Labor-Market Heterogeneity, Aggregation, and the Policy-(In)variance of DSGE Model Parameters

Yongsung Chang
University of Rochester
Yonsei University

Sun-Bin Kim
Yonsei University

Frank Schorfheide *
University of Pennsylvania
CEPR and NBER

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Abstract

Data from a heterogeneous-agents economy with incomplete asset markets and indivisible labor supply are simulated under various fiscal policy regimes and an approximating representative-agent model is estimated. Preference and technology parameter estimates of the representative-agent model are not invariant to policy changes and the bias in the representative-agent model's policy predictions is large compared to predictive intervals that reflect parameter uncertainty. Since it is not always feasible to account for heterogeneity explicitly, it is important to recognize the possibility that the parameters of a highly aggregated model may not be invariant with respect to policy changes.

KEY WORDS: Aggregation, DSGE Models, Fiscal Policy, Heterogeneous-Agents Economy, Policy Predictions, Representative-Agent Models.

JEL: C11, C32, E32, E62.

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[*] Correspondence: Y. Chang: yongsung.chang@gmail.com; S.B. Kim: sunbin.kim@yonsei.ac.kr; F. Schorfheide: schorf@ssc.upenn.edu. We thank Patrick Feve, Tony Smith, as well as participants at various seminars and conferences for helpful comments. We also thank Jinwook Hur for his excellent research assistance. Schorfheide gratefully acknowledges financial support from the National Science Foundation under Grant SES 0617803. Chang and Kim acknowledge the support from the Korean National Research Foundation (NRF-2011-32A-B00028).

1 Introduction

The Lucas (1976) critique of econometric policy evaluation states that if econometric models do not capture the primitive parameters of preferences and technology, their coefficients may vary with changes in policy regimes. The quantitative work inspired by the Lucas critique has proceeded by replacing econometric models that were parameterized in terms of agents' decision rules with representative-agent dynamic stochastic general equilibrium (DSGE) models in which parameters characterize the objective functions and constraints faced by representative economic agents. In recent years, estimated DSGE models have been widely used to study the effects of monetary policy changes (e.g., Smets and Wouters, 2003, 2007; Christiano, Eichenbaum, and Evans, 2005) and fiscal policy effects (e.g., Forni, Monteforte, and Sessa, 2009; Leeper, Plante, and Traum, 2010). The tacit assumption underlying the DSGE model-based policy analysis is that the parameters that characterize the preferences of a representative agent and the production technologies of a representative firm as well as the exogenous structural shocks are policy invariant. However, to the extent that macroeconomic time series on variables such as output, consumption, investment, and hours are constructed by aggregating across heterogeneous households and firms, the assumption of policy invariance is not self-evident. In fact, more than two decades ago, Geweke (1985, p.206), referring to the newly emerging rational expectations models in macroeconomics, criticized that while the treatment of expectations and dynamic optimization was careful, potential problems due to aggregation were usually ignored. We are taking a fresh look at this issue in the context of DSGE models.

The goal of this paper is to assess the quantitative importance of biases in policy predictions due to the potential lack of invariance of preference and technology parameters in representative-agent models. To do so, we simulate data under various fiscal policy regimes from a heterogeneous-agents economy in which households have to insure themselves against idiosyncratic income risks (e.g., Bewley, 1983; Huggett, 1993; Aiyagari, 1994). Following Chang and Kim (2006), our model economy extends Krusell and Smith's (1998) heterogeneous-agents model with incomplete capital markets (Aiyagari, 1994) to indivisible labor supply (Rogerson, 1988).[1] The equilibrium outcomes depend on the cross-sectional

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[1] Both the theoretical and the empirical importance of incomplete asset markets and indivisible labor supply are by now widely recognized. See, for instance, Krusell and Smith (1998), Chang and Kim (2006), Ljungqvist and Sargent (2007), Nakajima (2011), Krusell, Mukoyama, Rogerson, and Sahin (2008), and

distributions of households' wealth and earnings, which in turn depend on the policy regime. Using aggregate times series on output, consumption, wages, and employment generated from the heterogeneous-agents model, we estimate a representative-agent model with state-of-the-art Bayesian methods (Schorfheide, 2000; An and Schorfheide, 2007) and examine the potential lack of policy invariance of the representative-agent model's parameters.

The quantitative analysis generates the following findings. First, if the representative-agent model is estimated with data from the heterogeneous-agents economy under different policy regimes, several important parameters vary considerably. For instance, the aggregate labor supply elasticity, often recognized as a crucial parameter for fiscal policy analysis (e.g., Auerbach and Kotlikoff, 1987; Judd, 1987; Prescott 2004), depends on the cross-sectional distribution of reservation wages, which in turn is a function of the fiscal policy regime. The average level of total factor productivity is also not policy invariant because fiscal policy affects labor-market participation and thereby the cross-sectional distribution of productivities of the employed workforce. It is important to note that using the standards of the DSGE model estimation literature, the estimated representative-agent model fits the aggregate time series data from the heterogeneous-agents economy well. A posterior odds comparison with a more flexible vector autoregression (VAR) favors the structural model by a substantial margin.

Second, to assess the quantitative implications of the lack of policy invariance, we construct predictive distributions for the effects of fiscal policy changes on output, consumption, employment, and aggregate welfare based on the estimated representative-agent model under the benchmark fiscal policy, assuming that the preference and technology parameters are unaffected by the policy shifts. We find that the prediction bias due to imperfect aggregation is substantially larger than the prediction intervals that reflect parameter estimation uncertainty. While in practice it may not always be feasible to account for various types of heterogeneity explicitly, it is important for the characterization of uncertainty to entertain the possibility that preference and technology parameters of an estimated model may shift in response to policy changes.

As a by-product we confirm a result from the previous literature: the effects of imperfect aggregation manifest themselves through the presence of preference shocks (the so-called labor-market wedge) in the representative-agent model. While it is common to include

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Rogerson and Wallenius (2009).

such preference shocks in the specification of estimable representative agent models, their interpretation is subject to controversy. Some researchers regard them as fundamental aggregate demand shocks that contribute to business cycle fluctuations (e.g., Smets and Wouters, 2007). Other authors view them as wedges in optimality conditions and thus as a sign of model misspecification (e.g., Chari, Kehoe, and McGrattan, 2007). According to a variance decomposition computed based on our estimated representative-agent model, the measured preference shocks explain jointly between 45% and 50% of the fluctuations in hours worked.

While there exists a fairly extensive body of research on aggregation issues and parameter stability in econometric models, we will only briefly discuss two strands of the literature that are most closely related to this paper. First, calibrated heterogeneous-agents economies similar to the one in this paper have been used to assess equilibrium conditions derived from a representative-agent model in Chang and Kim (2006, 2007) and An, Chang, and Kim (2009). However, none of the three papers considers the (fiscal) policy invariance of the parameters in an estimated representative-agent model. Second, our analysis focuses on the cross-sectional heterogeneity on the household side and fiscal policies that distort households' labor supply and savings decision.[2] Much of the literature on policy analysis with estimated DSGE models, however, focuses on monetary policy analysis in the context of New Keynesian models. For the propagation of monetary policy shifts, the heterogeneity on the firm side, in particular with respect to pricing decisions, plays an important role.

Until now the literature on New Keynesian DSGE models has mostly focused on the question of whether the cost of changing nominal prices is invariant to, say, changes in the target inflation rate. Fernández-Villaverde and Rubio-Ramiréz (2008) estimate a model in which both monetary policy rule parameters and nominal rigidity parameters are allowed to vary over time. They find that during high inflation episodes, the estimated cost associated with nominal price changes is lower and interpret the negative correlation between policy and price-adjustment parameters as evidence against policy invariance. Cogley and Yagihashi (2010) conduct the following experiment. They simulate data under two monetary policy regimes from an economy in which firms are heterogeneous with respect to their price setting history and face some menu costs of nominal price adjustments. Based on the sim-

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[2] Earlier work by Geweke (1985) and Altissimo, Siviero, and Terlizzese (2002) considered heterogeneity on the firm side, but only in simple partial equilibrium frameworks.

ulated data, the authors then use Bayesian methods to estimate an approximating model that assumes that firms are able to re-optimize their nominal prices with a fixed probability in every period as in Calvo (1983). Cogley and Yagihashi (2010) find that some of the preference and technology parameters of the approximating model are not policy invariant. However, in their setting policy recommendations derived from the approximating model still lead to good outcomes under the data-generating economy.

The remainder of the paper is organized as follows. Section 2 lays out the heterogeneous-agents economy that features incomplete capital markets and indivisible labor. We calibrate the model economy to match salient features of the cross-sectional income and wealth distribution in the U.S. as well as some key business cycle properties. Section 3 introduces a representative-agent model through which we will interpret the equilibrium outcome of the heterogeneous-agents economy. In Section 4, we estimate the representative-agent model using data generated from the heterogeneous-agents economy. We also present the quantitative results on the (lack) of policy invariance – changes in the estimates of representative-agent model parameters in response to changes in the underlying heterogeneous-agents economy. In Section 5 we repeat the quantitative analysis for model economies with divisible labor supply and complete asset markets, in order to distinguish the separate roles played by the two frictions considered in our benchmark model. Section 6 concludes. Detailed derivations for the representative-agent model, data sources, additional information on the calibration of the heterogeneous-agents economy, and additional robustness exercises can be found in the online Appendix.

2 Heterogeneous-Agents Economy

We begin by providing a description of the heterogeneous-agents economy that serves as a data-generating mechanism for the quantitative analysis. The model economy is based on Chang and Kim (2007), who extend Krusell and Smith's (1998) heterogeneous-agents model with incomplete capital markets (Aiyagari, 1994) to indivisible labor supply (Rogerson, 1988). Due to the indivisible nature of labor supply the aggregate labor supply depends on the shape of the cross-sectional reservation wage distribution, which in turn is affected by the policy regime.

2.1 Economic Environment

Households: The model economy consists of a continuum (measure one) of worker-households who have identical preferences but different productivities *ex post*. Household-specific idiosyncratic productivity x_t varies exogenously according to a stochastic process with a transition probability distribution function $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. A household maximizes its utility by choosing consumption c_t and hours worked h_t:

$$\begin{aligned} \max \quad & \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left\{ \ln c_{t+s} - B \frac{h_{t+s}^{1+1/\gamma}}{1+1/\gamma} \right\} \right] \\ \text{s.t.} \quad & c_t + a_{t+1} = a_t + (1-\tau_H) W_t x_t h_t + (1-\tau_K) R_t a_t + \bar{T} \\ & a_{t+1} \geq \underline{a}, \quad h_t \in \{0,\ \bar{h}\}. \end{aligned} \tag{1}$$

Households trade assets a_t that yield the real rate of return R_t. These assets are either claims to the physical capital stock or IOUs, which are in zero net supply. Both asset types generate the same return R_t, which is subject to the capital tax τ_K.

Households face a borrowing constraint, $a_{t+1} \geq \underline{a}$, and supply their labor in an indivisible manner, that is, h_t either takes the value 0 or $\bar{h}$. We normalize the endowment of time to one and assume $\bar{h} < 1$. If a household supplies $\bar{h}$ units of labor, labor income is $W_t x_t \bar{h}$, where W_t is the aggregate wage rate for an efficiency unit of labor. Labor income is subject to the tax τ_H and $\bar{T}$ denotes lump-sum taxes or transfers. Ex post households differ with respect to their productivity and asset holdings. The joint distribution of productivity, x_t, and asset holdings, a_t, is characterized by the probability measure $\mu_t(a_t, x_t)$.

Firms: A representative firm produces output Y_t according to a constant-returns-to-scale Cobb-Douglas technology in capital, K_t, and efficiency units of labor, L_t:

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^{\alpha} K_t^{1-\alpha}, \tag{2}$$

We assume that workers are perfect substitutes for each other. While this assumption abstracts from reality, it greatly simplifies the labor-market equilibrium because we only need to clear the labor market through the total efficiency units of labor. The exogenous process λ_t shifts aggregate productivity and has the transition probability distribution function $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)$. The representative firm's profit function is:

$$\Pi_t = Y_t - W_t L_t - (R_t + \delta) K_t. \tag{3}$$

The first-order conditions for profit maximization are

$$W_t = \alpha Y_t / L_t \quad \text{and} \quad (R_t + \delta) = (1-\alpha) Y_t / K_t. \tag{4}$$

The return on capital (net of depreciation), R_t, is subject to capital tax. The physical capital stock evolves according to

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{5}$$

where I_t is aggregate investment and δ is the depreciation rate. The total factor productivity process λ_t is the only aggregate disturbance. While this feature of the model economy does not necessarily reflect our views about the sources of business cycle fluctuations, it makes the quantitative analysis more transparent. Since the aggregation error will show up as a preference shift in the representative-agent model, we intentionally exclude shocks that shift households' preferences, e.g., labor supply shocks, from the heterogeneous-agents economy.

Fiscal Policy: Fiscal policy in the model economy are characterized by labor and capital tax rates (τ_H and τ_K) as well as the level of lump-sum transfers ($\bar{T}$). We assume that transfers are constant over time and the government maintains a balanced budget in each period. The fiscal authority collects the revenue from income tax and spends it on fixed lump-sum transfers to households $\bar{T}$ or purchases of goods for its own consumption G_t:

$$\bar{T} + G_t = \tau_H W_t \int x_t h(a_t, x_t; \lambda_t, \mu_t)\, d\mu_t(a_t, x_t) + \tau_K R_t \int a_t d\mu_t(a_t, x_t). \tag{6}$$

In order to obtain total tax revenues we have to integrate over the distribution of household types using the measure $\mu_t(a_t, x_t)$. For simplicity, we assume that government purchases G_t do not affect the household's marginal utility from private consumption or leisure nor the production function of the representative firm. The lump-sum transfers are a constant fraction χ of the steady-state tax revenue, that is,

$$\bar{T} = \chi \left(\tau_H \bar{W} \int x h(a, x; \bar{\lambda}, \bar{\mu})\ d\bar{\mu}(a, x) + \tau_K \bar{R} \int a\, d\bar{\mu}(a, x) \right). \tag{7}$$

Government expenditures, G_t, adjust to maintain a balanced budget. This specification simplifies the solution of the model.

Market Clearing and Aggregate Resource Constraint: Since IOUs are in zero net supply, the net supply of assets has to equal the capital stock. Moreover, in equilibrium the labor hired by the firms has to equal the total supply of efficiency units by the households:

$$K_t = \int a_t d\mu_t(a_t, x_t), \quad L_t = \int x_t h(a_t, x_t; \lambda_t, \mu_t) d\mu_t(a_t, x_t). \tag{8}$$

The aggregate resource constraint can be expressed as

$$Y_t = \int c(a_t, x_t; \lambda_t, \mu_t) d\mu_t(a_t, x_t) + I_t + G_t. \tag{9}$$

A detailed description of the computational procedure that is used to approximate the competitive equilibrium fluctuations of the model economy can be found in Chang and Kim (2007).

2.2 Calibration

In order to simulate data from the heterogeneous-agents economy we have to specify parameter values for preferences, technology, as well as fiscal policy. Table 1 summarizes the parameter values.

Firm Parameters: The unit of time is a quarter. On the production side of the economy, we let capital depreciate at the rate $\delta = 0.025$ and set the capital share parameter $\alpha = 0.64$ to generate a labor share that is consistent with post-war U.S. data. The aggregate productivity shock, λ_t, is a discrete approximation of a continuous AR(1) process:

$$\ln \lambda_t = \rho_\lambda \ln \lambda_{t-1} + \sigma_\lambda \epsilon_{\lambda,t}, \quad \epsilon_{\lambda,t} \sim \mathcal{N}(0,1). \tag{10}$$

We set $\rho_\lambda = 0.95$ and $\sigma_\lambda = 0.007$. These parameter values are obtained by fitting an AR(1) process to de-trended measured TFP (e.g., Kydland and Prescott, 1982).[3]

Household Parameters: On the household side, we assume that the idiosyncratic productivity x_t follows an AR(1) process:

$$\ln x_t = \rho_x \ln x_{t-1} + \sigma_x \epsilon_{x,t}, \quad \epsilon_{x,t} \sim N(0,1). \tag{11}$$

The values of $\rho_x = 0.939$ and $\sigma_x = 0.287$ reflect the persistence and standard deviation of innovation to individual wages estimated from the PSID.[4] According to the Michigan Time-Use survey, an employed individual spends one-third of his discretionary time $\bar{h} = 1/3$ on

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[3] According to our heterogenous-agents economy, the measured TFP reflects the composition effect of workforce as well as productivity shocks. As explained in detail in Section 4.1, the measured TFP tends to be slightly less volatile than the productivity shocks. However, increasing the volatility of λ_t does not alter the quantitative results with respect to the accuracy of the representative-agent model's policy predictions.

[4] Chang and Kim (2007) restrict the household sample to those with a household head between 35 and 55 years of age with a high school education to avoid the fixed effect in wages. With this restricted sample, the estimates are $\rho_x = 0.929$ and $\sigma_x = 0.227$. Here, however, we use the whole sample of PSID, ages 18 to 65, to encompass the overall distribution of wages and obtain a larger shock for idiosyncratic productivity.

work-related activities. We set the intertemporal substitution elasticity of hours worked equal to $\gamma = 0.4$. Given all other parameters, we set the preference parameter B such that the steady-state employment rate is 60%, the average employment in our sample period. The discount factor β is chosen so that the quarterly rate of return to capital is 1% in the steady state. Finally, we let the borrowing constraint $\underline{a} = -2$. In our model this corresponds to half of the annual earnings of the household with average productivity, which is consistent with the average unsecured credit-limit-to-income ratio of U.S. households – 28% in 1992 and 47.5% in 1998 – reported by Narajabad (2010) based on data from the Survey of Consumer Finances.

Fiscal Policies: Chen, Imrohoroglu, and Imrohoroglu (2009) construct U.S. labor and capital tax rates. The capital tax rate fell from 45% to roughly 32% over the period from 1950 to 2003. Over the same time span the labor tax rate rose from about 22% to 30%. The ratio of transfer in total government expenditure, $\chi = T/(T + G)$, has shown a strong trend in the last half century. It rose from 22% in 1960 to 47% in 2010.[5] For the benchmark calibration we choose fiscal policy in 1984, the midpoint of our sample ($\tau_H = 0.29$, $\tau_K = 0.35$, $\chi = 0.36$). In addition to the benchmark fiscal policy, 5 alternative fiscal policy regimes are considered in Section 4: (i) low labor income tax ($\tau_H = 0.22$), (ii) high capital income tax ($\tau_K = 0.47$), (iii) higher ratio of lump-sum transfer in government expenditure ($\chi = 0.5$), (iv) the 1960 fiscal policy ($\tau_H = 0.229$, $\tau_K = 0.443$, $\chi = 0.224$), and (v) the 2004 fiscal policy ($\tau_H = 0.269$, $\tau_K = 0.327$, $\chi = 0.417$). These values, respectively, correspond to the lower or upper bound or the beginning or end point of U.S. fiscal policy during the sample period.

Implications: We now briefly comment on some of the key quantitative properties of the calibrated model economy. Further details are provided in the online Appendix. First, the benchmark calibration of the model economy generates a reasonable degree of heterogeneity. According to the Panel Study of Income Dynamics (PSID), the share of wealth of the poorest 20% of families is negative, indicating that they are net borrowers, potentially constrained in their consumption. This feature is matched by our calibrated model economy. More specifically, the PSID found that households in the 1st, 2nd, 3rd, 4th, and 5th quintiles

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[5] We compute this ratio based on the government consumption (NIPA3.1 Line 16) and net government social benefits to persons (NIPA3.1 Line 19 - Line 13) with the caveat that in reality government transfer payments are not made in a lump-sum fashion and distributed equally to all households.

own -0.5, 0.5, 5.1, 18.7, and 76.2% of total wealth, respectively. The corresponding shares in the model economy are -1.6, 3.3, 11.4, 24.7, and 62.2%, respectively. Households in the 1st, 2nd, 3rd, 4th, and 5th quintiles of the wealth distribution earn, respectively, 7.5, 11.3, 18.7, 24.2, and 38.2% of total earnings, according to the PSID. The corresponding groups earn 9.7, 15.8, 20.0, 23.7, and 30.8%, respectively, in the model.

Second, the cyclical features of the data generated from the heterogeneous-agents economy are similar to those of a neoclassical stochastic growth model. Since the model economy allows for an aggregate productivity shock only, the aggregate output of the model exhibits only about three-quarters of the volatility of U.S. output. Simulated consumption is as volatile as in the actual data. A striking difference is the standard deviation of hours. It is three times more volatile in the U.S. data than it is in the simulated data. This is in part due to the low-frequency movement in labor supply, not captured in the model economy. The correlations between output and hours as well as between consumption and hours are slightly stronger in the simulated data than they are in the U.S. data.

3 A Representative-Agent Model

In this section we describe a representative-agent model through which we will interpret the equilibrium outcome of the heterogeneous-agents economy.

3.1 Model Specification

We now replace the heterogeneous, borrowing-constrained households of Section 2 with a stand-in representative household that solves the following problem:

$$\begin{aligned} \max \quad & \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^{t+s}Z_{t+s}\left\{\ln C_{t+s}-\frac{(H_{t+s}/B_{t+s})^{1+1/\nu}}{1+1/\nu}\right\}\right] \\ \text{s.t.} \quad & C_t+K_{t+1}=K_t+(1-\tau_H)W_tH_t+(1-\tau_K)R_tK_t+\bar{T}. \end{aligned} \tag{12}$$

The representative household owns the capital stock and its budget constraint resembles that of the households at the micro-level. As in Section 2, the return R_t is defined in excess of the depreciation rate δ and the evolution of the capital stock is given by (5). The aggregate (Frisch) labor supply elasticity is denoted by $\nu(>0)$. We anticipate that ν is different from the micro elasticity of household labor supply γ, that appears in (1). Our representative-agent model nests both inelastic labor supply (e.g., ν being close to zero as in the micro labor-supply literature) and highly elastic labor supply (e.g., $\nu = \infty$ as in Hansen (1985) and Rogerson (1988)).

Chang and Kim (2007) document that the lack of exact aggregation leads to a wedge between the marginal product of labor and the marginal rate of substitution. This labor-market wedge is also well documented in the U.S. data, e.g., Hall (1997), and often interpreted as an intratemporal aggregate labor supply shock, which we denote as B_t in (12). Nakajima (2005), Scheinkman and Weiss (1986), Krüger and Lustig (2010), and Liu, Waggoner, and Zha (2009) show that capital market incompleteness can lead to a stochastic term in aggregate preferences that affects the intertemporal first-order condition of the stand-in representative household. Thus, we introduce a second preference shock Z_t in (12). As is common in the literature on estimated DSGE models (e.g., Smets and Wouters 2003, 2007), we assume that both preference shifters follow independent autoregressive processes:

$$\begin{aligned} \ln(B_t/\bar{B}) &= \rho_B \ln(B_{t-1}/\bar{B}) + \sigma_B \epsilon_{B,t}, \quad \epsilon_{B,t} \sim N(0,1) \\ \ln Z_t &= \rho_Z \ln Z_{t-1} + \sigma_Z \epsilon_{Z,t}, \quad \epsilon_{Z,t} \sim N(0,1). \end{aligned} \tag{13}$$

It is important to note that the laws of motion in (13) are not derived from the underlying aggregation problem, but rather reflect a commonly made assumption in the empirical literature.

The production technology in the representative-agent model is of the Cobb-Douglas form, identical to the one used in the heterogeneous-agents economy:

$$Y_t = A_t H_t^{\alpha} K_t^{1-\alpha}, \tag{14}$$

where technology evolves according to the AR(1) process

$$\ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \sigma_A \epsilon_{A,t}, \quad \epsilon_{A,t} \sim N(0,1). \tag{15}$$

The first-order conditions for the firm's static profit maximization are identical to (4) except that L_t needs to be replaced by H_t. The produced output is either consumed by the representative household, invested to accumulate capital, or consumed by the government. Thus, the aggregate resource constraint takes the form

$$Y_t = C_t + I_t + G_t \tag{16}$$

and resembles (9). As in the heterogeneous-agents economy the government uses its tax revenues for transfers $\bar{T}$ and purchases G_t, maintaining a balanced budget:

$$\bar{T} + G_t = \tau_H W_t H_t + \tau_K R_t K_t. \tag{17}$$

To construct an approximate solution to the representative-agent model, we log-linearize the equilibrium conditions around the deterministic steady state and apply a standard solution method for a linear rational expectations model.

3.2 Econometric Analysis

We use Bayesian techniques developed in Schorfheide (2000) and surveyed in An and Schorfheide (2007) in Section 4 to estimate the representative-agent model based on aggregated data from the heterogeneous-agents economy. As observables we use log levels of output Y_t, consumption C_t, and hours H_t, where

$$C_t = \int c(a_t, x_t; \lambda_t, \mu_t) d\mu_t(a_t, x_t), \quad H_t = \int h(a_t, x_t; \lambda_t, \mu_t) d\mu_t(a_t, x_t).$$

Bayesian inference combines a prior distribution with a likelihood function to obtain a posterior distribution of the model parameters. Since α and δ are easily identifiable based on long-run averages of the labor share and the investment-capital ratio, we fix these parameters in the estimation using the “true” values reported in Table 1. Moreover, we assume that the econometrician knows the “true” fiscal policy parameters (τ_H, τ_K, and χ). We also fix the autocorrelation of the intertemporal preference shock process (ρ_Z) to 0.99 because preliminary estimates seemed to drift toward one.[6] Marginal prior distributions for the remaining parameters of the representative-agent model are provided in the first columns of Table 2. Our prior is diffuse with respect to the coefficients determining the law of motion of the exogenous shocks and assigns a high probability to the event that the annualized real interest rate lies between 0 and 8% and the aggregate labor supply elasticity falls into the interval from 0 to 2. The joint prior distribution for all DSGE model parameters is obtained simply by taking the product of the marginals.

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[6] We also conducted Bayesian inference based on non-dogmatic priors elicited from beliefs about steady-state relationships as in Del Negro and Schorfheide (2008). The results were essentially the same as the ones reported below.

4 Quantitative Results

For the presentation of the quantitative findings it is useful to distinguish between two groups of parameters of the representative-agent model, namely, policy parameters $\theta_{(p)} = [\tau_H, \tau_K, \chi]'$ and non-policy (or preference and technology) parameters $\theta_{(np)}$. The linearized representative-agent model has the state-space representation

$$\begin{aligned} y_t &= \Psi_0(\theta_{(p)}, \theta_{(np)}) + \Psi_1(\theta_{(p)}, \theta_{(np)}) s_t \\ s_t &= \Phi_1(\theta_{(p)}, \theta_{(np)}) s_{t-1} + \Phi_\epsilon(\theta_{(p)}, \theta_{(np)}) \epsilon_t, \end{aligned} \tag{18}$$

where y_t is a vector of observables, s_t is a vector of latent state variables, and $\epsilon_t = [\epsilon_{A,t}, \epsilon_{B,t}, \epsilon_{Z,t}]'$ is a vector of structural shock innovations. We consider three main questions in the subsequent quantitative analysis.

First, in Section 4.1 the representative-agent model is estimated based on aggregate times series generated from the heterogeneous-agents economy under the benchmark calibration. Roughly speaking, conditional on the actual policy $\theta^0_{(p)}$ we determine an estimate $\hat{\theta}_{(np)}(\theta^0_{(p)})$ such that the means and autocovariances implied by the representative-agent model match the sample means and autocovariances of the data generated from the heterogeneous-agents economy. Because of the inclusion of shocks to the preferences of the representative household, we are able to obtain a good match.

Second, we study to what extent the parameters of the representative-agent model are invariant to changes in fiscal policy. We do so from two complementary perspectives. In Section 4.2 the representative-agent model is re-estimated based on data generated from the heterogeneous-agents model under different policy regimes $\tilde{\theta}_{(p)}$. Thus, we examine whether the parameter estimate $\hat{\theta}_{(np)}$ obtained in Section 4.1 is approximately invariant to changes in the policy parameter $\theta_{(p)}$. The answer is negative, indicating that at a minimum the coefficient matrices in (18) as functions of $\theta_{(p)}$ are misspecified. Notice that a misspecification of $\partial\Psi_i(\theta_{(p)}, \theta_{(np)})/\partial\theta_{(p)}$ or $\partial\Phi_i(\theta_{(p)}, \theta_{(np)})/\partial\theta_{(p)}$ need not be detectable from the time series fit of (4.2) if the policy does not change during the sample period.

In order to assess the economic magnitude of this lack of policy-invariance of the non-policy parameters $\theta_{(np)}$ the following experiment is conducted in Section 4.3. We use the representative-agent model parameter estimates obtained under the benchmark policy to predict the effect of new policies assuming that taste and technology parameters are policy-invariant. In a nutshell, using (18) we condition on $\hat{\theta}_{(np)}$ obtained in Section 4.1 and replace $\theta^0_{(p)}$ with counterfactual policy parameters $\tilde{\theta}_{(p)}$ to generate predictions of the policy effects. We assess the accuracy of these predictions by comparing them to the true equilibrium outcomes from the heterogeneous-agents models. We focus on the prediction of steady-state effects, thereby evaluating how well the function $\Psi_0(\theta_{(p)}, \hat{\theta}_{(np)})$ captures mean shifts in the ergodic distribution associated with the heterogeneous-agents model. Section 4.4 discusses potential remedies for inaccuracies of policy predictions.

4.1 Benchmark Estimates of the Representative-Agent Model

We begin by fitting the representative-agent model using the aggregate output, consumption, and employment generated from the heterogeneous-agents economy under the benchmark fiscal policy. To be clear, we are not conducting a Monte Carlo simulation in a frequentist sense. We are computing posterior estimates for two samples only. One sample contains aggregate time series of 200 observations and the other sample 2,500 observations. The sample of 200 would correspond to 50 years of quarterly observations. The posterior distribution computed from the short sample embodies a degree of parameter uncertainty that is commensurate with posteriors computed from actual data. This will become important subsequently, because we will compare the magnitude of the policy prediction errors of the representative-agent model – which are essentially caused by the lack of policy invariance of the preference and technology parameters due to imperfect aggregation rather than by the sampling variability of the estimators – to the magnitude of posterior uncertainty. In addition we consider the unrealistic sample size of 2,500 observations because the consistency property of Bayes estimators implies that the resulting parameter estimates are very close to the pseudo-true representative-agent model parameters that minimize the information-theoretic Kullback-Leibler distance between the approximating representative-agent model and the data-generating heterogeneous-agents economy.

Posterior means and 90% credible intervals are reported in Table 2. Our subsequent discussion of the estimation highlights the following four findings: (i) the estimation of the representative-agent model detects sizeable preference shocks. (ii) With these preference shocks the estimated representative-agent model fits the aggregate output, consumption, and employment data well in comparison with a VAR. (iii) The estimated aggregate labor supply elasticities are related to the slope of the reservation wage distribution in the heterogeneous-agents economy. (iv) Due to a composition effect of the labor force, measured total factor productivity A_t in the representative-agent model differs from the underlying technology shock λ_t in the heterogeneous-agents economy. Findings (iii) and (iv) will be very important for understanding the outcomes of the subsequent policy experiments.

Preference Shocks: Although there are no aggregate preference shocks in the underlying heterogeneous-agents economy, the representative-agent model estimation detects both intratemporal (B_t) and intertemporal (Z_t) preference shocks. For example, for the sample of 2,500 observations the estimated $\ln B_t$ has an autocorrelation coefficient of 0.92 with a standard deviation of innovation of 0.3%. According to a variance decomposition based on the estimated representative-agent model, the two preference shocks jointly account for between 12% to 15% of the variation in output and consumption and between 45% and 50% of the variation in hours worked.[7] While it is difficult to make direct comparisons with the literature that estimates richer DSGE models on aggregate U.S. data, a substantial variation of preference shocks for employment or hours worked seems broadly in line with recent studies by Hall (1997) and Chari, Kehoe, and McGrattan (2007), and Justiniano, Primiceri, and Tambalotti (2010). Our results suggest exercising caution when interpreting preference shocks measured from aggregate time series data: they may reflect a specification error (e.g., aggregate error) rather than a fundamental driving force behind business cycles.

Time Series Fit: For policy makers to regard experiments with a representative-agent model as compelling, it is important that the model be able to track the aggregate time series reasonably well. This claim is supported by the surge in attention that central banks paid to New Keynesian DSGE models after Smets and Wouters (2003, 2007) introduced modifications to an emerging canonical medium-scale DSGE model that led to a time series fit comparable to that of VARs. Thus, as is common in the literature (for references see, for instance, Del Negro, Schorfheide, Smets, and Wouters (2007)), we compute the posterior odds of the representative-agent model relative to a VAR. We find that the posterior odds based on the sample of 200 observations favor the structural representative-agent model

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[7] These estimates complement the findings in Chang and Kim (2007), who construct a time series for $\ln B_t$ directly as the wedge between the marginal product of labor and the marginal rate of substitution and then study its cyclical properties.

over a VAR(4) with the Minnesota prior[8] by e^{18}. While the posterior odds do not imply that the representative-agent model is correctly specified in some absolute sense, the result indicates that its cross-equation restrictions are well enough specified such that the resulting reduction in dimensionality in the DSGE model outweighs the improvement in the in-sample fit attainable with a less restrictive VAR. The inclusion of the preference shocks essentially enables the representative-agent model to match the autocovariances of the data generated from the heterogeneous agent economy.

Aggregate Labor Supply Elasticity: The 90% credible interval for the aggregate labor supply elasticity of a representative household, ν, ranges from 1.43 to 1.85 for the sample of 200 observations and from 2.02 to 2.24 for the sample of 2,500 observations. As emphasized by Chang and Kim (2006), in our heterogeneous-agents economy the aggregate elasticity is determined by the shape of the reservation wage distribution rather than by the willingness of individual households to substitute consumption and leisure.

Compositional Effect: In the heterogeneous-agents model, the means of the aggregate log productivity process, $\ln \lambda_t$, and the average level of the log of idiosyncratic productivity, $\ln x_t$, are zero. In equilibrium the workers who are not working tend to be the less productive ones. Due to this composition effect the estimated steady-state log productivity in the representative-agent model, $\ln \bar{A}$, is 0.45. Closely related, the point estimates of ρ_A and σ_A imply that the (unconditional) standard deviation of the aggregate technology process in the representative-agent model is about 1.2%. The standard deviation of the productivity process λ_t in the heterogeneous-agents model, on the other hand, is about 2.2%. In the heterogeneous-agents economy, newly hired workers during the expansion are, on average, less productive than existing workers, lowering the average productivity of the workforce. Vice versa, it is the low-productivity workers who leave the workforce during the recession. This composition effect of the workforce makes the measured aggregate productivity less volatile than the true aggregate technology. It also contributes to a larger estimate of aggregate labor supply because the measured hours worked (e.g., employment) exhibit a larger volatility than the total labor input in efficiency units. The composition effect is also

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[8] The particular version of the Minnesota prior used in this paper is described in Del Negro and Schorfheide (2010). The Minnesota prior tilts the maximum likelihood estimates (MLE) of the VAR coefficients toward univariate unit root representations. From a non-Bayesian perspective this procedure is attractive even if the time series do not follow unit processes because it reduces the sampling variance of the MLE while simultaneously offsetting some of its small-sample bias.

well documented for actual U.S. data. For instance, Bils (1985) estimates, based on PSID data, that the average wage of newly hired workers is 19% lower than the average wage of existing workers.

4.2 Policy (In)variance of Model Parameters

We now investigate whether the parameters of the representative-agent model are invariant with respect to policy changes. To do so, the heterogeneous-agents economy is simulated under the alternative fiscal policies listed in Table 1. In these simulations the sequences of aggregate and idiosyncratic shocks are kept identical to those used for the benchmark analysis. The representative-agent model is then re-estimated based on the newly generated data sets. If the representative-agent parameters were truly "structural," the parameter estimates should be the same (up to some estimation uncertainty), regardless of the policy regime. The resulting posterior mean parameter estimates and 90% credible intervals for samples of 2,500 observations are reported in the top panel of Table 3. There is considerable variation in the estimates of average log productivity, $\ln \bar{A}$, the aggregate labor supply elasticity ν, and the implicit steady-state interest rate r_A, which determines the discount factor β. The 90% credible intervals for these parameters do not overlap. Moreover, the estimated aggregate shock processes (not shown in the table) are also sensitive to the policy regime. We now consider the labor tax cut, the rise in the capital tax rate, and the increase in transfers in more detail. The 1960 and the 2004 policy lead to a combination of the effects described subsequently.

Labor Tax Cut: In order to shed light on the instability of the parameter estimates, the second panel of Table 3 provides long-run averages of employment, capital, output, labor productivity, and interest rates associated with the stochastic steady states of the heterogeneous-agents economy under the various fiscal policies. When the labor income tax rate is lowered to $\tau_H = 0.22$, the employment rate increases by almost 7%. Because of the tax cut, the total tax revenue, however, decreases. Given the fixed proportion of lump-sum transfers ($\chi = 0.36$) a decrease in tax revenues implies that each household receives fewer lump-sum transfers, which increases the precautionary savings motive. Higher labor input also reinforces the accumulation of capital given the complementarity between capital and labor. As a result, the aggregate capital stock rises by 6%, lowering an equilibrium annual interest rate from 4% to 3.68%.

A higher capital stock is reflected in a low discount rate in the estimated representative-agent model. Aggregate output increases about 4% (from 1.48 to 1.53). The measured average labor productivity decreases by 3% (from 2.46 to 2.39) due to the compositional effect discussed in Section 4.1. In order for the representative-agent model to capture the compositional effect and the precautionary savings, the estimates of the discount rate, r_A, and average productivity $\ln \bar{A}$ have to fall, as in the second column of Table 3. As the average employment rate rises with the labor tax cut, the economy moves toward a thinner part of the reservation wage distribution, requiring the labor supply elasticity ν, of the representative-agent model to decrease.[9]

Rise in Capital Tax Rate: When we increase the capital income tax rate from $\tau_K = 0.35$ to $\tau_K = 0.47$, the equilibrium employment rate remains essentially unchanged. Thus, the workforce composition effect is not operational. A high capital tax, however, decreases savings and results in a decrease in the capital stock of 8% (from 15.2 to 14.0). A decreased capital-labor ratio raises the equilibrium interest rate from 4% to 4.76%. Unlike the case of the labor tax cut, the estimates of the representative-agent parameters are more or less unaffected as the employment rate remains approximately constant.

More Transfers: The increase in the ratio of lump-sum transfers in government expenditures from $\chi = 0.36$ to 0.5 generates a negative income effect on the labor supply, decreasing the employment rate to 57%. A larger transfer discourages the precautionary motive of savings, decreasing aggregate capital stock by 3% (from 15.2 to 14.76). As both capital and labor decrease, the equilibrium interest rate is virtually unaffected. Labor productivity, however, increases as the employment rate decreases because less-productive workers retreat from the labor market. The changing workforce composition is captured in the representative-agent model by a higher estimated value of $\ln \bar{A}$. Finally, the heterogeneous-agents economy moves toward a thicker part of the reservation wage distribution, requiring a larger elasticity of labor supply for the representative-agent model.

4.3 Accuracy of Policy Predictions

In order to assess the quantitative importance of the policy dependence of the parameters of the representative-agent model, we now examine the accuracy of the policy predictions

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[9] A detailed discussion is provided in the online Appendix

that the representative-agent model delivers under the assumption that its parameters are unaffected by policy interventions. To do so, we construct posterior predictive distributions for the effects of the policy changes based on the model estimated under the benchmark fiscal policy. We consider the percentage change in long-run aggregate output, consumption, and employment as well as the overall welfare effect induced by the policy change.

Welfare Measure: An important advantage of DSGE models over reduced-form models, such as vector autoregressions, is the welfare analysis. Following Aiyagari and McGrattan (1998), we define social welfare as:[10]

$$\mathcal{W} = \int V(a,x) d\mu(a,x), \tag{19}$$

where $\mu(a,x)$ is the steady-state joint distribution of asset holdings and idiosyncratic productivity and $V(a,x)$ is the value function associated with the optimal decisions. This is a utilitarian social welfare function that measures the *ex ante* welfare in the steady state—i.e., the welfare of an individual before the realization of initial assets and productivity, which is drawn from the steady-state distribution $\mu(a,x)$. We measure the welfare gain or loss due to a policy change by the constant percentage change in consumption each period for all individuals which is required to equate social welfare before and after the policy change. We compare the welfare measures based on the steady-state ergodic distributions only, not including the transition dynamics. In the representative-agent model the distribution $\mu(a,x)$ is degenerate and the computation of the welfare effect simplifies considerably. The equilibrium of the representative-agent model is approximated with a first-order log-linearization, which is known to be fairly accurate for the stochastic growth model considered in this paper. Under this approximation the welfare effect can be calculated directly from the steady-state levels of consumption and hours. Further details are provided in the online Appendix.

Quantitative Findings: The quantitative results for the policy predictions are summarized in Table 4. For now, we focus on the columns with the heading "Estimation Uncertainty." The entries in the table refer to percentage changes relative to the benchmark values. The "true" policy effect is computed based on the new ergodic distribution of

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[10] This measure of social welfare or its variants have been widely used in the literature. Examples include Domeij and Heathcote (2004), Young (2004), Pijoan-Mas (2005), Heathcote, Storesletten and Violante (2008) and Rogerson (2009). Detailed justifications for this welfare measure are provided in Aiyagari and McGrattan (1998).

the heterogeneous-agents economy. The "90% interval" entries correspond to 90% predictive intervals computed based on the posterior distribution of the parameters of the representative-agent model obtained from 200 observations under the benchmark fiscal policy. These intervals reflect the uncertainty with respect to the "structural" parameters of the representative-agent model. With the widespread adoption of Bayesian methods in empirical macroeconomics, such predictive distributions are frequently used to conduct policy analysis under parameter (and model uncertainty) as, for instance, in Levin, Onatski, Williams, and Williams (2006). Moreover, the use of the predictive distribution allows us to relate the magnitude of the prediction biases due to lack of parameter invariance to the overall level of uncertainty associated with the predictions. We also report "p-values," which indicate how far in the tails of a Gaussian approximation of the predictive distribution the realization of the policy effect lies.

Across the entries in Table 4 we find that the "true" effects of the policies, both with respect to the average level of output, consumption, and hours as well as with respect to households' welfare, lie almost always far outside the 90% intervals. Almost all of the p-values are essentially zero. Among the three "single-instrument" policy changes, the prediction of the effect of a capital tax increase is the most accurate. This is consistent with our previous finding that the parameter estimates under the high-capital-tax regime are very close to the ones under the benchmark fiscal regime. If the representative-agent model is used to rank the five alternative policies, its welfare predictions imply that the labor tax cut is the most beneficial and the capital tax increase is the worst policy. The welfare ranking based on the actual effects in the heterogeneous-agents economy, however, is quite different. The most favorable policy is the increase in transfers. The "1960 policy" is the worst, leading to a larger welfare loss than the pure capital tax increase. Thus, the parameter instability is sufficiently large to render predictions from the representative-agent model inaccurate and the predicted rankings of policies incorrect.

To gain a better understanding of the results, we now consider the prediction of the effect of changing the fraction of lump-sum transfers (raising χ from 36% to 50%) in more detail. Due to the income effect, total hours worked decrease by 5.45% in the heterogeneous-agents economy. The representative-agent model predicts, under the assumption of policy-invariant preference and technology parameters, a decrease of only 3.04%, to 3.22%, with 90% probability. The employment effect is under-predicted because the constant aggregate supply elasticity of the representative-agent model does not capture the increased slope of the reservation wage distribution at higher employment levels. The representative-agent model also under-predicts the rise in consumption. The consumption effect in the heterogeneous-agents model is stronger, because the transfer increase reduces the need for precautionary savings for households near the borrowing constraint.

Aggregate output decreases by 2.19% in the heterogeneous-agents economy, whereas, according to the representative-agent model, it is predicted to fall by 3.04% to 3.22%. The representative-agent model overpredicts the fall in output for two reasons. First, it misses the effect of workforce composition on labor productivity. Second, the aggregate capital stock decreases by more than the representative-agent model predicts, due to less need for precautionary savings. In sum, the average welfare of households increases by 5.80% in the heterogeneous-agents economy. The welfare effect predicted by the representative-agent model, on the other hand, ranges only from 3.10% - 3.18% because the effect of the additional insurance provided by the transfers is not captured.

4.4 Potential Remedies

The DSGE model research agenda seemingly promises that one can predict the effects of policy changes without having any observed variation in the policy instrument. In our simulation, we obtained predictions that were qualitatively reasonable yet quantitatively imprecise. The first-best approach to addressing the prediction inaccuracy is to work with a better model. In practice, of course, “true” models remain elusive and the best response is to model and measure the policy-relevant mechanisms and trade-offs as well as possible. For instance, while a careful modeling of labor-market heterogeneity was not particularly important for the assessment of the capital tax change, the representative-agent model’s inability to capture the effects of labor-market heterogeneity rendered its predictions with regard to labor tax and transfer changes grossly misleading.

Better Measures of Labor Inputs: One of the important lessons we learned from the heterogeneous-agents model is that the aggregate labor supply elasticity is not iso-elastic. We will subsequently show in Section 5.1 that replacing raw hours (i.e., the employment rate) by the efficiency unit of labor alleviates the composition bias in the estimates of productivity and labor supply elasticity of the representative-agent model. With the time series of efficiency units of labor, both estimates of average productivity, $\bar{A}$, and labor supply elasticity, ν, become much more stable across policy regimes. While it is not easy to obtain the exact measure of efficiency units because it is very difficult to capture *all* the heterogeneity using observed characteristics, it appears important to measure labor input more accurately.[11]

Accounting for Parameter Instability Risk: From the perspective of a policy maker who has to make decisions based on imperfect models, our analysis indicates that the parameter uncertainty reflected in the formal Bayesian estimation of the representative-agent model captures only a small aspect of the policy maker's "risk." The results in Table 4 could be interpreted as the predictive intervals being too small because the possibility that preference and technology parameters may shift in response to a policy change is not being entertained. Let $\theta_{(np)}$ denote the non-policy-related preference and technology parameters of the representative-agent model. Moreover, let $\Delta\theta_{(np)}$ denote an intervention-induced shift in the preference and technology parameters. In order to account for the possibility of a parameter change, a policy maker could combine the posterior distribution of $\theta_{(np)}$ with a conditional distribution of $\Delta\theta_{(np)}$ given $\theta_{(np)}$ to characterize beliefs about post-intervention values of $\theta_{(np)}$.[12]

For concreteness, assume the discrepancy distribution is independent normal and the standard deviations for our parameters r_A, ν, $\ln \bar{A}$, and $\ln \bar{B}$ are 0.09, 0.09, 0.002, and 0.003, respectively. These numbers correspond to the posterior standard deviations of the four parameters associated with the $T = 200$ estimates in Table 2. In the rightmost columns of Table 4 we report the predictive intervals obtained with accounting for the possibility of a parameter shift (under the heading "Invariance Uncertainty"). While the mean predictions do not change much, the predictive intervals become a lot wider in the latter case and encompass the "true" effects on consumption and output. The p-values now range from 0.06 to 0.32.

Exploiting Information from Policy Variation if Available: If an econometrician has access to observations from different policy regimes, statistical techniques such as the estimation of structural break, regime-switching, or time-varying parameter models could be

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[11] For the composition effect in aggregate measures of wages and hours, see Bils (1985) and Hansen (1993).

[12] Mechanically, we let the post-intervention non-policy parameters be $\tilde{\theta}_{(np)} = \theta_{(np)} + \Delta\theta_{(np)}$, where draws of $\theta_{(np)}$ are generated from the posterior distribution obtained under the historically observed policy regime and $\Delta\theta_{(np)}$ is drawn independently from the above-specified "prior" distribution.

used to quantify the magnitude of potential parameter shifts. To examine the detectability of coefficient changes across policy regimes in our heterogeneous-agents environment we conduct the following experiment. We construct three new data sets by combining 100 observations from the benchmark policy regime with 100 observations from one of the following alternative regimes: labor tax cut, capital tax raise, and more transfers. The "true" policy coefficients for the benchmark and the alternative policy regime are treated as known and two versions of the representative-agent model are estimated. In the first version, $\mathcal{M}_0$, the non-policy parameters are assumed to be identical across regimes, whereas in the second version, $\mathcal{M}_1$, the non-policy parameters are allowed to differ. $\mathcal{M}_1$ is estimated under a prior distribution that restricts potential changes in the non-policy parameters to be small. A more detailed discussion of the structural break tests is provided in the online Appendix.

For each of the three data sets we compute log marginal data densities for models $\mathcal{M}_0$ and $\mathcal{M}_1$. If the alternative policy is either a labor tax cut or an increase in transfers, the switching coefficient model, $\mathcal{M}_1$, is favored by the posterior odds. If the alternative policy is a capital tax increase, the constant coefficient model is preferred. These results are consistent with our earlier result that the representative-agent model delivers relatively accurate predictions of the effects of a capital tax change, but has difficulties capturing labor market effects.

The estimation results from the structural break model provide some information about the invariance of the non-policy parameters. Empirical evidence about the sensitivity of non-policy parameters to changes in the policy coefficient could in principle be used to inform the conditional distribution of $\Delta\theta_{(np)}$ given $\theta_{(np)}$ used in the simulations that underlie the results reported in the rightmost columns of Table 4. The practical problem is that the estimation of the structural break model provides only very few observations that could be used to approximate $\partial\theta_{(np)}/\partial\theta_{(p)}$.

In our particular application we know that the level of total factor productivity as well as the slope of the aggregate labor supply schedule depends on the reservation wage distribution. Thus, it might be fruitful to make total factor productivity as well as the aggregate labor supply elasticity dependent upon the level of employment. However, providing an operational procedure is beyond the scope of this paper and we leave it as a topic for future research.

5 Alternative Measurements and Model Economies

We now modify the empirical analysis in two dimensions. First, in Section 5.1 the representative-agent model is re-estimated based on three alternative data sets. Two data sets contain the real interest rate, and in the third data set, hours worked is replaced by a measure of efficiency-adjusted labor. Second, we examine the role of the two frictions, indivisible labor and incomplete capital markets, that prevent the aggregation of individual households' optimality conditions in our benchmark heterogeneous-agents economy. In Sections 5.2 and 5.3 we repeat the analysis of Section 4 for an economy with incomplete capital markets but divisible labor and for an economy in which asset markets are complete but labor is indivisible, respectively.

5.1 Estimating the Representative-Agent Model with Other Observables

The econometric analysis in Section 4 uses output, hours worked, and consumption as observables in the estimation of the representative-agent model. Since the Cobb-Douglas production function implies that real wages are proportional to average labor productivity, the use of output and hours data implies that our estimation is also implicitly using information from a measure of real wages that is commensurate with our measure of hours worked. However, we have not used any information about the real interest rate. Thus, we repeat the econometric analysis based on the following two alternative data sets: (i) real interest rates, hours, and consumption; (ii) output, hours, and real interest rates. It turns out that the findings with respect to parameter stability as well as the inaccuracy of policy predictions are quantitatively similar to those reported in Section 4. Detailed results are reported in the online Appendix.

When we replace the hours worked series in the benchmark output-hours-consumption data set with the efficiency units of labor, the estimates of ν and $\ln \bar{A}$ are more stable across policy regimes compared to the benchmark analysis (Table 5). For example, with efficiency units of labor, the estimate for the aggregate elasticity of labor supply ($\hat{\nu}$) ranges from 0.4 to 0.62, implying a smaller aggregate labor supply elasticity.[13] The average level of aggregate productivity, $\ln \bar{A}$, remains close to zero across policies. Given the approximate invariance of

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[13] Hansen (1993) obtains a similar result. He finds that hours in efficiency units (measured by demographic variables of the households from the CPS) move less than actual hours over the business cycle.

parameters, the use of efficiency units might appear to be a promising alternative. However, in practice, it is often difficult, if not impossible, to obtain the efficiency unit measures of quantity (hours) and prices because it is almost impossible to capture *all* the heterogeneity using observed characteristics – a typical cross-sectional wage regression barely reaches an R^2 of 0.4.

5.2 Divisible Labor

The first alternative model economy we consider allows for divisible labor supply, but capital markets remain incomplete. This is essentially the same specification as in Krusell and Smith (1998) with endogenous hours choice. The equilibrium of this economy can be defined similarly to that of the benchmark model with the worker's value function with divisible labor, $V^D(a, x; \lambda, \mu)$:

$$V^D(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}, h \in (0,1)} \left\{ \ln c - B \frac{h^{1+1/\gamma}}{1 + 1/\gamma} + \beta E \Big[V^D(a', x'; \lambda', \mu') \big| x, \lambda \Big] \right\}$$

subject to

$$c = (1 - \tau_H) w(\lambda, \mu) x h + (1 + (1 - \tau_K) r(\lambda, \mu)) a + \bar{T} - a', \quad a' \geq \bar{a}, \quad \mu' = \mathbf{T}(\lambda, \mu).$$

We estimate the parameters of the representative-agent model using output, hours, and consumption data simulated from this economy. As Table 5 shows, the aggregate labor supply elasticity ν of a stand-in household is 0.37, very close to the elasticity of individual households, $\gamma = 0.4$. Moreover, the estimated standard deviations for the two preference shocks (not reported in the table) are very close to zero. This is consistent with a "near perfect" aggregation result by Krusell and Smith (1998) – a representative-agent model is a good approximation of the heterogeneous-agents economy with incomplete markets. A comparison of the entries in Table 5 and Table 2 indicates that for the divisible-labor economy the parameter estimates are much less sensitive to the tax policy than in our benchmark economy. For instance, the estimate of $\ln \bar{A}$ is not at all affected by the policy regime, indicating that the divisibility of labor essentially eliminated the labor-force composition effect.

5.3 Complete Asset Markets

Our second auxiliary model economy has complete capital markets but labor supply is indivisible. Due to perfect risk sharing, agents enjoy the same level of consumption regardless of their employment status, productivity, or asset holdings.[14] The equilibrium of this economy is identical to the allocation made by a social planner who maximizes the equally weighted utility of the population. The planner chooses the sequence of consumption $\{C_t\}_{t=0}^{\infty}$ and the cut-off productivity $\{x_t^*\}_{t=0}^{\infty}$ for labor-market participation. To ensure an efficient allocation, the planner assigns workers who have a comparative advantage in the market (more productive workers) to work. If a worker's productivity is above x_t^*, he supplies $\bar{h}$ hours of labor. It turns out that under complete markets, the first-order condition for the choice between hours and consumption is *exactly* defined in terms of effective units of labor and wages at the aggregate level.

In theory, our estimation of a representative-agent model should reveal the preference of a social planner. At the estimated parameters the intratemporal and intertemporal first-order conditions of the representative household hold almost exactly, and the estimated standard deviations of the preference shocks are very close to zero. However, since we use actual hours worked instead of efficiency units of labor in our estimation, the estimated parameters are still subject to a composition bias. According to Table 5 the aggregate labor supply elasticity, ν, is estimated to be 1.42. The aggregate labor supply elasticity, ν, depends on the cross-sectional distribution of productivity, analogous to the model with incomplete asset markets. In response to a policy change, the estimates of ν and $\ln \bar{A}$ change, but not as drastically as under the benchmark economy. The change of the $\ln \bar{A}$ estimate from -1.45 to -1.44 under the more-transfers policy indicates the presence of the labor-force composition effect.

6 Conclusion

Representative-agent dynamic stochastic general equilibrium models are widely used for economic policy analysis. A key assumption in policy experiments is that fundamental

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[14] The distribution of workers is no longer a state variable in the individual optimization problem. Moreover, because of the ergodicity of the stochastic process for idiosyncratic productivity, the cross-sectional distribution of workers is always stationary.

parameters of the model such as taste and technology are invariant with respect to policy changes. We demonstrate that this is not always the case. We construct a heterogeneous-agents economy in which equilibrium outcomes depend on the distributions of wealth and earnings, which in turn depend on the policy regime. We estimate a representative-agent model that best approximates the aggregate times series generated from the heterogeneous-agents model. We find that (i) taste and technology parameters in the representative-agent model are not policy invariant; and (ii) fiscal policy predictions from the representative-agent model are often inaccurate. Moreover, as has been pointed out in previous papers, we document that the aggregation error manifests itself as a preference shift of a representative household.

We demonstrate that the representative-agent model that abstracts from cross-sectional heterogeneity on the household side can potentially mislead fiscal policy predictions. Of course there are other forms of heterogeneity, e.g. on the firm side, that may lead to even stronger biases in policy predictions. Thus, our study should not be interpreted as a claim that household-level heterogeneity is the most important one or that the use of representative-agent models should be abandoned. Instead, we conclude that it is important to account for the possibility that the preference and technology parameters of an estimated model may shift in response to a policy change. To the extent that an econometrician has access to observations from different policy regimes, statistical techniques such as the estimation of time-varying coefficients or regime-switching models could be useful to quantify the magnitude of potential parameter shifts.

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Table 1: PARAMETERIZATION OF THE HETEROGENEOUS-AGENTS ECONOMY

Preference and Technology Parameters

Firm parameters	$\alpha = 0.64$, $\delta = 0.025$
	$\rho_\lambda = 0.95$, $\sigma_\lambda = 0.007$
Household parameters	$\beta = 0.983$, $\gamma = 0.4$, $B = 101$, $\overline{h} = 1/3$, $\underline{a} = -2.0$
	$\rho_x = 0.939$, $\sigma_x = 0.287$

Fiscal Policy Parameters

	Bench-mark	Labor Tax Cut	Capital Tax Rise	More Transfers	1960 Policy	2004 Policy
τ_H	0.29	0.22			.229	.269
τ_K	0.35		0.47		.443	.327
χ	0.36			0.50	.224	.417

Table 2: PARAMETER ESTIMATES

	Prior			Posterior (Benchmark Calibr.)			
				$T = 200$		$T = 2,500$	
	Density	Mean	S.D.	Mean	90% Intv.	Mean	90% Intv
r_A	Gamma	4.00	2.00	2.83	[2.69, 2.97]	2.77	[2.71, 2.83]
ν	Gamma	1.00	0.50	1.65	[1.43, 1.85]	2.13	[2.02, 2.24]
$\ln \bar{A}$	Normal	0.00	10.0	0.44	[0.44, 0.45]	0.45	[0.44, 0.45]
$\ln \bar{B}$	Normal	0.00	10.0	-1.43	[-1.44, -1.42]	-1.41	[-1.42, -1.41]
ρ_A	Beta	0.50	0.25	0.90	[0.89, 0.91]	0.91	[0.91, 0.92]
ρ_B	Beta	0.50	0.25	0.74	[0.57, 0.90]	0.92	[0.91, 0.93]
σ_A	Inv. Gamma	.012	.007	.005	[.005, .006]	.005	[.005, .006]
σ_B	Inv. Gamma	.012	.007	.003	[.002, .003]	.003	[.003, .003]
σ_Z	Inv. Gamma	.012	.007	.003	[.002, .003]	.003	[.003, .003]

Notes: During the estimation the parameters δ, τ_H, τ_K, and χ are fixed to the values in Table 1 and ρ_Z is set to 0.99. r_A is the annualized discount rate $r_A = 400 \times (1/\beta - 1)$. As parameter estimates we report posterior means and 90% credible intervals (in brackets).

Table 3: ESTIMATES UNDER ALTERNATIVE POLICIES AND "TRUE" STEADY STATES

	Bench-mark	Labor Tax Cut	Capital Tax Rise	More Transfers	1960 Policy	2004 Policy
	Parameter Estimates, $T = 2,500$					
r_A	2.77	2.55	2.74	2.84	2.52	2.75
	[2.71, 2.83]	[2.49, 2.62]	[2.66, 2.82]	[2.78, 2.91]	[2.46, 2.58]	[2.69, 2.81]
ν	2.13	1.44	2.23	3.60	1.22	2.09
	[2.02, 2.24]	[1.38, 1.50]	[2.10, 2.35]	[3.32, 3.88]	[1.17, 1.27]	[1.98, 2.21]
$\ln \bar{A}$	0.45	0.42	0.45	0.47	0.40	0.45
	[0.44, 0.45]	[0.42, 0.42]	[0.45, 0.45]	[0.47, 0.47]	[0.40, 0.41]	[0.44, 0.45]
$\ln \bar{B}$	-1.41	-1.41	-1.41	-1.40	-1.41	-1.42
	[-1.42, -1.41]	[-1.42, -1.41]	[-1.42, -1.41]	[-1.41, -1.40]	[-1.41, -1.41]	[-1.42, -1.41]
	Steady States in Heterogeneous-Agents Economy					
E	0.60	0.64	0.60	0.57	0.66	0.60
K	15.2	16.1	14.0	14.8	15.4	15.5
Y	1.48	1.53	1.43	1.44	1.51	1.49
Y/H	7.38	7.17	7.17	7.62	6.87	7.44
R_A	4.00	3.68	4.76	4.04	4.16	3.80

Notes: See Table 2.

Table 4: Predictions of Policy Effects, $T = 200$

		Estimation Uncertainty					Invariance Uncertainty	
		Labor Tax Cut	Capital Tax Rise	More Transfers	1960 Policy	2004 Policy	Labor Tax Cut	More Transfers
H	"True"	6.06	-0.23	-5.45	9.44	-0.21	6.06	-5.45
	90% Intv.	[2.96, 3.15]	[-0.31, -0.28]	[-3.22, -3.04]	[5.18, 5.51]	[-0.21, -0.20]	[2.26, 3.78]	[-4.12, -2.32]
	p-Value	2.2E-308	1.5E-013	2.2E-308	2.2E-308	3.3E-002	5.3E-011	1.8E-005
C	"True"	7.33	-2.73	3.04	1.73	3.86	7.33	3.04
	90% Intv.	[7.84, 8.03]	[-3.63, -3.37]	[1.79, 1.98]	[2.25, 2.65]	[3.66, 3.71]	[6.53, 8.78]	[0.67, 3.10]
	p-Value	3.2E-025	2.5E-021	9.7E-089	8.1E-009	3.5E-035	3.2E-001	5.5E-002
Y	"True"	3.44	-2.89	-2.19	2.57	0.81	3.44	-2.19
	90% Intv.	[2.96, 3.15]	[-4.07, -3.84]	[-3.22, -3.04]	[2.28, 2.63]	[0.36, 0.41]	[1.51, 4.47]	[-4.70, -1.56]
	p-Value	1.3E-011	5.9E-052	9.0E-066	1.7E-001	1.2E-178	3.2E-001	1.5E-001
$\mathcal{W}$	"True"	4.51	-2.61	5.80	-3.09	4.07	4.51	5.80
	90% Intv.	[6.60, 6.68]	[-3.52, -3.25]	[3.10, 3.18]	[0.16, 0.44]	[3.75, 3.79]	[5.70, 7.58]	[2.18, 4.11]
	p-Value	2.2E-308	3.8E-020	2.2E-308	2.2E-308	1.2E-120	9.7E-005	3.4E-006

Notes: The benchmark policy is $\tau_H = 0.29$, $\tau_K = 0.35$, $\chi = 0.36$. The entries in the table refer to percentage changes relative to the benchmark policy. The last two rows ($\mathcal{W}$) contain welfare gains (if positive) or costs (if negative) in percentage terms due to the policy change. "True" effects are computed from the means of the ergodic distributions of the heterogeneous-agents economy. 90% Intv. are predictive intervals computed from the posterior of the representative-agent model based on observations under the benchmark policy.

Table 5: Parameter Estimates for Alternative Model Economies and Measurements, $T = 2,500$

	Divisible Labor			Complete Markets			Efficiency Hours		
	Bench-mark	Labor Tax Cut	More Transfers	Bench-mark	Labor Tax Cut	More Transfers	Bench-mark	Labor Tax Cut	More Transfers
r_A	2.72	2.45	2.81	2.50	2.50	2.48	2.75	2.54	2.82
	[2.68, 2.77]	[2.40, 2.50]	[2.76, 2.86]	[2.47, 2.54]	[2.46, 2.53]	[2.44, 2.52]	[2.70, 2.80]	[2.48, 2.60]	[2.74, 2.88]
ν	0.37	0.37	0.38	1.42	1.34	1.52	0.64	0.54	0.80
	[0.37, 0.38]	[0.36, 0.37]	[0.37, 0.38]	[1.42, 1.43]	[1.33, 1.35]	[1.51, 1.53]	[0.62, 0.67]	[0.52, 0.56]	[0.77, 0.84]
$\ln \bar{A}$	0.34	0.34	0.34	0.46	0.45	0.47	0.01	0.00	0.01
	[0.34, 0.34]	[0.33, 0.34]	[0.34, 0.35]	[0.46, 0.46]	[0.45, 0.45]	[0.47, 0.47]	[0.00, 0.01]	[0.00, 0.01]	[0.00, 0.01]
$\ln \bar{B}$	-1.53	-1.53	-1.54	-1.45	-1.45	-1.44	-0.81	-0.82	-0.79
	[-1.53, -1.53]	[-1.53, -1.53]	[-1.54, -1.53]	[-1.45, -1.44]	[-1.45, -1.45]	[-1.44, -1.44]	[-0.81, -0.81]	[-0.82, -0.82]	[-0.79, -0.79]

Notes: See Table 2.

Labor-Market Heterogeneity, Aggregation, and the Policy-(In)variance of DSGE Model Parameters

Online Appendix

Yongsung Chang, Sun-Bin Kim, and Frank Schorfheide

A Aggregate Data Sources

Aggregate capital and labor tax rates are obtained from Chen, Imrohoroglu, and Imrohoroglu (2009). As a measure of hours we use the Aggregate Hours Index (PRS85006033) published by the Bureau of Labor Statistics. The remaining data series are obtained from the FRED2 database maintained by the Federal Reserve Bank of St. Louis. Consumption is defined as real personal consumption expenditures on non-durables (PCNDGC96) and services (PCESVC96). Output is defined as the sum of consumption, consumption expenditures on durables (PCDGCC96), gross private domestic investment (GPDIC), and federal consumption expenditures and gross investment (FGCEC96).

For the estimation of the representative agent model based on U.S. data (see Table E-3 below), output, consumption, and hours are converted into per capita terms by dividing by the civilian non-institutionalized population (CNP16OV). The population series is provided at a monthly frequency and converted to quarterly frequency by simple averaging. Finally we take the natural logarithm of output, consumption, and hours. We restrict the sample to the period from 1965:I to 2006:IV, using observations from 1964 to initialize lags. We remove linear trends from the log output and consumption series and demean the log hours series. To make the log levels of the U.S. data comparable to the log levels of the data simulated from the heterogeneous-agents economy, we adjust (i) detrended log output by the steady-state output level in the heterogeneous-agents economy under the benchmark tax policy, (ii) detrended log consumption by the steady state output level in the heterogenous agent economy plus the log of the average consumption-output ratio in the U.S. data, and (iii) demeaned hours by the steady state of log employment.

B Derivations for the Representative-Agent Model

In this section, we collect the first-order conditions (and their log-linear approximation around the steady state) of the representative-agent model we use to fit the time series generated from the heterogeneous-agents economy.

First-Order Conditions: The first-order conditions (FOCs) associated with the Household Problem are:

$$\begin{aligned}
\lambda_t &= \frac{Z_t}{C_t} \\
\lambda_t &= \beta \mathbb{E}_t[\lambda_{t+1}(1+(1-\tau_K)R_{t+1})] \\
H_t^{1/\nu} &= (1-\tau_H)\frac{\lambda_t}{Z_t} W_t B_t^{1+1/\nu}
\end{aligned}$$

Notice that the preference shock Z_t drops out of the labor supply function:

$$H_t^{1/\nu} = (1-\tau_H)\frac{1}{C_t} W_t B_t^{1+1/\nu}.$$

The FOCs of the firms problem are provided in (4).

Steady States: We subsequently denote the deterministic steady-state values by

$$\bar{H}, \bar{K}, \bar{\lambda}, \bar{C}, \bar{Y}, \bar{A}, \bar{B}, \bar{W}, \bar{G}, \bar{R}.$$

The steady state value of Z_t is equal to one. It is convenient to express the model in terms of ratios relative to steady-state hours worked. The first-order conditions in the steady state become

$$\begin{aligned}
\bar{R} &= \frac{1/\beta - 1}{1-\tau_K}, \quad \left(\frac{\bar{H}}{\bar{B}}\right)^{\frac{1}{\nu}} = (1-\tau_H)\frac{\bar{B}}{\bar{C}}\bar{W}, \\
\frac{\bar{K}}{\bar{H}} &= \left(\frac{\bar{A}(1-\alpha)}{\bar{R}+\delta}\right)^{\frac{1}{\alpha}}, \quad \bar{W} = \alpha \bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha}.
\end{aligned}$$

Hence,

$$\frac{\bar{H}}{\bar{B}} = \left(\frac{(1-\tau_H)\bar{W}}{\bar{C}/\bar{H}}\right)^{\frac{\nu}{1+\nu}}.$$

Moreover, the production function can be expressed as

$$\frac{\bar{Y}}{\bar{H}} = \bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha}.$$

The government budget constraint leads to

$$\frac{\bar{T}}{\bar{H}} = \chi\left(\tau_H \bar{W} + \tau_K \bar{R}\frac{\bar{K}}{\bar{H}}\right), \quad \frac{\bar{G}}{\bar{H}} = (1-\chi)\left(\tau_H \bar{W} + \tau_K \bar{R}\frac{\bar{K}}{\bar{H}}\right)$$

and the market clearing condition can be written as

$$\frac{\bar{Y}}{\bar{H}} = \frac{\bar{C}}{\bar{H}} + \delta\frac{\bar{K}}{\bar{H}} + \frac{\bar{G}}{\bar{H}}.$$

We can now write the consumption-hours ratio as

$$\begin{aligned}
\frac{\bar{C}}{\bar{H}} &= \bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} - \delta\frac{\bar{K}}{\bar{H}} - (1-\chi)\left(\tau_H\bar{W} + \tau_K\bar{R}\frac{\bar{K}}{\bar{H}}\right) \\
&= \bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} - (\delta + (1-\chi)\tau_K\bar{R})\frac{\bar{K}}{\bar{H}} - (1-\chi)\tau_H\alpha\bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} \\
&= [1-(1-\chi)\tau_H\alpha]\bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} - (\delta + (1-\chi)\tau_K\bar{R})\frac{\bar{K}}{\bar{H}}.
\end{aligned}$$

Hence, the steady state of hours worked is given by

$$\begin{aligned}
\bar{H} &= \bar{B}\left(\frac{(1-\tau_H)\alpha\bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha}}{[1-(1-\chi)\tau_H\alpha]\bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} - (\delta + (1-\chi)\tau_K\bar{R})\frac{\bar{K}}{\bar{H}}}\right)^{\frac{\nu}{1+\nu}} \\
&= \bar{B}\left(\frac{(1-\tau_H)\alpha}{[1-(1-\chi)\tau_H\alpha] - (\delta + (1-\chi)\tau_K\bar{R})\bar{A}^{-1}\left(\frac{\bar{K}}{\bar{H}}\right)^{\alpha}}\right)^{\frac{\nu}{1+\nu}} \\
&= \bar{B}\left(\frac{(1-\tau_H)\alpha}{[1-(1-\chi)\tau_H\alpha] - [\delta/(\bar{R}+\delta) + (1-\chi)\tau_K(\bar{R}/(\bar{R}+\delta))](1-\alpha)}\right)^{\frac{\nu}{1+\nu}}
\end{aligned}$$

Log-Linear Approximation: Denote the percentage gap from the steady-state value of each variable by

$$\widehat{H}_t, \widehat{K}_{t+1}, \widehat{\lambda}_t, \widehat{C}_t, \widehat{Y}_t, \widehat{A}_t, \widehat{B}_t, \widehat{W}_t, \widehat{G}_t, \widehat{Z}_t, \widehat{R}_t.$$

We obtain the following equations:

$$\begin{aligned}
[\bar{R}/(\bar{R}+\delta)]\widehat{R}_t &= \widehat{A}_t + \alpha\widehat{H}_t - \alpha\widehat{K}_t \\
\widehat{W}_t &= \widehat{A}_t + (\alpha-1)\widehat{H}_t + (1-\alpha)\widehat{K}_t \\
\widehat{\lambda}_t &= -\widehat{C}_t + \widehat{Z}_t \\
\widehat{\lambda}_t &= \mathbb{E}_t[\widehat{\lambda}_{t+1} + (1-\beta)\widehat{R}_{t+1}] \\
\nu^{-1}\widehat{H}_t &= -\widehat{C}_t + \widehat{W}_t + (1+\nu^{-1})\widehat{B}_t \\
\bar{Y}\widehat{Y}_t &= \bar{C}\widehat{C}_t + \bar{K}\widehat{K}_{t+1} - (1-\delta)\bar{K}\widehat{K}_t + \bar{G}\widehat{G}_t \\
(1-\chi)\widehat{G}_t &= \frac{\tau_H\alpha[\widehat{W}_t + \widehat{H}_t] + \tau_K(1-\alpha)[\bar{R}/(\bar{R}+\delta)]\widehat{Y}_t}{\tau_H\alpha + \tau_K(1-\alpha)[\bar{R}/(\bar{R}+\delta)]} \\
\widehat{Y}_t &= \widehat{A}_t + \alpha\widehat{H}_t + (1-\alpha)\widehat{K}_t \\
\widehat{A}_t &= \rho_A\widehat{A}_{t-1} + \sigma_A\epsilon_{A,t} \\
\widehat{B}_t &= \rho_B\widehat{B}_{t-1} + \sigma_B\epsilon_{B,t} \\
\widehat{Z}_t &= \rho_Z\widehat{Z}_{t-1} + \sigma_Z\epsilon_{Z,t}.
\end{aligned}$$

If $\chi = 0$ then $\bar{G} = 0$ and we compute the level of government spending rather than percentage deviations from a steady state that is zero.

The return on capital R_t is before taxes and net of depreciation. We can define

$$R_t^\delta = R_t + \delta.$$

Its steady state is given by

$$\bar{R}^\delta = \frac{1/\beta - 1}{1-\tau_k} + \delta.$$

The steady state ratio can be expressed as

$$\frac{\bar{R}}{\bar{R}^\delta} = \frac{\bar{R}}{\bar{R}+\delta} = \frac{1/\beta - 1}{1/\beta - 1 + (1-\tau_K)\delta} = \frac{1-\beta}{1-\beta+\beta(1-\tau_K)\delta}.$$

In terms of percentage deviations from the steady state

$$\hat{R}_t^\delta = \frac{\bar{R}}{\bar{R}^\delta}\hat{R}_t.$$

Thus, the log-linearized equilibrium conditions involving R_t can be rewritten as

$$\begin{aligned}
\hat{R}_t^\delta &= \hat{A}_t + \alpha\hat{H}_t - \alpha\hat{K}_t \\
\hat{\lambda}_t &= \mathbb{E}_t\left[\hat{\lambda}_{t+1} + (1-\beta)\frac{\bar{R}^\delta}{\bar{R}}\hat{R}_t^\delta\right] \\
&= \mathbb{E}_t\left[\hat{\lambda}_{t+1} + (1-\beta[1-(1-\tau_K)\delta])\hat{R}_t^\delta\right].
\end{aligned}$$

In the procedure *dsgess*$(\cdot)$ the variable *rmallst* corresponds to $\bar{R}$ and *rst* corresponds to $\bar{R}^{\delta}$. In the procedure *dsgesolv*$(\cdot)$ the variable R corresponds to $\hat{R}^{\delta}_t$. The measurement equation is set up under the assumption that we observe R^{δ}_t.

C Welfare Measures

The social welfare is defined as:

$$\mathcal{W} = \int V(a,x) d\mu(a,x),$$

where $\mu(a,x)$ is the steady-state joint distribution of asset holdings and idiosyncratic productivity and $V(a,x)$ is the value function associated with the optimal decisions, i.e.,

$$V(a,x) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \bigg\{ \ln c(a_t,x_t) - B\frac{h(a_t,x_t)^{1+1/\gamma}}{1+1/\gamma} \bigg\}.$$

$c(a,x)$ and $h(a,x)$ are the optimal decision rules for an individual whose asset holdings are a and idiosyncratic productivity is x. This is a utilitarian social welfare function that measures the *ex ante* welfare in the steady state—i.e., the welfare of an individual before the realization of initial assets and productivity, which is drawn from the steady-state distribution $\mu(a,x)$. We measure the welfare gain or loss due to a policy change by the constant percentage change in consumption each period for all individuals which is required to equate the social welfare before and after the policy change. Specifically, we compute Δ that solves

$$\begin{aligned}\int \Bigg\{ \mathbb{E}_0 \Bigg[\sum_{t=0}^{\infty} \beta^t \bigg\{ \ln\big((1+\Delta)c_0(a_t,x_t)\big) - B\frac{h_0(a_t,x_t)^{1+1/\gamma}}{1+1/\gamma} \bigg\} \Bigg] \Bigg\} d\mu_0(a_t,x_t) \\ = \int \Bigg\{ \mathbb{E}_0 \Bigg[\sum_{t=0}^{\infty} \beta^t \bigg\{ \ln c_1(a_t,x_t) - B\frac{h_1(a_t,x_t)^{1+1/\gamma}}{1+1/\gamma} \bigg\} \Bigg] \Bigg\} d\mu_1(a_t,x_t)\end{aligned}$$

where c_0, h_0, and μ_0 are consumption, labor supply, and steady-state distribution before the policy change and c_1, h_1, and μ_1 are those after the policy change. A positive Δ implies that average welfare improves upon a policy change. With the logarithmic utility, the welfare gain Δ can be expressed as

$$\Delta = \exp\big((\mathcal{W}_1 - \mathcal{W}_0)(1-\beta)\big) - 1,$$

where $\mathcal{W}_0$ and $\mathcal{W}_1$ represent social welfare before and after the policy change, respectively. In the representative-agent model the distribution $\mu(a, x)$ is degenerate and the computation of the welfare effect simplifies considerably:

$$\Delta = \exp\left(\ln\left(\bar{C}_1/\bar{C}_0\right) - B\frac{\bar{H}_1^{1+1/\gamma} - \bar{H}_0^{1+1/\gamma}}{1+1/\gamma}\right) - 1,$$

where $\bar{C}_0$ and $\bar{H}_0$ are the steady-state values of consumption and labor supply in the benchmark economy, while $\bar{C}_1$ and $\bar{H}_1$ are those in an economy with a different policy.

D Structural Break Tests

To examine the detectability of coefficient changes across policy regimes we conduct the following experiment. Suppose an econometrician has access to 100 observations from the benchmark policy regime as well as 100 observations from one of the following alternative regimes: labor tax cut, capital tax raise, and more transfers. The econometrician knows the "true" policy coefficients for the benchmark and the alternative policy regime and estimates two versions of the representative agent model.

In the first version, $\mathcal{M}_0$, the non-policy parameters are assumed to be identical across regimes, whereas in the second version the non-policy parameters are allowed to differ. The second version of the model, $\mathcal{M}_1$, is estimated under a prior distribution that restricts potential changes in the non-policy parameters to be small. Let

$$r_A,\ \nu,\ \ln \bar{A},\ \ln \bar{B},\ \rho_A,\ \rho_B,\ \sigma_A,\ \sigma_B,\ \sigma_\zeta$$

denote the non-policy parameters under the benchmark regime. Then the parameters under the alternative policy regime are given by

$$r_A e^{\delta_r},\ \nu e^{\delta_\nu},\ \ln \bar{A} + \delta_A,\ \ln \bar{B} + \delta_B,\ \Phi(\Phi^{-1}(\rho_A) + \delta_{\rho_A}),\ \Phi(\Phi^{-1}(\rho_B) + \delta_{\rho_B}),\ \sigma_A e^{\delta_{\sigma_A}},\ \sigma_B e^{\delta_{\sigma_B}},\ \sigma_\zeta e^{\delta_{\sigma_\zeta}}.$$

Here we use $\Phi(\cdot)$ to denote the cumulative density function of a standard normal random variable. Note that for $\delta = 0$ the parameters are identical across regimes. According our prior, all δ's are independent. Moreover, δ_A and δ_B are normally distributed according to $N(0, 0.05^2)$. The prior for the remaining discrepancies is $N(0, 0.1^2)$. The following table provides the log marginal likelihood values for the specifications $\mathcal{M}_1$ and $\mathcal{M}_2$:

Policy Change	$\mathcal{M}_0$	$\mathcal{M}_1$
None	2796.19	2789.68
Labor Tax Cut	2724.08	2787.15
Capital Tax Raise	2728.52	2724.03
More Transfers	2753.42	2801.42

If the alternative policy is either a labor tax cut or and increase in transfer, the switching coefficient model $\mathcal{M}_1$ is favored by the posterior odds. If the alternative policy is a capital tax raise, the constant coefficient model is preferred. These results are consistent with our earlier result that the representative agent model delivers relatively accurate predictions of the effects of a capital tax change, but has difficulties capturing labor market effects.

E Additional Tables and Figures

Table E-1 compares the quintiles of the wealth distribution in the U.S. data (Panel Study of Income Dynamics, PSID) to the quintiles of the wealth distribution in the data simulated from the heterogenous agent economy under the benchmark calibration. Family wealth in the PSID reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. For each quintile group of the wealth distribution, we calculate the wealth share, ratio of group average to economy-wide average, and the earnings share. The household sample in the PSID cannot capture the right tail of the wealth distribution of the U.S. economy. Despite this shortcoming, the wealth share held by the top 20% of the distribution in the PSID, 76.2%, is fairly close to that in the Survey of Consumer Finance (SCF), 79.6%. See Chang and Kim (2006) for the detailed comparison of the wealth distributions between the PSID and SCF.

Table E-2 compares second moments of selected U.S. post-war time series to moments of the corresponding series in the simulated data from the heterogeneous agent economy. Data definitions for the U.S. time series are provided in Section B of this Appendix. Since the representative-agent model accommodates a deterministic balanced-growth path, we remove a linear trend from the U.S. time series of log output and consumption. Since the model economy allows for an aggregate productivity shock only and our calibration of the technology shock probably underestimates its variability, the aggregate output of the model exhibits only about three-quarters of the volatility of actual output. Consumption is as volatile as that in the data. A striking difference is the standard deviation of hours. It is three times more volatile in the actual data than it is in the simulated data. This is in part due to the low-frequency movement in labor supply, not captured in the model economy. In fact, the volatility of hours in the model-generated data is about half as volatile as the standard deviation of actual Hodrick-Prescott-filtered hours, which removes the low frequency variation. Output, consumption, and hours are all positively correlated. The correlations between output and hours as well as between consumption and hours are slightly stronger in the simulated data than they are in the U.S. data.

Table E-3 displays posterior estimates for the parameters of the representative agent model obtained from U.S. data. We remove a linear trend from the output and consumption data, normalize mean output such that it corresponds to mean output in the heterogeneous agents economy, and adjust the level of consumption such that we maintain the average consumption-output ratio in the U.S. data. It turns out that the estimated aggregate labor supply elasticity ($\hat{\nu} = 0.38$) based on U.S. data is much smaller than the estimates obtained from the simulated data.[15] Two salient features of the aggregate labor market of the U.S. economy are a high volatility of quantities (hours) relative to prices (productivity) and a lack of systematic correlation between hours and productivity. These features lead to estimates that imply a low aggregate labor supply elasticity and fairly large preference shocks. A variance decomposition based on the estimated (with U.S. data) DSGE model parameters implies that almost all of the variation in hours worked is due to preference shocks.

Figure E-1 plots time series of U.S. labor income and capital tax rates.

Table E-4 provides a variance decomposition of output, consumption, and hours based on the representative agent model that is estimated with data from the heterogeneous agent economy.

In order to shed light on how policy changes affect the aggregate labor supply estimates, Figure E-2 depicts pseudo aggregate labor supply schedules based on the steady-state reservation wage distribution, i.e., the inverse function of the cumulative reservation wage distribution, for the various fiscal policy regimes. Each curve represents the employment rate (on the x-axis) at a given wage rate (y-axis). The vertical line denotes the steady-state level of

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[15] A more detailed empirical analysis based on post-war U.S. data can be found in Rios-Rull, Schorfheide, Fuentes-Albero, Kryshko, and Santaeulalia-Llopis (2009).

employment under each policy regime. The panels of Figure E-2 illustrate how the elasticity around each steady state varies with the fiscal policy. The aggregate labor supply schedule in the heterogeneous-agents economy becomes steeper toward the full employment level, as the economy moves toward the right tail of the reservation wage distribution. This pattern is mirrored in the labor-supply elasticity estimates generated with the representative-agent model.[16]

Tables E-5 to E-8 provide results that are obtained when the real interest rate is used as an observable.

Additional References

Rios-Rull, Jose-Victor, Frank Schorfheide, Cristina Fuentes-Albero, Maxym Kryshko, and Raul Santaeulalia-Llopis (2009): "Methods versus Substance: Measuring the Effects of Technology Shocks," *NBER Working Paper*, **15375**.

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[16] The representative-agent-based estimate of the labor supply elasticity is not identical to the slope of the reservation wage distribution in the heterogeneous-agents economy. The calculation based on the slope of the reservation wage distribution assumes that the entire wealth-earnings distribution remains unchanged, whereas the aggregate productivity shock shifts the wealth-earnings distribution over time.

Table E-1: Characteristics of Wealth Distribution

	Quintile					
	1st	2nd	3rd	4th	5th	Total
PSID						
Share of wealth	-.52	.50	5.06	18.74	76.22	100
Group average/population average	-.02	.03	.25	.93	3.81	1
Share of earnings	7.51	11.31	18.72	24.21	38.23	100
Benchmark Model						
Share of wealth	-1.56	3.27	11.38	24.74	62.17	100
Group average/population average	-.08	.16	.57	1.24	3.11	1
Share of earnings	9.74	15.76	19.97	23.72	30.81	100

Notes: The PSID statistics reflect the family wealth and earnings levels published in the 1984 survey. Family wealth in the PSID reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets.

Table E-2: SECOND MOMENTS OF SIMULATED AND U.S. DATA

	Model 3000 obs.	U.S. Data 1964-2006
$\sigma(\ln Y)$	.033	.041
$\sigma(\ln C)$	.020	.021
$\sigma(\ln H)$	.013	.042
$\sigma((\ln H)_{HP})$	.007	.018
$\text{corr}(\ln Y, \ln C)$	0.84	0.83
$\text{corr}(\ln Y, \ln H)$	0.80	0.56
$\text{corr}(\ln C, \ln H)$	0.37	0.51

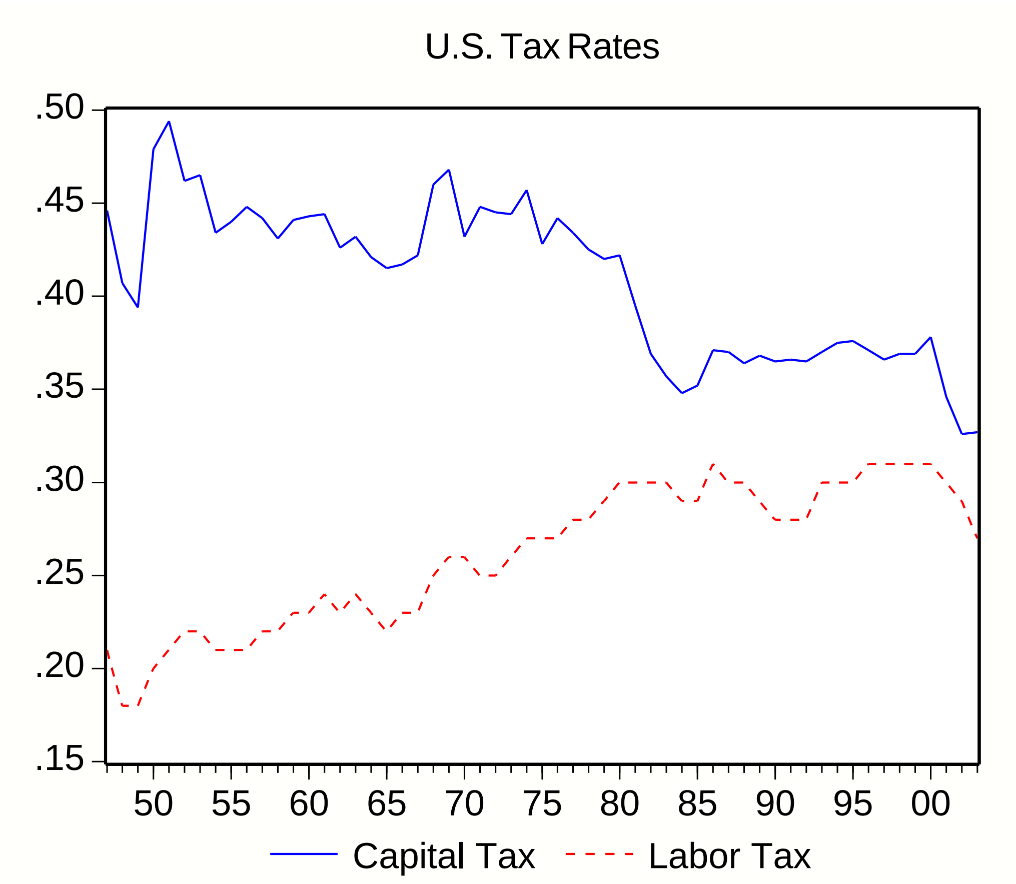
Notes: $\sigma(\cdot)$ is sample standard deviation, $\text{corr}(\cdot)$ is sample correlation, and $(\ln H)_{HP}$ denotes HP-filtered (smoothing parameter 1,600) log hours. Unless noted otherwise, we extract a linear trend from the U.S. data before computing the sample moments.

Table E-3: PARAMETER ESTIMATES OBTAINED FROM U.S. DATA

		Prior		Posterior	
	Domain	Mean	S.D.	Mean	90% Intv
r_A	Gamma	4.00	2.00	7.18	[5.60, 9.11]
ν	Gamma	1.00	0.50	0.38	[0.14, 0.61]
$\ln \bar{A}$	Normal	0.00	10.0	0.60	[0.58, 0.63]
$\ln \bar{B}$	Normal	0.00	10.0	-1.49	[-1.57, -1.42]
ρ_A	Beta	0.50	0.25	0.97	[0.95, 0.99]
ρ_B	Beta	0.50	0.25	0.98	[0.97, 0.99]
σ_A	Inv. Gamma	.012	.007	.006	[.006, .007]
σ_B	Inv. Gamma	.012	.007	.007	[.007, .008]
σ_Z	Inv. Gamma	.012	.007	.019	[.010, .029]

Notes: The following parameters are fixed during the estimation: $\delta = 0.025$, $\rho_Z = 0.99$, $\tau_H = 0.2$, $\tau_K = 0.2$, and $\chi = 0.5$. r_A is the annualized discount rate $r_A = 400 \times (1/\beta - 1)$. The estimation sample ranges from 1965:Q1 to 2006:Q4 ($T = 168$).

Figure E-1: U.S. Capital and Labor Tax Rates



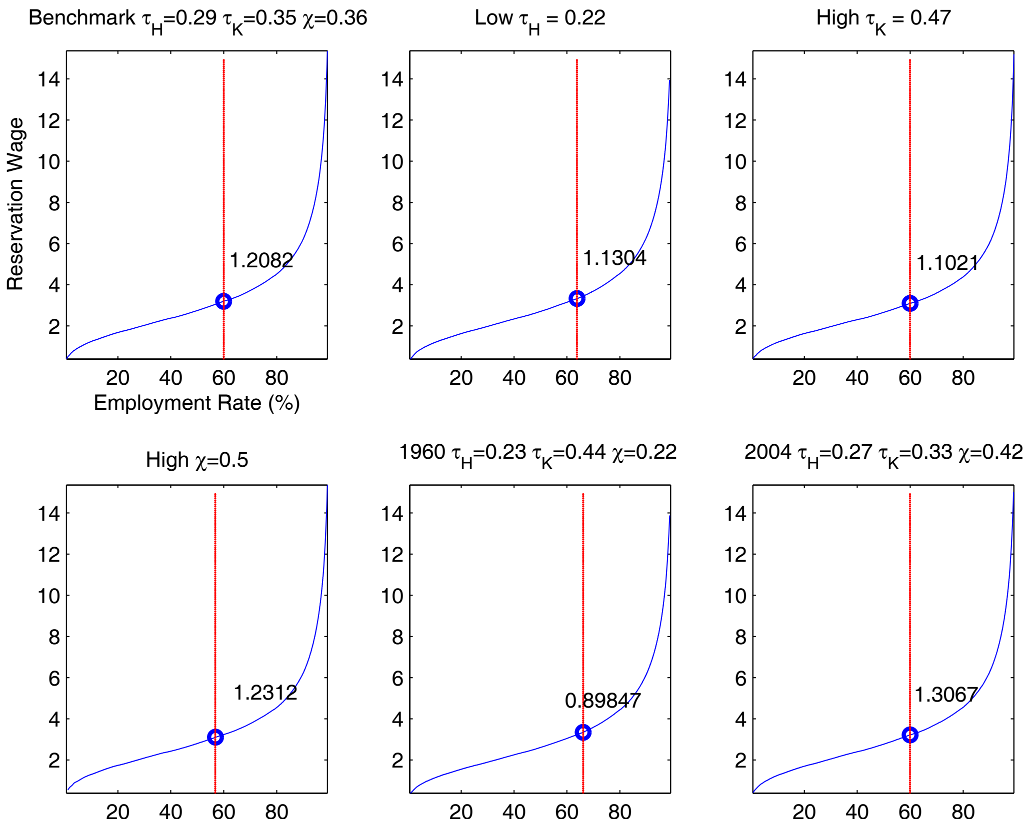
Notes: The data are taken from Chen, Imrohoroglu, and Imrohoroglu (2009).

Table E-4: RELATIVE IMPORTANCE OF PREFERENCE SHOCKS

	B		Z	
	Mean	90% Intv.	Mean	90% Intv.
Benchmark Economy, $T = 200$				
Output	5	[2, 8]	5	[4, 6]
Consumption	3	[0, 7]	6	[4, 7]
Hours	33	[18, 45]	5	[3, 7]
Benchmark Economy, $T = 2,500$				
Output	9	[8, 10]	5	[4, 5]
Consumption	9	[8, 10]	4	[4, 5]
Hours	43	[41, 46]	4	[4, 4]
U.S. Data				
Output	43	[20, 68]	13	[3, 24]
Consumption	46	[20, 75]	10	[3, 18]
Hours	98	[96, 99]	1	[0, 3]

Notes: The entries correspond to percentages.

Figure E-2: Aggregate Labor Supply based on Reservation Wage Distribution



Notes: Each curve represents the employment rate (on the x-axis) at a given wage rate (y-axis). The vertical line denotes the steady-state level of employment under the benchmark and the no-transfer policy regimes. The numbers in the plots indicate the elasticity of employment with respect to wages around the steady-state employment rate.

Table E-5: ESTIMATES UNDER ALTERNATIVE POLICIES: $H - C - R$ DATA SET

	Bench-mark	Labor Tax Cut	Capital Tax Raise	More Transfers	1960 Policy	2004 Policy
	Parameter Estimates, $T = 200$					
r_A	2.63	2.42	2.55	2.66	2.37	2.59
	[2.56, 2.71]	[2.33, 2.52]	[2.44, 2.64]	[2.60, 2.74]	[2.28, 2.46]	[2.50, 2.68]
ν	1.73	1.13	1.69	2.67	1.07	1.71
	[1.40, 2.05]	[0.92, 1.33]	[1.34, 2.03]	[2.09, 3.28]	[0.86, 1.27]	[1.37, 2.05]
$\ln \bar{A}$	0.44	0.41	0.44	0.47	0.40	0.44
	[0.44, 0.45]	[0.41, 0.42]	[0.44, 0.45]	[0.46, 0.47]	[0.39, 0.41]	[0.44, 0.45]
$\ln \bar{B}$	-1.43	-1.43	-1.43	-1.42	-1.42	-1.43
	[-1.44, -1.42]	[-1.44, -1.42]	[-1.45, -1.42]	[-1.44, -1.41]	[-1.43, -1.41]	[-1.45, -1.42]
ρ_A	0.90	0.95	0.93	0.92	0.95	0.94
	[0.89, 0.91]	[0.94, 0.95]	[0.92, 0.93]	[0.92, 0.93]	[0.94, 0.95]	[0.93, 0.94]
ρ_B	0.84	0.91	0.89	0.90	0.92	0.93
	[0.75, 0.91]	[0.88, 0.94]	[0.83, 0.93]	[0.89, 0.92]	[0.90, 0.94]	[0.90, 0.95]
σ_A	.005	.006	.006	.005	.006	.006
	[.005, .006]	[.006, .006]	[.005, .006]	[.005, .006]	[.005, .006]	[.005, .006]
σ_B	.003	.003	.003	.003	.003	.003
	[.003, .003]	[.002, .003]	[.003, .003]	[.003, .003]	[.002, .003]	[.003, .003]
σ_ζ	.003	.003	.003	.002	.002	.003
	[.002, .003]	[.002, .003]	[.003, .004]	[.002, .002]	[.002, .002]	[.002, .003]

Notes: The following parameters are fixed during the estimation of the representative-agent model: τ_H, τ_K, χ, $\delta = 0.025$, $\rho_Z = 0.99$. r_A is the annualized discount rate $r_A = 400 \times (1/\beta - 1)$. As parameter estimates we report posterior means and 90% credible intervals (in brackets).

Table E-6: PREDICTIONS OF POLICY EFFECTS, $T = 200$: $H - C - R$ DATA SET

		Labor Tax Cut	Capital Tax Raise	More Transfers	1960 Policy	2004 Policy
H	"True"	6.06	-0.23	-5.45	9.44	-0.21
	90% Intv.	[2.78, 3.21]	[-0.34, -0.29]	[-3.40, -2.95]	[4.80, 5.52]	[-0.22, -0.19]
	Score	8.8E-127	3.9E-010	2.3E-063	1.1E-083	2.8E-001
C	"True"	7.33	-2.73	3.04	1.73	3.86
	90% Intv.	[7.44, 7.86]	[-3.45, -3.29]	[1.63, 2.08]	[2.23, 2.96]	[3.57, 3.61]
	Score	5.8E-003	1.3E-040	1.2E-018	6.8E-005	9.9E-133
Y	"True"	3.44	-2.89	-2.19	2.57	0.81
	90% Intv.	[2.78, 3.21]	[-3.94, -3.80]	[-3.40, -2.95]	[2.15, 2.87]	[0.33, 0.37]
	Score	2.7E-004	3.7E-114	1.6E-013	4.2E-001	2.2E-308

Notes: The benchmark policy is $\tau_H = 0.29$, $\tau_K = 0.35$, $\chi = 0.36$. The entries in the table refer to percentage changes relative to the benchmark policy. "True" effects are computed from the means of the ergodic distributions of the heterogeneous-agents economy. 90% Intv. are predictive intervals computed from the posterior of the representative-agent model based on observations under the benchmark policy.

Table E-7: ESTIMATES UNDER ALTERNATIVE POLICIES: $Y - H - R$ DATA SET

	Bench-mark	Labor Tax Cut	Capital Tax Raise	More Transfers	1960 Policy	2004 Policy
	Parameter Estimates, $T = 200$					
r_A	2.56	2.37	2.47	2.59	2.29	2.52
	[2.45, 2.67]	[2.25, 2.49]	[2.34, 2.61]	[2.49, 2.69]	[2.16, 2.42]	[2.40, 2.64]
ν	2.79	1.55	3.01	3.71	1.65	2.56
	[2.14, 3.46]	[1.20, 1.89]	[2.14, 3.85]	[2.65, 4.75]	[1.23, 2.06]	[1.90, 3.17]
$\ln \bar{A}$	0.45	0.42	0.45	0.47	0.41	0.45
	[0.42, 0.47]	[0.39, 0.44]	[0.42, 0.47]	[0.45, 0.49]	[0.38, 0.43]	[0.42, 0.47]
$\ln \bar{B}$	-1.40	-1.41	-1.40	-1.40	-1.40	-1.41
	[-1.42, -1.38]	[-1.43, -1.39]	[-1.42, -1.37]	[-1.42, -1.37]	[-1.41, -1.38]	[-1.43, -1.38]
ρ_A	0.98	0.98	0.98	0.98	0.98	0.98
	[0.98, 0.98]	[0.97, 0.98]	[0.98, 0.98]	[0.98, 0.98]	[0.98, 0.98]	[0.98, 0.98]
ρ_B	0.97	0.98	0.98	0.98	0.98	0.98
	[0.97, 0.98]	[0.97, 0.98]	[0.98, 0.99]	[0.97, 0.98]	[0.97, 0.99]	[0.98, 0.99]
σ_A	.006	.006	.006	.005	.006	.006
	[.005, .006]	[.006, .007]	[.005, .006]	[.005, .006]	[.006, .007]	[.005, .006]
σ_B	.003	.003	.003	.003	.003	.003
	[.003, .003]	[.002, .003]	[.003, .003]	[.003, .004]	[.002, .003]	[.003, .003]
σ_ζ	.003	.003	.004	.002	.003	.003
	[.003, .003]	[.003, .003]	[.003, .004]	[.002, .002]	[.003, .003]	[.003, .003]

Notes: The following parameters are fixed during the estimation of the representative-agent model: τ_H, τ_K, χ, $\delta = 0.025$, $\rho_Z = 0.99$. r_A is the annualized discount rate $r_A = 400 \times (1/\beta - 1)$. As parameter estimates we report posterior means and 90% credible intervals (in brackets).

Table E-8: PREDICTIONS OF POLICY EFFECTS, $T = 200$: $Y - H - R$ DATA SET

		Labor Tax Cut	Capital Tax Raise	More Transfers	1960 Policy	2004 Policy
H	"True"	6.06	-0.23	-5.45	9.44	-0.21
	90% Intv.	[3.24, 3.71]	[-0.40, -0.35]	[-3.94, -3.45]	[5.61, 6.41]	[-0.26, -0.22]
	Score	5.8E-074	4.5E-020	1.1E-031	4.5E-045	1.0E-003
C	"True"	7.33	-2.73	3.04	1.73	3.86
	90% Intv.	[7.92, 8.38]	[-3.46, -3.24]	[1.09, 1.58]	[3.09, 3.90]	[3.52, 3.57]
	Score	2.9E-009	2.1E-020	1.9E-030	3.5E-013	1.1E-120
Y	"True"	3.44	-2.89	-2.19	2.57	0.81
	90% Intv.	[3.24, 3.71]	[-3.96, -3.76]	[-3.94, -3.45]	[3.00, 3.80]	[0.28, 0.33]
	Score	3.8E-001	3.2E-057	3.8E-024	2.8E-004	6.3E-268

Notes: The benchmark policy is $\tau_H = 0.29$, $\tau_K = 0.35$, $\chi = 0.36$. The entries in the table refer to percentage changes relative to the benchmark policy. "True" effects are computed from the means of the ergodic distributions of the heterogeneous-agents economy. 90% Intv. are predictive intervals computed from the posterior of the representative-agent model based on observations under the benchmark policy.