

Why Don't Rich Countries Default? Explaining Debt/GDP and Sovereign Debt Crises

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Abstract

Incentives for default are different for a rich sovereign than for a poor one. Rich countries have well-developed financial systems with government debt as a central anchor. Strategic default would destroy the assets and trust upon which the financial system is based, inflicting a massive punishment. We introduce a debt contract, which explicitly incorporates the different incentives faced by a rich sovereign. The implicit contract contains the threat of massive punishment for a sovereign who fails to pay what she is able, but no punishment, even in default, for a sovereign who pays what she is able. The central planner uses this debt contract to smooth consumption in the face of persistent output with stochastic shocks. We calibrate to the default experience of Greece in its 2010 debt crisis. This alternative debt contract can explain why: 1) countries with debt/GDP ratios higher than the value of standard default punishments do not default; 2) a sovereign in default always repays something; 3) crises follow an increase in debt which sometimes ends in a sudden stop; 4) debt becomes risky for different countries at different levels of debt/GDP; 5) haircuts and default duration are highly heterogeneous across default events.

Keywords: Ability to Pay, Fiscal Limits, Willingness to Pay, Sovereign Default, Sudden Stop, Strategic Default

1 Introduction

Following the Great Recession of 2008, government debt/GDP in most industrial countries has increased substantially, in many cases to unprecedented levels. The literature on sovereign default¹ shows that a sovereign will optimally choose default when the gains from non-repayment of debt exceed the cost of punishment to default (Arellano 2008). Empirically, punishments suffered by defaulting sovereigns include temporary exclusion from credit markets and output loss. However, the value of these punishments is small on both counts. Sovereigns regain access to credit markets quickly after debts are settled. The fall in output which empirically coincides with default might not actually be caused by default, does not always occur, and is temporary and the order of magnitude of a recession (Yeyati and Panizza 2011).

In light of these small costs of punishment and the large benefits of non-repayment of debt/GDP, why don't more countries optimally choose default? And why did Greece, with debt/GDP less than Belgium has experienced historically, default? Will other rich sovereigns, who have recently been threatened with fiscal crises (Portugal, Spain, Italy), also succumb to default, and why has default not already occurred? Additionally, if default triggers an identical punishment independent of the magnitude of default, why does a defaulting sovereign make any debt repayments?

To answer these questions, we focus on the different incentives to repay debt faced by rich versus poor sovereigns. Poor sovereigns constitute the bulk of actual sovereign defaults and are the focus of the strategic default literature.² However, recent events in Europe demonstrate the need for a model of sovereign default for rich countries. Rich countries are different because they have well-developed financial systems in which government debt plays a central anchoring role, often serving as a risk-free asset. Strategic default, whereby a solvent government refuses to pay, would destroy significant financial assets and the trust upon which these financial systems are based,³ inflicting tremendous economic damage. Think about the EMU's threat to destroy Greek banks if Greece refuses to comply with German terms, or the destruction of the world financial system should the US give in to calls from some in Congress, and even the president, to default on US debt. A poor country with a less-developed financial system does not face the same cost of a strategic default.

¹We define default as failure to repay contractual debt obligations.

²Arellano (2008) provides the baseline model. Aguiar and Amador (2013) and Aguiar, Chatterjee, Cole, and Stangeby (2016) provide extensions of this model and survey extensions offered by others.

³See Gennaioli et al (2014) and Bolton and Jeanne (2011) for a discussion of the risk of sovereign default to banks.

The primary contribution of this paper is to design a debt contract based on these different incentives. This paper builds on the seminal papers by Eaton and Gersovitz (1982) and Arellano (2008). We follow their lead and assume that the sovereign acts as a central planner, maximizing expected utility of the representative agent when endowment income is subject to stochastic shocks. To focus on rich country incentives, we make two primary departures from their assumptions. First, we assume that a sovereign has limited ability to repay and that this ability is increasing in current and expected future income. The ability-to-pay is related to the government's ability to extract tax revenue from the population to use for debt repayment. Therefore, ability is related positively to national wealth and to the efficiency of the tax collection process, and negatively to corruption and political aversion to tax payment. Quality of institutions matters, and quality tends to increase with income. This ability to repay is less than the Aiyagari (1994) debt limit because no government of a modern economy could impose zero consumption, or even subsistence consumption, on its constituents forever. Davig, Leeper, and Walker (2011), Bi (2012), Daniel and Shiamptanis (2013) refer to the ability to pay as a fiscal limit.

Second, we modify the punishment to default to account for the damage that strategic default would inflict on the financial system. Specifically, we assume that a defaulting sovereign enters into debt renegotiation with the creditor. The idea borrows from Grossman and Van Huyck's (1988) concept of "excusable default," whereby lenders forgive some portion of debt in some states, and from Bulow and Rogoff (1989), in which renegotiation in some states is part of the implicit contract. There is no explicit punishment to default outside of this debt renegotiation. In the debt renegotiation, the creditor threatens a massive punishment if the debtor refuses to pay what she is able. Refusal to pay what she is able, equivalently strategic default, would be an inexcusable debt repudiation and would elicit maximum punishment. This massive punishment involves destabilization of the financial sector and is relevant for countries with a well-developed financial sector, primarily developed and prosperous emerging market countries, but not for very poor countries. Faced with an implicit contract with this massive punishment, the sovereign defaults only when debt exceeds ability to pay and never chooses strategic default.

We derive the implications of the contract for the dynamic behavior of debt and for risk-taking. Our debt contract has very different incentives for risk-taking from those in the model of strategic default. In our model, as debt increases, there are more future income states in which the sovereign will replace contractual payments with the lower ability-to-pay amount. Therefore, for states in which contractual repayments are not made, there is no cost to increasing debt, increasing the equilibrium amount of debt. We

show analytically that in the absence of any dead-weight loss to default, the increased incentive to take on risk dominates the response of the interest rate to debt, such that the sovereign raises debt in low-income states. This risk-taking behavior implies that debt is rising just before a crisis in contrast to the strategic default model in which debt is falling. Stiglitz (1981) first demonstrated that the possibility of default cuts off the lower portion of the risk distribution, thereby incentivizing risk-taking.

Our second contribution is quantitative. We calibrate the model to match the Greek crisis, which culminated in the first quarter of 2010. Following a period of about seven years of relative tranquility, with GDP at or slightly below its trend value and debt relatively constant at a little less than 100 percent of GDP, Greece was struck by the financial crisis. GDP fell, and debt rose. In the model, the sovereign borrows to smooth consumption in the wake of the fall in GDP. The crisis comes when output falls so much that Greece is unable to pay the debt she has accumulated. Our model predicts default with an orderly settlement in which Greece agrees to pay what she is able and reattains access to financial markets. Therefore, at the point of crisis, our market-based model departs from the reality of large official intervention. However, our model is able to capture the timing of the crisis, together with rising debt prior to the crisis, as the optimal response of sovereign borrowing to stochastic changes in GDP. In contrast, the strategic default model would have predicted falling debt as the crisis approached.

We use the calibrated model to answer the question about why Greece defaulted when other high-debt European countries did not. If we begin in 2005Q1 and compute the probability of a crisis for Greece over the next ten years, we obtain an estimate of about ten percent. This implies that Greek debt was risky. The financial crisis was the realization of that risk, with the outcome that Greece became insolvent. However, other European countries also experienced the financial crises beginning with similar levels of debt/GDP – in 2005Q1 Belgium had debt/GDP of 100.6 percent while Italy had 103.7 percent – and these countries did not default. A major reason for the difference in behavior is a difference in ability to pay, as evidenced by the ability to raise government surpluses. Historically, Belgium and Italy have experienced considerably higher surpluses than Greece has, implying higher ability to pay. And higher ability to pay reduces the probability of default.

Additionally, we use this calibrated model to explore the characteristics of financial crises under the debt contract we propose. We find that crises tend to occur after a period of relatively low output during which the sovereign has been borrowing to smooth consumption. Therefore, defaults occur when output is low and debt is high and possibly

rising. After default, the sovereign agrees to pay what she is able and borrows again based on expected future ability to pay. If output is high enough next period, she repays her debt and emerges from the default. If not, she again pays what she is able and borrows based on expected ability to pay. Most crises are resolved quickly, but some can take a very long time. Emergence from a crisis often comes when output rises, increasing ability to pay. We compute the magnitude of haircuts in default and find high variability. The longest lasting defaults have the largest haircuts. Both characterizations are consistent with empirical evidence.⁴

The paper is organized as follows. Section 2 presents the theoretical model, including the debt contract and optimizing behavior by the sovereign. Section 3 provides the calibration to the Greek crisis. Section 4 provides a quantitative description of financial crises, including the Greek crisis, using the calibrated model. Section 5 addresses the questions posed above, and Section 6 concludes.

2 Theoretical Model

The domestic economy is small and open and subject to stochastic endowment shocks. We assume that endowment income on each date is drawn from a bounded distribution, indexed by $j \in \{1, \bar{j}\}$. The bounded distribution approximates a distribution in which income is determined by

$$\ln y_t = \ln \bar{y} + \rho \ln y_{t-1} + \epsilon_t \quad 0 < \rho < 1 \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

such that high income today implies high expected future income. The value of j determines the value of income and therefore the income state of the economy.

We assume that the sovereign is a benevolent dictator who maximizes the expected present value of utility of the representative agent, given by

$$U = E \sum_{t=1}^{\infty} \beta^t u(c_t).$$

The sovereign can trade in a limited set of financial contracts with risk-neutral international creditors, allowing consumption-smoothing and consumption-tilting based on the country's rate of time preference relative to the world's. The characteristics of the

⁴Sturzenegger and Zettelmeyer (2008), Benjamin and Wright (2008), and Cruces and Trebesch (2011) provide evidence.

financial contract we propose deviate from those in the literature and constitute the contribution of the present paper.

2.1 Debt Contract

We modify the standard debt contract with two assumptions, first, a limit on ability to pay, and second the threat of a massive punishment for a sovereign who fails to pay what she is able. Davig, Leeper and Walker (2011), and Bi (2012) define the maximum level of debt the country can repay as the "fiscal limit", and they motivate the limit by the top of the Laffer curve for distortionary taxes. If the government's attempt to raise taxes sufficiently to service debt causes output to fall proportionately more than debt falls, then the country has hit its fiscal limit on debt. However, the concept can be more general (Daniel and Shiamptanis 2012). The fiscal limit can be based on the maximum level of the primary surplus that a country could raise. It can include the inability to reduce government spending, perhaps due to the dependence of economic activity on the provision of public goods, together with the inability to raise taxes for other reasons, including political difficulties (as in Bi, Leeper, and Leith 2013) and tax evasion (as in Daniel 2014).

We assume that the ability to repay in endowment state h is given by A_h and is determined by the expected present value of the country's endowment income net of a minimum level of consumption, \bar{c} , according to

$$A_h = \sum_{t=0}^{\infty} [E(y(t) | y(0) = y_h) - \bar{c}] \left(\frac{\psi}{1 + r^*} \right)^t, \quad (1)$$

where y_h represents current endowment income, r^* represents the risk-free interest rate, and E is notation for the rational expectation. We assume $\psi \leq 1$, such that the sovereign is not necessarily able to make repayments equal to the excess of expected income over minimum consumption forever. And the minimum consumption should be interpreted as politically feasible, and not as consumption at an Aiyagari (1994) debt limit. The assumption of declining ability to extract a surplus for repayments over time is motivated by statements like that of Greece in the spring of 2016 that it cannot sustain a primary surplus of 3.5 percent of GDP indefinitely, as required by German demands. Note that the autoregressive behavior of income implies that A_h is increasing in y_h , such that higher income today yields higher ability-to-pay. Additionally, the specification implies that

ability-to-pay relative to GDP is increasing in GDP.⁵ We assume that ability-to-pay, conditional on current income, is known, although we realize this is an important assumption to relax in future work.

Our second modification is that failure to repay, when able, triggers a massive punishment. Think about Europe's implicit threat to destroy Greece's banking system if Greece does not cooperate in repayments or the destruction of the world financial order if the US followed some members of Congress and the president and chose to default on US debt. The punishment does not necessarily rely on an explicit act by the creditor, but could be the endogenous response of the economic system to sovereign default of an "inexcusable" magnitude, an action which would destroy both the trust and the assets on which the financial sector is based. This follows the "excusable" default literature (Grossman and Van Huyck 1988), where default provides insurance against bad outcomes for the borrower. A poor sovereign with a less developed financial system would not face the same kind of punishment. As long as the sovereign pays what she is able in default, there is no explicit default punishment. And the punishment is so costly that it would never be chosen in equilibrium.⁶

2.2 Equilibrium in Financial Markets with Default

We assume that the domestic economy is small and open. Additionally, it has access to a risk-neutral international creditor. We characterize equilibrium for a small open economy with an exogenous foreign interest rate in two steps. First, we characterize equilibrium in financial markets and second we characterize equilibrium jointly in goods and financial markets.

Financial Market Equilibrium: Given the terms of the debt contract, the sovereign chooses repayment or default optimally, and the interest rate assures that the risk neutral lender receives an expected rate of return equal to the risk-free rate.

Proposition 1: Default occurs when the ability to repay, conditional on current income, is less than outstanding debt.

⁵Expected future output relative to current GDP falls less with an increase in current GDP than does minimum consumption relative to GDP.

⁶We could also motivate this with a Nash bargain where the surplus to be divided between the players includes the massive punishment. The borrower then agrees to the Nash bargain, the division of this very large surplus, subject to his ability to repay. With a large enough punishment, the ability-to-pay binds, and she repays what she is able.

As long as the sovereign makes debt payments in default equal to ability to pay, there are no punishments to default. And there is a massive punishment when the sovereign fails to make repayments up to her ability to pay. Therefore, the sovereign optimally chooses to repay whenever contractual debt repayments are less than her ability to repay and to default otherwise.

Corollary: There is no strategic default whereby a sovereign who is able to pay optimally chooses default.

Given the terms of the contract, a sovereign who is able to repay optimally chooses repayment.

Optimal behavior with respect to default and repayment implies that the budget constraint for the country is effectively

$$q'D' = c + \min \{A_h, D\} - y. \quad (2)$$

We follow Arellano (2008) and determine cutoff values for income as a function of the face value of debt, above which the sovereign makes contractual repayments and below which she defaults. We let \hat{y} be the cutoff value, given by the income state in which the face value of debt equals ability to pay. States with income below the cutoff ($y_{\hat{y}}$) are default states, and states above are repayment states. We assume that repayment occurs with income equal to $y_{\hat{y}}$. For values of $D < A_1$, debt is safe and $\hat{y} = 1$, its lower support. For higher values of debt, the cutoff state is implicitly defined by

$$D = A_{\hat{y}} \text{ for } D \geq A_1. \quad (3)$$

As debt rises, ability-to-pay is equal to debt only if the income state rises, allowing the increase in $A_{\hat{y}}$. Therefore, \hat{y} , defined as the lowest income state in which the sovereign repays, is increasing in debt.

Proposition 2: The size of the "haircut" in default depends on the ability to repay relative to outstanding debt.

The sovereign, currently in state h with face value of debt D , optimally chooses to repay

$$\min \{A_h, D\}.$$

Therefore, the size of the "haircut" in state h (H_h) is given by

$$H_h = \frac{D - A_h}{D}.$$

Proposition 3: When debt is large enough to be risky, the price of debt is decreasing in debt.

The return on debt is determined such that the international creditor expects to receive the risk-free interest rate. Define \hat{j} as the lowest income state in which repayment occurs. The arbitrage relationship governing the interest rate set in the current period (r') in state h for next period's debt (d') is given by

$$(1 + r^*)d' = (1 + r')d'[1 - F(\hat{j}|h)] + \int_{j=1}^{\hat{j}} A_j f(j|h) dj, \quad (4)$$

where r^* is the world risk-free interest rate, $f(j|h)$ is the density function for the distribution of income levels indexed j , conditional on beginning in state h , and

$$F(\hat{j}|h) = \int_{j=1}^{\hat{j}} f(j|h) dj$$

is the cumulative distribution in state \hat{j} , conditional on beginning in state h . The probability of repayment is given by $[1 - F(\hat{j}|h)]$. The arbitrage relation in equation (4) requires that the value of debt (d') multiplied by the gross risk-free interest rate ($1 + r^*$) equal contractual repayments $[(1 + r')d']$, multiplied by the probability of repayment $[1 - F(\hat{j}|h)]$, plus repayments in each default state, $(A_j|j < \hat{j})$, multiplied by their probabilities $(f(j|h) dj)$.

Defining the price of debt (q) as

$$q \equiv \frac{1}{1 + r},$$

and the face value of debt (D) as

$$D \equiv (1 + r)d, \quad (5)$$

equation (4) implies that the price of debt is

$$q' = \frac{D'[1 - F(\hat{j}|h)] + \int_{j=1}^{\hat{j}} A_j f(j|h) dj}{(1 + r^*)D'}. \quad (6)$$

The derivative of the price of debt with respect to its face value is given by

$$\frac{\partial q'}{\partial D'} = - \left[\frac{D' - A_j}{D'} \right] \frac{f(\hat{j}|h)}{1 + r^*} \frac{\partial \hat{j}}{\partial D'} - \frac{\int_{j=1}^{\hat{j}} A_j f(j|h) dj}{(1 + r^*) (D')^2}. \quad (7)$$

Recognizing that $A_j = D'$ from equation (3) and simplifying yields

$$\frac{\partial q'}{\partial D'} = - \frac{\int_{j=1}^{\hat{j}} A_j f(j|h) dj}{(1 + r^*) (D')^2} \leq 0. \quad (8)$$

When the face value of debt is low enough that it is less than ability to pay in the worst state ($D' < A_1$), all debt is safe ($\hat{j} = 1; F(1|h) = 0$), and $q' = \frac{1}{1+r^*}$. Since the integral has unity as the upper and lower limit, the derivative is zero. However, once the face value of debt rises above A_1 , \hat{j} rises, and the price of debt falls as debt rises.

It is useful to compare the effect of an increase in debt on the interest rate in this model with that in the model of strategic default in which either there are no debt repayments in default (Arellano 2008) or the repayments are some fixed fraction of debt. In the strategic default model, default occurs only if the gains to default, based on the difference between what the sovereign owes and what she repays, exceed the value of the punishment. With this alternative contract, the value in equation (7) for $D' - A_j$, where we interpret A_j as debt repayments, must be positive and large enough to exceed the punishment, justifying default. Therefore, the first term in equation (7) is large and positive, instead of zero, implying that an increase in borrowing creates a larger rise in the interest rate (and a larger fall in the price of debt) than the debt contract we specify. This large increase in the interest rate is responsible for the result that the sovereign saves when there is a positive probability of default, even though consumption-smoothing would require borrowing. Since the interest rate rises less with the ability-to-pay contract than with strategic default, the sovereign will be able to borrow smooth consumption even in the neighborhood of default.

Corollary: When debt is risky, an increase in the face value of debt increases resources from borrowing ($q'D'$) by less than the price of debt.

Multiplying equation (6) by D' and taking the derivative with respect to D' yields

$$\frac{\partial (q'D')}{\partial D'} = \frac{[1 - F(\hat{j}|h)] + f(\hat{j}|h) [A_j - D'] \frac{\partial \hat{j}}{\partial D'}}{1 + r^*}. \quad (9)$$

Noting that $A_{\hat{j}} = D'$ from equation (3) and simplifying yields

$$\frac{\partial (q'D')}{\partial D'} = \frac{[1 - F(\hat{j}|h)]}{1 + r^*} \geq 0. \quad (10)$$

When $D' < A_1$, all debt is safe and $F(\hat{j}|h) = 0$. The effect of an increase in the face value of debt on the proceeds from borrowing is the inverse of the gross risk-free interest rate, equivalently the price of debt. However, once debt is large enough to be risky, implying that the probability of default is positive ($F(\hat{j}|h) > 0$), an increase in D' requires a decrease in q' such that the proceeds from borrowing rise by less than $\frac{1}{1+r^*}$.

The foregoing implies that there is an upper bound on borrowing ($q'D'$). From equation (10), $q'D'$ is increasing in D' until D' reaches the ability to pay in the highest state possible next period, conditional on the current state. Define this state as \tilde{j}_h . Using equation (6) with \tilde{j} replacing \hat{j} , and $F(\tilde{j}|h) = 1$, the upper bound on sovereign borrowing is determined by the expected present-value of repayments in default, conditional on income in the initial state.

$$q'D' \leq (q'D')_{ub} = \frac{\int_{j=1}^{\tilde{j}} A_j f(j|h) dj}{(1 + r^*)}, \quad (11)$$

where h is the initial state. Higher initial income implies a higher upper bound due to the autoregressive assumption about the behavior of income.

The upper bound on $q'D'$ also implies an endogenous upper bound on D' . Once the face value of debt rises so much that $q'D' = (q'D')_{ub}$, the sovereign will not choose further increases in D' . Larger D' would be accompanied by a proportionate fall in q' such the increase in future debt obligations would not be accompanied by an increase in borrowing proceeds and current consumption, a suboptimal move.

When the probability of default is positive, the domestic interest rate carries a default-risk premium, given by

$$r - r^* = \frac{1}{q'} - (1 + r^*) = \frac{(1 + r^*) \int_{j=1}^{\hat{j}} (D' - A_j) f(j|h) dj}{D' - \int_{j=1}^{\hat{j}} (D' - A_j) f(j|h) dj}, \quad (12)$$

where the second equality uses equation (6). Note that an increase in debt, which causes \hat{j} to rise, creates a discrete jump in the interest premium by increasing the number of states with default repayments.

Proposition 4: The probability of default is increasing in debt and decreasing in income.

A sovereign optimally chooses default when ability to pay is less than debt. Ability to pay is increasing in income. Therefore, for a given level of debt, the probability of default is the probability that future income falls sufficiently to reduce ability to pay below debt owed (D'), equivalently the probability of transiting from the current state h to a state below \hat{j} , where the value of \hat{j} is determined by the value of D' . This probability is higher, the lower is income, due to the autoregressive nature of shocks, and the higher is debt, since \hat{j} is increasing in debt.

2.3 Optimization Problem

2.3.1 Value Function

The dynamic behavior of debt, in response to shocks to income, is determined by the optimizing behavior of the sovereign. We represent the expected present value of utility for the sovereign with a value function, which depends on the exogenous state given by income (y), and on the face value of debt (D), according to

$$V(y, D) = u(c) + \beta EV'(y', D').$$

Since the sovereign defaults in states $j < \hat{j}$, and repays in others, we can rewrite the value function as

$$V(y, D) = u(c) + \beta \left[\int_{j=1}^{\hat{j}} V'(y', A(y')) f(j) dj + \int_{j=\hat{j}}^1 V'(y', D') f(j) dj \right].$$

The only distinction between repayment states and default states is initial debt, implying different arguments for the future value functions in repayment versus default states, but not different functions.

Maximization is subject to a budget constraint, given by equation (2), which depends on the current income state h , and which allows default with repayments equal to ability to repay, whenever ability is less than contractual debt repayments.

The derivative of the value function with respect to debt differs in default and repayment states. In a repayment state

$$\frac{\partial V(y, D | j \geq \hat{j})}{\partial D} = - \left(\frac{\partial u(c)}{\partial c} \right).$$

However, for values of j putting the system into default states, the derivative of the value function with respect to debt is zero since, in default, the sovereign pays its ability

irrespective of actual debt.

The first order condition for the choice of next period's debt is given by⁷

$$\frac{\partial u(c)}{\partial c} \frac{\partial (q'D')}{\partial D'} - \beta \int_{j=\hat{j}}^{\bar{j}} \left(\frac{\partial u(c')}{\partial c'} \right) f(j|h) dj = 0$$

where c' should be understood as depending on j . Substituting from equation (10) yields

$$\frac{\partial u(c)}{\partial c} = \beta(1+r^*) \left[\frac{\int_{j=\hat{j}}^{\bar{j}} \left(\frac{\partial u(c')}{\partial c'} \right) f(j|h) dj}{\int_{j=\hat{j}}^{\bar{j}} f(j|h) dj} \right] = \beta(1+r^*) E \left\{ \left(\frac{\partial u(c')}{\partial c'} \right) | (j > \hat{j}) \right\} \quad (13)$$

The right hand side of equation (13) is the expected marginal utility of consumption next period, conditional on obtaining states in which repayment occurs. Since repayment in default states is not related to the amount borrowed, states below \hat{j} are not included in the integral for expected future marginal utility of consumption. At the optimum, the marginal utility of current consumption equals the expected marginal utility of future consumption, conditional on repayment, multiplied by $\beta(1+r^*)$. Since consumption is higher in states in which repayment occurs, the marginal utility of expected future consumption, conditional on repayment, is lower than unconditional marginal utility of expected future consumption. Therefore, when default is possible, the marginal utility of current consumption must be lower and current consumption higher.

2.3.2 Equilibrium

Definition of Equilibrium: Equilibrium is a set of policy functions for consumption $c(D, y)$ and government debt holdings $D'(D, y)$, a cutoff value for states determining repayment $\hat{j}(D)$, and a price function for debt $q'(D', y)$ such that the policy functions satisfy the optimization criteria and the budget constraint, and bond prices assure risk-neutral lenders the exogenous expected risk-free rate of return.

Proposition 5: Consumption and the choice of debt next period are higher when the probability of default is positive.

A positive probability of default next period ($\hat{j} > 1$) decreases the right hand side of the Euler equation (13) because expected future marginal utility is included only for repayment states, and consumption is higher in those states than in default states. The lower expected marginal utility of future consumption requires that the marginal utility

⁷The term multiplying $\frac{\partial \hat{j}}{\partial D'}$ vanishes since at \hat{j} , $A(y') = D'$.

of current consumption also fall, thereby increasing current consumption. Therefore, when the economy enters states low enough that default is possible, it raises consumption through an increase in borrowing. The possibility of paying only what the sovereign is able and not actual debt repayments cuts off the lower portion of the risk distribution encouraging the sovereign to increase consumption and debt, thereby taking on more risky behavior. This is Stiglitz’s (1981) classic result that the availability of bankruptcy increases risk-taking behavior. The result is opposite that in the strategic default model, in which the sovereign saves in all states for which the probability of default is positive.

3 Calibrated Model

We solve the model by creating a grid for the face value of debt with 2000 points, ranging logarithmically from -10 to 1.67.⁸ We use value function iteration with the choice variables being the decision to default or repay current debt and next period’s debt, conditional on the current value of debt, the current output state, and the equilibrium price of new debt.

3.1 Standard Values

For our quarterly calibration, the external interest rate (r^*), and the coefficient of relative risk aversion (σ) take on standard values: $r^* = 0.017$, based on a long average of US real stock returns; and $\sigma = 2$, based on current convention in macroeconomic calibration models.⁹ We choose Greece and its crisis as the focus of our calibration. We estimate the autoregressive parameter for real Greek GDP and its standard error using quarterly OECD data from 1960Q1 to 2008Q2. We detrend and demean the log of the data and obtain values of $\rho = 0.965$ and $\sigma_\epsilon = 0.028$. We approximate the behavior of the data using a discrete approximation with eleven output states based on Tauchen’s (1986) method of approximating an autoregressive series with a Markov chain.

⁸The exact value we use is the ability to pay in state 9 with parameter values \bar{c} and ψ , chosen below. The sovereign never wants debt this high. The lower bound binds only for a sovereign beginning with high income and very near the lower bound on debt. We use a logarithmic grid to place relatively more points in the region with positive debt, the region of interest in default.

⁹Arellano (2008) used both values in her calibration to Argentina.

3.2 Deadweight Cost of Default

Our model has no deadweight loss in default, a simplifying assumption that is not realistic. Therefore, we add a small deadweight loss to default to offset some of the Stiglitz-type risk-taking in the neighborhood of default.

In the model, the sovereign already pays the maximum she is able in default, implying that we cannot add anything to these repayments. Therefore, we assume that, in default, the sovereign continues to pay her full ability, but the lender receives only a fraction ω of this repayment. This requires revision of equation (6) to yield

$$q' = \frac{D' [1 - F(\hat{j}|h)] + \int_{j=1}^{\hat{j}} \omega A_j f(j|h) dj}{(1 + r^*) D'}. \quad (14)$$

We view this deadweight loss as the administrative cost of the default and not as an explicit punishment to default. With the deadweight loss, the interest premium becomes

$$r - r^* = \frac{1}{q'} - (1 + r^*) = \frac{(1 + r^*) \int_{j=1}^{\hat{j}} (D' - \omega A_j) f(j|h) dj}{D' - \int_{j=1}^{\hat{j}} (D' - \omega A_j) f(j|h) dj}.$$

This revision changes the derivative of the price of debt and current borrowing with respect to the face value of debt, equations (8) and (10), and the Euler equation (13), to yield

$$\frac{\partial q'}{\partial D'} = -\frac{\int_{j=1}^{\hat{j}} \omega A_j f(j|h) dj}{(1 + r^*) (D')^2} - [1 - \omega] \frac{f(\hat{j}|h)}{1 + r^*} \frac{\partial \hat{j}}{\partial D'} < 0 \quad (15)$$

$$\frac{\partial (q' D')}{\partial D'} = \frac{[1 - F(\hat{j}|h)] - f(\hat{j}|h) A_{\hat{j}} (1 - \omega) \frac{\partial \hat{j}}{\partial D'}}{1 + r^*} \geq 0, \quad (16)$$

$$\frac{\partial u(c)}{\partial c} = \beta (1 + r^*) \left[\frac{\int_{j=\hat{j}}^{\bar{j}} \left(\frac{\partial u(c')}{\partial c'} \right) f(j|h) dj}{[1 - F(\hat{j}|h)] - f(\hat{j}|h) A_{\hat{j}} (1 - \omega) \frac{\partial \hat{j}}{\partial D'}} \right], \quad (17)$$

where we have used $A_j = D'$. With $\omega < 1$, repayments in default per unit of debt are lower, implying a lower price of debt and a smaller reduction in the price when debt rises. For increases in next period's debt which raise the value of \hat{j} ($\frac{\partial \hat{j}}{\partial D'} > 0$), reducing the number of repayment states, deadweight loss ($\omega < 1$) implies that the price of debt takes a downward jump, with the interest premium taking a corresponding upward jump. Additionally, the proceeds from additional borrowing ($q' D'$) do not rise as much.

In equation (17), the term multiplying $\frac{\partial \hat{j}}{\partial D'}$ implies a discrete increase in the cost of raising debt beyond the next critical bound at which \hat{j} increases. At such a bound, a small

increase in debt does not yield significant future debt relief in default because ability-to-pay in the next higher state almost matches the debt. However, since the creditor suffers a deadweight loss in default, he requires a discrete reduction in the price of debt with the increase in \hat{j} . Therefore, due to the fall in the price of debt, the sovereign receives little additional consumption from increasing debt just beyond the critical barrier, and she shoulders additional debt in repayment states. Together these incentives act to keep debt below the bounds at which \hat{j} changes, and they are larger, the larger the deadweight loss (the smaller is ω). The deadweight loss mitigates Stiglitz risk-taking.

The deadweight cost of default adds an additional parameter, ω , for calibration.

3.3 Remaining Parameters

There are four remaining parameter values, ψ , \bar{c} , ω , and β which we calibrate to match five features of the data: (1) the timing of the crisis, (2) the value of average debt over the full business cycle preceding the crisis,¹⁰ the values of debt/GDP on two dates: (3) pre-crisis (2009Q4) and (4) crisis (2010Q1), and (5) a spike in the interest rate premium with little debt reduction in the initial crisis period.

We obtain data on the values of debt using Eurostat data on quarterly values of debt relative to GDP, beginning in 2006Q1, and annual values for 2001-2005. We convert these values to our measure of debt, which is debt relative to GDP in the median state, by multiplying the Eurostat data by actual GDP relative to mean GDP, using our detrended and demeaned OECD data on real GDP.¹¹ For output, we convert our detrended and demeaned data on real Greek GDP into the 11 output states of the model by choosing the output state closest to the detrended and demeaned value. Our data on the Greek interest rate premium is the difference between the interest rate on ten year government bonds for Greece and Germany from the ECB Statistical Data Warehouse.

Our model defines the crisis date as the first period in which Greece's ability to pay is less than debt. In 2010Q1, Greece suffered a reduction in output, reducing ability to pay. Greece did not have scheduled debt repayments in this period, implying that there were no observations on repayments, either missed or made. However, Greece began austerity programs and the ECB softened rules on collateral for ECB loans, implying that Greece expected financing difficulties once maturity dates arrived. This evidence implies that the

¹⁰We measure the business cycle preceding the crisis using the discretized states. Greek output enters state 5 in 2005Q1 after a period of being in state 6 and its last period in state 6 before returning to state 5 is 2007Q4. Therefore, we measure the business cycle as the first period of recession (2005Q1) through the last period of the subsequent boom (2007Q1).

¹¹This requires that we use quarterly interpolations of the annual data for the years 2001-2005.

first period in which Greek debt exceeded ability to pay was 2010Q1, leading us to use this date as the first period of the crisis.

To obtain model values, we generate a time series on the sovereign's choice of debt and default conditional on the initial value of debt/GDP given by the data and on output states from the data. We choose the start date as 2005Q1, the first date of the previous recession.¹² This requires that the sovereign choose debt over the entire business cycle preceding the one which created the crisis, as well as over the beginning of the business cycle created by the financial crisis.

Our first step in matching model values with the data is to narrow the choices of the parameter values to those which exactly match the timing of the crisis. Therefore, our calibration strategy requires that beginning on the start date (2005Q1), the sovereign chooses to repay in all periods leading up to the crisis and chooses not to repay in the crisis period. Candidate parameter values must imply that the sovereign choose next period's debt on each date beginning with 2005Q1 and ending with 2010Q1, such that debt is below realized ability to pay through 2009Q4 and above realized ability in 2010Q1. This requires that the sovereign choose debt consistent with the actual repayment and default decisions for a total of 21 periods.

Second, we choose from a set of parameter values which yields a crisis equilibrium with characteristics actually observed for Greece. As we explain later, there are two discretely different types of crises, one with an large increase in the interest premium and no debt reduction, and another with no increase in the interest premium and substantial debt reduction. Greece did experience a large increase in the interest premium in 2010Q1 and little debt reduction between 2010Q1 and 2010Q2. Therefore, the Greek crisis seems to be of the first type, and we require parameter values which generate this type of crisis. However, the actual interest premium and the value of debt reduction are not targets of the calibration.

Given these restrictions, we finalize the choices for ψ , \bar{c} , ω , and β by matching three additional features of the data: the average value of debt over the previous business cycle, the value of debt in the period prior to the crisis, and the value of debt in the crisis period. Parameter values and the sources for their calibration are given in Table 1.

¹²This choice of beginning date requires that we use an interpolated value for Debt/GDP in 2005Q1. Eurostat's quarterly debt data for Greece begins in 2005Q4.

Table 1: Parameter Values

Parameter	Value	Source
σ	2	standard value
r^*	0.017	standard value
σ_ϵ	0.028	regression estimate using real GDP data (1960Q1:2008Q2)
ρ	0.965	regression estimate using real GDP data (1960Q1:2008Q2)
ψ	0.41	crisis timing, crisis type, and three data targets
\bar{c}	0.21	crisis timing, crisis type, and three data targets
β	0.982	crisis timing, crisis type, and three data targets
ω	0.9965	crisis timing, crisis type, and three data targets

Consider how the four different parameter values affect the five features of the data we were trying to match. All are important in matching crisis timing. The requirement that we match debt data on the eve of the crisis and in the period of the crisis requires that ψ and \bar{c} be chosen such that ability-to-pay was above actual Greek debt on the eve of the crisis but was below debt on the date of the crisis. The value for β partially determines the sovereign's propensity to take on debt in alternative states and is important in determining the average value of debt. The values for ψ and \bar{c} and β are jointly responsible for determining average debt over the previous business cycle and values for debt as the crisis unfolds. The value for ω is important in determining the type of crisis. The closer ω is to unity, the smaller the deadweight loss and the greater is the Stiglitz-type risk-taking. With more risk-taking, debt and the interest premium are both larger, creating the type of crisis equilibrium Greece experienced.¹³

It is useful to note that our calibration does not require a particularly impatient sovereign. The inverse of the gross risk-free interest rate in our calibration of β is 0.983, compared with our calibrated value for β of 0.982. Our model does not require much impatience due to the Stiglitz-type risk-taking created with the high value of ω , implying a deadweight loss in financial markets of only 0.35 percent of the value of repayments. Additional risk-taking, as a result of debt accumulation due to impatience, is unnecessary.

Table 2 compares model values with those in the data as a test of model fit.

¹³The value for ω is reported to the fourth decimal place because very small changes can cause the economy to switch between types of equilibria. A value of $\omega = 0.9963$ implies the alternative type of crisis equilibrium.

Table 2: Model Fit

	Timing	Average Debt	Pre-crisis Debt	Crisis Debt
Model	2010Q1	1.02	1.11	1.12
Data	2010Q1	1.02	1.11	1.13

The model fits the data well matching values in the data within 0.01 percentage points.

4 Quantitative Results

To describe debt dynamics and the characteristics of default crises, we create a time series of 5,010,000 values for output based on our calibrated model. We use a random number generator to create values between zero and unity, and then use the transition matrix, generated from the Tauchen approximation for output with parameters ρ and σ_ϵ , to place each value into one of the eleven output states. Beginning with an initial value of debt, we use the calibrated solution of the model to solve for optimal decisions on default and repayment and on next period's debt, conditional on output and on the preceding value of debt. We drop the first 10,000 simulations. We collect instances of default, together with experience prior to and after the default. When we have a period between defaults of one year or less, we aggregate the subsequent crises into a single longer one. We have a total of 3,453 separate default-crisis events.

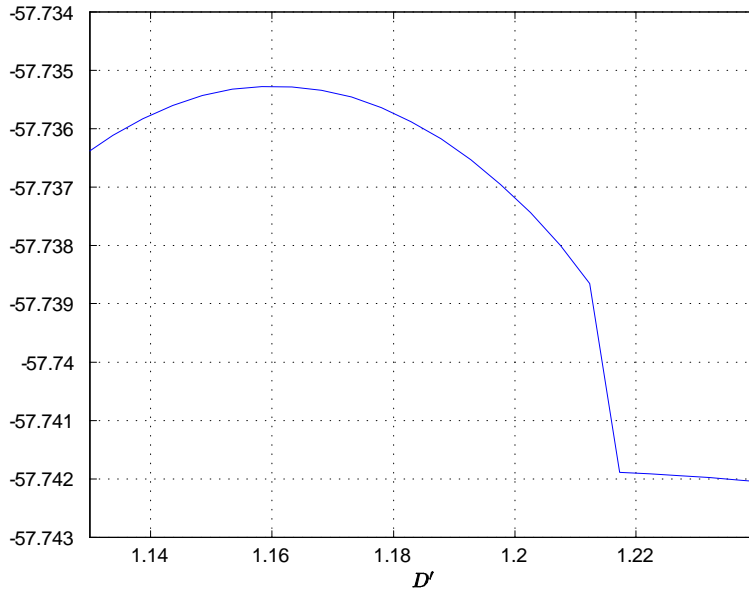
We use the simulations to characterize properties of fiscal policy and debt dynamics, given the possibility of debt crises with default.

4.1 Fiscal Policy is Counter-Cyclical

The first result from the simulations is that fiscal policy is counter-cyclical. When a sovereign is in an income state below the median and has debt lower than ability to pay in the same state, she generally chooses a modest increase in debt. There are exceptions to this behavior when debt is very close to ability-to-pay, which we discuss below.

Figure 1 plots the value function as a function of the sovereign's choice of debt next period (D'). The country is in state 5, one state below the median. Initial debt equal to 1.13, the starting value on the horizontal axis, and is less than ability to pay, given by 1.21.

Figure 1: Value Function in State 5



Value function is concave in debt

The peak of the value function occurs with debt a little higher than its current value, but still below ability-to-pay in the current state. Figure 5 illustrates that with debt lower than ability-to-pay, the sovereign chooses a moderate increase in debt. This choice of increasing debt generalizes to all states below the median when initial debt is sufficiently¹⁴ below ability to pay in the current state.

When output is above the median, the sovereign tends to smooth consumption by saving.¹⁵ Together, this behavior describes a counter-cyclical fiscal policy, which is generally consistent with fiscal policy in advanced countries, and opposite to much fiscal policy in developing countries. (Frankel et al 2013).

4.2 Endogenous Debt Limits

The solution of the model reveals that the sovereign has endogenous debt limits conditional on the state. For the median income state and below, the endogenous debt limit is

¹⁴We show below that in some income states, as debt approaches ability to pay, the increase in D' can be large instead of moderate.

¹⁵When debt takes on large negative values, debt decumulation does end for most states. And for output states near the median, there is an equilibrium value of debt, conditional on a given state, implying that with debt low enough and output in a state above but near the median, the sovereign could borrow.

one of two types. One type is a limit on debt equal to ability to pay, conditional on output remaining in the same state next period. With debt at this limit, if output remains in the same state, then debt is safe from default. We label a limit of this type a safe-in-state limit. However, there is a second type of debt limit, which equals ability-to-pay, conditional on output increasing by one state next period. With this limit, the debt is not safe unless output rises by one state. Therefore, we label this debt limit, safe-in-next-state. The existence of the debt limits implies that if output remains in a state below the median for a long period of time, then the sovereign will accumulate debt up to a debt limit determined either by (1) safe-in-state, or (2) safe-in-next-state.

Let $J \in \{1, \bar{j}\}$ be the largest state with a safe-in-next-state debt limit. Simulations reveal that all states below J have save-in-next-state debt limits, while all states above have safe-in-state debt limits.

We illustrate the two types of debt limits with graphs of the value functions. In Figures 2 and 3, we measure the value function on the vertical axis and the choice of next period's debt on the horizontal axis. The value functions are drawn for specific values of the output state and initial debt. Figure 2 illustrates a safe-in-state debt limit while Figure 3 is a safe-in-next-state limit.

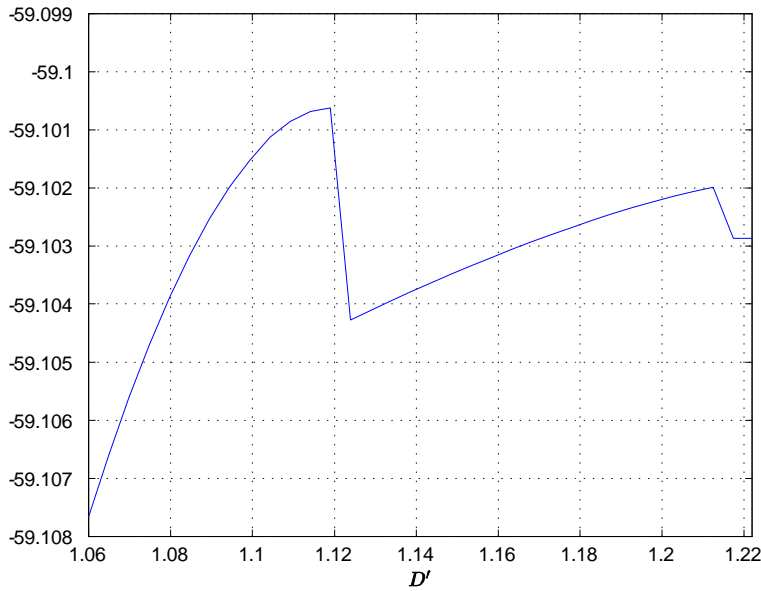
Figure 2 illustrates the sovereign's choice of debt in state 4 when initial debt is given by 1.1043, the starting value on the horizontal axis. The value function has a local peak at a higher value of next period's debt equal to the state 4 ability to pay. For an increase in next period's debt beyond this point, the value function takes a discrete downward jump. The jump is a consequence of two factors. First, the small increase in debt beyond ability to pay in the current state triggers a downward jump in the price of debt (upward jump in the interest rate), due to increased probability of default and the deadweight cost (equation 15). The reduced price of debt mitigates the increase in current consumption created by additional borrowing. Second, since the small increase in debt leaves debt almost identical to ability to pay in state 4, default in state 4 would provide little debt relief. These two factors create the downward jump in the value function and incentivize the sovereign to keep debt at ability to pay in the current state 4. On the other hand, the sovereign can obtain substantial debt relief in default, if it raises debt to ability to pay in the next state 5, the second local peak in the value function. This is the Stiglitz risk-taking incentive to take on more debt because additional debt increases consumption without increasing the debt burden in default states.

For debt above ability to pay in state 5 (beyond the second peak), the value function takes a another discrete downward jump as debt increases beyond ability to pay in state 5,

and then becomes flat. With the high autoregressive coefficient on output, the probability of transiting from the current state 4 to state 6 in one period is very close to zero. Debt has become so high that any increase in debt is offset fully by a fall in its price so that the value of borrowing does not increase. Additionally, since the agent will default almost surely, additional debt does not increase the debt burden for the future. The value function becomes flat.

The sovereign's choice for debt can be narrowed to a choice between the two local peaks in the diagram, where the local peaks represent the value function at safe-in-state and safe-in-next state values of debt. The first peak with debt safe-in-state is higher, implying that the sovereign chooses debt next period equal to its safe-in-state limit. The disincentive to borrow, due to the rise in the interest rate, together with little debt relief in default, keeps the sovereign from going over the first peak. And the sovereign is not drawn to the second peak because the incentives for Stiglitz risk-taking are not strong enough. While the sovereign could increase consumption by taking on sufficient debt to raise the probability of default and would not have to repay the additional debt in the event of default, the sovereign would have to repay if output were to rise. The expected cost of the extra debt is high enough that the sovereign chooses not to take on the extra risk.

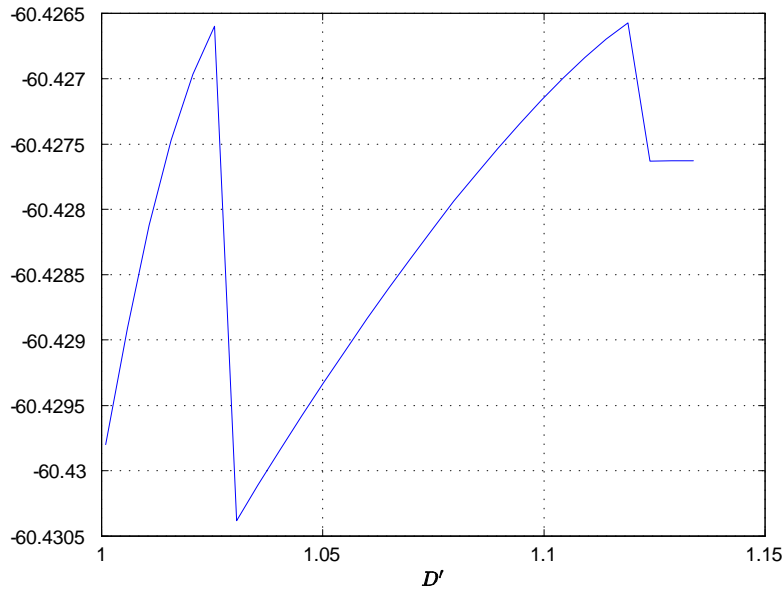
Figure 2: Value Function with Safe-in-State Debt Limit



Local peaks at safe-in-state and safe-in-next state debt and global peak at safe-in-state

Figure 3 illustrates the same decision for a sovereign initially in state 3. The function is drawn for initial debt equal to state 3 ability to pay. The first peak is at ability to pay in state 3, while the second peak is at ability to pay in state 4. The second peak is higher. Therefore, a sovereign, whose debt has reached ability to pay in state 3, has the incentive to choose a large increase in debt, large enough for debt to reach ability to pay in state 4. Stiglitz-type risk-taking dominates the risk-moderating effect of the increase in the interest rate. This is the second type of endogenous debt limit, in which the limit is determined by ability to pay in the next higher state. For income states below state 3, the relative height of the second peak is more pronounced, implying these states also have safe-in-next-state type debt limits.

Figure 3: Value Function with Safe-in-Next-State Debt Limit



Local peaks at safe-in-state and safe-in-next-state debt and global peak at safe-in-next-state debt

4.3 Duration of Defaults and Haircuts

In the model, defaults last a single period because the sovereign settles her debt by either paying what she is able or repaying the contractual obligation. However, in many cases, the initial default is immediately followed by a succession of future defaults. Therefore, to bring the model to the data, we view the period in which the country is engaged in successive defaults as a period of renegotiation. Exit from this default

period requires that the sovereign meet two criteria. First, the sovereign must make a full repayment. And second, she must not default again for the next four quarters. If she defaults again within the one-year time-frame, we aggregate the two crises into one with the exit date of the second. This timing concept aggregates successive single-period defaults in the model.

During an aggregated default episode, we assume that behavior continues as in the model. That is, following default, whereby the sovereign pays her current ability-to-pay in lieu of contractual debt repayments, the sovereign borrows either her safe-in-state or safe-in-next-state debt limit. In the subsequent period, if ability to pay is less than contractual debt payments, she defaults again. We view each period's repayment and reborrowing during the period of default as debt renegotiation. Crisis duration is the time between entering the crisis with default and exiting the crisis with the first full payment of contractual debt which is not followed by another default within one year.

Given that a period of default includes repayments as well as additional lending, it is not obvious how to measure the haircut. We use the concept of excess return, measured by one minus actual payments relative to contractual payments. In the absence of default, actual payments equal contractual payments implying that excess return is zero. With default, actual payments are less than contractual payments implying that excess return is negative.

For example, if the default lasts only one period, then actual payments relative to contractual payments imply a gross excess return of $A_{h(1)}/D(1) < 1$ for the single period. For a two-period default, we measure the gross two-period return as the product of the first period excess return and the second period excess return. In general, for an asset whose default period is given by n , the haircut is one minus the gross n -period excess return yielding

$$HC = 1 - \frac{[A_{h(1)}] \times [\min(A_{h(2)}, D(2))] \times \dots \times [\min(A_{h(n)}, D(n))]}{D(1) \times D(2) \times \dots \times D(n)},$$

where integers in parentheses represent the period of default, and where the default ends once $\min(A_{h(n+1)}, D(n+1)) = D(n+1)$ and is not followed by another default for four consecutive quarters. Note that net repayments are given by the minimum of the state-specific ability to pay ($A_{h(i)}$), and contractual debt obligations $D(i)$.

An outside observer of such an aggregated period of default might observe only two features of the default episode. First, the sovereign fails to repay anything in the period of default. The model claims that she defaults to her ability to pay, and borrows either to

a safe-in-state or a safe-in-next-state debt limit. When she borrows to a safe-in-next-state debt limit, debt does not change, implying that there are no observable transactions. Second, the sovereign makes a final payment in the exit period, and the creditor accepts. In the context of the model, the creditor accepts the exit payment because it is a full repayment of debt agreed to at that point, even though the repayment could be less than the initial contractual debt obligation. The possibility of borrowing based on income in the default exit period, can imply that debt with which the sovereign exits default equals that with which it entered, as in Benjamin and Wright (2008). And the longer the period of default, the more opportunities there are for additional shortfalls of repayments relative to contractual debt, implying that the magnitude of haircuts should be positively related to the duration of defaults.

4.4 Two Types of Debt Crises

The two types of debt limits imply that there are two different types of debt crises with very different characteristics. We demonstrate that one type is associated with a short default period and modest haircut while the other has a longer default period and a more severe haircut.

4.4.1 Safe-in-State Debt Limit

When output is in a state below the median with a safe-in-state limit, debt climbs slowly up to its limit. If output stays in that particular state long enough, and with the high autocorrelation coefficient on output, this is reasonably likely, then debt reaches its endogenous limit. If debt reaches its endogenous limit prior to a crisis, the time pattern for debt appears to be a sudden stop. If output subsequently falls, then the sudden stop is followed by a crisis. If output eventually rises, then the sudden stop does not foretell a crisis. And if output falls before debt reaches its safe-in-state limit, then there is no sudden stop and rising debt precedes the crisis.

As debt approaches and eventually reaches its endogenous safe-in-state limit, the probability of a crisis is positive but low. This is because, with debt close to its safe-in-state limit, the probability of a crisis is the probability that output falls below its current value. The fall in output would reduce ability to pay below current debt, triggering default. With output below the median, the high autoregressive parameter on output implies that the probability of a fall in output is low. And with crisis probability low, the interest rate premium on debt is low.

A crisis occurs if output does fall. If the debt limit for the lower output state is also the safe-in-state type, then the sovereign defaults to its ability to pay, given by the safe-in-state debt limit, and does not raise debt beyond this limit. Another crisis would occur only if output falls again. Since this is a low-probability event, the probability of another crisis is also low. Therefore, the probability that the sovereign exits default after a single period is high. The exit is with reduced debt. And the size of the haircut is determined by ability to pay in the lower output state relative to ability in the initial higher state. Since ability-to-pay is similar in the two states, the haircut is small. Alternatively, if the debt limit for the lower output state is safe-in-next-state, the default duration and haircut are substantially different, as described below.

4.4.2 Safe-in-Next-State Debt Limit

For output states in which the debt limit is safe-in-next-state, debt climbs slowly toward the current ability-to-pay. At some value of debt, Stiglitz-type risk-taking takes over and debt jumps to its safe-in-next-state limit. Once debt reaches this limit, a crisis can be avoided only if output rises. Due to the high autoregressive parameter on output, this is unlikely, implying that both the probability of a crisis and the default-risk premium on the interest rate are high.

If output does not rise in the period after debt reaches its safe-in-next-state limit, then a crisis occurs. Repayments are determined by the value of safe-in-state debt with reborrowing determined by safe-in-next-state debt. Therefore, following the initial crisis, both the probability of another crisis and the interest premium are high. The default crisis does not end until output actually rises, increasing ability-to-pay to the value of outstanding debt. But since output is very unlikely to rise by two or more states in a single period, the debt limit most likely remains the safe-in-next-state type. Therefore, the probability of a subsequent crisis and the interest rate premium both remain high. Once output rises sufficiently for the debt limit to become a safe-in-state limit, the probability of a crisis and the interest premium fall. Since the probability of subsequent crises remains high as long as output is in a state with the safe-in-next-state type of debt limit, exit is most likely to occur after output has risen sufficiently to have a safe-in-state type limit. Benjamin and Wright (2008) find that recovery from default tends to occur after output has risen. And the value of debt upon exiting the crisis is likely to equal its value upon entering since the safe-in-state debt limit in the next higher output state equals the safe-in-next-state limit preceding the crisis.

4.4.3 Simulations for Haircuts and Default Duration

The above discussion suggests that the duration of defaults and value of haircuts should be highly skewed and variable, and simulations confirm this.¹⁶ About thirty-nine percent of defaults resolve in a single period with a modest haircut. However, there are defaults which last a long time. There are two ways for a default event to persist even after debt is settled with the sovereign paying what she is able. If she is in a state with a debt limit determined by current ability-to-pay, a default event persists only if output falls. This is because the sovereign reborrows to her safe-in-state debt limit, implying that she will not default again as long as output does not fall. And since the probability of output falling is lower than the probability that it remains constant or rises when output is below the mean, these defaults tend to resolve quickly. With few periods in which actual repayments fall short of contractual repayments, haircuts are small.

Alternatively, when the sovereign is in a state for which the debt limit is safe-in-next-state, the sovereign reborrows the safe-in-next-state debt limit, implying that she cannot repay and exit default unless output rises. Output rising is less probable than output remaining constant or falling, implying that these defaults tend to be prolonged. And since the sovereign defaults to safe-in-state debt and reborrows to the higher safe-in-next-state as the default crisis persists, the magnitude of the haircut grows over time. Therefore, the longer a default lasts, the larger the haircut, consistent with evidence in Benjamin and Wright (2008).

The mean and standard deviation for default duration and haircuts are given in Table 3.

Table 3: Default

	duration	haircut
mean	9.4	0.19
standard deviation	13.2	0.19

The result that the standard deviations are large relative to the means reflects the fact that so many crises are settled quickly with small haircuts, while some take a long time to settle and involve large haircuts.

¹⁶Sturzenegger and Zettelmeyer (2008), Benjamin and Wright (2008), and Cruces and Trebesch (2011) find high variability in the size of haircuts for sample of defaults with primarily developing countries.

4.5 Defaults Require High Debt and Low Output

With the modest amount of impatience in our calibration, debt accumulates with output below trend up to a limit, and tends to fall with output above trend. Therefore, debt is high only following a period of low output. A default occurs when a sovereign has borrowed so much that realized ability to pay is below debt. This cannot happen with low debt or with high output. If debt is low enough, then ability to pay would exceed debt for any output. Alternatively, if output is high enough, then ability to pay would exceed any debt the sovereign could have chosen to borrow during recessions. Tomz and Wright (2007) find that most (but not all) historical defaults have occurred with low output.

Using our simulated crises we find that the mean value of output in the beginning of a debt crisis is 1.6 standard deviations below trend, and the mean value of debt relative to trend GDP is 111 percent.

4.6 Greek Crisis

We can use the calibrated model to explain how the Greek crisis unfolded. Over the business cycle from 2005Q1 to 2007Q4, Greek debt relative to trend GDP averaged about one hundred percent of GDP. The period began with four quarters of recession with output in state 5, one state below the median, during which debt was rising, and ended with eight quarters of boom with output exactly at trend (state 6) and debt falling. This relatively tranquil period ceased with the worldwide financial crisis which sent Greek GDP back to state 5, below trend, in 2008Q1. With output below trend, Greek debt increased, as it had in the previous recession.

The difference is that, this time, after four quarters with output in state 5, output fell in 2009Q1 to state 4 instead of rising to state 6, as it had in the previous recession. Output remained in state 4 through 2009Q4, accelerating the increase in debt. New borrowing in 2009Q4 increased debt to its state 4 safe-in-state limit. Figure 2 illustrates this choice for Greece with initial debt given by its 2009Q4 value, slightly below the state 4 safe-in-state limit. Even with debt at its safe-in-state limit, the probability of a crisis was low since the probability of output falling from state 4 was low, at less than ten percent.

In 2010Q1, output fell further to state 3. Output this low, together with Greece's debt accumulated over eight quarters of worsening recession, meant that ability to pay was lower than debt, dating the crisis.¹⁷ Therefore, our model implies a crisis with output

¹⁷Greece did not actually have scheduled debt repayments in this period, implying that there were no observations on repayments, either missed or made. However, Greece began austerity programs and the ECB softened rules on collateral for ECB loans, implying that Greece expected financing difficulties once

1.6 standard deviations below trend and with debt relative to trend GDP equal to 111 percent, values identical to the mean values predicted by the model. We calibrated the model to match the value of debt in default, but not the value of output, implying that the later is a positive test of model fit.

The interest rate premium for next quarter's debt, together with the change in debt, determine whether state 3 was characterized by a safe-in-state debt limit or by a safe-in-next-state limit. This distinction is important because it determines the expected size of both the haircut and the duration of default. If state 3 had a safe-in-state debt limit, then Greece would have defaulted to the state 3 ability-to-pay and would have set next period's debt equal to the same value, that is, its safe-in-state limit. Therefore, debt would have fallen and the interest premium would have remained low. If state 3 had a safe-in-next-state limit, then Greece would have defaulted to its ability to pay in state 3 and reborrowed to its safe-in-next-state limit, implying no change in debt. With debt at its safe-in-next-state limit, both the probability of a crisis and the interest rate premium would have spiked. In the calibrated model, interest premium with a safe-in-state limit would have been about 85 basis points and with a safe-in-next-state limit would have been about 850 basis points.

The data show that debt relative to median GDP was almost constant between the first and second quarters¹⁸, providing evidence for a safe-in-next-state debt limit for state 3. The inference based on the behavior of the interest rate spread is less clear-cut. The model claims that for a safe-in-state equilibrium, the interest rate premium would have risen from 81 basis points to 85, virtually no change. For a safe-in-state equilibrium, the interest rate premium would have risen to 850 basis points, a steep spike. The data show the spread on ten year German government bonds rising from 235 basis points in December 2009 to 314 basis points in March. Increases continued with the interest premium exceeding 800 basis points by August. We must determine whether a flat interest rate premium, implying a safe-in-state equilibrium, or a steep spike, implying a safe-in-next-state equilibrium, is more consistent with the actual rise in the premium from 235 to 314, with continuing sharp increases shortly thereafter. Given that the interest rate in the data continued to rise sharply, our interpretation is that the model equilibrium requiring a spike is more consistent with the outcome, although the model predicted the spike earlier than it actually occurred. Additionally, the safe-in-next-state equilibrium forecasts a crisis with a large haircut and long duration, consistent with events which followed. Together,

maturity dates arrived.

¹⁸Debt relative to mean GDP (before converting to grid values) in 2010Q1 was 1.11 and in 2010Q2 was 1.10.

these outcomes support calibrating the value for ω to deliver a safe-in-next-state type of equilibrium for state 3.

This scenario makes the Greek crisis sound entirely like bad luck instead of bad policy, and the bad luck of the financial crisis did contribute. However, Belgium and Italy were also high-debt countries whose output took sharp decreases with the financial crisis. We compute the probability of crisis for Greece over the next ten years from our start date of 2005Q1, and obtain a probability of 10.1 percent over the next ten years. The relatively high probability of a crisis is due to large debt relative to ability to pay, enabling a period of recession to send it into crisis. Either greater ability to pay or parameters delivering lower debt would have been required to reduce Greek risk.

A model of strategic default does not fit the Greek crisis. Greece was accumulating debt during the year prior to the crisis when output was low and the probability of a crisis one quarter ahead was greater than zero at about ten percent. The model of strategic default would have Greece repaying debt during each quarter of this year since the probability of a crisis one period ahead was positive.

Attributing the Greek default to ability to pay instead of to a strategic decision carries a policy implication for creditors. With strategic default, Greece would be refusing to pay when she is able. Threats of additional punishments would be warranted to force Greece to pay something closer to her ability. However, if the Greek default is due to inability to pay, additional threats and/or additional punishments can extract no additional payments.

5 Initial Questions

We are now in a position to address the questions posed at the beginning of the paper.

Why do rich countries with debt/GDP ratios higher than the value of standard default punishments not default? There are two components to the answer of the title question. First, we justify the immense punishment to failure to pay what a sovereign is able as an implicit or explicit threat to destroy the country's financial system. Rich countries are vulnerable to this threat while poor countries are not. This implies that countries for which this threat is meaningful avoid strategic default while others might not.

The second component is ability to pay. Institutions seem to improve as income rises. Better institutions imply higher ability to pay. And higher ability to pay reduces the probability of a crisis.

Why does a sovereign in default always repay something? This question is related to the first. If she fails to repay what she is able, she faces an immense punishment. The large cost of the punishment implies that she prefers to pay what she is able.

Why do some crises appear to follow a rise in debt which ends in a sudden stop, while others do not? For output states with a safe-in-state debt limit, as long as output remains in the same state, debt accumulates slowly to the limit and then stops accumulating. This appears as a sudden stop. If output falls subsequently, then the country experiences a crisis, while if output rises, there is no crisis.

For output states with a safe-in-next-state debt limit, once debt reaches a particular value, Stiglitz-type risk-taking implies that debt surges to its limit. If output does not rise, then default occurs. Therefore, the crisis is preceded by a surge in debt, but since the crisis occurs immediately thereafter, there is no time for a sudden stop. And if output rises, there is no crisis.

Why does debt become risky at different levels of debt/GDP for different countries? There are two components to this answer. We have posited an ability-to-pay function for which ability to pay is increasing in GDP. Therefore, the value of debt/GDP a country could repay at low output is less than the value she could repay at higher output. One reason Greece defaulted on its debt/GDP of 130 percent is that this occurred in an output state 1.6 standard deviations below the mean.

The second component of the answer is that countries have different abilities to raise tax revenue above spending and, therefore, different abilities to pay as a fraction of income. Institutions do matter in determining ability to pay and therefore ability to borrow.

Why are haircuts and the duration of defaults heterogeneous across crises? Debt limits in low output states are riskier than debt limits in higher output states. And the two types of debt limits imply different crises with different characteristics. Safe-in-state crises tend to have small haircuts and short durations, while safe-in-next state crises are more likely to have large haircuts and long durations.

6 Conclusion

The motivation for this paper was to offer an answer to the title question "Why Don't Rich Countries Default" when standard punishments to default are much smaller than the

value of their debt/GDP. We propose that rich sovereigns face an implicit debt contract with immense punishment for failing to pay what she is able. This punishment involves severe damage to the financial system and therefore applies only to countries with well-developed financial systems. When the punishment is relevant, the sovereign chooses not to engage in strategic default, and instead chooses to pay what she is able.

We set up a small open economy model in which a sovereign uses this debt contract to smooth consumption in the face of stochastic endowment shocks. We calibrate the model to the Greek crisis, matching crisis timing, average debt over the preceding business cycle, and debt/GDP on pre-crisis and crisis dates. Our calibrations imply that the probability of a Greek crisis over the ten years beginning in 2005Q1 was about ten percent. The crisis was due to debt accumulation in response to multiple reductions in Greek GDP, following the world-wide financial crisis. Countries, like Italy and Belgium, had debt/GDP at similar levels, but avoided default, presumably due to higher ability to pay as evidenced by higher historical values of government surpluses.

Finally, we use the calibrated model to describe general characteristics of crises when countries have access to this debt contract. We find that crises occur when output is low and debt is high. In the periods approaching a crisis, debt is rising unless it has reached an endogenous debt limit. However, crises and recoveries do not all look alike. Some have sudden stops and others do not. Some have long durations and large haircuts while others do not. The difference in the types of crises is due to the type of endogenous debt limit the sovereign chooses. When the endogenous debt limit is safe-in-state, debt accumulates gradually until it reaches its limit. If output does not rise, the end of debt accumulation looks like a sudden stop. If output falls, then the country experiences a debt crisis. Recovery is fast and haircuts are small. With the alternative debt limit, safe-in-next-state, the crisis is preceded by a surge in debt, followed immediately by a crisis if output does not rise. Recovery does not occur until output rises, and haircuts are large.

7 Data Appendix

For Belgium and Italy Eurostat (gov_10q_ggdebt) provides quarterly data on government debt as a percentage of GDP from 2000Q4. For Greece, this data is available annually from 2000Q4 and quarterly from 2006Q1. We use linear interpolation to estimate quarterly values for Greece for the earlier years.

The OECD (VPVOBARSA: US dollars, volume estimates, fixed PPPs, OECD reference year, annual levels, seasonally adjusted) provides quarterly values for real GDP from 1960Q1 to 2016Q2. We use data up to 2008Q2 to estimate behavior of GDP prior to the financial crisis. We obtain demean and detrend the data by regressing logged values on a constant and a linear time trend. We estimate the autocorrelation coefficient and standard error by regressing the residuals on one lag. We obtain standardized measures of output by taking the exponential of our demeaned and detrended data

The debt variable in our model is debt relative to mean output. In our demeaned and detrended data, mean output is unity. In the data, debt is government debt relative to actual GDP, not relative to mean GDP. We convert data to our model variable by multiplying the data variable by real actual GDP relative to mean real GDP.

Data for interest rate premiums is the difference between interest rates on Greek and German ten-year government bonds from the ECB Statistical Data Warehouse.

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