Hiring Through Referrals

Manolis Galenianos*

Pennsylvania State University

March 2011

Abstract

An equilibrium search model of the labor market is combined with a social network. The key features are that the workers' network transmits information about jobs and that wages and entry of firms are determined in equilibrium. In the baseline model workers are homogeneous and referrals are used to mitigate search frictions. When worker heterogeneity is added referrals also facilitate the hiring of better workers. Consistent with empirical evidence, access to referrals decreases unemployment probability and increases wages for workers while hiring through referrals yields more productive workers for firms. The aggregate matching function exhibits decreasing returns to scale.

*I would like to thank Steven Davis, Steven Durlauf, Sanjeev Goyal, Ed Green, Philipp Kircher, Nobu Kiyotaki, Guido Menzio, Alex Monge, Theodore Papageorgiou, Nicola Persico, Andres Rodriguez, Rob Shimer, Giorgio Topa, Neil Wallace, Randy Wright and Ruilin Zhou as well as many seminar and conference participants for helpful comments and the National Science Foundation for financial support (grant SES-0922215).

1 Introduction

Social networks are an important feature of labor markets (Granovetter (1995)). Approximately half of all American workers report learning about their job through their social network (friends, acquaintances, relatives etc.) and a similar proportion of employers report using the social networks of their current employees when hiring (the evidence is summarized in Section 1.1 and is extensively surveyed in Ioannides and Loury (2006) and Topa (2010)).

Surprisingly, however, social networks are typically not included in the equilibrium models that are used to study labor markets. For instance, in their survey of search-theoretic models of the labor market, Rogerson, Shimer and Wright (2005) do not cite a single paper that includes social networks or referrals. On the other hand, a large literature uses graph theory to study social networks (Jackson (2008)). When applied to labor markets, however, these models usually restrict attention to partial equilibrium analyses where, for instance, wages or the demand for labor are exogenous (e.g. Calvo-Armengol and Jackson (2004)).

This paper proposes to bridge this gap by combining an equilibrium search model with a network structure that is simple enough to preserve tractability but also rich enough to deliver a large number of predictions that can be compared with the empirical evidence. The model's two key features are: first, the workers' network transmits information about jobs; second, wages and the entry of firms are determined in equilibrium and depend non-trivially on the workers' network.

In the baseline model workers are homogeneous in terms of their productivity and network. Each worker is linked with a measure of other workers and the network is exogenous. Vacancies are created both through the free entry of new firms and through the expansion of producing firms. A firm and a worker meet either through search in the frictional market or through a referral, which occurs when a producing firm expands and asks its current employee to refer a link. Each firm hires one worker and vacancies created through expansion are immediately sold off. The flow surplus of a worker-firm match is equal to output plus the value of the referrals and the wage is determined by Nash bargaining.

Referrals affect the labor market in two ways in the baseline model. First, they mitigate search frictions which unambiguously reduces unemployment. Second, under certain conditions they discourage the entry of new firms which makes it harder for workers to find a job through the market. The second effect is driven by the model's equilibrium nature.

The model is then extended to allow for worker heterogeneity. There are two worker types which represent heterogeneity beyond the workers' observable characteristics. A worker's type (high or low) determines his productivity and network. Conditional on type, every worker has the same measure and composition of links and, in accordance with evidence from the sociology literature, a worker is assumed to have more links with workers of his own type (homophily; see McPherson, Smith-Lovin and Cook (2001)). Firms act similarly to the baseline model. In the context of worker heterogeneity, referrals facilitate the hiring of high type workers in addition to mitigating search frictions.

Despite the model's simplicity, it yields predictions that are consistent with a number of stylized facts about the interaction between social networks and labor markets. The first two empirical observations are that a worker with better access to referrals (say, due to a larger network) is less likely to be unemployment and enjoys higher wages (Bayer, Ross and Topa (2008)) and that a worker's job-finding rate increases in the employment rate of his links (Topa (2001), Weinberg, Reagan and Yankow (2004), Cappellari and Tatsiramos (2010)). In the model, an unemployed worker's job finding rate increases in the number of workers that are linked with him as well as their employment rate, which is consistent with the above.

Three stylized facts which are relevant for the extension to worker heterogeneity are that, conditional on observable worker characteristics, referred candidates are more likely to be hired (Fernandez and Weinberg (1997), Castilla (2005)), they receive higher wages (Dustmann, Glitz and Schoenberg (2010)) and they are more productive on the job (Castilla (2005)). There three facts are consistent with the model's prediction that a referred worker is more likely to be of a high type than a non-referred worker. The model delivers that prediction because high productivity workers are more likely to be employed and therefore more likely to act as referrers; the recipients of referrals are therefore more likely to also be high types due to the network's homophily.[1]

Additionally, the model predicts that in a wage regression one should not necessarily expect a positive effect on a dummy variable for finding the job through a referral. In labor markets where worker heterogeneity is not very important, for which the baseline model is a good approximation, referrals are only used to alleviate search frictions, meaning that there is no wage premium for finding a job through a referral. It is only in labor markets where heterogeneity is important that one would expect to find such a premium. This observation gives some context as to why finding a wage premium for referrals has occasionally been difficult (Pistaferri (1999), Pelizzarri (2010), Bentolila, Michelacci and Suarez (2010)).

Finally, a novel prediction of the baseline model is that, once referrals are included, the aggregate matching function exhibits decreasing returns to scale. The reason is that when a worker loses his job, this reduces the flow of referrals in addition to increasing the pool of unemployed workers. This generates persistence of labor market variables and implies that the aggregate job finding rate is decreasing in the unemployment rate even after conditioning on the vacancy-unemployment ratio (which is the only variable that affects job-finding in the standard search and matching model). The last prediction is consistent with the findings of the business cycle accounting performed in Cheremukhin and Restrepo (2010).

1.1 Empirical evidence about social networks and labor markets

Numerous studies have documented that both workers and firms use referrals extensively when searching for a job or trying to fill a vacancy, respectively. More than 85% of worker use informal contacts when searching for a job according to the National Longitudinal Survey

[1]This prediction is based on selection and is similar in spirit to Montgomery (1991) who considers a two-period model of the labor market with heterogeneous workers and a homophilous network among them. That model has no implications about employment rates and does not address the possibility of using referrals when there is no informational advantage concerning the worker's productivity.

of Youth (NLSY) (Holzer (1988)). In terms of outcomes, more than 50% of all workers found their job through their social network according to data from the Panel Study of Income Dynamics (PSID) (Corcoran, Datcher and Duncan (1980)) while the 24 studies surveyed by Bewley (1999) put that figure between 30% and 60%.

On the firm side, between 37% and 53% of employers use the social networks of their current employees to advertise jobs according to data from the National Organizations Survey (NOS) (Mardsen (2001)) and the Employment Opportunity Pilot Project (EOPP) (Holzer (1987)), respectively. According to the EOPP 36% of firms filled their last opening through a referral (Holzer (1987)).

From the workers' side, referrals lead to faster job-finding. Using census data Bayer, Ross and Topa (2008) find that when a male individual's access to social networks improves by one standard deviation (say, by moving to a city block where more people have children of the same age) this raises his labor force participation by 3.3 percentage points, hours worked by 1.8 hours and earning by 3.4 percentage points, and these figures are even higher for females. The employment status of the individuals in the network is also important. Topa (2001) finds strong evidence of local spillovers in employment rates across different census tracks in the Chicago area. Weinberg, Reagan and Yankow (2004) find that an increase of one standard deviation in a neighborhood's social characteristics increases annual hours by 6.1% with confidential NLSY data. Using data from the British Household Panel Survey (BHPS) Cappellari and Tatsiramos (2010) show that an additional employed friend is associated with a increase in the probability of finding a job of 3.7 percentage points and a 5% increase in wages which they interpret as evidence for better access to referrals.

Referred applicants are statistically different from non-referred ones. Fernandez and Weinberg (1997) and Castilla (2005) find in their firm-level studies that referred applicants are more likely to be hired after controlling for their observable characteristics. This is consistent with the finding of Holzer (1987) and Blau and Robbins (1990) that referrals have a greater "hire yield" for firms than searching in the market, using EOPP data. Castilla (2005) has direct measures of worker productivity and reports that referred workers are more productive after controlling for observable characteristics. Using German data Dustmann, Glitz and Schoenberg (2010) find that referred candidates receive higher wages and have lower lay-off rates after controlling for worker observables and firm fixed effects.

Pistaferri (1999), Pelizzarri (2010) and Bentolila, Michelacci and Suarez (2010) document that using the job-finding method as one of the explanatory variables in a wage regression may lead to an insignificant or even negative coefficient of referrals on wages. These studies do not control for firm or job fixed effects, unlike the studies cited in the previous paragraph. This suggests there is selection either on the firm side or on the type of jobs that are found through referrals. Studying this effect, however, is beyond the scope of the present paper.

The interaction between social contacts and labor markets has been extensively studied by the sociology literature. One finding is that the social ties that are most useful for transmitting information about job opportunities are the more numerous "weak" ties, e.g. acquaintances, as opposed to the "strong" ties, such as close friends (Granovetter (1973)). A second very robust finding is that social interactions tend to feature *homophily*: individuals who socialize together are more likely to share many characteristics, such as race and religion but also educational and professional characteristics (see McPherson, Smith-Lovin and Cook (2001) for an exhaustive survey). Both findings inform the modeling choices of the present paper.

2 The Labor Market with Homogeneous Workers

This Section adds referrals to a standard equilibrium search model of the labor market.

2.1 The Model

Time runs continuously, the horizon is infinite and the labor market is in steady state. There is free entry of firms and each firm hires one worker, is risk-neutral, maximizes expected discounted profits and discounts the future at rate r. A firm is either filled and producing or vacant and searching and the flow profit when vacant is 0.

There is a unit measure of workers who are homogeneous, risk-neutral, maximize expected discounted utility and discount the future at rate r. A worker is either employed or unemployed and the flow utility of unemployment is b. Every worker is linked with a *measure* ν of other workers, where $\nu \leq 1$.

Modeling a worker's network as a continuum of links preserves the model's tractability and is consistent with the (spirit of the) sociology literature's finding that it is a person's more numerous weak ties that help most with finding a job (Granovetter (1973)). A worker's employment opportunities will in general depend on how many of his links are employed at any point in time which necessitates keeping track of each link's time-varying employment status. Having a continuum of links means that the aggregate (un)employment rate of a worker's social contacts does not change over time due to the law of large numbers, thereby greatly simplifying the analysis.[2] It is certainly true that some of the richness in the predictions generated by graph-theoretic models of networks is lost by the assumption of a continuum of links; however, given the available data it seems that many of these additional predictions would be difficult to empirically verify or refute. Last, note that introducing workers who are heterogeneous in their networks is fairly straightforward (see Section 3).

Vacancy creation occurs in two ways, both of which cost K: a new firm enters the market or an existing firm expands which occurs at exogenous rate ρ (the position that is created by the expansion is immediately sold off which keeps firms' employment at one). A firm and a worker meet either through search in the market or through a referral, which occurs when a firm expands and asks its current employee to refer a link. The rate of meeting through

[2] In Calvo-Armengol and Zenou (2005) each worker has a finite number of links and he is assumed to draw a new network every period so as to avoid keeping track of transitions in the network's employment rate. In Fontaine (2008) every worker belongs to a finite network, each network is isolated from the others and the analysis focuses on the steady state distribution of network employment rates. Wages are determined by Nash bargaining and change every time the network's employment rate changes. Furthermore, vacant firms are not allowed to target their search in low-employment networks.

the market is determined by a matching function. The rate of meeting through referrals is determined by the rate at which firms expand.

An expansion can be interpreted in (at least) a couple of different ways. At rate ρ, the firm meets an entrepreneur who wants to enter the market at which point the firm expands and sells him the new position (that entrepreneur would otherwise create a new firm through free entry). Alternatively, at rate ρ the firm identifies a business opportunity and expands to take advantage of it. However, it is subject to decreasing returns and finds it profitable to sell the new position to some new entrepreneur. For this paper's purposes it makes little difference which interpretation is adopted, though that choice is certainly important to endogenize the expansion rate.

The flow value of a match is given by the worker's productivity, y, and the value of the referrals that he generates. The worker and the firm split the surplus according to the Nash bargaining solution where the worker's bargaining power is denoted by $\beta \in (0, 1)$.[3] Matches are exogenously destroyed at rate δ and there is no on the job search.

A producing firm expands at exogenous rate ρ, where $\delta > \rho$.[4] When an expansion occurs, one of the links of the incumbent worker is contacted at random. If the link is employed then the referral opportunity is lost and search in the market begins; if the link is unemployed then he is hired by the firm. To summarize, both ways of creating a vacancy bear the same cost but expansion could lead to an immediate hire while entry of a new firm is necessarily followed by time-consuming search in the market.[5]

The assumption that the referrer contacts one of his links at random regardless of that link's employment status captures the frictions which are present when the referral channel is used. One justification is that the referring employee does not know which of his links

[3] It is assumed that all payoff-relevant information, including the worker's network, are common knowledge within the match.

[4] This assumption guarantees that entry of new firms is necessary for steady state: absent entry of new firms, the stock of producing firms will decline.

[5] Note that vacancy creation through expansion dominates the entry of a new firm. Since entry occurs in steady steady state, every firm that is given the opportunity to expand will choose to do so.

is currently looking for a job, which is consistent with the weak ties interpretation of the network, and starts calling them at random to find out if they are interested in the job. Because this is costly, he will only try a finite number of calls and with positive probability he will not find anyone interested in the job. In this paper it is assumed that the referring worker stops calling after a single try, but this is only for simplicity.

Denote the expected surplus generated during an expansion by E. When a firm expands, it pays K and creates a vacancy, whose value is denoted by V. The incumbent worker contacts one of his links and a match is created if that worker is unemployed, the probability of which is denoted by u. Letting the firm's value of a match be J we have

$$E = -K + V + u(J - V). \tag{1}$$

The new position is immediately sold off and the incumbent firm receives share $\gamma \in [0, 1]$ of that surplus (the remaining $(1 - \gamma)E$ is captured by the buyer). Therefore a match's flow value is given by $y + \rho\gamma E$.

To determine the referral rate focus on some worker j who is linked to ν^j workers, each of whom is in turn linked with ν workers. The number of employed links of worker j is equal to $(1-u)\nu_j$. The employer of each link expands at rate ρ in which case one of the incumbent employee's ν links receives the referral at random. Therefore, the rate at which worker j is referred to a job is $\alpha_R^j = \rho\nu^j(1 - u)/\nu$ and the network's homogeneity ($\nu^j = \nu,\ \forall j$) implies

$$\alpha_R \quad = \quad \rho(1 - u).$$

Note that the exact measure of links each worker has does not affect the characterization of equilibrium.

Consider the rate of meeting in the market and let v denote the number of vacancies. The flow of meetings in the market between a vacancy and a worker is given by a Cobb-Douglas function

$$M(v,u) = \mu v^{\eta} u^{1-\eta},$$

where $\mu > 0$ and $\eta \in (0,1)$.

The rate at which a firm meets with a worker is

$$\alpha_F = \frac{M(v,u)}{v} = \mu(\frac{u}{v})^{1-\eta}$$

and the rate at which a worker meets a firm through the market is

$$\alpha_M = \frac{M(v,u)}{u} = \mu(\frac{v}{u})^{\eta}.$$

The aggregate matching function, which includes both meetings through referrals and meetings through the market, is given by

$$\mathcal{M}(v,u) = \mu v^{\eta} u^{1-\eta} + \rho u(1-u) \tag{2}$$

The second term is derived from noting that when the number of producing firms is $1-u$, the rate of vacancy creation through expansion is equal to $\rho(1-u)$ and each referral leads to a new match with probability u.

The steady state condition is that the flows in and out of unemployment are equal:

$$u(\alpha_M + \alpha_R) = (1-u)\delta. \tag{3}$$

The agents' value functions are now described. When vacant, a firm searches in the market and meets with a worker at rate α_F. When producing, the firm's flow payoffs are $y + \rho\gamma E - w$ where w denotes the wage. The match is destroyed at rate δ. The firm's value of a vacancy (V) and production (J) are given by:

$$rV = \alpha_F(J - V),$$
$$rJ = y + \rho\gamma E - w - \delta J.$$

When unemployed, a worker's flow utility is b and job opportunities appear at rate $\alpha_M + \alpha_R$. When employed, the worker's flow utility is equal to the wage and the match is destroyed at rate δ. The worker's value of unemployment (U) and employment (W) are given by:

$$rU = b + (\alpha_M + \alpha_R)(W - U),$$
$$rW = w + \delta(U - W).$$

The wage solves the Nash bargaining problem

$$w = \text{argmax}_w (W - U)^\beta (J - V)^{1-\beta}. \tag{4}$$

We are ready to define the Labor Market Equilibrium.

Definition 2.1 *A Labor Market Equilibrium is the steady state level of unemployment u and the number of vacancies v such that:*

- *The labor market is in steady state as described in* (3).
- *The surplus is split according to* (4).
- *There is free entry of firms: $V = K$.*

2.2 Labor Market Equilibrium

The characterization of equilibrium is fairly standard.

The condition that describes the steady state can be rewritten as follows:

$$
\begin{aligned}
& u[\mu(\tfrac{v}{u})^{\eta}+\rho(1-u)]=(1-u)\delta \\
\Rightarrow \qquad & v=[\tfrac{1-u}{\mu}(\tfrac{\delta}{u^{1-\eta}}-\rho u^{\eta})]^{1/\eta}. \qquad (5)
\end{aligned}
$$

Equation (5) shows that the steady state rate of unemployment is uniquely determined given v and it is strictly decreasing in v.[6] As a result, in steady state α_M and α_R are strictly increasing in v while α_F is strictly decreasing in v.

The surplus of a match is given by $S = W + J - U - V$. Nash bargaining implies

$$
\begin{aligned}
W-U &= \beta S, \\
J-V &= (1-\beta)S.
\end{aligned}
$$

The value functions can be rearranged to yield

$$
(r+\delta)S = y+\rho\gamma E-b-(\alpha_M+\alpha_R)\beta S-(r+\delta)V. \qquad (6)
$$

Combining equation (6) with the definition of E (equation (1)) and the free entry condition ($V = K$) and going through some algebra yields an expression that only depends on the number of vacancies in the market:

$$
S = \frac{y-b-(r+\delta)K}{r+\delta+(\alpha_M+\alpha_R)\beta-\rho\gamma u(1-\beta)}.
$$

The denominator of the right-hand side is strictly increasing in v which means that when the steady state and free entry conditions hold we have $dS/dv < 0$.

[6]It is more convenient mathematically to write v as a function of u, although conceptually the measure of unemployed workers is the dependent variable –determined through the steady state condition for a given v– and the measure of vacancies is the independent variable –eventually determined through free entry.

The value function of a newly-created firm is

$$rV = \alpha_F(1-\beta)S. \tag{7}$$

Since α_F and S are strictly decreasing in v, there is a unique measure of vacancies such that the value of creating a vacancy is equal to K.

The proposition summarizes the previous statements:

Proposition 2.1 *An equilibrium exists and it is unique.*

To study the way that referrals affect the labor market, a comparative statics exercise is performed with respect to the rate that referrals are generated (ρ) and the resulting equilibrium is examined. The following propositions state the results which are proven in the Appendix.

Proposition 2.2 *An increase in the rate that referrals are generated (ρ) leads to a decrease in the equilibrium rate of unemployment (u).*

Proposition 2.3 *An increase in the rate that referrals are generated (ρ) leads to a decrease in the market job-finding rate (α_M) if $\beta \geq (1-\beta)\gamma$ and $u \leq 1/2$. A sufficient condition for $u \leq 1/2$ is $K \leq \overline{K}$ where $\overline{K}$ is a function of the parameters (the explicit form is in the Appendix).*

Increasing the rate at which referrals are generated affects the labor market in two ways. First, referrals mitigate search frictions. From the worker's point of view meeting a firm through referrals or through the market are perfect substitutes. Therefore, more referrals increase the worker's contact rate which unambiguously decreases unemployment. Second, referrals affect a potential firm's decision of whether to enter the market. Since referrals create an additional channel for getting workers into jobs, they reduce the worker-finding rate of newly-created vacancies. Under certain conditions, this might discourage firm entry to such an extent that the vacancy-unemployment ratio falls despite the fact that unemployment itself has dropped. When that happens, it becomes more difficult for workers to meet firms in the market, although when including referrals the overall firm-meeting rate has increased.

The reason why the second result is interesting is that it provides a natural way to think about insiders and outsiders in a labor market context. Such an analysis is beyond the scope of the present paper, but one could easily extend the model so that agents have differential access to networks and consider the effect of referrals in that context. For instance, if a new arrival to some location has no network to refer him to a job then he would be strictly worse off if the referral rate is high despite the fact that it leads to lower unemployment. In such a context with non-trivial network heterogeneity, the friction-mitigating effect of referrals would be counterbalanced by the potentially negative effects on individuals with less access to networks.

2.3 Testable Predictions

Turning to the model's testable predictions, we have:

Prediction 1: Ceteris paribus, increasing the size of a worker's network leads to a drop in the probability that he is unemployed and an increase in his wage.

Prediction 2: Ceteris paribus, increasing the employment rate of a worker's network leads to a drop in the probability that he is unemployed and an increase in his wage.

These predictions are straightforward: increasing the size of a worker's network or the employment rate of his links lead to a higher rate of meeting a firm through a referral. This reduces his unemployment probability and raises his wage by increasing his value of unemployment which is consistent with the finding of Bayer, Ross and Topa (2008) about network size and Topa (2001), Weinberg, Reagan and Yankow (2004) and Cappellari and Tatsiramos (2010) about the network's employment rate.

This model has implications about the aggregate matching function.

Prediction 3: The aggregate matching function exhibits decreasing returns to scale.

Consider the effect an increase in the measure of unemployed workers and vacancies by a factor $\xi > 1$ on the aggregate matching function (equation (2)):[7]

$$\begin{aligned}\mathcal{M}(\xi v, \xi u) &= \mu(\xi v)^{\eta}(\xi u)^{1-\eta} + \rho(\xi u)(1-(\xi u)) \\ &= \xi[\mu v^{\eta} u^{1-\eta} + \rho u(1-\xi u)] \\ &< \xi \mathcal{M}(v, u)\end{aligned}$$

This prediction is consistent with the findings of Cheremukhin and Restrepo (2010) in a business cycles accounting exercise on the US labor market. They find that in the aftermath of recessions fewer matches are created than what one would expect given the number of searchers in the market (vacancies and unemployed workers) which is interpreted by their model as a decline in the efficiency of the matching function. The present paper has the same qualitative prediction: when unemployment is high, few jobs are filled through referrals which is equivalent to an increase in matching frictions. A quantitative exploration, though certainly desirable, is well beyond the scope of the current paper.

3 The Labor Market with Heterogeneous Workers

This Section introduces worker heterogeneity.

3.1 The Model

Firms are identical to Section 2. Workers are heterogeneous and each worker belongs to a high or a low type (H or L). The measure of each type is equal to one and the two types differ in

[7] Note that decreasing returns occur regardless of the frictions of the referral channel. If every referral leads to a hire, i.e. $\hat{\mathcal{M}}(v, u) = \mu v^{\eta} u^{1-\eta} + \rho(1-u)$, we still have that $\hat{\mathcal{M}}(\xi v, \xi u) < \xi \hat{\mathcal{M}}(v, u)$ when $\xi > 1$.

terms of their productivity and their network. The different types capture heterogeneity that remains after worker observables have been controlled for. The relevant modeling assumption is that a firm cannot post a type-specific vacancy and both types search for jobs in the same market (though a worker's type is observable to the firm when they meet, i.e. there is no private information).

Conditional on his type, every worker has the same network. The network of a worker of type $i \in \{H, L\}$ is fully described by the measure of other workers that he is linked with, ν_i, and the proportion of these links that are with workers of his own type, ϕ_i. Consistency requires that the measure of links that high type workers have with low types is equal to the measure of links that low type workers have with high types: $\nu_H(1 - \phi_H) = \nu_L(1 - \phi_L)$.

Two assumptions will be maintained about the network structure: a worker has more links with workers of the same type (homophily) and this is weakly more prevalent for high type workers: $\phi_i \geq \frac{1}{2}$ for $i \in \{H, L\}$ and $\phi_H \geq \phi_L$. These two assumptions are not required for proving the existence of equilibrium but are helpful in deriving some characterization results.

As earlier, a worker and a firm meet either through a referral or through the market. When a worker and a firm meet, the match-specific productivity is drawn from a distribution that depends on the worker's type and remains constant for the duration of the match. All payoff-relevant variables (match-specific productivity, worker's type and network) become common knowledge and the pair decides whether to consummate the match.

More precisely, with probability p_i a worker of type i is productive and flow output is given by y_i; with probability $1 - p_i$ he is unproductive and the match is not formed.[8] It is assumed that $p_H > p_L$ and $y_H > y_L$ so that high type workers draw from a productivity distribution that first order stochastically dominates that of the low type workers.[9] It is also

[8] Alternatively, and equivalently, the worker's flow output is a large negative number when unproductive.

[9] A high-type worker is more likely to be hired when meeting a firm and has higher productivity conditional on being employed. A previous version of this paper delivered the same qualitative results with a distribution of match-specific productivities that was continuous and log-concave and had a higher mean for the high type

assumed that $y_L > b + (r+\delta)K$ which guarantees that low type workers are hired when productive.[10]

As in Section 2, a referral occurs when some firm expands and it is sent at random to one of the incumbent worker's links. When a firm that employs a type-i worker expands, it meets a type-i worker with probability $\phi_i u_i$ and a type-k ($\neq i$) worker with probability $(1-\phi_i)u_k$, where u_j is the unemployment rate of a type-j worker. In addition to the possibility of instantaneous matching, a referred worker is drawn from a different pool than a random draw of unemployed workers.

Denoting the value of employing a type-j worker by J_j, the value of expanding when employing a type i worker is equal to:

$$E_i = -K + V + \phi_i u_i p_i (J_i - V) + (1-\phi_i) u_k p_k (J_k - V). \tag{8}$$

The flow value to a match between a firm and a type-i worker is $y_i + \rho\gamma E_i$.

A worker of type i is referred to a firm when the employer of one of his links expands and this worker is chosen among the referrer's links. A type i worker has $\nu_i\phi_i$ links of type i and $\nu_i(1-\phi_i)$ links of type k. Each link of type j is employed with probability $1-u_j$ and gets the opportunity to refer at rate ρ. A referrer of type j has ν_j links and each of them is equally likely to receive the referral. Therefore, our worker is referred to a job at rate

$$\begin{aligned} \alpha_{Ri} &= \frac{\rho\nu_i\phi_i(1-u_i)}{\nu_i} + \frac{\rho\nu_i(1-\phi_i)(1-u_k)}{\nu_k} \\ &= \rho\phi_i(1-u_i) + \rho(1-\phi_k)(1-u_k), \end{aligned}$$

where the consistency condition $\nu_H(1-\phi_H) = \nu_L(1-\phi_L)$ was substituted in the second term.

Three types of agents search in the same market: measure v of vacancies, measure u_H

workers. That specification complicated the analysis without adding further insights.

[10]Introducing a probability $1-p$ that the worker is unproductive to the baseline model of Section 2 yields results that are identical to a rescaling of the matching efficiency parameters to $\tilde{\mu} = \mu * p$ and $\tilde{\rho} = \rho * p$.

high-type unemployed workers and measure u_L low-type unemployed workers.[11] The flow of meetings in the market between a vacancy and a worker of either type is given by a Cobb-Douglas function

$$M(v, u_H, u_L) = \mu v^{\eta}(u_H + u_L)^{1-\eta},$$

where $\mu > 0$ and $\eta \in (0, 1)$.

When a firm meets a worker, the worker is drawn at random from the unemployed population. The rate at which a firm meets with a type i worker is

$$\alpha_{Fi} = \frac{M(v, u_H, u_L)}{v} \frac{u_i}{u_H + u_L} = \mu(\frac{v}{u_H + u_L})^{\eta} \frac{u_i}{v}.$$

The rate at which a type i worker meets a firm through the market is

$$\alpha_{Mi} = \frac{M(v, u_H, u_L)}{u_H + u_L} = \mu(\frac{v}{u_H + u_L})^{\eta}.$$

Since this rate does not depend on the worker's type, the i subscript is henceforth dropped.

The steady state conditions are that each type's flows in and out of unemployment are equal:

$$u_H(\alpha_M + \alpha_{RH})p_H = (1 - u_H)\delta, \tag{9}$$

$$u_L(\alpha_M + \alpha_{RL})p_L = (1 - u_L)\delta. \tag{10}$$

The agents' value functions are now described. Consider a firm. When vacant, it searches in the market and meets with a type-i worker at rate α_{Fi}. With probability p_i the worker is productive and the match is formed. When producing, the firm's flow payoffs are $y_i + \rho\gamma E_i - w_i$

[11] Since there is a unit measure of each type, u_j denotes both the proportion and the measure of type-j unemployed.

where w_i denotes the wage. The match is destroyed at rate δ. The firm's value of a vacancy (V) and production with a type-i worker (J_i) are given by:

$$\begin{aligned} rV &= \alpha_{FH}p_H(J_H - V) + \alpha_{FL}p_L(J_L - V). \\ rJ_i &= y_i + \rho\gamma E_i - w_i - \delta J_i. \end{aligned}$$

Consider a worker of type i. When unemployed his flow utility is b. Job opportunities appear at rate $\alpha_M + \alpha_{Ri}$ and a match is formed with probability p_i. When employed, the worker's flow utility is equal to the wage and the match is destroyed at rate δ. The worker's value of unemployment (U_i) and employment (W_i) are given by:

$$\begin{aligned} rU_i &= b + (\alpha_M + \alpha_{Ri})p_i(W_i - U_i). \\ rW_i &= w_i + \delta(U_i - W_i). \end{aligned}$$

The wage solves the Nash bargaining problem

$$w_i = \text{argmax}_w (W_i - U_i)^\beta (J_i - V)^{1-\beta}. \tag{11}$$

Definition of Equilibrium: The Labor Market Equilibrium is defined as follows.

Definition 3.1 *A Labor Market Equilibrium is the steady state unemployment levels* $\{u_H, u_L\}$ *and the number of vacancies* v *such that:*

- *The labor market is in steady state as described in* (9) *and* (10).
- *The surplus is split according to* (11).
- *There is free entry of firms:* $V = K$.

3.2 Labor Market Equilibrium

This Section's analysis mirrors that of Section 2.2.

The following lemmata characterize the steady state labor market flows and their proofs are in the Appendix. It should be noted that although these results are conceptually straightforward they are non-trivial to prove. The source of the complication is that the unemployment rates are implicitly defined by equations (9) and (10) and one type's unemployment rate affects the other's meeting rate through both the referral and the market channel. Therefore a change in v affects u_H both directly, through the steady state condition of high-type workers, and indirectly, through its effect on u_L.

Lemma 3.1 *In steady state, the unemployment rates for the two worker types* $\{u_H, u_L\}$ *are uniquely determined given any number of vacancies,* v*. Furthermore, the unemployment rate of both types is monotonically decreasing in* v*.*

The unemployment rate for the two types is characterized as follows:

Lemma 3.2 *If* $\phi_H \geq \phi_L$ *then the high productivity workers have lower unemployment rates than the low types in a steady state* ($u_H < u_L$)*.*

The rate at which a firm contacts workers is characterized as follows:

Lemma 3.3 *If* $\phi_H \geq \phi_L \geq 1/2$ *and* $1 - \eta - \eta^2 \geq 0$ *then in steady state the rate at which a firm meets with a type* i *worker* (α_{Fi}) *is decreasing in* v*.*

To see why a condition on η is needed note that a change in v affects both the flow of matches and the proportion of unemployed workers who belong to each type. When η is high, v affects the flow of matches less and, consequently, the proportion plays a more important role. For instance, when $\eta = 1$ the arrival rate of a certain type only depends on that type's proportion in the unemployed population. This implies that if α_{Fi} is decreasing in v then α_{Fk} must be increasing in v. For this reason, η needs to be bounded away from 1 for the (reasonable) requirement that the worker-meeting rate is declining in v. The bound derived in Lemma 3.3 is equivalent to $\eta \leq 0.62$. Empirically, the coefficient of vacancies has been estimated to be within 0.3-0.5 (Petrongolo and Pissarides (2001)) which suggests that the bound is not very restrictive.

The surplus of a match between a firm and a type-i is given by $S_i = W_i - U_i + J_i - V$. Nash bargaining implies that

$$\begin{aligned} W_i - U_i &= \beta S_i, \\ J_i - V &= (1-\beta) S_i, \end{aligned}$$

and the value functions can be rearranged to yield

$$(r+\delta)S_i = y_i + \rho\gamma E_i - b - (\alpha_M + \alpha_{Ri})\beta S_i - (r+\delta)V.$$

Combine the above with equation (8) and the free entry condition to arrive at:

$$S_i = \frac{y_i - b - (r+\delta)K + \rho\gamma(1-\beta)(1-\phi_i)u_k p_k \; S_k}{r + \delta + (\alpha_M + \alpha_{Ri})p_i\beta - \rho\gamma(1-\beta)\phi_i u_i p_i}. \tag{12}$$

Equation (12) illustrates that the dependence between S_i and S_k is due to the fact that a type-i worker may refer a type-k in the case of an expansion. If $\phi_i = 1$ then i types only refer workers of the same type and the term multiplying S_k drops out.

The value of a vacancy is given by

$$rV = \alpha_{FH} p_H (1-\beta) S_H + \alpha_{FL} p_L (1-\beta) S_L \tag{13}$$

The following proposition states the main result.

Proposition 3.1 *An equilibrium exists. The equilibrium is unique if* $1 - \eta - \eta^2 \geq 0$ *and* $\phi_H \geq \phi_L \geq 1/2$.

Proof. See the Appendix. ■

When including heterogeneity, referrals help firms find high type workers, in addition to mitigating search frictions. The intuition is quite straightforward. High productivity workers are more likely to be employed at any point in time and therefore they are more likely to refer one of their links. The assumption of homophily ($\phi_H \geq \frac{1}{2}$) implies that the recipients of these referrals are more likely to be other high-type workers. Formally:

Proposition 3.2 *When a firm and a worker meet, it is more likely that the worker is of high type if the meeting is through a referral rather than through the market if* $\phi_H \geq \phi_L \geq 1/2$.

Proof. See the Appendix. ■

3.3 Testable Predictions

In this section we present predictions that the model delivers and compare them with empirical evidence.

Prediction 4: When a worker and a firm meet, the match is more likely to be formed if they meet through a referral.

Prediction 5: When a worker and a firm meet, the match is more productive in expectation if they meet through a referral.

Prediction 6: When a worker and a firm meet, the wage is higher in expectation if they meet through a referral.

The above predictions are direct consequences of Proposition 3.2 and, as detailed in Section 1.1, there is ample evidence supporting them. Fernandez and Weinberg (1997) and Castilla (2005) present evidence from their field surveys that supports the prediction 4. Regarding prediction 5, Castilla (2005) finds that, conditional on being hired, referred candidates have higher productivity while Dustmann, Glitz and Schoenberg (2010) find that referred candidates receive higher wages, after controlling for worker observables and firm fixed effects.

Prediction 7: A referred worker receives a higher wage only in markets where worker heterogeneity is an important feature.

In Section 2, where workers are homogeneous, every worker receives the same wage. In Section 3, where heterogeneity plays an important role, referred worker receive a higher wage, on average. This observation clarifies that in a wage regression one should not necessarily expect a positive effect on a dummy variable for referrals. In labor markets where worker heterogeneity is not very important, referrals are only used to alleviate search frictions which means that there is no wage premium for finding a job through a referral. In labor markets where heterogeneity is important, one would expect to find such a premium. One could try to distinguish between the two cases by, for instance, using a measure of the job's complexity as a proxy for the importance of heterogeneity and having a dummy on the *interaction* between a referral and that proxy.

4 Conclusions

The aim of this paper is to combine social networks, which have long been recognized as an important feature of labor markets, with the equilibrium models that are used to understand labor markets. This is done in a tractable model which, despite its simplicity, yields predictions that are consistent with a large number of stylized facts about the interaction between social networks and labor markets.

The model's tractability makes it amenable to extensions to study issues that this paper abstracted from. One possibility is to model the firm decision of using formal versus informal means of hiring. For instance, there is evidence that smaller and less productive firm use informal means of hiring to a larger extent than their more productive counterparts (Dustmann, Glitz and Schoenberg (2010), Pelizzarri (2010)).

Another path is to introduce social networks in the study of individuals' migration decisions as there is ample evidence to suggest that social networks affect these decisions. For instance, Munshi (2003) finds that Mexican migrants are more likely to move to locations with more people from their region of origin and this helps them with finding employment while Belot and Ermisch (2009) show that an individual is less likely to move if he has more friends at his current location.[12] Therefore, it seems natural to combine the decision to migrate with an explicit model of how the social network helps a worker to find a job.

Finally, this paper's focus is on the positive implications of introducing social networks inside a labor market model. Having provided a theoretical framework, one can move towards answering normative questions regarding whether or how labor market policies should change once the importance of social networks is taken into account. This is left for future work.

[12] Belot and Ermisch (2009) focus on the number of close friends and they interpret their findings to reflect the intrinsic value of friendship. To the extent that the number of one's close friends in some location reflects the overall ties to that location, close friends can be used as a proxy for the overall number of one's contacts.

5 Appendix

Proposition 2.2: An increase in the rate that referrals are generated (ρ) leads to a decrease in the equilibrium rate of unemployment (u).

The steady state condition implies that

$$\frac{v}{u} = \frac{1}{\mu^{1/\eta}}[(1-u)(\frac{\delta}{u}-\rho)]^{1/\eta}$$

The free entry condition can therefore be rearranged as follows:

$$\frac{(y-b-(r+\delta)K)(1-\beta)}{rK} = \frac{1}{\mu^{1/\eta}}[(1-u)(\frac{\delta}{u}-\rho)]^{\frac{1-\eta}{\eta}}(r+\delta+\frac{(1-u)\delta\beta}{u}-\rho\gamma(1-\beta)u).$$

In this expression u is the only endogenous variable and each u uniquely defines the vacancy rate according to the steady state condition. In equilibrium:

$$\begin{aligned} Q(\rho,u) &= C, \quad \text{where} \\ Q(\rho,u) &\equiv [(1-u)(\frac{\delta}{u}-\rho)]^{\frac{1-\eta}{\eta}}[r+(1-\beta)\delta+\frac{\delta\beta}{u}-\rho\gamma(1-\beta)u] \\ C &\equiv \frac{y-b-(r+\delta)K](1-\beta)\mu^{1/\eta}}{rK}. \end{aligned}$$

It is easy to verify that Q is decreasing both in u and in ρ and the implicit function theorem implies:

$$\frac{du}{d\rho} = -\frac{\partial Q/\partial\rho}{\partial Q/\partial u} < 0$$

Proposition 2.3: An increase in the rate that referrals are generated (ρ) leads to a decrease in the *market* job-finding rate (α_M) if $\beta \geq (1-\beta)\gamma$ and $u \leq 1/2$. A sufficient condition for $u \leq 1/2$ is $K \leq \overline{K}$ where $\overline{K}$ is a function of the parameters.

To find how a change in ρ affects v/u start with:

$$\frac{d(v/u)}{d\rho} \quad = \quad \frac{[(1-u)(\delta/u-\rho)]^{1/\eta-1}}{\eta\mu^{1/\eta}}[-\frac{du}{d\rho}(\frac{\delta}{u^2}-\rho)-(1-u)],$$

which implies

$$\frac{d(v/u)}{d\rho} < 0 \Leftrightarrow -\frac{du}{d\rho} < \frac{1-u}{\delta/u^2-\rho}. \tag{14}$$

It is straightforward (though tedious) to use the implicit function theorem and arrive at:

$$\frac{du}{d\rho} = -\frac{1+\gamma(1-\beta)u(\delta/u-\rho)/\Xi}{(\delta/u^2-\rho)/(1-u)+(\rho\gamma(1-\beta)+\beta\delta/u^2)(\delta/u-\rho)/\Xi} \tag{15}$$

where $\Xi = \frac{1-\eta}{\eta}[r+\delta(1-\beta)-\rho\gamma(1-\beta)u+\frac{\beta\delta}{u}]$.

Combining (15) with (14) and going through the algebra yields:

$$\frac{d(v/u)}{d\rho} < 0 \Leftrightarrow$$

$$\rho\gamma(1-\beta)+\frac{\delta}{u^2}((1-u)\beta-\gamma(1-\beta)u) > 0$$

Sufficient conditions for this inequality to hold are $\beta \geq (1-\beta)\gamma$ and $u \leq 1/2$.

Finally, note that C is strictly decreasing in K which implies that $u \leq 1/2$ if $K \leq \overline{K}$ where

$$\overline{K} = \frac{(y-b)(1-\beta)\mu^{1/\eta}}{rQ(\rho,0.5)+(r+\delta)(1-\beta)\mu^{1/\eta}}$$

Lemma 3.1: In steady state, the unemployment rates for the two worker types $\{u_H, u_L\}$ are uniquely determined given any number of vacancies, v. Furthermore, the unemployment rate of both types is monotonically decreasing in v.

Proof. Define

$$
\begin{aligned}
H(v,u_H,u_L) &\equiv u_H\mu(\frac{v}{u_H+u_L})^{\eta} + u_H\rho(\phi_H(1-u_H)+(1-\phi_L)(1-u_L)) - \frac{\delta}{p_H}(1-u_H)\\
L(v,u_H,u_L) &\equiv u_L\mu(\frac{v}{u_H+u_L})^{\eta} + u_L\rho(\phi_L(1-u_L)+(1-\phi_H)(1-u_H)) - \frac{\delta}{p_L}(1-u_L)
\end{aligned}
$$

and note that in a steady state we have $H(v,u_H,u_L) = L(v,u_H,u_L) = 0$. From now on, let $H_x(v,u_H,u_L) \equiv \partial H(v,u_H,u_L)/\partial x$ where $x \in \{v,u_H,u_L\}$, and similarly for $L(v,u_H,u_L)$. Define $h^H(v,u_L)$ and $h^L(v,u_L)$ to be the set of $\{u_H\}$ that satisfy $H(v,u_H,u_L) = 0$ and $L(v,u_H,u_L) = 0$, respectively, for every $v > 0$ and $u_L \in [0,1]$.

The proof proceeds by showing that (1) $h^H(v,u_L)$ and $h^L(v,u_L)$ include at most one point for any given (v,u_L) (i.e. they are functions); (2) they are strictly increasing in u_L and strictly decreasing in v; (3) for every $v > 0$ there is a unique $u_L(v) \in (0,1)$ such that $h^H(v,u_L(v)) = h^L(v,u_L(v)) \equiv h(v,u_L(v))$ and $h(v,u_L(v)) \in (0,1)$; (4) $h(v,u_L(v))$ and $u_L(v)$ are decreasing in v. The steady state unemployment levels for high and low type workers are then given by $h(v,u_L(v))$ and $u_L(v)$, respectively.

Observe that

$$
\begin{aligned}
H(v,0,u_L) &= -\frac{\delta}{p_H} < 0\\
H(v,1,u_L) &= \mu(\frac{v}{1+u_L})^{\eta} + \rho(1-\phi_L)(1-u_L) > 0\\
H_{u_H}(v,u_H,u_L) &= \mu(\frac{v}{u_H+u_L})^{\eta}(1-\frac{\eta u_H}{u_H+u_L}) + \rho(\phi_H(1-u_H)+(1-\phi_L)(1-u_L)) + \frac{\delta}{p_H} - u_H\rho\phi_H > 0
\end{aligned}
$$

The above equations imply that $h^H(v,u_L)$ is uniquely defined and belongs to $(0,1)$ given any $v > 0$ and $u_L \in [0,1]$. Furthermore,

$$
\begin{aligned}
H_{u_L}(v,u_H,u_L) &= -\frac{\eta u_H}{u_H+u_L}\mu(\frac{v}{u_H+u_L})^{\eta} - u_H\rho(1-\phi_L) < 0\\
H_v(v,u_H,u_L) &= \frac{\eta u_H}{v}\mu(\frac{v}{u_H+u_L})^{\eta} > 0
\end{aligned}
$$

Therefore $h^H(v, u_L)$ is strictly increasing in u_L and strictly decreasing in v.

Turning to $h^L(v, u_L)$, note that

$$\begin{aligned} L(v, u_H, 1) &= \mu(\frac{v}{u_H + 1})^\eta + \rho(1 - \phi_H)(1 - u_H) > 0, \quad \forall\, u_H \in [0, 1] \\ L(v, u_H, 0) &= -\frac{\delta}{p_L} < 0, \quad \forall\, u_H \in [0, 1] \\ L_{u_L}(v, u_H, u_L) &= \mu(\frac{v}{u_H + u_L})^\eta \frac{u_H + (1-\eta)u_L}{u_H + u_L} + \rho(\phi_L(1 - u_L) + (1 - \phi_H)(1 - u_H)) + \frac{\delta}{p_L} - u_L \rho \phi_L > 0 \end{aligned}$$

The first equation shows that $L(v, u_H, u_L) = 0$ has no solution for u_L "close enough" to 1. The second equation shows that $L(v, u_H, u_L) = 0$ has no solution for u_L "close enough" to 0. The third equation implies that a solution to $L(v, u_H, u_L) = 0$ with $u_H \in [0, 1]$ only exists if $u_L \in [\underline{u}_L(v), \overline{u}_L(v)]$ where $\underline{u}_L(v) > 0$ and $\overline{u}_L(v) < 1$.

Furthermore,

$$L_{u_H}(v, u_H, u_L) = -\frac{\eta u_L}{u_H + u_L} \mu(\frac{v}{u_H + u_L})^\eta - u_L \rho(1 - \phi_H) < 0$$

implies $h^L(v, \underline{u}_L(v)) = 0$, $h^L(v, \overline{u}_L(v)) = 1$ and $0 < \underline{u}_L(v) < \overline{u}_L(v) < 1$.

To complete the analysis of $h^L(v, u_L)$, note that $L_{u_L}(v, u_H, u_L) > 0 > L_{u_H}(v, u_H, u_L)$ and

$$L_v(v, u_H, u_L) = \frac{\eta u_L}{v} \mu(\frac{v}{u_H + u_L})^\eta > 0$$

imply that given any $v > 0$ and $u_L \in [\underline{u}_L(v), \overline{u}_L(v)]$, $h^L(v, u_L)$ is uniquely defined and is strictly decreasing in v and strictly increasing in u_L.

The next step is to examine the intersection of $h^H(v, u_L)$ and $h^L(v, u_L)$. Observing that $h^L(v, \underline{u}_L(v)) = 0 < h^H(\underline{u}_L(v))$ and $h^L(v, \overline{u}_L(v)) = 1 > h^H(\overline{u}_L)$ implies that there is some $u_L(v) \in (0, 1)$ such that $h^H(v, u_L(v)) = h^L(v, u_L(v))$. To show that the intersection is unique it suffices to show

$$\frac{\partial h^H(v,u_L)}{\partial u_L} < \frac{\partial h^L(v,u_L)}{\partial u_L}$$
$$\Leftrightarrow$$
$$-\frac{H_{u_L}(v,u_H,u_L)}{H_{u_H}(v,u_H,u_L)} < -\frac{L_{u_L}(v,u_H,u_L)}{L_{u_H}(v,u_H,u_L)}$$

Noting that

$$\begin{aligned} L_{u_L}(v,u_H,u_L) + H_{u_L}(v,u_H,u_L) &= \mu(\frac{v}{u_H+u_L})^\eta(\frac{(1-\eta)(u_H+u_L)}{u_H+u_L}) + \rho\phi_L(1-u_L) \\ &+\rho(1-\phi_H)(1-u_H) + \frac{\delta}{p_L} - \rho(u_L\phi_L + (1-\phi_L)u_H) > 0 \end{aligned}$$

and

$$\begin{aligned} H_{u_H}(v,u_H,u_L) + L_{u_H}(v,u_H,u_L) &= \mu(\frac{v}{u_H+u_L})^\eta(\frac{(1-\eta)(u_H+u_L)}{u_H+u_L}) + \rho\phi_H(1-u_H) \\ &+\rho(1-\phi_L)(1-u_L) + \frac{\delta}{p_H} - \rho(u_H\phi_H + (1-\phi_H)u_L) > 0 \end{aligned}$$

proves that the intersection is unique.

Finally, $H_v(v,u_H,u_L) > 0$ and $L_v(v,u_H,u_L) > 0$ imply that the steady state u_H and u_L decrease in v. ■

Lemma 3.2: If $\phi_H \geq \phi_L$ then the high productivity workers have lower unemployment rates than the low types in a steady state ($u_H < u_L$).

Proof. The aim is to prove that $u_L(v) > h(v,u_L(v))$. Define f^H and f^L by $h^H(v,f^H) = f^H$ and $h^H(v,f^L) = f^L$ (of course, f^H and f^L depend on v but since v will be kept constant throughout this proof this is omitted for notational brevity). Let $T^H(v,u) \equiv H(v,u,u)$ and $T^L \equiv L(v,u,u)$ and note that $T^i(v,u) = 0 \Leftrightarrow u = f^i$. The proof's steps are to prove that (1) f^H and f^L are uniquely defined; (2) $f^H < f^L \Leftrightarrow h(v,u_L(v)) < u_L(v)$; (3) $\phi_H \geq \phi_L \geq 1/2$

suffices for $f^H < f^L$.

The following proves that f^i exists and is unique:

$$\begin{aligned}
T^i(v,u) &= \mu u^{1-\eta}(\frac{v}{2})^\eta + u(1-u)\rho(\phi_i + 1 - \phi_k) - \frac{\delta}{p_i}(1-u) \\
T^i(v,0) &= -\frac{\delta}{p_i} < 0 \\
T^i(v,1) &= \mu(\frac{v}{2})^\eta > 0 \\
\frac{\partial T^i(v,u)}{\partial u} &= (1-\eta)\mu(\frac{v}{2u})^\eta - u\rho(\phi_i + 1 - \phi_k) + \frac{\delta}{p_i} > 0
\end{aligned}$$

Define $\overline{f} = \max\{f^H, f^L\}$ and $\underline{f} = \min\{f^H, f^L\}$. Recall that $h^H(v,0) > 0$ and therefore $u_L < f^H \Leftrightarrow h^H(v,u_L) > u_L$. Similarly, $h^L(v, \underline{u}_L(v)) = 0 < \underline{u}_L(v)$ implies $u_L < f^L \Leftrightarrow h^L(v,u_L) < u_L$.

In steady state we necessarily have $u_L(v) \in [\underline{f}, \overline{f}]$ because $u_L < \underline{f} \Rightarrow h^H(v,u_L) > h^L(v,u_L)$ and $u_L > \overline{f} \Rightarrow h^H(v,u_L) < h^L(v,u_L)$. If $f^L < f^H$ then the intersection between $h^H(u_L)$ and $h^L(u_L)$ occurs above the 45 degree which implies that $h(v,u_L) > u_L$; and the opposite happens if $f^L > f^H$. We have shown that $f^H < f^L \Leftrightarrow u_H < u_L$.

Perform the following monotonic transformation: $\tilde{T}^i(u) = \frac{T^i(u)}{1-u}$ which preserves $T^i(u) = 0 \Leftrightarrow \tilde{T}^i(u) = 0$ and therefore $\tilde{T}^i(f^i) = 0$. Note that $0 > \tilde{T}^H(0) = -\delta/p_H > -\delta/p_L = \tilde{T}^L(0)$. For $f^H < f^L$ it is necessary that $\tilde{T}^L$ "overtakes" $\tilde{T}^H$ before the latter reaches zero. To examine whether this happens define

$$\begin{aligned}
\Delta T(u) &\equiv \tilde{T}^H(u) - \tilde{T}^L(u) \\
&= 2u\rho(\phi_H - \phi_L) - \frac{\delta}{p_H} + \frac{\delta}{p_L}
\end{aligned}$$

and note that $\phi_H \geq \phi_L$ implies that $f^H < f^L$. ■

Lemma 3.3: If $\phi_H \geq \phi_L \geq 1/2$ and $1 - \eta - \eta^2 \geq 0$ then in steady state the rate at which a firm meets with a type i worker (α_{Fi}) is decreasing in v.

Proof. We prove that the rate at which firms meet workers of type i decreases in v. Recall:

$$\begin{aligned} \alpha_{Fi} &= u_i(u_H+u_L)^{-\eta}v^{-1+\eta} \\ \frac{d\alpha_{Fi}}{dv} &= v^{-1+\eta}(u_H+u_L)^{-\eta-1}[(1-\eta)u_i(\frac{du_i}{dv}-\frac{u_H+u_L}{v})+\frac{du_i}{dv}u_k-\eta\frac{du_k}{dv}u_i] \end{aligned} \qquad (16)$$

In steady state u_H and u_L are defined by $H(v,u_H,u_L)=0$ and $L(v,u_H,u_L)=0$ which defines an implicit system of two equations and two unknowns. Using the implicit function theorem we have

$$\begin{aligned} \frac{du_H}{dv} &= -\frac{L_{u_L}H_v-H_{u_L}L_v}{D} \\ \frac{du_L}{dv} &= -\frac{H_{u_H}L_v-L_{u_H}H_v}{D} \\ D &= H_{u_H}L_{u_L}-H_{u_L}L_{u_H} \end{aligned}$$

Recall that

$$\begin{aligned} H_{u_H}(v,u_H,u_L) &= \alpha_M(1-\frac{\eta u_H}{u_H+u_L})+\alpha_{RH}+\frac{\delta}{p_H}-u_H\rho\phi_H>0 \\ H_{u_L}(v,u_H,u_L) &= -\frac{\eta u_H}{u_H+u_L}-\rho u_H(1-\phi_L)<0 \\ H_v(v,u_H,u_L) &= \frac{\eta u_H\alpha_M}{v}>0 \end{aligned}$$

and similarly for $L(v,u_H,u_L)$.

We can rewrite (16) for the high types as:

$$\begin{aligned} \frac{d\alpha_{FH}}{dv} &= -\frac{v^{-1+\eta}(u_H+u_L)^{-\eta-1}}{Dv}\{(1-\eta)u_H[\eta\alpha_M(L_{u_L}u_H-H_{u_L}u_L)+(u_H+u_L)(H_{u_H}L_{u_L}-H_{u_L}L_{u_H})] \\ &\quad +u_L\eta\alpha_M[L_{u_L}u_H-H_{u_L}u_L]-\eta^2u_H\alpha_M[H_{u_H}u_L-L_{u_H}u_H]\} \end{aligned}$$

To prove that the above expression is negative, it suffices to show that the term in the braces is positive. The terms can be re-grouped as follows:

$$\begin{aligned}
P_{H1} &= u_H(1-\eta)[\eta\alpha_M u_H L_{u_L} + (u_H(1-\eta)+u_L)(\frac{\delta}{p_L} + \alpha_{RL} - \rho\phi_L u_L) - \eta\rho u_L(1-\phi_H)] \\
&\geq u_H(1-\eta)[(u_H+u_L)\frac{\delta}{p_L} - \rho\phi_L u_L - \eta\rho u_L(1-\phi_H)] > 0 \\
P_{H2} &= u_H(1-\eta)[-\eta\alpha_M u_L H_{u_L} + (u_H+u_L)\frac{\eta\alpha_M u_L}{u_H+u_L} H_{u_L}] = 0 \\
P_{H3} &= u_H(1-\eta)(u_H+u_L)[(\frac{\delta}{p_H} + \alpha_{RH} - \rho u_H\phi_H)(\frac{\delta}{p_L} + \alpha_{RL} - \rho u_L\phi_L) - \rho^2 u_H u_L(1-\phi_H)(1-\phi_L)] > 0 \\
P_{H4} &= u_H u_L \alpha_M H_{u_H}(1-\eta-\eta^2) > 0 \\
P_{H5} &= \eta\alpha_M[L_{u_L} u_H u_L - H_{u_L} u_L^2 + L_{u_H}\eta u_H^2] > 0
\end{aligned}$$

The grouping of terms and resulting calculations for α_{FL} are very similar and therefore omitted (but are available upon request). ■

Proposition 3.1: An equilibrium exists. The equilibrium is unique if $1-\eta-\eta^2 \geq 0$ and $\phi_H \geq \phi_L \geq 1/2$.

Proof. We can express the S_i's as follows:

$$S_H = \frac{B_H + B_L C_H/A_L}{A_H - C_H C_L/A_L},$$

$$S_L = \frac{B_L + B_H C_L/A_H}{A_L - C_H C_L/A_H}.$$

where

$$\begin{aligned}
A_i &= r + \delta + (\alpha_M + \alpha_{Ri})p_i\beta - \rho\gamma(1-\beta)\phi_i u_i p_i, \\
B_i &= y_i - b - (r+\delta)K, \\
C_i &= \rho\gamma(1-\beta)(1-\phi_i)u_k p_k.
\end{aligned}$$

Note that an increase in v leads to an increase A_i and a fall in C_i and so S_i is decreasing in v. The value of a vacancy is given by:

$$rV = \alpha_{FH}(1-\beta)S_H + \alpha_{FL}(1-\beta)S_L \tag{17}$$

The steady state equations imply that

$$\begin{aligned} v \to 0 \quad &\Rightarrow \quad (u_H, u_L) \to (1,1) \Rightarrow \alpha_{Fi} \Rightarrow \infty, \\ v \to \infty \quad &\Rightarrow \quad (u_H, u_L) \to (0,0) \Rightarrow \alpha_{Fi} \Rightarrow 0. \end{aligned}$$

These observations, together with the fact that S_i is strictly decreasing in v, means that a vacancy's value is above K for v near zero and below K for v very large and, therefore, an equilibrium exists.

If $1-\eta-\eta^2 \geq 0$ and $\phi_H \geq \phi_L \geq 1/2$ then α_{Fi} is monotonically decreasing in v and therefore the right-hand side of equation (17) is strictly decreasing in v. As a result, in that case, the equilibrium is unique. ■

Proposition 3.2: When a firm and a worker meet, it is more likely that the worker is of high type if the meeting is through a referral rather than through the market if $\phi_H \geq \phi_L \geq 1/2$.

Proof. In a meeting through the market the probability that the worker is of high type is given by

$$P[H|\text{market}] = \frac{u_H}{u_H + u_L}$$

In a meeting through referrals the probability that the worker is of high type is given by:

$$\begin{aligned} P[H|\text{referral}] &= \frac{P[\text{ref. from } H] * P[H|\text{ref. from } H] + P[\text{ref. from } L] * P[H|\text{ref. from } L]}{P[\text{referral from } H] + P[\text{referral from } L]} \\ &= \frac{[(1-u_H)\phi_H + (1-u_L)(1-\phi_L)]u_H}{[(1-u_H)\phi_H + (1-u_L)(1-\phi_L)]u_H + [(1-u_H)(1-\phi_H) + (1-u_L)\phi_L]u_L} \end{aligned}$$

Noting that

$$(1-u_H)\phi_H + (1-u_L)(1-\phi_L) \geq (1-u_H)(1-\phi_H) + (1-u_L)\phi_L$$

completes the proof. ■

References

[1] Bayer, Patrick, Stephen L. Ross and Giorgio Topa (2008). "Place of work and place of residence: informal hiring networks and labor market outcomes." Journal of Political Economy 116 (6), 1150-1196.

[2] Belot, Michele and John Ermisch (2009). "Friendship ties and geographical mobility: evidence from Great Britain." Journal of the Royal Statistical Society 172, 427-442.

[3] Bentolila, Samuel, Claudio Michellaci and Javier Suarez (2010). "Social contacts and occupational choice." Economica 77, 20-45.

[4] Bewley, Truman (1999). Why wages don't fall during a recession. Cambridge, MA. Harvard University Press.

[5] Blau, David and Philip Robbins (1990). "Job search outcomes for the employed and the unemployed." Journal of Political Economy.

[6] Calvo-Armengol, Antoni and Matthew Jackson (2004). "The effects of social networks on employment and inequality." American Economic Review 94, 426-454.

[7] Calvo-Armengol, Antoni and Yves Zenou (2005). "Job matching, social networking and word-of-mouth communication." Journal of Urban Economics 57, 500-522.

[8] Cappellari, Lorenzo and Konstantinos Tatsiramos (2010). "Friends' networks and transitions into employment." Mimeo.

[9] Castilla, Emilio (2005). "Social networks and employee performance in a call center." American Journal of Sociology 110 (5), 1243-1283.

[10] Cheremukhin, Anton and Paulina Restrepo (2010). "The labor wedge as a matching friction." Mimeo.

[11] Corcoran, Mary, Linda Datcher and Greg Duncan (1980). "Information and influence networks in labor markets," in *Five Thousand American Families: Patterns of Economic Progress*, edited by Greg Duncan and JAmes Morgan, vol 7, 1-37, Ann Arbor, MI: Institute of Social Research.

[12] Dustmann, Christian, Albrecht Glitz and Uta Schoenberg (2010). "Referral-based job search networks." Mimeo.

[13] Fernandez, Roberto and Nancy Weineberg (1997). "Sifting and sorting: personal contacts and hiring in a retail bank." American Sociological Review 62, 883-902.

[14] Fontaine, Francois (2008). "Why are similar workers paid differently? the role of social networks." Journal of Economic Dynamics and Control 32, 3960-3977.

[15] Granovetter, Mark (1973). "The strength of weak ties." American Journal of Sociology 78 (6), 1360-1380.

[16] Granovetter, Mark (1995). Getting a job: a study of contacts and careers. 2nd edition Cambridge, MA. Harvard University Press.

[17] Holzer, Harry (1987). "Hiring procedures in the firm: their economic determinants and outcomes." NBER WP 2185.

[18] Holzer, Harry (1988). "Search method use by unemployed youth." Journal of Labor Economics 6(1), 1-20.

[19] Ioannides, Yannis and Linda Datcher Loury (2006). "Job information networks, neighborhood effects, and inequality." Journal of Economic Literature XLII, 1056-1093.

[20] Jackson, Matthew O. (2008). Social and economic networks. Princeton University Press.

[21] Mardsen, Peter V. (2001). "Interpersonal ties, social capital, and employer staffing practices," in *Social capital: theory and research*, ed. Nan Lin, Karen Cook and Ronald S. Burt, Mew Brunswick, NJ, Transaction Publishers.

[22] McPherson, Miller, Lynn Smith-Lovin and James M. Cook (2001). "Birds of a feather: Homophily in social networks." Annual Review of Sociology 27, 415-444.

[23] Montgomery, James D. (1991). "Social networks and labor market outcomes." American Economic Review 81, 1408-1418.

[24] Munshi, Kaivan (2003). "Networks in the modern economy: Mexican migrants in the US labor market." The Quarterly Journal of Economics 118 (2), 549-599.

[25] Pellizzari, Michele (2010). "Do friends and relatives really help in getting a good job?" Industrial and Labor Relations Review 63 (3), 494-510.

[26] Petrongolo, Barbara and Christopher A. Pissarides (2001). "Looking into the black box: a survey of the matching function." Journal of Economic Literature XXXIX, 390-431.

[27] Pistaferri, Luigi (1999). "Informal networks in the Italian labor market." Giornale degli Economisti 58 (3-4), 355-375.

[28] Rogerson, Richard, Robert Shimer and Randall Wright (2005). "Search-Theoretic Models of the Labor Market: A Survey." Journal of Economic Literature XLIII, 959-988.

[29] Topa, Giorgio (2001). "Social interactions, local spillovers and unemployment." Review of Economic Studies 68, 261-295.

[30] Topa, Giorgio (2010). "Labor Markets and Referrals." The Handbook of Social Economics, forthcoming.

[31] Weinberg, Bruce A., Patricia Reagan B. and Jeffrey J. Yankow (2004). “Do neighborhoods affect hours worked? Evidence from longitudinal data.” Journal of Labor Economics 22 (4) 891-924.