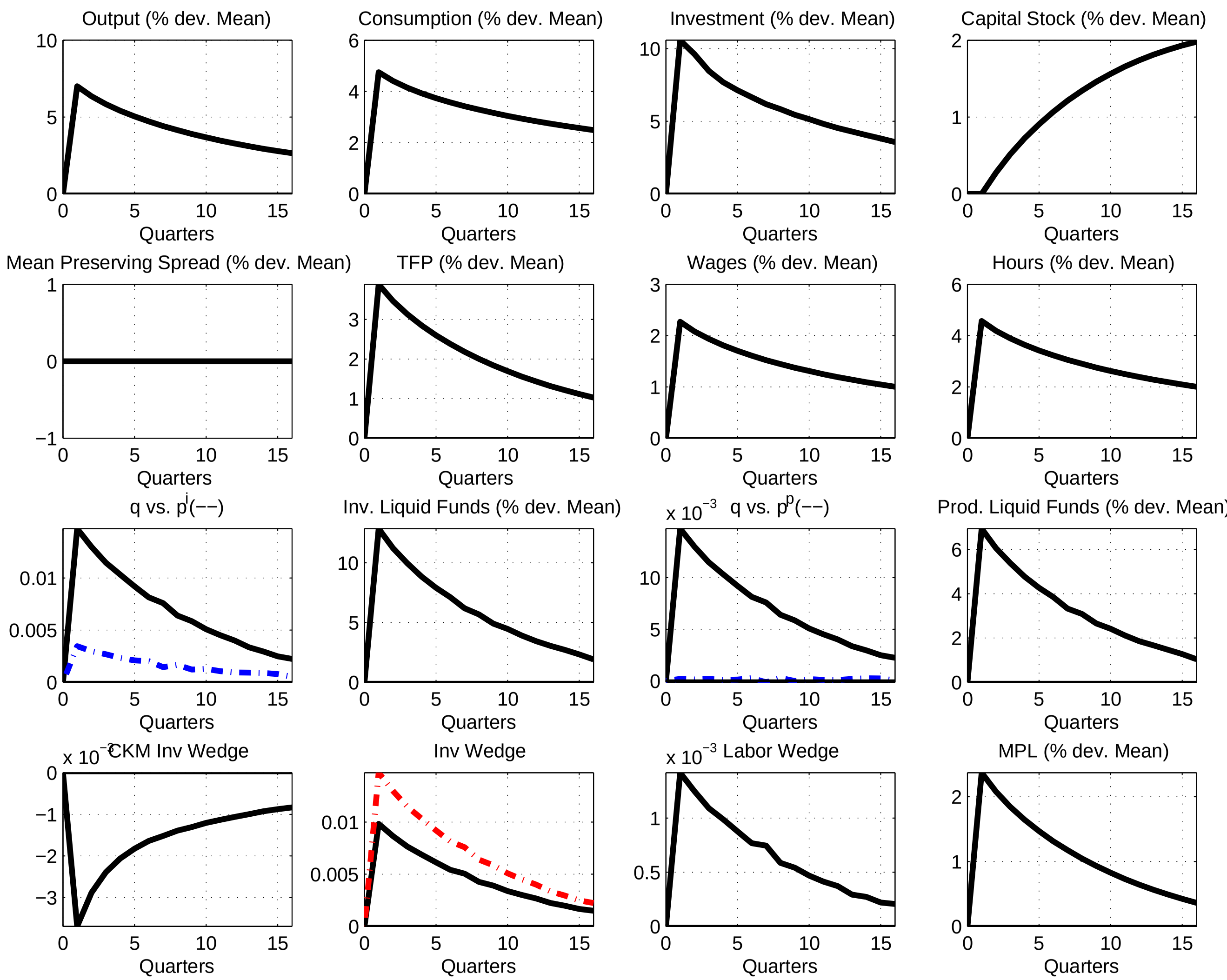

Figure 11: **Response to a Positive Technology Shock**. The figure plots the response of aggregate variables to a positive TFP shock.