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Financial Markets and Fluctuations in Uncertainty*

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ABSTRACT

Researchers have documented that in the recent financial crisis the large decline in economic activity and credit has been accompanied by a large increase in the dispersion of growth rates across firms. We build a quantitative general equilibrium model in which financial frictions interact with increases in uncertainty at the firm level to generate a contraction in economic activity and a large increase in the dispersion of growth rates across firms. We find that our model can generate about 67% of the decline in output of the Great Recession of 2007. A promising feature of our model is that it generates large labor wedges, a feature of the recent data on business cycles.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

The recent financial crisis has been accompanied by severe contractions in economic activity and credit as well as large increases in the cross section dispersion of firm growth rates (Bloom, Floetotto and Jaimovich 2009). Motivated by these observations we build a quantitative general equilibrium model with heterogeneous firms and financial frictions in which increases in uncertainty at the firm level lead to an increase in the cross section dispersion of firm growth rates and a contraction in economic activity.

The key idea in the model is that hiring inputs to produce output is a risky endeavor. The reason is that there is a separation between the time firms hire inputs to produce and the time they receive the revenues from their sales of that production. By hiring inputs, the firm takes on financial obligations to pay for those inputs. Because of this separation any idiosyncratic shocks that occur between time of production and the receipt of revenues, such as demand shocks, makes hiring inputs risky. When financial markets are imperfect, firms have only limited means to insure against such shocks and hence they must bear this risk. This risk has real consequences if when firms cannot meet their financial obligations they must experience a costly default. In such an environment, an increase in the uncertainty arising from an increase in the volatility of idiosyncratic shocks leads firms to pull back on their hiring of inputs.

We quantify our model and ask, Can an increase in the volatility of firm level idiosyncratic shocks that generates the observed increase in the cross section dispersion in the recent recession lead to a sizable contraction in economic activity? We find that our model can generate about 67% of the decline in output seen in the data. More generally, we find that the model generates labor fluctuations that are large relative to those in output. Generating such a pattern has been a major goal of the business cycle literature.

Recently, Chari, Kehoe and McGrattan (2007) have decomposed business cycles into the components due to various wedges and have argued that labor wedges can account for 2/3 of the fluctuations in output. A striking feature of the Great Recession that started in 2007 is that it was associated mainly with a worsening of the labor wedge rather than a large fall in productivity. A promising feature of our model is that it can generate a substantial worsening of the labor wedge during the episode. More generally, in our model with financial frictions changes in the volatility of firm level shocks manifest themselves in aggregate data

as movements in the labor wedge.

Our model has a continuum of heterogeneous firms that produce differentiated products. The demand for these products is subject to idiosyncratic shocks. The volatility of demand shocks are stochastically time-varying and these *volatility* shocks are the only aggregate shocks in the economy. A continuum of identical households supply labor to firms and lend to firms using uncontingent debt through financial intermediaries.

The model has three key ingredients. First, firms hire their inputs, here labor, and produce before they know their demand. Second, financial markets are imperfect in that the loans to firms cannot depend on their shocks and firms default if they have insufficient funds to pay for their debt. Third, since firms must pay a fixed cost to enter, in equilibrium they make positive expected profits in each period that they do not default. The cost of default is the loss of future expected profits.

Given these ingredients, when firms choose their inputs they face a trade off between expected return and risk. As firms increase their employment they increase the expected return conditional on not defaulting but they also increase the probability of default. For a given variance of demand idiosyncratic shocks they choose their optimal employment to balance off the increase in expected return against the losses from default. The potential losses from default is an extra cost of increasing labor and acts as a labor wedge in the firm's first order condition. When the variance of the idiosyncratic shocks increases, at a given level of employment, the probability of default increases, and thus so does the labor wedge. In equilibrium, in the face of such an increase in variance, firms become more cautious and decrease employment. At the aggregate level, these firm level responses imply that when the dispersion of idiosyncratic shocks increases, aggregate output and employment both fall.

The result that firms decrease employment when the variance of demand shocks increase depends critically on our assumption of imperfect financial markets. If firms had access to complete financial markets there is no tradeoff between expected return and default risk. Thus, an increase in the variance of these shocks leads to no change in their employment since firms simply restructure the pattern of payments across states so that they never default.

In our model firms only have access to uncontingent debt and can default. They optimally time the purchases and sales of such debt to help meet their financial obligations

in the presence the stochastic revenue stream generated by the demand shocks. In this sense, firms have a precautionary motive to use debt to self-insure. Since firms have only limited means to repay their debt they face upward-sloping interest rate schedules and a *credit limit*, namely the maximum amount they can borrow. Firms typically maintain a *buffer stock* of unused credit. By running up and down this buffer stock firms can partially dampen the fluctuations in labor input in response to volatility shocks.

We consider a quantitative version of the model in which we choose the parameters of idiosyncratic firm demand shock process so that the model produces the time variation in cross section dispersion of the growth rate of sales observed in a panel of Compustat firms. To illustrate the workings of the model we consider the impulse response after an increase in the volatility of firm level shocks. When the volatility shock hits, firms pull back on their employment and their debt to avoid default and, in equilibrium, leave the default rate constant. This increase in volatility leads firms' credit limits to tighten, which in turn tends to amplify the reduction in employment. Since this increase in volatility leads to greater distortions in firms' employment decisions, it makes entry by firms less attractive and hence entry falls.

The pattern of both financial responses and employment to this increase in volatility is heterogeneous across firms. Firms with relatively low demand shocks and high existing debt run their buffer stocks down to zero and decrease their employment the most. At the same time, firms with higher levels of the demand shock tend to increase their buffer stocks and decrease their employment less. These heterogeneous financial responses in the aggregate result in both a large increase in the fraction of firms with zero buffer and at the same time an increase in the aggregate buffer stock. In this sense, our model simultaneously produces tighter credit market conditions in which more firms are constrained in their borrowing while in the aggregate firms are sitting on larger buffers.

We then show that if the volatility shocks in the model are set to reproduce the observed increase in the dispersion of cross-section growth rates, the model can account for 67% of the fall in output and 73% of the fall in employment in the Great Recession.

In terms of the literature, this work is related to a growing literature that studies time varying volatility. Bloom, Floetotto, and Jaimovich (2009) and Bloom (2009) show that in

the presence of adjustment costs, firms drop their investment and hiring when hit by a high uncertainty shock. Other papers that study the effects of uncertainty on investment in the presence of adjustment costs include Bernanke (1983), Abel and Eberly (1996), and Caballero and Engel (1999). A key difference between our approach and that of Bloom et al. is that in our work the financial frictions manifest themselves as labor wedges and the fixed cost frictions in the Bloom et al. paper manifest themselves as TFP shocks. Christiano, Motto, and Rostagno (2009) also explore the business cycle implications of uncertainty shocks. They show that in a DSGE model with nominal rigidities and financial frictions uncertainty shocks to investment account for a significant portion of the fluctuations in output.

Our work is also related to the work on heterogenous firms and financial frictions. For example, Cooley and Quadrini (2001) develop a model of heterogenous firms with incomplete financial markets and default risk and explore its implications for the dynamics of firms investment growth and exit. A series of papers have used similar heterogenous firm models to help account for the relation between financial frictions and the level of development (see, for example, the work of Buera and Shin (2008) and Buera, Kaboski, and Shin (forthcoming)). Most related to our work is Gilchrist et al. (2010) who study the implications of time varying volatility for firms investment in an environment with financial frictions. They find that high volatility hinders firms' ability to invest and worsens the allocation of capital which generates a decline in measured TFP.

In our model volatility shocks lead credit constraints to endogenously tighten. Recently, a literature has developed business cycle models in which the exogenous shock is directly to the credit constraint. See for example, the work of Guerrieri and Lorenzoni (2010), Perri and Quadrini (2011), and Jermann and Quadrini (2012). This approach is complementary to our approach.

1. Model

Consider a dynamic model of a continuum of identical households, a continuum of heterogeneous intermediate goods firms, final goods firms, and financial intermediaries. The households have preferences over consumption and leisure, they provide labor services to intermediate goods firms and lend to these firms through the financial intermediaries.

The households own all firms and pay lump sum taxes. The final goods firms are competitive and have a technology that converts intermediate goods into a final good. This technology is subject to idiosyncratic shocks, referred to as *demand* shocks, which affect the relative demand of the final goods firms for different types of intermediate goods.

The monopolistically competitive intermediate goods firms pay a fixed entry cost and then produce differentiated products using labor. The shocks to the final goods firms' technology makes the demand for their good stochastic. The intermediate goods firms can only borrow state-uncontingent debt and hence cannot insure away their fluctuations in demand that they face. These are allowed to default and if they do they exit.

The volatility of demand shocks are stochastically time-varying and these *volatility* shocks are the only aggregate shocks in the economy.

The timing of decisions is as follows. In the beginning of each period households decide on the amount of labor to supply to intermediate goods firms. The wage rate is determined so that the labor market clears and production of intermediate goods takes place. Next, the current demand and volatility shocks are realized. After the shocks are realized all other decisions are taken simultaneously. The intermediate goods firms make all their decisions: they set their prices, sell their products to final goods firms, pay their workers, choose whether to repay their existing debts to financial intermediaries, distribute dividends, and choose new borrowing and a plan for employment. The final goods firms buy the intermediate goods and sell their final good to households and new entrants. New entrants decide on entry and buy some final goods in order to pay their entry costs. Households consume, receive payments on existing funds lent to intermediaries, and lend new funds to intermediaries.

A. Intermediate Goods and Final Goods Firms

Intermediate goods firms produce differentiated products that are subject to idiosyncratic demand shocks z_t which follow a Markov process with transition function $\pi_z(z_t|z_{t-1}, \sigma_{t-1})$ where σ_{t-1} is an aggregate shock to the standard deviation of idiosyncratic demand shocks. The aggregate shock σ_t follows a Markov process with transition function $\pi_\sigma(\sigma_t|\sigma_{t-1})$.

These firms are monopolistically competitive and produce at the beginning of the period before the idiosyncratic demand shocks and the aggregate shock are realized. Firms have

access to one period debt contracts and enter period t with debt b_t . They then produce output y_t using the technology $y_t = \ell_t^\alpha$ where ℓ_t is the labor input. After production, demand shocks are realized. At this point, each firm is indexed by *idiosyncratic state* $x_t = (\ell_t, b_t, z_t)$ which records their labor input used in production, their debt due, and their current idiosyncratic demand shock. We let Υ_t denote the measure of firms indexed across x_t .

At this stage, the aggregate state for the economy S_t includes the beginning of period aggregate state S_{bt} together with the current aggregate shock σ_t , so that $S_t = (S_{bt}, \sigma_t)$. As we explain below the beginning-of-period aggregate state $S_{bt} = (\sigma_{t-1}, \Upsilon_t, B_t)$. This state records the beginning of the period information on aggregate shocks, namely σ_{t-1} , the measure of firms Υ_t , and the contingent assets B_t of consumers discussed below. We find it convenient to record the shock z_t in the beginning-of-period aggregate state even though an individual firm's z_t is not realized until the middle of the period. This is permissible since there is a continuum of firms of each type (ℓ_t, b_t) at the beginning of the period, so the fraction of these firms that will receive experience each level of z_t is known.

Final goods firms buy the products from intermediate goods firms. The final good is used for consumption and to pay the fixed cost of starting a new firm. The final good Y_t is produced from the intermediate goods $y_t(x)$ via the technology

$$(1) \quad Y_t \leq \left(\int z y_t(x)^{\frac{\gamma-1}{\gamma}} d\Upsilon_t(x) \right)^{\frac{\gamma}{\gamma-1}}.$$

where $\gamma > 1$ is the elasticity of substitution across goods and z is an element of the firm state $x = (\ell, b, z)$. The final goods firms choose the intermediate goods $\{y_t(x)\}$ to solve

$$(2) \quad \max_{\{y_t(x)\}} Y_t - \int_x p_t(x) y_t(x) d\Upsilon_t(x)$$

subject to (1), where $p_t(x)$ is the price of good x relative to the aggregate price index. This problem yields that the demand $y_t(x)$ for any good with idiosyncratic state $x = (\ell, b, z)$ and price $p_t(x)$ is

$$(3) \quad y_t(x) = \left(\frac{z}{p_t(x)} \right)^\gamma Y_t$$

where $Y_t = Y(S_t) = \left(\int z y_t(x)^{\frac{\gamma-1}{\gamma}} d\Upsilon_t(x) \right)^{\frac{\gamma}{\gamma-1}}$.

Let us turn now to the details of the problem faced by intermediate goods firms. These firms have access to one period debt in the form of discount bonds that are not contingent on either the idiosyncratic or the aggregate shocks. After shocks are realized each firm decides on the price of its product. They also decide on whether to repay their debt or default, denoted $\phi = 1$ or $\phi = 0$ respectively. Firms that repay continue while firms that default exit.

Continuing firms choose new debt contracts and its labor input ℓ_{t+1} .¹ The debt contract pays off b_{t+1} at $t + 1$ as long the firm chooses not to default at $t + 1$ and gives the firm $q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1}$ at t . The price $q(S_t, z_t, \ell_{t+1}, b_{t+1})$ reflects the compensation for the loss in case of default and depends on the current aggregate state S_t , the firms' current idiosyncratic shock z_t and decisions of the firm, namely, its labor input ℓ_{t+1} and its borrowing level b_{t+1} .

The dividends d_t for a continuing firm are restricted to be nonnegative

$$(4) \quad d_t = p_t \ell_t^\alpha - w(S_{bt}) \ell_t - b_t + q(S_t, z_t, \ell_{t+1}, b_{t+1}) b_{t+1} \geq 0.$$

Here p_t is the relative price of this firm's product relative to the aggregate price index which is the numeraire of this economy.

In our model firms only default if they are forced to do so because their budget set is empty. For a firm in state (x_t, S_t) , the budget set is defined as $\Gamma(x_t, S_t) = \{d_t \mid d_t \geq 0\}$, where d_t is given by (4). Clearly, firms with large enough debt have an empty budget set which forces them to default. That is, given the bond price schedules $q(S_t, z_t, \ell_{t+1}, b_{t+1})$ for new

¹In this timing, when firms are borrowing b_{t+1} at the end of period t they commit to their plan of production ℓ_{t+1} for period $t + 1$. This production actually occurs in period $t + 1$. An alternative timing is one without commitment in which firms borrow b_{t+1} at the end of period t and then are free to choose whatever production level they want at the beginning of period $t + 1$, before the shocks are realized. Notice that in both scenarios there are no shocks realized in between the end of period $t + 1$ and the beginning of period t so the only difference is the differences in implied commitment. Here the firms prefer the commitment outcomes because under commitment the firms confront price schedules that reward them for choosing less aggressive labor schedules. Mechanically, with commitment when firms choose their labor take the derivative of the price schedule for bonds. When firms do not have commitment then at the beginning of period $t + 1$ they have already received the proceeds of the bond sales and when they choose their labor they take no such derivative. Note that this subtlety only arises because of the possibility of default.

borrowing b_{t+1} , there is a large enough inherited debt b_t at t such that no new debt contract can deliver non-negative dividends. For such a configuration the only option for the firm is to default. Such a firm then exits. We capture this formally by requiring that if $\Gamma(x_t, S_t) = \emptyset$ then firms default by setting $\phi(x, S_t) = 0$.

All firms choose prices and produce, even those that default. Defaulting firms that make enough revenues to cover their wage bill, namely those with $p_t \ell_t^\alpha - w_t \ell_t \geq 0$, pay this wage bill and pay the residual revenues to debt holders. If defaulting firms have insufficient revenues to cover current wages, so that $p_t \ell_t^\alpha - w_t \ell_t < 0$, we assume the firm pays out all its revenues to the worker and the government pays off the residual wages by levying lump sum taxes on households and bondholders receive zero.

Let $V(x_t, S_t)$ denote the value of firm after demand shocks are realized in period t . For any state (x_t, S_t) such that the budget set is empty this value is zero. For all other states in which the budget set is nonempty, firms continue their operations: they hire labor ℓ_{t+1} and produce output y_{t+1} , choose new loans b_{t+1} , and dividends d_t . The value for such continuing firms equals

$$(5) \quad V(x_t, S_t) = \max_{\{p_t, \ell_{t+1}, b_{t+1}, d_t\}} d_t + \sum_{z_{t+1}, \sigma_{t+1}} \delta \pi_z(z_{t+1}|z_t, \sigma_t) Q(\sigma_{t+1}|S_t) V(x_{t+1}, S_{t+1})$$

subject to the production technology $y_{t+1} = \ell_{t+1}^\alpha$, the demand for their product (3), the non-negative dividend condition (4), and the law of motion for aggregate states which evolves according to

$$(6) \quad S_{t+1} = H(S_t).$$

In the firm's problem (5), the aggregate price function $Q(\sigma_{t+1}|S_t)$ is the state contingent price of final goods at $t+1$ in units of final goods at t . This problem gives the decision rules for prices $p(x_t, S_t)$, employment $\ell(x_t, S_t)$, borrowing $b(x_t, S_t)$, dividends $d(x_t, S_t)$, and repayment $\phi(x_t, S_t)$. Note that since the elasticity of demand γ is larger than one continuing and defaulting intermediate goods firms set their prices so as to sell all of their output.

Since firms make their choices of labor and current borrowing before they know the

realization of their current shocks, they realize that once these shocks occur they can use new borrowing to cover their wage bill and existing debt obligations. We can think of each firm as having a *credit line* $\bar{B}(S_t, z_t)$, which is the maximum amount of resources it can borrow at the end of a period

$$(7) \quad \bar{B}(S_t, z_t) = \max_{\ell_{t+1}, b_{t+1}} [q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1}].$$

and maintaining a *buffer stock* of potential funds, defined as

$$(8) \quad \bar{B}(S_t, z_t) - q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1}$$

which is the unused portion of their credit line. In our model, firms find it optimal to maintain a buffer stock so as to guard against the possibility of receiving a very low demand shock and being forced to default on existing obligations. As we shall see, when volatility increases firms raise their buffer stock because they have a higher incentive to guard against default.

We assume the parameter δ in (5) is less than one. We think of this assumption as a simple reduced-form way of capturing the tensions between shareholders and managers discussed by Jensen (1986). The idea is that if firms have large amounts of retained earnings then managers will often use these funds in ways that benefit their private interests relative to the interests of the shareholders. Since shareholders understand the incentives of managers to inefficiently use such funds, the shareholders design the contracts of the manager to compensate them highly from paying out funds immediately rather than retaining them. In this context, δ stands in for the attempts of the shareholders to discipline the managers by rewarding them for paying out profits as dividends rather than keeping as retained earnings.

The parameter δ plays an important role in our model. In it the combination of the lack of insurance against idiosyncratic shocks and the nonnegative dividend condition restricts the ability of the firm to choose the size of employment so as to maximize expected profit. Because of this restriction firms have an incentive to build up a large amount of savings. Such savings would allow the firm to avoid the possibility of default and achieve higher expected profits. By assuming $\delta < 1$ we make it attractive for firms to borrow rather than to build up large levels of savings.

We note that most dynamic models of financial frictions face a similar issue. The financial frictions, by themselves, makes internal finance through retained earnings more attractive than external finance. Absent some other force, firms build up their savings and circumvent these frictions. Such forces include finite lifetimes (as in Bernanke, Gertler, and Gilchrist (1999), Gertler and Kiyotaki (2011)) impatient entrepreneurs (as in Kiyotaki and Moore (1998)), the tax benefits of debt (as in Jermann and Quadrini (2011)). For a survey of these forces and the role they play see Quadrini (2011).

Consider next firm entry. There is a continuum of potential entrant firms every period. To enter, firms have to pay an entry cost ξ in period t and decide on the labor input ℓ_{t+1}^e for the following period. The entry costs are paid for by consumers and gives the consumers the claims to all future dividends of the firm. The idiosyncratic demand shocks of new entrants z_{t+1} are drawn from a distribution with transition function $\pi_e(z_{t+1}|\sigma_t)$. The value function of entrants is given by

$$(9) \quad V^e(S_t) = \max_{\{\ell_{t+1}^e\}} -\xi + \sum_{z_{t+1}, \sigma_{t+1}} \delta \pi_e(z_{t+1}|\sigma_t) Q(\sigma_{t+1}|S_t) V(\ell_{t+1}, 0, z_{t+1}, S_{t+1})$$

subject to the evolution of the aggregate states. This problem gives project sizes for new entrants $\ell_{t+1}^e(S_t)$. Let $M(S_t)$ denote the measure of new entrants.

B. Financial Intermediaries

Competitive financial intermediaries borrow from consumers and lends to firms. At time t , the intermediary borrows from consumers by selling them a vector of state contingent bonds $\{B_{t+1}(\sigma_{t+1})\}$ at prices $\{Q(\sigma_{t+1}|S_t)\}$ and lends these funds to firms. We now derive the bond price schedules offered to firms. To do so we use the fact that competition between financial intermediaries implies that every contract that the intermediary offers earns zero profits.

To develop the expression for the value of a contingent loan to a firm, suppose the current aggregate state is S_t and imagine that a firm with current idiosyncratic shock z_t , and labor input ℓ_{t+1} promises, conditional on not defaulting, to pay the intermediary b_{t+1} at $t+1$. The intermediary realizes that at $t+1$ when the aggregate shock is σ_{t+1} and the idiosyncratic

shock is z_{t+1} , if the repayment indicator $\phi(\ell_{t+1}, b_{t+1}, z_{t+1}, S_{t+1})$ is one this firm will repay completely and if this indicator is zero it will partially default by repaying only its operating profits, $\max\{p_{t+1}\ell_{t+1}^\alpha - w_{t+1}\ell_{t+1}, 0\}$. The intermediary values these repayments using the price for contingent claims $Q(\sigma_{t+1}|S_t)$ on the funds obtained from consumers. Hence, the value for such a default-contingent loan is given by

$$(10) \quad q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1} = \sum_{z_{t+1}, \sigma_{t+1}} Q(\sigma_{t+1}|S_t) \pi_z(z_{t+1}|z_t, \sigma_t) \phi(x_{t+1}, S_{t+1}) b_{t+1} \\ + \sum_{z_{t+1}, \sigma_{t+1}} Q(\sigma_{t+1}|S_t) \pi_z(z_{t+1}|z_t, \sigma_t) (1 - \phi(x_{t+1}, S_{t+1})) \max\{p(x_{t+1}, S_{t+1})\ell_{t+1}^\alpha - w(S_{t+1})\ell_{t+1}, 0\}$$

where ℓ_{t+1} is part of the state x_{t+1} .

C. Households

At the beginning of period t , households provide labor L_t to firms. After the aggregate shock σ_t and the idiosyncratic shocks are realized, the households choose their consumption C_t and state-contingent asset holdings $\{B_{t+1}(\sigma_{t+1})\}$, get paid their wages w_t , receive aggregate dividends D_t from their ownership of the intermediate goods firms and pay a lump sum tax T_t .

The state of the household includes the distribution of firms Υ_t , the current aggregate shock σ_{t-1} , and the vector of state-contingent debts $B_t = \{B_t(\sigma_t)\}$. The recursive problem for households is the following

$$(11) \quad V^H(S_{bt}) = \max_{L_t} \left\{ \sum_{\sigma_t} \pi_\sigma(\sigma_t|\sigma_{t-1}) \max_{C_t, \{B_{t+1}(\sigma_{t+1})\}} [U(C_t, L_t) + \beta V^H(S_{bt+1})] \right\}$$

subject to their budget constraint

$$(12) \quad C_t + \sum_{\sigma_{t+1}} Q(\sigma_{t+1}|S_t) B_{t+1}(\sigma_{t+1}) = w_t(S_{bt})L_t + B_t(\sigma_t) + D_t(S_t) - T_t(S_t)$$

and the aggregate law of motion for S_t given in (6) where we recall that $S_t = (S_{bt}, \sigma_t)$. The aggregate dividend that households receive each period is the sum of all the dividends from

incumbent intermediate goods firms net of the entry costs from all new entrant firms so that

$$(13) \quad D_t(S_t) = \int d(x, S_t) d\Upsilon_t(x) - M(S_t)\xi.$$

The consumer's problem (11) gives the decision rule for labor $L(S_{bt})$ and the decision rules for consumption and bond holdings $C(S_t)$ and $B(\sigma_{t+1}|S_t)$.

D. Equilibrium

Market clearing in the labor market requires that

$$(14) \quad \int \ell(x, S_{t-1}) d\Upsilon_{t-1}(x) = L(S_{bt})$$

where $\ell(x_{t-1}, S_{t-1})$ is the labor input demand for period t committed to by the firm with state x_{t-1} at $t-1$ and $L(S_{bt})$ is the labor supplied by the household. Market clearing in the final goods market requires that the total consumption of households plus the total investment in new entrant firms equals the total final good output

$$(15) \quad C(S_t) + M(S_t)\xi = Y(S_t).$$

The government budget constraint requires that the lump-sum taxes levied on households pay any wages not paid for by the defaulting firms

$$T_t(S_t) = \int (1 - \phi(x, S_t)) \max\{w(S_{bt})\ell - p(x, S_t)\ell^\alpha, 0\} d\Upsilon_t(x)$$

where ℓ is an element of the firm's state $x = (\ell, b, z)$. Next, bond market clearing requires that the repayments by firms to the intermediary equals the payments by the intermediary on the bonds it purchased from the households so that

$$(16) \quad \int [\phi(x, S_t)b + (1 - \phi(x, S_t)) \max\{p(x, S_t)\ell^\alpha - w(S_t)\ell, 0\}] d\Upsilon_t(x) = B_t(\sigma_t)$$

where b and ℓ are elements of $x = (\ell, b, z)$. Finally, the free entry condition for new intermediate goods firms is that $V^e(S_t)M(S_t) = 0$.

The transition function for the measure of firms is $\Upsilon_{t+1} = H(S_t) = (H(x_{t+1}; S_t))$ where

$$(17) \quad H(x_{t+1}; S_t) = \int \Lambda(x_{t+1}, x|S_t)\Upsilon_t(x)dx + M(S_t)\Lambda^e(x_{t+1}|S_t).$$

where the probability that a firm with some $x = (\ell, b, z)$ transits to $x_{t+1} = (\ell_{t+1}, b_{t+1}, z_{t+1})$ in aggregate state S_t is given by $\Lambda(x_{t+1}, x|S_t) = \pi_z(z_{t+1}|z_t, \sigma_t)$ if $\ell_{t+1} = \ell(x, S_t)$, $b_{t+1} = b(x, S_t)$, and $\phi(x, S_t) = 1$ and zero otherwise. Likewise the probability that a new entrant has a state equal to $x_{t+1} = (\ell_{t+1}, b_{t+1}, z_{t+1})$ is $\Lambda^e(x_{t+1}|S_t) = \pi_e(z_{t+1}|\sigma_t)$ if $\ell_{t+1} = \ell^e(S_t)$ and $b_{t+1} = 0$ and zero otherwise.

We now define the equilibrium of this economy. Given the initial distribution Υ_0 and an initial aggregate shock σ_0 , a recursive equilibrium consists of policy and value functions of intermediate goods firms $\{d(x_t, S_t), b(x_t, S_t), \ell(x_t, S_t), \phi(x_t, S_t), V(x_t, S_t)\}$, households policy functions for consumption $C(S_t)$, hours $L(S_{bt})$ and savings $B(\sigma_{t+1}, S_t)$, the wage rate $w(S_{bt})$ and discount bond price $Q(\sigma_{t+1}, S_t)$, bond price schedules $q(S_t, z_t, \ell_{t+1}, b_{t+1})$, the mass of new entrants $M(S_t)$, and the evolution of aggregate states Υ_t governed by the transition function $H(S_t)$, such that for all t : (i) The policy and value functions of intermediate firms satisfy their optimization problem, (ii) households decisions are optimal, (iii) loan contracts reflect the expected default losses such that every contract breaks even in expected value, (iv) domestic good, labor, and credit markets clear, (v) the free entry condition holds, and (vi) the evolution of the measure of firms is consistent with the policy functions of firms, households and shocks.

We turn now to discussing the definitions for real GDP and the aggregate labor wedge in our economy. In our model final good output Y_t is the sum the output of intermediate goods with prices that vary over time. We define GDP as the sum of output of intermediates in base period prices $p_0(x)$

$$(18) \quad GDP_t = \int_x p_0(x)y_t(x)d\Upsilon_t(x).$$

(Notice the the final goods producer has no value added and is a simple device to aggregate the output of the intermediate goods firms into a single value.) In practice we consider a base period in which $p_0(x) = 1$ for all x .

2. Simple Examples

In this section we construct a simple example to illustrate how fluctuations in volatility of demand shocks gives rise to fluctuations in the labor wedge in the presence of default risk. To do so we compare the optimal labor choice of firms in an environment where they can fully insure against shocks to one in which they cannot insure at all.

Consider a one period stripped-down version of our model. Firms begin the period with some debt obligations and immediately choose to hire the amount of labor input ℓ . Each firm produces a differentiated product using the technology $y = \ell^\alpha$ before its idiosyncratic demand shock z is realized. These shocks are drawn from a continuous distribution $\pi_z(z)$. Given the demand shock z and the aggregate output Y firms choose the prices p for their products.

If the firm has sufficient revenues it pays its wage bill $w\ell$ and its debt obligations and receives a continuation value V , here simply modeled as a positive constant. If the firm cannot pay its wage bill and debt it defaults and receives a continuation value of 0.

Consider first the case when financial markets are complete. We imagine that firm chooses the state-contingent pattern of repayments $b(z)$ to meet its total debt obligations b and hence faces the constraint

$$(19) \quad \int_0^\infty b(z)\pi_z(z)dz = b$$

The firm chooses labor and state contingent debt to solve the following problem

$$\max_{\ell, b(z)} \int_0^\infty [p(z)\ell^\alpha - w\ell - b(z)]\pi_z(z)dz + V$$

subject to (19) and the nonnegative dividend condition

$$(20) \quad z\ell^\alpha - w\ell - b(z) \geq 0$$

where $p(z) = zY^{1/\gamma}\ell^{-\alpha/\gamma}$ is the price the firm sets to sell all of its output and is derived from (3) and $y = \ell^\alpha$. We assume that the debt b is small enough so that it can be paid for by the profits of the firm. Hence, with perfect financial markets the firm can guarantee positive

cash flows in every state in period 1 by using state contingent debt $b(z)$ and the dividend constraint is not binding.

The optimal labor choice ℓ^* is such that the expected marginal product of labor is a constant markup over the wage

$$(21) \quad E p(z) \alpha \ell^{*\alpha-1} = \frac{\gamma}{\gamma-1} w.$$

This first order condition shows that with perfect financial markets, fluctuations in the volatility of the idiosyncratic shock z that does not affect its mean will have no impact on firms' labor wedge or its labor choice.

Consider now the case in which the existing debt is state-uncontingent so that firms have no way to insure against demand shocks. Here firms with large employment have to liquidate when they experience low demand shocks due to insufficient cash flow to cover the wage bill plus debt repayments. Effectively, the firm chooses its labor input ℓ as well as a cutoff productivity \hat{z} below which it liquidates where for any ℓ , \hat{z} is the lowest z such that $p(z)\ell^\alpha \geq w\ell + b$ where $p(z)$ is described above. Thus, the firm solves the following problem

$$\max_{\ell, \hat{z}} \int_{\hat{z}}^{\infty} [p(z)z\ell^\alpha - w\ell] \pi_z(z) dz + \int_{\hat{z}}^{\infty} V \pi_z(z) dz$$

subject to $p(\hat{z})\ell^\alpha - w\ell - b = 0$. This last condition defines the cutoff productivity \hat{z} below which the firm liquidates because for any $z < \hat{z}$ the firm would have negative cash flow. The larger the scale ℓ the larger is the probability of liquidation for the firm.

In this environment, the optimal choice of labor maximizes not only period 1 profits (as in the case of perfect financial markets), but also future profits by preventing liquidation. The choice of ℓ^* satisfies:

$$(22) \quad E(p(z)|z \geq \hat{z}) \alpha \ell^{*\alpha-1} = \frac{\gamma}{\gamma-1} \left[w + V \frac{\pi_z(\hat{z})}{1 - \Pi_z(\hat{z})} \frac{d\hat{z}}{d\ell^*} \right]$$

where $p(\hat{z})\ell^{*\alpha} - w\ell^* - b = 0$.

When financial markets are imperfect and firms face liquidation risk, the choice of ℓ equates the effective marginal product of labor in the states in which the firm is operative to

the marginal cost of labor which includes the wage and the loss in future value. Condition (22) illustrates the distortion in the firm’s first order condition arising from default risk that makes the marginal product of labor larger than the wage.

In contrast to the case of perfect financial markets, fluctuations in the volatility of idiosyncratic shocks affect the choice of labor in this case. Increases in volatility typically increase the hazard rate $\pi_z(\hat{z})/(1 - \Pi_z(\hat{z}))$ which in turn leads to a larger distortion and a smaller labor input. Intuitively, a rise in volatility increases the risk of liquidation; firms then have incentives to lower this risk by reducing their scale.

3. Quantitative Analysis

We turn now to the quantitative analysis. We begin with the parameterization. We use the impulse responses to show how an increase in volatility leads to a drop in output and employment. We illustrate the importance of the financial structure and the source of shocks by contrasting our results to those of alternative specifications with complete markets or with productivity shocks. Then we evaluate the ability of our model to account for the patterns of aggregates during the Great Recession of 2007. Finally, we discuss the business cycle moments implied by the model.

A. Parametrization

Many of the parameters of preferences and technology are fairly standard and are chosen to reflect commonly used values. We use features of the time variation in the cross section distribution of firms in the United States to help inform the choice of some key parameters of the intermediate goods firms.

Consider first the setting of some standard parameters. The utility function is assumed to take the form $u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - \xi \frac{h^{1+\nu}}{1+\nu}$. We set $\sigma = 2$ which is a common estimate in the business cycle literature. We set $\nu = 0.5$ which implies a labor elasticity of 2. This elasticity is in the range of elasticities used in macroeconomics as reported by Rogerson and Wallenius (2009). The exponent of the production function α is set to the labor share of 0.70. We choose the elasticity of substitution parameter $\gamma = 7.7$ so as to generate a markup of 15%, which is in the range estimated by Basu and Fernald (1997).

Consider next the parameterization of the Markov processes over idiosyncratic demand

shocks and aggregate shocks to volatility. We want the parameterization to allow for an increase in the volatility of the idiosyncratic demand shock z while keeping fixed the mean level of this shock. We choose a discrete process for idiosyncratic shocks that approximates one that is autoregressive in the log of z , namely,

$$(23) \quad \log z_t = \mu_t + \rho_z \log z_{t-1} + \sigma_{t-1} \varepsilon_t$$

where the innovations $\varepsilon_t \sim N(0, 1)$ are independent across firms. We choose $\mu_t = -\sigma_{t-1}^2/2$ so as to keep the mean level of z (as opposed to its log) across firms unchanged as σ_{t-1} varies. The discrete process for the aggregate shocks approximates the continuous process

$$\log \sigma_t = (1 - \rho_\sigma) \log \mu_\sigma + \rho_\sigma \log \sigma_{t-1} + v_t$$

where $v_t \sim N(0, \varphi^2)$.

Our discrete Markov chains have two aggregate shocks and five discrete set of values for demand shocks for each of the two aggregate shocks. We also want to have an additional demand shock low enough such that when financial markets are imperfect, firms default when this shock occurs. When we do so, firms default only at this additional low demand shock and at none of the other levels of the demand shock. We choose the probability of the additional low demand shock to be 2.5%. With this level the model reproduces failure rates similar to the mean failures since 2000 reported in Campbell, Hilscher, and Szilagyi (2008). Our approximations follow the methods in Tauchen and Hussey (1991).

We set the rest of the parameters so that the model reproduces salient features of the micro data on firms. We choose the serial correlation of idiosyncratic shocks to be $\rho_z = 0.7$ which is in line with Foster et al. (2008) estimated value. We choose the rest of the parameters governing the aggregate and idiosyncratic shocks $\mu_\sigma, \varphi, \rho_\sigma$, the firms' discount factor δ , and the entry cost ξ to target 5 moments. Four of these moments use Compustat data and are features of the distribution of firms: the mean, standard deviation and autocorrelation of the cross section interquartile range of annual sales growth and the mean liabilities over sales ratio. Annual sales growth is computed using quarterly data from 1970 to 2010 as $(sales_t - sales_{t-4})/0.5(sales_{t-4} + sales_t)$ with sales deflated by CPI for firms in Compustat

with at least 100 quarters of observations. The last moment is the fraction of labor employed in entering firms from the Bureau of Labor Statistics. The resulting parameters from the calibration are $\mu_\sigma = 0.18$, $\varphi = 0.13$, $\rho_\sigma = 0.85$, $\delta = 0.7$ and $\xi = 1.57$. Table 1 summarizes the targeted moments in the model and the data.

B. Impulse Response to a Volatility Shock

Here we discuss the impulse response of the aggregate economy to an increase in volatility. We then use the impulse responses of individual firms in this economy as well as firms' decision rules to provide intuition for the model's mechanism.

To set the initial conditions before the shock, we consider a long enough sequence of realizations in which the volatility shock is at its mean so that all aggregates do not change from one period to the next. We then use as an initial condition the resulting measure over individual states. Starting from this distribution we suppose that in period 1 the volatility shock increases by one standard deviation and stays there from then onwards. To help interpret the magnitude of the shock, the IQR of sales growth increases from 18% to 23% with this shock.

Impulse Responses for the Aggregate Economy

We start with the impulse responses at the aggregate level. In Figure 1A we plot the impulse response of the main aggregates: output, labor and consumption for 10 quarters. In the period when the shock hits, the *impact* period, output and labor do not change because firms produce before shocks. In the period after the shock hits, output falls about 2.4% and labor falls more than 3.2%. The reason why aggregate output and labor fall is that incumbent firms reduce their employment and there is less entry of new firms.

The dynamics of consumption differ from those of output and labor. On impact, consumption rises about 0.3% and then falls. Consumption rises on impact because investment in new firms falls more than output. If we continued these impulse responses, we would find that in the long run, consumption follows output and they both settle at about 1.6% and 2.5% lower respectively and the labor slowly rises and eventually returns to almost its initial level. In Figure 1B, we plot the aggregate debt of firms and the measure of firms. We see that higher volatility leads firms to deleverage by decreasing their debt. This increase in volatility

also leads to fewer firms to enter.

We turn now to two commonly used measures in business cycle analysis: labor productivity and the labor wedge. Labor productivity is simply the ratio of GDP to aggregate employment, GDP_t/L_t while our notion of the labor wedge is the ratio of the marginal rate of substitution between consumption and leisure to labor productivity and is given by

$$(24) \quad 1 - \tau_t^L = -\frac{U_{L_t}}{U_{C_t}} / \frac{GDP_t}{L_t}.$$

Note that our definition (24) of the labor wedge corresponds, up to an irrelevant constant, to the one used Chari, Kehoe, and McGrattan (2007), who assumed a Cobb-Douglas aggregate production function. In Figure 2A we graph the labor wedge and labor productivity. We see that the labor wedge falls about 2.2% after the volatility shock is realized while labor productivity increases a modest amount, about 0.9%. In Figure 2B we see that wage falls on impact, about 1.4%, and continue to fall thereafter. Interest rates fall modestly by about 0.5% (from 2.1% to 1.6%) on impact and then stay slightly depressed.

Consider now the intuition for these patterns. When volatility rises, firms realize that at their same level of employment and debt they would be forced to undergo costly default more often. In response they decrease both their employment and their debt. This *pull back effect* on employment thus leads to lower output and higher marginal product of labor (and thus higher labor productivity). Note here that firms' labor input is falling even though the wage is falling. Loosely, the increase in volatility causes aggregate labor demand to shift inward. This inward shift shows up as a labor wedge. These same forces lead firms to shift in their demand for debt so the decrease in debt occurs despite the fall in interest rates. Finally, the increase in volatility leads the financial frictions to become more severe, which manifest themselves in part as an increased labor wedge. This increase in financial frictions makes it less attractive for new firms to enter and hence leads to a fall in entry, despite the decrease in wages and interest rates.

Impulse Responses of Individual Firms

We now turn to the responses for the individual firms to shed light on the mechanisms driving the aggregate responses above. Recall that each firm's state (z, ℓ, b) include its shock,

employment and debt. We plot the responses for labor and debt for three firms that happen to have a sequence of low (z_L) demand realizations, medium (z_M) demand realizations, and high (z_H) demand realizations. Here we set z_M at the mean level of z and set the levels of z_L and z_H to be one standard deviation below and above z_M . The initial employment and debt states for each of the firms are set to the median levels within each z group. When volatility increases, the mean demand shock stays the same and the standard deviation increases. Thus, after the increase in volatility we adjust the levels of z_L and z_H so that they continue to be one standard deviation below and above the mean, thus lowering z_L , raising z_H , and leaving z_M unchanged.

In Figure 3 we plot the employment ℓ and in Figure 4 we plot the debt b and the buffer stock $\bar{D} - qb'$ defined in (8) for each firm for 10 quarters. The employment and debt levels are plotted relative to the levels in period 0 for the firm with z_M and the buffer stock is reported relative to the contemporaneous level of output for each firm.

Consider first the responses for the firm with z_M . When volatility increases, on impact, the firm decreases its employment by about 1.2%, decreases its debt by 0.5% and it increases its buffer stock by 0.3% of its output. The intuition is that at the original employment and debt level, when volatility increases firms would be forced into default more often. When firms default they lose the future stream of positive expected profits. To avoid losing this stream the firm with z_M takes the precautionary actions of lowering its employment and debt in order to reduce its financial obligations to workers and debt holders due at the end of the period and thereby reduce default risk. This firm also chooses to build up its buffer stock to help ensure that it can remain solvent in face of a more volatile distribution of z .

High volatility also shrinks all firms' credit lines (7) because higher default risk restricts lending. This effect further amplifies the desire of firms to reduce their employment and debt and to increase their buffer stock. After the impact period, the firm starts increasing its employment because at that point it has been able to increase its buffer and because of the general equilibrium effect of lower wages.

The figures also shows responses for firms with z_L and z_H . On impact the firms with z_L and z_H take the same precautionary actions of contracting their employment and decreasing their debt as did the firm with z_M . Since the conditional means of z vary for these firms,

however, the magnitude of their responses differ. For example, since the conditional mean of the demand shock for the z_L firm falls as volatility increases, this firm decreases employment by more than the other two types of firms. Likewise, since the conditional mean of the demand shock for the z_H firm increases as the volatility increases, this firm decreases employment by less than the other two types of firms. These differential effects persist beyond the impact period. After the impact period the employment for firms with z_L remains depressed while the employment of firms with z_H increase.

Consider next the behavior of debt and the buffer stock for the firms with z_L and z_H . Recall that the credit line available to all firms shrink with the shock. As shown in Figure 4B, the firms with z_L start with a low buffer stock and when the shock hits such firms reduce their debt and exhaust their credit line. Firms with z_H have a larger buffer stock before the shock and increase it by reducing their debt when the shock hits.

In Figure 5 we show that aggregate buffer stock and the fraction of firms with zero buffer. We see that the increase in volatility leads to an increase in the aggregate buffer stock at the same time as it leads to an increase in the fraction of firms with zero buffer. The reason that both of these responses occur simultaneously is that while most firms increase their buffer stocks, the higher volatility leads to a fatter tail of low shocks. This fatter tail in turn leads to an increase in the number of firms that experience relatively low shocks. Such firms end up running down their buffer to zero.

Debt Overhang and Liquidity

Our model has a type of *debt overhang*, in that all else equal, highly indebted firms choose smaller employment. To illustrate this phenomenon in Figure 6A we plot the decision rules for employment as a function of the inherited debt for a firm with z_M , when the volatility is low and when the volatility is high. Clearly, firms with larger existing debt choose smaller employment. The reason is since firms find it optimal to rollover most of their debt, firms that inherit larger amounts of debt also take on more new debt. Firms that have relatively higher new debt obligations find it optimal to reduce the risk of default by decreasing their level of employment to a more conservative level. As the figure shows, when debt is large enough, firms default and exit. Note that the negative relation between firm inputs and debt

that holds in our model also holds in many model of firm dynamics and financial frictions such Albuquerque and Hopenhayn (2003), Cooley and Quadrini (2001), and Arellano, Bai, and Zhang (2010). Note finally that high debt is disproportionately disruptive in times of high volatility as the level of debt for which the firm shrinks its employment and defaults is lower with high dispersion.

Our model generates default because of *liquidity* as opposed to *solvency* problems. To see that default is due to liquidity problems, note that default happens when firms cannot roll-over their debt even though the firm has a positive value. Figure 6B shows the value of the firm as a function of debt for the two aggregate shocks. Clearly, the higher is the debt of a firm, the lower its value and once the debt reaches a critical size, the firm's value discretely jumps down to zero. At this critical value the firm is just able to borrow enough to pay off its existing debt. Hence, for slightly higher values of debt the firm cannot borrow enough and must default. The reason the value function jumps at this critical value is that by defaulting the firm loses a strictly positive discounted stream of expected future profits.

This analysis leads naturally to the question, Why can the firm with a positive present value of dividends not borrow more? The reason is that the firm cannot borrow freely at the contingent prices used in the valuation of the future dividends. In particular, the firm cannot borrow against future dividends and repay different amounts contingent on its idiosyncratic shocks. Because of this friction in asset markets, the firm cannot pledge resources based only on the expected stream of profits. Hence, it is possible for a firm to be illiquid, in that it cannot borrow, even though it is solvent, in that it has positive value.

C. Impulse Responses in Two Reference Models

To help assess the importance of imperfect financial markets and the source of aggregate shocks, we contrast the impulse responses in our model with those in two reference models. The first has volatility shocks but complete markets. The second has imperfect financial markets, but aggregate TFP shocks.

Volatility Shocks and Complete Markets

To get a feel for the importance of financial frictions at the firm level we compare our model to one with *complete markets*, in which we add to the model state-contingent claims

that payoff on the realization of both idiosyncratic and aggregate shocks. When firms can issue state-contingent claims, their employment choices ℓ are undistorted and solve

$$\sum_{z_{t+1}, \sigma_{t+1}} Q(\sigma_{t+1}|S_t) \pi_z(z_{t+1}|z_t, \sigma_t) p(z_{t+1}, S_t) \alpha \ell^{\alpha-1} = \frac{\gamma}{\gamma-1} w_t(S_{bt})$$

where $p(z_{t+1}, S_t) = z_t Y(S_t)^{1/\gamma} \ell^{-\alpha/\gamma}$. Complete markets also eliminates default because firms can structure the state-contingent payoffs such that their budget sets are never empty. By eliminating default, complete markets prevents inefficient liquidations and delivers a constant measure of firms in the long-run.

Figure 7A plots the aggregate responses to an increase volatility in complete markets model. The difference in responses between the benchmark economy and the complete markets economy is striking. Volatility shocks have very minor effects on aggregates in the complete markets economy in contrast to the benchmark economy. Aggregate output decreases slightly, about 0.2%, aggregate employment is unchanged, and consumption increases slightly.

Productivity Shocks and Imperfect Financial Markets

To get a feel for the importance of the source of aggregate shocks, we compare our model to one in which we replace the aggregate volatility shocks with aggregate TFP shocks. Figure 7B plots the aggregate responses to an increase volatility in the *TFP shock* model. Even though this model has imperfect financial markets, it produces a larger decline in output than in labor. This pattern, which is a typical problem in standard real business cycle models, contrasts sharply with the pattern in Figure 1A of our model.

Several papers, including Bernanke, Gertler, and Gilchrist (1999) and Mendoza (2010), have analysed the effects of productivity shocks in economies with imperfect financial markets. They have shown that imperfect financial markets can amplify the effects of shocks on aggregates. Nonetheless, as Mendoza (2010) shows it is difficult for the model to generate declines in employment that are larger than the declines in output, as our model does.

D. The Great Recession of 2007

So far we have investigated the implications for our model following a one time shock to volatility. Here we ask how much of the movement in aggregates in the current recession can be accounted for by our model.

In this experiment we let the initial number of firms be the one that arises in the limit after a long sequence of volatility levels such that the IQR equals the one at the start of the recession in 2007:4. We then choose a sequence of shocks so that the IQR of sales growth that the model produces is similar to that in the data. In Figure 8 we show the IQR of sales growth in the model and the data. The IQR increased substantially during the recession, from 0.17 to 0.31. We think of this procedure as using the data (and the model) to back out the realized sequence of volatility shocks. Given our initial condition and this sequence of shocks, we simulate the model.

Figure 9 show the resulting movements in output and labor. From Figure 9A we see that over the period 2007:4 to 2009:3 the model generates a decline in output of 6.5% whereas in the data, output declines by 9.7%. From Figure 9B we see that the dynamics of labor are similar to those for output: the model produces about an 8% decline in labor whereas in the data labor declines by about 10%. At the end of the recession, the model predicts a slow recovery, whereas in the data employment remains depressed. Mechanically the reason that the model produces a slow recovery towards the end of the recession is that as Figure 8 shows, the increase in the IQR tapers off and hence so do our backed out volatility shocks. We summarize the overall contraction in both output and labor as the cumulative decline in these variables during this whole event. Using this measure we find that the model can explain 67% of the overall contraction of output and 73% of the contraction in labor during the recent recession.

Figure 10 shows the series for labor wedges and productivity during this event. From Figure 10A we see that the labor wedge in the model worsens about 5% whereas in the data it worsens more, by about 13%. From Figure 10B we see that the model produces a fairly flat productivity profile for the recession. In the data the productivity falls and then rises. More importantly, note that both in the model and in the data productivity fluctuations are modest and that productivity at the end of this event is essentially unchanged from what it

was at the beginning of this event even though output has fallen by 10% from beginning to end.

Our model produces a tight connection between the volatility shocks and the resulting aggregate movements. As the end of the Great Recession experiment shows perhaps this connection is too tight. One reason for this tight connection is that agents know exactly when the volatility shifts. A more elaborate stochastic structure on information in which agents only receive noisy signals of the volatility would allow the model to break the tight connection.

Note that the response of productivity in our model helps to contrast the mechanism in our model with the one in Bloom, Floetotto and Jaimovich (2009). They show that in a model with adjustment costs for capital and labor, high volatility generates a large productivity decline. Both models generate a contraction in aggregate output in response to high volatility but they do so through different margins. In their environment, the contraction in output is accounted by an endogenous decline in productivity, while in our model the contraction is accounted mainly by a decline in the labor wedge.

E. Other Implications

We have focused our quantitative analysis on the impulse responses to a one-time shock and on the implications of our model for the Great Recession. We are also interested in briefly exploring the second moment implications of our theory. To do so we consider the business cycle statistics that the model generates for benchmark model. To highlight the importance of financial frictions and volatility shocks, we compare these statistics to those generated by the complete markets version of our model with volatility shocks and by an our model with aggregate TFP shocks but constant volatility.

In examining these statistics it is important to keep in mind that in our benchmark results we have purposefully abstracted from other shocks to highlight the quantitative importance of volatility shocks, and hence our model should not be thought of as complete model of the business cycle. In order to remind the reader what a standard range is for business cycle statistics we also report some statistics from the U.S. data. Specifically, we use quarterly data from 1970:1 to 2011:2 and log and detrend each series with linear trends.

In Table 2 we see that even though our model has only volatility shocks, it generates highly volatile business cycles. The model generates a volatility of output of 2.4 which is nearly 75% of the volatility observed in the data. The model also generates a relative volatility of labor relative to output which is similar to that in the data (1.26 in the model versus 1.27 in the data). The model generates a lower relative volatility of consumption to output than is in the data. Part of what accounts for this is that we have no adjustment costs on new entrants, so even though their share in output is small, their investment easily adjusts so as to smooth consumption.

Consider now the model's implications for the labor wedge. To put these implications in perspective, recall that classic frictionless business cycle models with TFP shocks do not generate the high volatility of labor relative to output observed in the data because in those models the labor wedge is constant. As noted above, Chari, Kehoe and McGrattan (2007) show that fluctuations in the labor wedge modeled as an exogenous stochastic process can account for about 2/3 of the fluctuations in output. In our model, financial frictions at the firm level together with volatility shocks increase the volatility of aggregate labor which results in a volatile labor wedge. Relative to the frictionless business cycle models, our model predicts a relatively high volatility of the labor wedge albeit one that is less volatile than that observed in the data.

In Table 2 we also report second moments for the complete markets model. Clearly when financial markets are complete, volatility shocks produce only minor fluctuations in aggregates. The volatility of output is tiny relative to the volatility of output in either our benchmark model or in the data. Moreover, even when measured in relative volatility terms, labor is quite unresponsive to volatility shocks. Although, labor wedge fluctuations relative to output are quite large, the co-movement between the labor wedge and output is very negative in the model, whereas they are very positive both in the data and in the benchmark model (.90 in data, .97 in benchmark, and -.96 in complete markets).

Finally, Table 2 reports second moments when the economy has constant volatility of idiosyncratic demand shocks and is hit by aggregate TFP shocks. We choose the volatility of TFP shocks such that the aggregate fluctuations in output in this experiment is equal to that in the benchmark. As in standard business cycles with aggregate TFP shocks, the

volatility of labor relative to output is much lower in this economy and is less than half of that observed in the data. Also, the labor wedge fluctuations are small and negatively correlated with output (-.57)

These last two experiments illustrate two points. First, a necessary ingredient for volatility shocks to have a large impact on output and employment is the presence of imperfect financial markets. Second, even with imperfect financial markets, standard TFP shocks do not generate much volatility in labor relative to output, but volatility shocks do.

4. Conclusion

We have developed a model in which fluctuations in the volatility of idiosyncratic shocks leads to quantitatively sizeable downturns in output. The model is promising parable for the Great Recession of 2007. In that recession the downturn in output was associated mainly with labor wedges rather than productivity shocks and was accompanied by a large increase in the cross-section dispersion of growth rates by firms. Finally, in the model an increase in volatility simultaneously leads to an increase in the fraction of firms that hit their financial constraints while at the same time the total buffer of firms in the economy increases. This feature captures the idea that even though financial constraints may be essential for the downturn, the shock itself makes firm cautious and leads them to optimally build up a pool of untapped funds.

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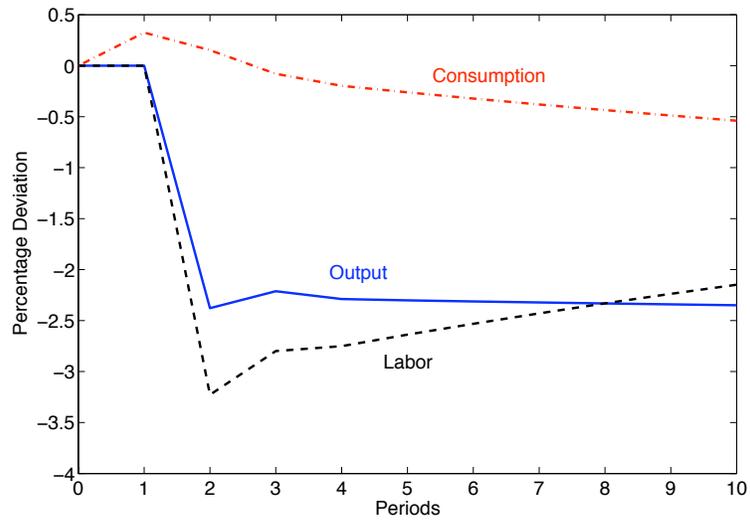
Table 1: Target Moments in Data and Model

Moments	Data	Model
Mean of IQR sales growth	.18	.18
Std. Deviation of IQR sales growth	4.8	4.7
Autocorrelation of IQR sales growth	.84	.84
Mean leverage (liabilities to sales)	5.5	5.6
Ratio of entry labor to aggregate labor	1.8	1.8

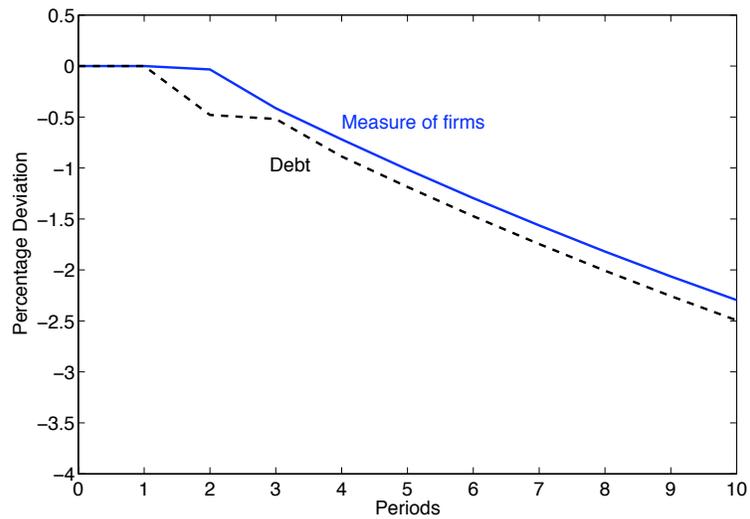
Table 2: Business Cycles Stastics

	Data		Benchmark Model		Complete Markets		TFP shocks	
	std(x)	$\frac{std(x)}{std(GDP)}$	Std(x)	$\frac{std(x)}{std(GDP)}$	Std(x)	$\frac{std(x)}{std(GDP)}$	Std(x)	$\frac{std(x)}{std(GDP)}$
GDP	3.2		2.4		0.15		2.4	
Labor	4.1	1.27	3.1	1.26	0.03	0.21	1.2	0.50
Consumption	2.7	0.83	1.2	0.48	0.21	1.35	0.9	0.38
Labor Wedge	5.4	1.69	2.3	0.95	0.36	2.38	0.8	0.32

Figure 1: Aggregate Impulse Responses to an Increase in Volatility

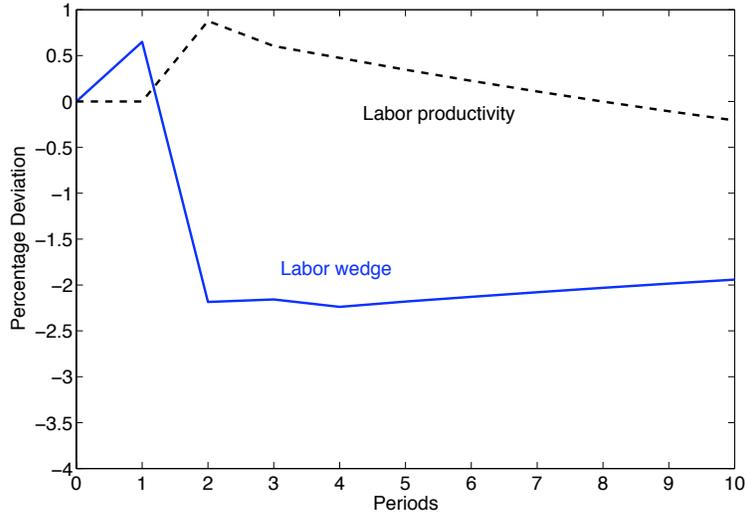


(a) Consumption, Output, and Labor

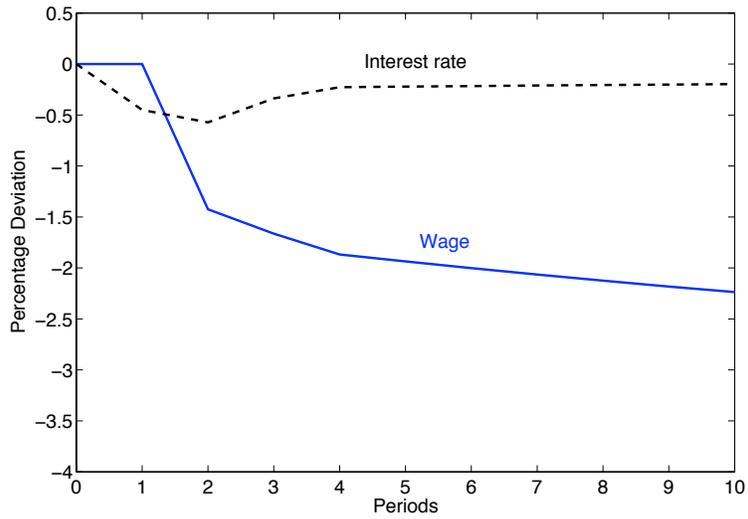


(b) Debt and Measure of Firms

Figure 2: Aggregate Impulse Responses to an Increase in Volatility

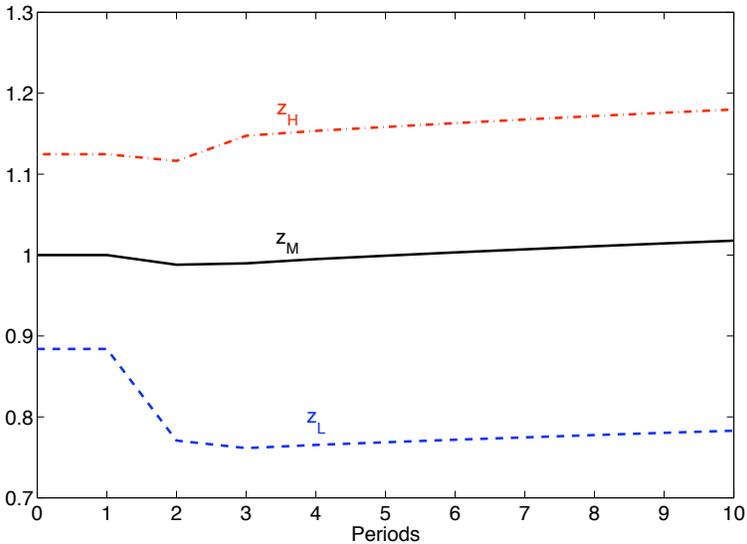


(a) Labor Productivity and the Labor Wedge



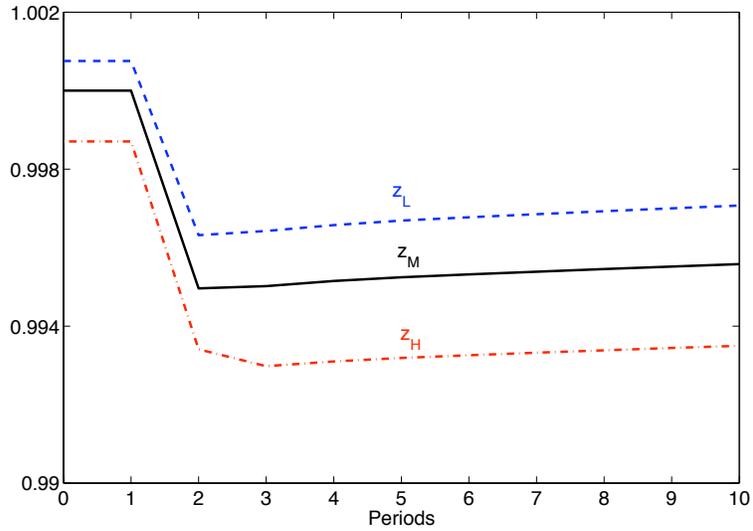
(b) Wage and Interest Rate

Figure 3: Firm Level Impulse Responses to an Increase in Volatility

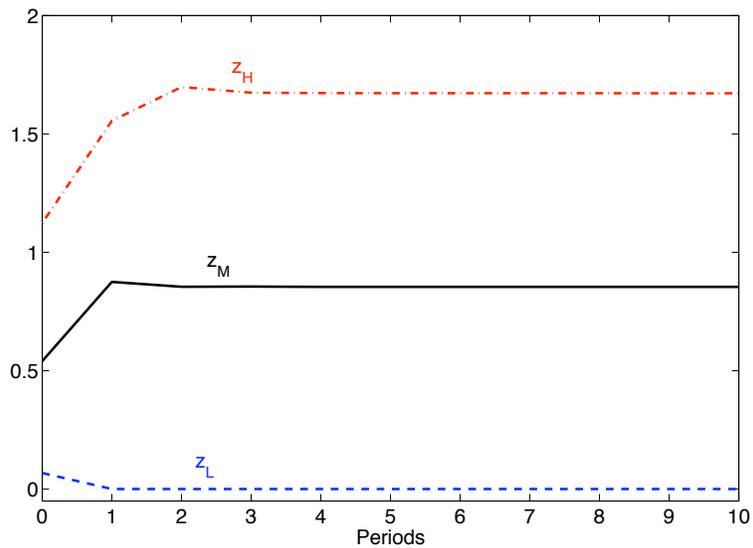


(a) Employment (Normalized by Initial Employment of Firms with Medium Shock)

Figure 4: Firm Level Impulse Responses to an Increase in Volatility

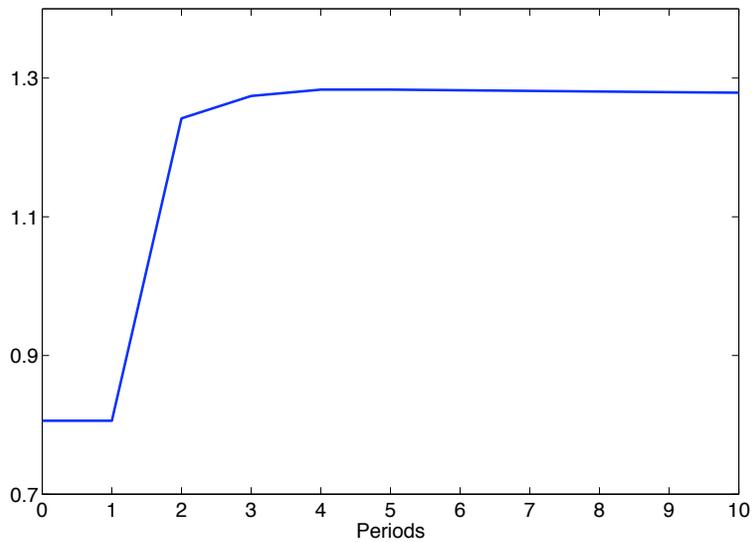


(a) Debt (Normalized by Initial Debt of Firms with Medium Shock)

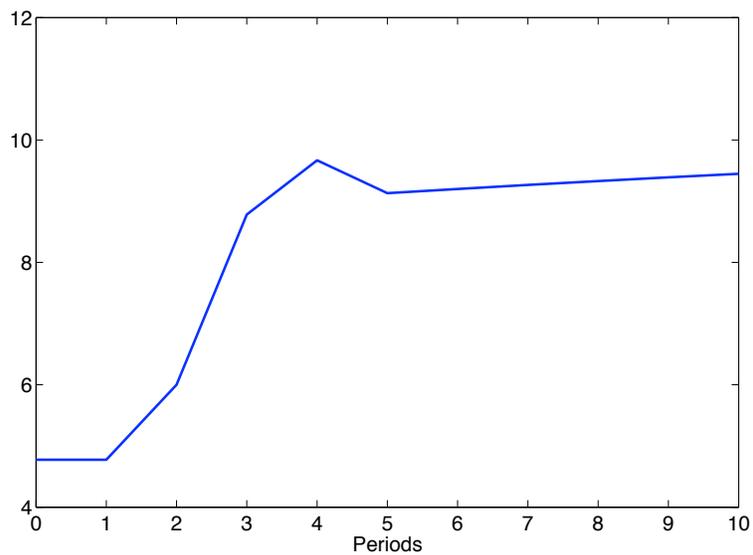


(b) Buffer Stock (As a Percentage of Firm Output)

Figure 5: Aggregate Impulse Responses to an Increase in Volatility

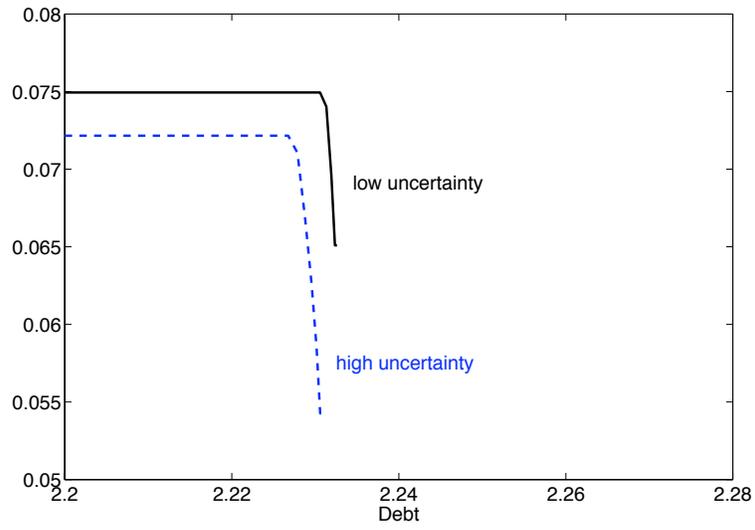


(a) Aggregate Buffer (As a Percentage of Aggregate Output)

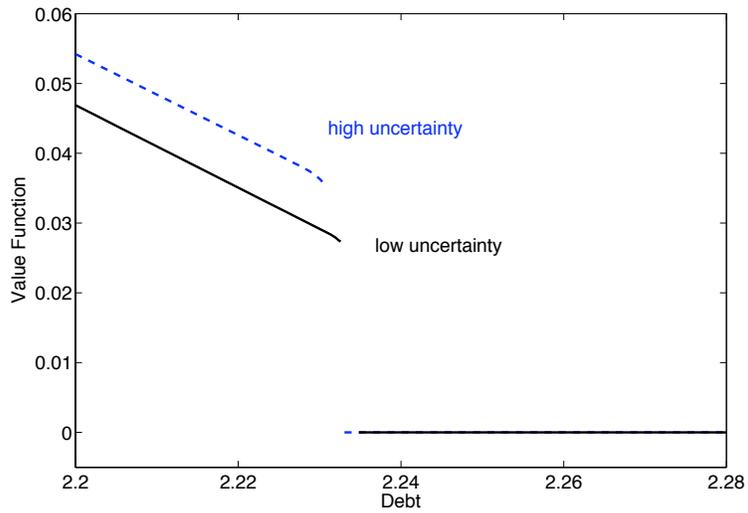


(b) Fraction of Firms with Zero Buffer

Figure 6: Policy and Value Functions

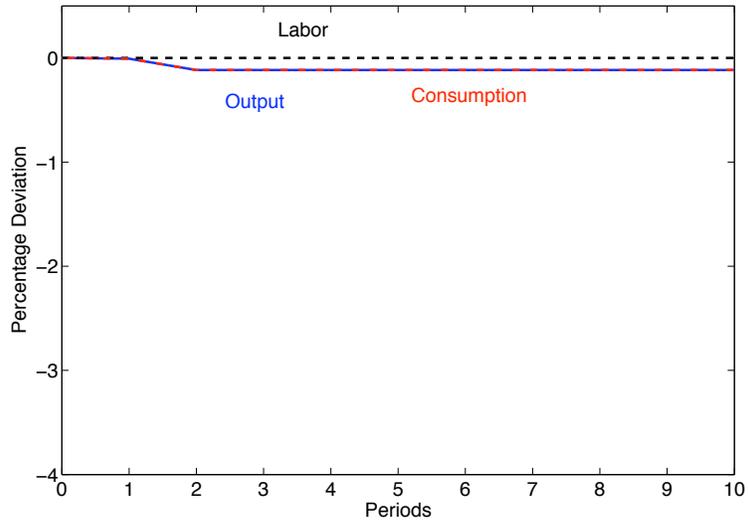


(a) Firm Employment versus Debt

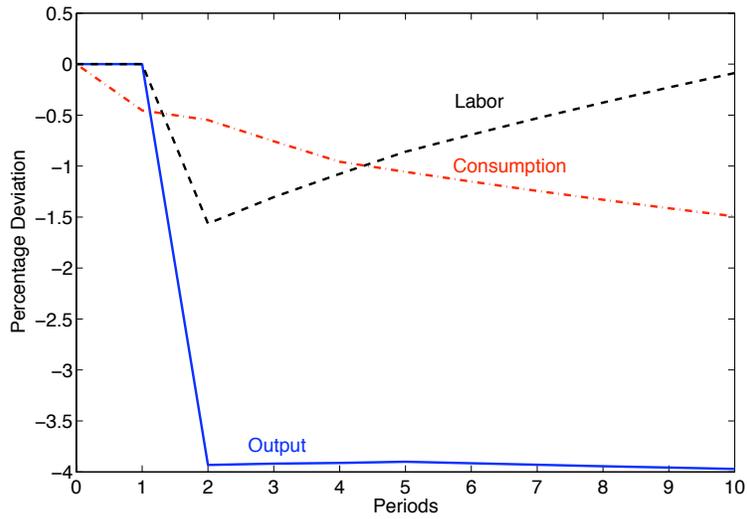


(b) Firm Value versus Debt

Figure 7: Aggregate Impulse Responses in Two Reference Models

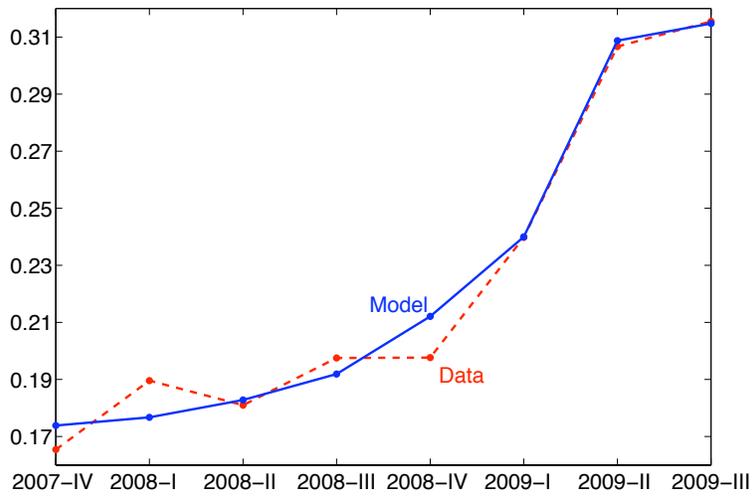


(a) Complete Market with Volatility Shocks



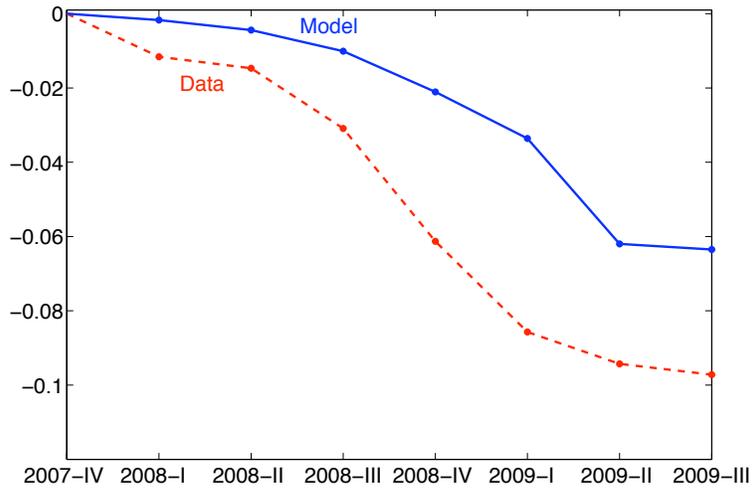
(b) Benchmark Model with Productivity Shocks

Figure 8: The Great Recession of 2007

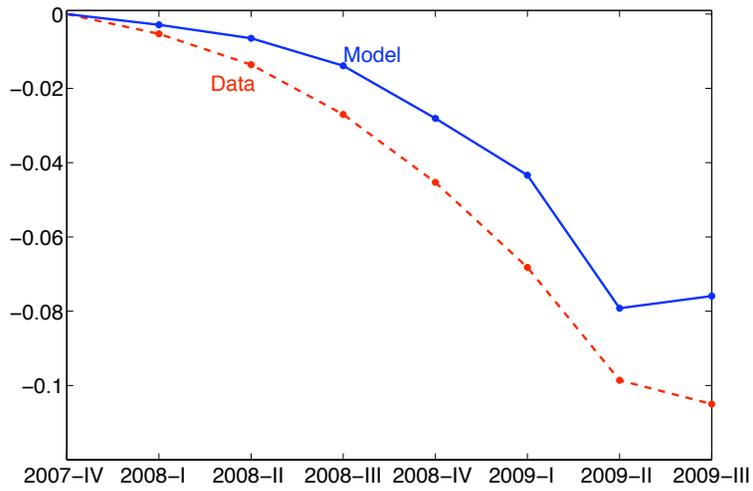


(a) Volatility: Interquartile Range of Sales Growth

Figure 9: The Great Recession of 2007

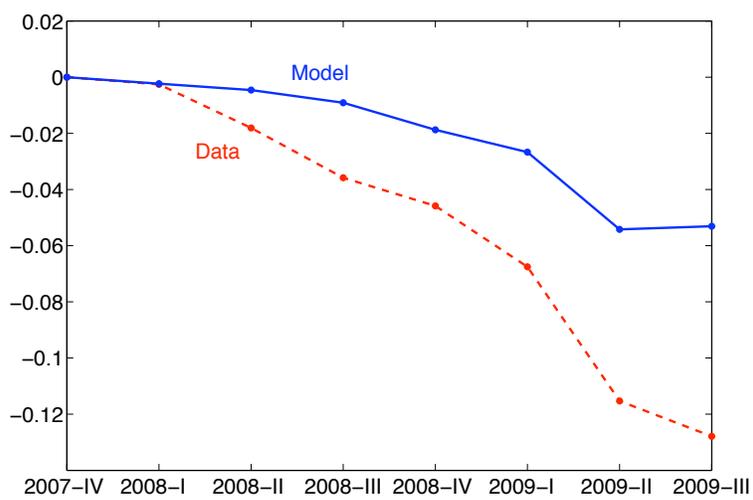


(a) Output

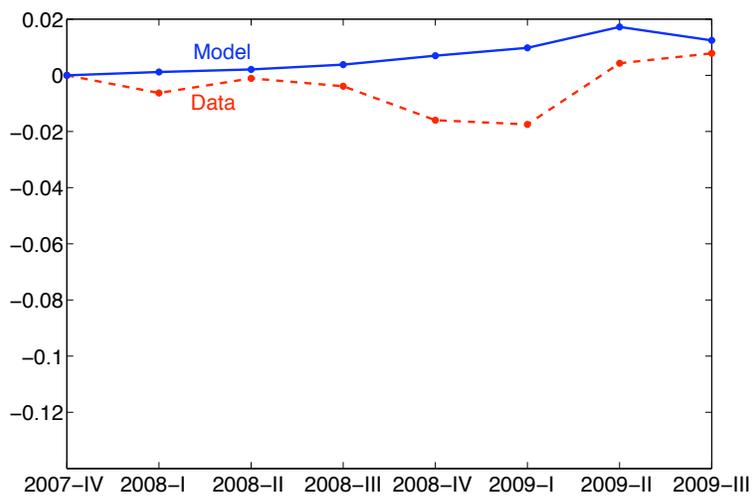


(b) Labor

Figure 10: The Great Recession of 2007



(a) Labor Wedge



(b) Labor Productivity