Marketmaking Middlemen[*]

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Very Preliminary

Abstract

This paper develops a model in which market structure is determined endogenously by the choice of intermediation mode. We consider two representative business modes of intermediation that are widely used in real-life markets: one is a *market-making mode* by which an intermediary offers a platform for buyers and sellers to trade by their own; the other is a *middleman mode* by which an intermediary holds inventories which he stocks from sellers for the purpose of reselling to buyers. In our model, buyers and sellers have an option of searching in an outside market as well as using the service offered by a monopolistic intermediary. We derive the conditions under which the mixture of the two intermediation modes is selected over an exclusive use of either of the modes.

Keywords: Middlemen, Marketmakers, Platform, Search

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1 Introduction

This paper considers a framework in which market structure is determined by the intermediation service offered to customers. We consider two representative business modes of intermediation that are widely used in real-life markets. In one mode, an intermediary acts as a *middleman* (or a *merchant*), who is specialized in buying and selling by his own account and typically operates by holding inventory (e.g. supermarkets, and traditional brick and mortar retailers). In the other mode, an intermediary acts as a *marketmaker*, who offers a marketplace or a platform where the participating buyers and sellers trade with each other (e.g. eBay).

In most real-life markets, however, intermediaries are not one of those extremes but operate both as a middleman and a marketmaker at the same time. This is what we call a marketmaking middleman.[1] For example, the well-known electronic intermediary Amazon, one of the largest marketmaking middlemen nowadays, started off as a pure middleman, buying and reselling products in its own name since its founding in 1994. In the early 2000s, Amazon moved toward a marketmaker, by allowing other suppliers to participate as independent sellers. In 2014, those independent sellers accounted for 50% of gross merchandise volume of Amazon. Similar operation patterns are used by both Rakuten and JD.com, which are Amazon's counterparts in Japan and China. Other examples of marketmaking middlemen include real-estate agents, dealer/brokers in financial markets, and department stores.[2]

We present a model in which the intermediated-market structure is determined endogenously as a result of the strategic choice of a monopolistic intermediary. In our model, there are two markets open to individuals, one is an intermediated market operated by the intermediary, and the other is a decentralized market where buyers and sellers search randomly. The intermediated market combines two business modes: acting as a middleman, the intermediary is prepared to serve many buyers at a time by holding inventories; acting as a marketmaker, the intermediary offers a platform. The intermediary's decision is essentially about how to allocate the attending buyers among these two business modes.

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[1] In the finance literature, different terminologies are used to classify the type of intermediaries: brokers refer to intermediaries who do not trade for their own account, but act merely as conduits for customer orders, akin to our marketmakers; dealers refer to intermediaries who do trade for their own account, akin to our middlemen/merchants. The marketmakers (or specialists) in financial markets correspond broadly to our market-making middlemen, since they quote prices to buy or sell the asset as well as take market positions.

[2] In particular, real-estate agents match buyers and sellers, and also stock properties themselves for sale; dealer/brokers in financial markets engage in trading securities on behalf of clients as well as for their own accounts; some department stores also rent shelf spaces to manufacturers. Further examples can be found in Hagiu and Wright (2015).

In either way of intermediated trade, we formulate the intermediated market as a directed search market in order to feature the intermediary's technology of spreading price and capacity information efficiently.[3] In addition, this approach enables us to highlight the middleman's advantage in the high selling capacity that mitigates search frictions and provides customers with proximity. Thus, the middleman mode outperforms the marketmaker mode in allocation efficiency. The decentralized market represents an individuals' outside option that determines the lower bound of their market utility.

With this set up, we consider two situations, *single-market search* versus *multiple-market search*. With the single market search, agents have to choose which market to search in advance, either the decentralized market or the intermediated market. This implies that the intermediary needs to subsidize either buyers or sellers with their expected value in the decentralized market, but once they participate, the intermediated market operates without fear of competitive pressure outside. In this situation, the intermediary can extract the individual surplus monopolistically for each realized transaction. This can be done either by setting a monopolistic intermediation fee as a marketmaker, or by charging a monopolistic price for the inventory as a middleman – in either way, the per-transaction profits are the same. Given the middleman mode is more efficient in realizing transactions, the intermediary uses the middleman-mode exclusively when agents search a single market.

When agents are allowed to search multiple markets, attracting buyers and sellers becomes less costly compared to the single-market search case — the intermediary does not need to subsidy either of the sides to induce participation. However, this time, the prices/fees charged in the intermediated market must be acceptable relative to the available option in the decentralized market. Thus, with the multiple-market search, the outside option creates competitive pressure to the overall intermediated market. On the one hand, a higher capacity of the middleman leads to a larger number of successful transactions in the intermediated market. This happens due to the demand stimulating effect: a higher capacity induces more buyers to buy from the middleman, and fewer buyers to search in the platform, which increases the intermediary's profit. On the other hand, the demand effect makes sellers less likely to be successful in the platform, so that more sellers are available when a buyer attempts to search in the decentralized market. Accordingly, the buyers expect a higher value from the decentralized market, which causes a competitive pressure on

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[3]Using the search function in the Amazon website, for example, one can receive instantly all relevant information such as prices, stocks of individual sellers and Amazon's inventories.

the price that the intermediary can charge. Hence, the intermediary trade-offs a larger quantity against lower price/fees to operate as a larger-scaled middleman. This trade-off pins down an optimal structure of intermediation modes.

In real-life economy, the single-market search may correspond to the traditional search technology for supermarkets or brick and mortar retailers. Over the course of shopping trip under such a market structure, consumers usually have to search, buy and transport the purchased items during a fixed amount of time. Given the time constraint, they visit a limited number of shops – typically one supermarket –, and appreciate the proximity provided by its inventory. In contrast, the multi-market search may be related to the advanced search technologies available in the digital economy. Such a technology may allow the online-customers to search with ease and convenience, and to compare various options. A similar can happen in the market for durable goods such as housing or expensive items where customers are usually exposed for a sufficiently long time period to ponder multiple available options thoroughly.

Given these technologies, our theory is in line with a casual observation that internet intermediaries such as Amazon and e-Bay often face massive offline competitors, and by organizing a platform, they attract offline sellers. Eventually, the burgeon of the online/mobile commerce sometimes leads to the decline of high street retailers.

The comparison of the above two cases delivers an interesting policy implication of intermediated markets. In our economy, the optimal business mode from the social planner's viewpoint is the middleman mode because it minimizes market frictions and can implement an efficient trade. In the presence of outside competition, however, using a platform can be profitable for the intermediary and it reduces its inventory from the efficient level. Hence, competition for intermediation can deteriorate efficiency and welfare.

This paper contributes to two strands of literature. One is the literature of middlemen developed by Rubinstein and Wolinsky (1987), Duffie, Garleanu, and Pedersen (2006), Lagos and Rocheteau (2009), Shevichenko (2004), Johri and Leach (2002), Masters (2007), and Wong and Wright (2014).[4] Using a directed search approach, Watanabe (2010, 2013) provide a model of an intermediated market operated by middlemen with high inventory holdings. The middlemen's high

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[4]Rubinstein and Wolinsky (1987) show that an intermediated market can be active under frictions, when it is operated by middlemen who have an advantage in the meeting rate over the original suppliers. Given some exogenous meeting process, two main reasons have been considered for the middlemens advantage in the rate of successful trades: a middleman may be able to guarantee the quality of goods (Biglaiser, 1993, Li, 1998), or to satisfy buyers demand for a varieties of goods (Shevichenko, 2004). While these are clearly sound reasons for the success of middlemen, the buyers search is modeled as an undirected random matching process, thereby the middlemens capacity cannot influence buyers search decisions in these models.

capacity enables them to serve many buyers at a time, thus to lower the likelihood of stock-out, which generates a retail premium of inventories. This mechanism is adopted by the middleman in our model as well. Hence, if there is no market-making mode, then our model is a simplified version of Watanabe where we added an outside market. It is worth mentioning that in Watanabe (2010, 2013), the middleman's inventory is modeled as a discrete unit, i.e., a positive integer, so that the middlemen face a non-degenerate distribution of their selling units as other sellers do. In contrast, we model the inventory as measured by mass, assuming more flexible inventory technologies, so that the middleman faces a degenerate distribution of sales. This simplification allows us to characterize the middleman's profit maximizing level of inventory – in Watanabe (2010) the inventory level of middlemen is determined by aggregate demand-supply balancing. Recently, Holzner and Watanabe (2015) study a labor market equilibrium using a directed search approach to model a job-brokering service (i.e., a market-making service), offered by Public Employment Agencies, but the choice of intermediation mode is not the scope of their paper.

The other related strand is the two-sided market literature, e.g. Rochet and Tirole (2003, 2006), Caillaud and Jullien (2003), Rysman (2004), Armstrong (2006), Hagiu (2006) and Weyl (2010).[5] The critical feature of platform is the presence of the cross-group externality, i.e. the participants' expected gains from a platform depend positively on the number of participants on the other side of it. When such a cross-group externality exists, the marketmaker can use "divide-and-conquer" strategies, namely, subsidizing one group of participants in order to attract another group and extract the ensuing externality benefit (see, Caillaud and Jullien, 2003). This strategy is adopted by the intermediary in our model as well. Broadly speaking, if there is no middleman mode, then our model is a directed search version of Caillaud and Jullien (2003) in combination with a decentralized market.

Finally, Hagiu and Wright (2015) study the profitability of intermediation modes as is determined by marketing activities. In their model, it is assumed that the owner of a product has private information on how effective their marketing activity will be. Then, they show that the optimal design of intermediation mode is determined, among others, by the cross-product spillovers of marketing activity, and the degree of owners' informational advantage. For each product, an intermediary only takes the preferred extreme mode instead of a hybrid one, and their theory

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[5]Closely related papers based on a similar spirit can be found in Baye and Morgan (2001), Rust and Hall (2003), Nocke et al. (2007), Loertscher (2007), Galeotti and Moraga-Gonzalez (2009) and Edelman and Wright (2015). For earlier contributions of this strand of literature, see, e.g., Stahl (1988), Gehrig (1993), Yavas (1994, 1996), Spulber (1996) and Fingleton (1997).

explains which products the intermediary should offer in which mode. In contrast, we show that even for a homogeneous product and homogeneous agents, a hybrid intermediation mode can occur in equilibrium. As the driving force of our result is the competitive thread, our theory explains how the intermediary should organize its intermediated structure depending on the competitive environment.

The rest of the paper is organized as follows. Section 2 presents the basic setup and section 3 studies the optimal strategy of a marketmaker mode. Section 4 brings the middleman mode into the benchmark model and characterizes the intermediary's optimal strategy under exclusivity. Section 5 extends the analysis to allow for non-exclusivity and gives the key finding of the paper. Section 6 discusses the welfare and other modeling issues. Section 7 concludes. Some proofs are in the appendix.

2   Setup

We consider a large economy with two types of agents: a mass B of buyers and a mass S of sellers. Agents of each type are homogeneous. Each buyer has unit demand of a homogeneous good, and each seller has a production technology of that good. The consumption value for the buyer is normalized to 1. The marginal production cost is denoted by $c \in [0, 1)$. Sellers are able to produce as much as they want but their selling/trading capability is limited so that each seller can sell only one unit of the good.

There are two retail markets, an intermediated market and a decentralized market (see Figure 1). The decentralized market (hereafter D market) is featured by random matching and bilateral bargaining. Without loss of generality, we assume that a buyer finds a seller with probability λ^b and a seller finds a buyer with probability λ^s, satisfying $B\lambda^b = S\lambda^s$ and $\lambda^b, \lambda^s \in (0, 1)$. This linear matching technology is extended to general non-linear matching functions in Section 6. Matched partners follow an efficient bargaining process, which yields a linear sharing of the total surplus, with $\beta \in (0, 1)$ as the share for the buyer, and $1 - \beta$ for the seller.

The intermediated market (hereafter I market) is operated by a monopolistic intermediary. The intermediary can perform two different intermediation activities. As a middleman, he purchases a good from sellers in a competitive wholesale market, and holds it as an inventory to resell to buyers. As a marketmaker, he does not buy and sell but instead provides a platform where buyers and sellers can trade with each other upon paying transaction fees and registration fees.

We assume that the I market is subject to directed search frictions. In a directed search environment, the prices and capacities of all the individual suppliers in the intermediated market are made publicly observable, before buyers start searching a good. The intermediary has technologies to spread this information. Still, given that individual buyers cannot coordinate their search activities, the limited selling capacity of individual sellers creates a possibility that some units remain unsold and some demands are not satisfied. This is the standard notion of directed search frictions, see e.g., Peters (1991, 2001), Moen (1997), Acemoglu and Shimer (1999), Burdett, Shi and Wright (2001), Albrecht, Pieter, and Vroman (2006), and Guerrieri, Shimer and Wright (2010), and in this sense, the platform in our economy is frictional. As will be detailed below, however, there is no such a friction in the middleman's trade since its inventory management technologies allow for the selling capability comparable to the population of potential buyers in magnitude.
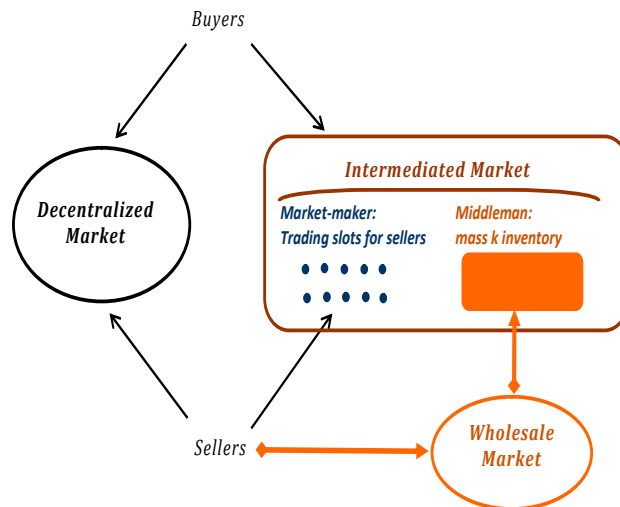

Figure 1: Overview

The timing of events is as follows.

1. The wholesale market opens, and the intermediary decides how many units to stock from

sellers. The amount of inventory is denoted by $k \in [0, B]$. There is no limit on sellers' production capacity, hence the inventory demand of the middleman can always be satisfied. We assume the wholesale market to be competitive so that the wholesale price equals the marginal production cost c. The wholesale market then closes.

2. Two retail markets open to buyers and sellers, an I market and a D market. The I market is operated by the intermediary, who announces its inventory k for the middleman sector, and a set of fees $F \equiv \{f^b, f^s, g^b, g^s\}$ for participation and transaction in the marketmaker sector. In particular, f^b, $f^s \in [0, 1]$ is a transaction fee charged to a buyer or a seller, respectively, and satisfies $f \equiv f^b + f^s \in [0, 1]$, g^b, $g^s \in [-1, 1]$ is a registration fee charged to a buyer or a seller, respectively.

3. Observing the fees F and inventory k, buyers and sellers simultaneously decide which market to participate in. In Section 3 and 4, we explore the exclusivity case where they can only attend one market. In Section 5, we consider the non-exclusivity case where they can attend both markets sequentially.

4. Once in the I market, buyers and sellers as well as the middleman are engaged in a standard directed search game that determines the equilibrium price for individual sellers p^s and the middleman p^m, as will be detailed shortly below.

3   Directed search model of marketmaker

Assuming for the moment $k = 0$, we start with the case where the middlemen option is absent so the only intermediation is a marketmaker mode. We also restrict the meeting technology to be exclusive so that agents are allowed to visit only one market, either the C or D market. Our derivation follows backward induction. We starts with an analysis of the directed search equilibrium in the C market, and then solve for the intermediary's optimal fees F.

Directed search equilibrium in the marketplace   Given an announced set of fees, suppose that a mass B^C of buyers and a mass S^C of sellers have decided to participate in the C market. Then, the C market has the following stages. In the first stage, all the participating sellers simultaneously post a price which they are willing to sell at. Observing the prices, all buyers simultaneously decide which seller to visit in the second stage. Each buyer can visit one seller. If

more than one buyer visits a seller, then the good is allocated randomly. Assuming buyers cannot coordinate their actions over which seller to visit, a symmetric equilibrium is investigated where all buyers use an identical visiting strategy for any configuration of the announced prices, and all sellers play the same strategy which in this case implies that they post an identical price denoted by p^s.

Each individual seller is characterized by an expected queue of buyers, denoted by x. The number of buyers visiting a given seller who has expected queue x is a random variable, denoted by N, which in a larger market has a Poisson distribution, $\text{Prob}[N = n] = \frac{e^{-x}x^n}{n!}$. In a symmetric equilibrium where x^s is the expected queue length of buyers at a seller, each buyer visits some seller with probability $S^C x^s$. They should satisfy the accounting identity,

$$S^C x^s = B^C, \tag{1}$$

requiring that the number of buyers visiting individual sellers sums up to the total population of buyers. The buyer's probability of being served by a seller depends on the expected queue length x^s and is denoted by $\eta^s(x^s)$. The equilibrium values of buyers and sellers, denoted by V^b and V^s respectively, should satisfy

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\begin{aligned}
V^b &= \eta^s\left(x^s\right)\left(1 - p^s - f^b\right) \\
V^s &= x^s \eta^s\left(x^s\right)\left(p^s - f^s - c\right).
\end{aligned}
$$

Given a symmetric equilibrium, suppose a seller deviates to a price $p^d \neq p^s$. If a buyer visits this deviating seller then the probability that it will get served is given by

$$\eta^s\left(x^d\right) = \frac{1 - e^{-x^d}}{x^d},$$

where x^d is the expected queue length for this deviating seller. To understand this probability, note that the probability of this seller to receive at least one buyer is given by $x^d \eta^s(x^d) = 1 - e^{-x^d}$ where $e^{-x^d} = \text{Prob}[N = 0]$. Since buyers must be indifferent between visiting any sellers (including the deviating seller), the buyer's market utility condition is

$$V^b = \eta^s\left(x^d\right)\left(1 - p^d - f^b\right).$$

Given V^b, this equation determines the relationship between x^d and p^d, which we denote by $x^d = x^d(p^d|V^b)$. Since in a large market, individual buyers and sellers cannot affect market utility, the deviant takes V^b as given and V^b does not depend on p^d.

We now determine the equilibrium price. Taking into account the relationship between the expected queue length x^d and price p^d, the optimal price p^{d*} has to maximize the expected payoff of the deviant,

$$p^{d*} = \mathrm{argmax}_{p^d} x^d(p^d) \eta^s \left(x^d(p^d) \right) (p^d - f^s - c).$$

Since there exists a one to one relationship between p^d and x^d, the problem can be written as if the seller selects a queue length x^d rather than a price p^d. Hence, substituting out the price $p^d = 1 - f^b - \frac{x^d}{1-e^{-x^d}} V^b$, the expected profit of the seller with queue length x^d is

$$\left(1 - e^{-x^d} \right) (1 - f - c) - x^d V^b,$$

where $f \equiv f^s + f^b$ is the sum of transaction fees. Taking V^b as given, the first order condition with respect to x^d yields $V^b = e^{-x^{d*}}(1 - f - c)$. In a symmetric equilibrium, the optimal x^{d*} coincides with the equilibrium expected queue length, $x^{d*} = x^s$. Hence,

$$V^b = e^{-x^s}(1 - f - c). \tag{2}$$

The second order condition is satisfied. Substituting it back into the price, the equilibrium price can be written as

$$p^s = 1 - f^b - \frac{x^s e^{-x^s}}{1 - e^{-x^s}}(1 - f - c). \tag{3}$$

Finally, the value for a seller is

$$V^s = \left(1 - e^{-x^s} - x^s e^{-x^s} \right)(1 - f - c). \tag{4}$$

Lemma 1 *Given an announced set of fees, $F = \{f^b, f^s, g^b, g^s\}$, suppose that $B^C > 0$ buyers and $S^C > 0$ sellers have decided to participate in the C market. Given that the intermediary has no inventory $k = 0$ and acts as a marketmaker, a directed search equilibrium in marketplace exists and is unique. It is described as a queue of buyers, $x^s \in (0, B^C/S^C)$, an equilibrium price $p^s \in (0,1)$, the value of buyers $V^b \in (0,1)$ and the value of sellers $V^s \in (0,1)$, satisfying (1), (3), (2) and (4), respectively.*

Define the matching elasticity as $\beta_C \equiv \frac{dx^s \eta^s(x^s)/dx^s}{x^s \eta^s(x^s)/x^s} = \frac{x^s e^{-x^s}}{1-e^{-x^s}} \in (0,1)$. In the directed search equilibrium, sellers are matched with probability $x^s \eta^s(x^s) = 1 - e^{-x^s}$, and get a revenue of $p^s - f^s - c = (1 - \beta_C)(1 - f - c)$ and buyers are matched with probability $\eta^s(x^s) = \frac{1-e^{-x^s}}{x^s}$ and gain $1 - f^b - p^s = \beta_C(1 - f - c)$. Hence, it is clear that whoever bears the transaction fee does

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not matter because the equilibrium price will adjust so that the total surplus $1 - f - c$ is always divided according to the share β_C. Hereafter, we assume that buyers pay all the transaction fees.

Marketmaker's problem To model how the intermediary should set fees, we follow the literature of two-sided markets (Caillaud and Jullien, 2003). In particular, we assume that agents hold pessimistic expectations on the participation decision of agents on the other side of the market. More precisely, in a bad-expectation market allocation, users coordinate on a distribution that yields minimal profits for the marketmaker. For instance, a pessimistic expectation of sellers means that sellers believe the number of buyers registering with the C market is zero whenever

$$\lambda^b \beta (1 - c) > -g^b,$$

where $\lambda^b \beta (1 - c)$ is the expected payoff of buyers in the D market and they believe that no sellers would be in the C market so that all buyers receive in the C market is $-g^b$ (it is a participation subsidy when $g^b < 0$).

Under those beliefs, a profitable strategy for the marketmaker is necessarily a divide-and-conquer strategy. To divide buyers and conquer sellers, referred to as a DbCs strategy, it is required that

$$\text{Db} \quad : \quad -g^b \geq \lambda^b \beta (1 - c), \tag{5}$$

$$\text{Cs} \quad : \quad V^s - g^s \geq 0. \tag{6}$$

The divide-condition Db tells us that the marketmaker should provide a negative entrance fee to buyers so that the participating buyers can be guaranteed at least what they would get in the D market, even if no sellers were operating in the C market. This makes sure that buyers will join the marketplace. The conquer-condition Cs guarantees the participation of sellers, by giving them a nonnegative payoff. Since under this strategy all buyers are in the C market, and the D market is empty, the expected payoff from the D market is zero. Here, the expected value of sellers in the C market V^s is as given by (4), and all the agents will participate in the C market, i.e., $B^C = B$ and $S^C = S$.

Similarly, a strategy to divide sellers and conquer buyers (DsCb) requires that

$$\text{Ds} \quad : \quad -g^s \geq \lambda^s (1 - \beta)(1 - c), \tag{7}$$

$$\text{Cb} \quad : \quad V^b - g^b \geq 0. \tag{8}$$

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Under either of the divide-and-conquer strategies, the marketmaker's expected profit is

$$\Pi = Bg^b + Sg^s + Tf,$$

where Bg^b and Sg^s are registration fees from buyers and sellers, Tf is the transaction fee, $T \equiv S(1 - e^{-x^s})$ is the expected amount of trades in the marketplace. Substituting the binding divide-and-conquer constraints, the expected profits with a DbCs strategy or a DsCb strategy, denoted by Π_{DbCs} and Π_{DsCb} respectively, are given by

$$\Pi_{DbCs} = -B\lambda^b \beta (1 - c) + S \left(1 - e^{-x^s}\right)(p^s + f - c), \qquad (9)$$

$$\Pi_{DsCb} = -S\lambda^s(1 - \beta)(1 - c) + S \left(1 - e^{-x^s}\right)(1 - p^s), \qquad (10)$$

where the expected queue length of buyers is $x^s = \frac{B}{S}$.

Substituting equilibrium price (3) and taking derivatives with respect to f, we have

$$\frac{\partial \Pi_{DbCs}}{\partial f} = Sx^s e^{-x^s} > 0,$$

$$\frac{\partial \Pi_{DsCb}}{\partial f} = S(1 - e^{-x^s} - x^s e^{-x^s}) > 0.$$

Thus, the optimal transaction fee is given by $f = 1 - c$, and

$$\Pi_{DbCs} = \left[-B\lambda^b \beta + S(1 - e^{-x^s})\right](1 - c),$$

$$\Pi_{DsCb} = \left[-S\lambda^s(1 - \beta) + S(1 - e^{-x^s})\right](1 - c).$$

The intuition is as follows. An increase in f merely implies transfers from buyers and sellers to the intermediary. A higher f certainly harms buyers and sellers, and the registration fees decline correspondingly. The result of the trade-off is determined by the fact that using a divide and conquer strategy only the value of one type of agents can be collected using registration fees. Therefore, in the perspective of the intermediary, the overall effect of f is positive. The intermediary would like to set f as high as possible. To summarize the analysis so far, we get the following proposition.

Proposition 1 *Given $k = 0$, the profit-maximizing transaction fee set by a monopolistic market-maker is $f = 1 - c$. When $\beta < 1 - \beta$, the marketmaker divides buyers and conquers sellers (DbCs) by setting $g^b = -\lambda^b \beta (1 - c)$ and $g^s = 0$. When $\beta > 1 - \beta$, the marketmaker divides sellers and conquers buyers (DsCb) by setting $g^s = -\lambda^s(1 - \beta)(1 - c)$ and $g^b = 0$.*

The profit-maximizing marketmaker subsidies one side by guaranteeing their outside value, and then profits from the other side by charging them the highest possible transaction fee. Since $B\lambda^b = S\lambda^s$, it is the value of bargaining power β that decides which side should be subsidized (divided). When $\beta < 1 - \beta$, it is less costly to divide buyers and conquer sellers, whereas when $\beta > 1 - \beta$, it is less costly to divide sellers and conquer buyers.

4 Single-market search

In this section, we introduce the middleman mode, that is $k \geq 0$, into the directed search framework set out in the last section and explore the optimal structure of a hybrid intermediary under exclusive service.

4.1 Directed search equilibrium in the marketplace

If the intermediary also works as a middleman by holding its own inventory, then in the centralized market there are three types of agents: a mass B^C of buyers, a mass S^C of sellers and the intermediary/middleman. The symmetric equilibrium is characterized by an expected queue length x^s per seller, and an expected queue length x^m of the middleman. They satisfy the accounting identity,

$$S^C x^s + x^m = B^C, \tag{11}$$

requiring that the number of buyers visiting individual sellers and the middleman sums up to the total population of buyers.

Since the middleman has a continuum inventory, total matched buyers at the middleman equals the minimum of inventory and the mass of visiting buyers, $\min\{k, x^m\}$. The probability for a buyer to be served at the middleman is

$$\eta^m(x^m) = \frac{\min\{k, x^m\}}{x^m}.$$

Given this probability, the equilibrium values of buyers at sellers and the middleman in the C market, denoted by $V^b_{(s)}$ and $V^b_{(m)}$ respectively, are given by

$$V^b_{(s)} = \eta^s(x^s)(1 - p^s - f),$$

$$V^b_{(m)} = \eta^m(x^m)(1 - p^m),$$

where $\eta^s(x^s) \equiv \frac{1 - e^{-x^s}}{x^s}$ as defined before. In equilibrium, buyers should be indifferent between visiting sellers and the middleman, thus $V^b_{(s)} = V^b_{(m)}$.

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Next, turn to the supply side. The value of an independent seller is

$$V^s = x^s \eta^s(x^s)(p^s - c).$$

The value of the intermediary consists of two parts, the profit of the middleman sector $x^m \eta^m (x^m) p^m - kc$, and the profit from organizing the marketplace $B^C g^b + S^C g^s + Tf$, where $T \equiv S^C \left(1 - e^{-x^s}\right)$ equals the expected platform transactions. Under pessimistic expectations, the intermediary must employ divide-and-conquer strategies to attract users. Using these strategies all the agents will participate in the C market, i.e., $B^C = B$ and $S^C = S$. Taking inventory k and the set of fees F as given, the intermediary's expected profit is

$$\Pi(p^m; k, F) = Bg^b + Sg^s + Tf + x^m \eta^m (x^m) p^m - kc.$$

Notice at the moment that sellers and the middleman post prices, registration fees g^b, g^s are paid and thus are taken as given.

Let's turn to the determination of p^s. p^s can be derived by considering a deviating seller setting $p^d \neq p^s$. This part is similar as in section 3. The expected queue length for this deviating seller is denoted by x^d. In the equilibrium, buyers must be indifferent between visiting this deviating seller and other sellers,

$$V^b = \eta^s(x^d)\left(1 - p^d - f\right).$$

Each seller is infinitely small, and can't influence the market utility of buyers, thus from the deviant's perspective, V^b can be treated as a constant. To determine the optimal price p^{d*}, substitute the price $p^d = 1 - f - \frac{x^d}{1 - e^{-x^d}} V^b$ into V^s

$$V^s \left(x^d\right) = \left(1 - e^{-x^d}\right)(1 - f - c) - x^d V^b.$$

The first order condition with respect to x^d is $e^{-x^{d*}}(1 - f - c) - V^b = 0$. In a symmetric equilibrium, the optimal arrival rate x^{d*} is equal to the equilibrium arrival rate, i.e. $x^{d*} = x^s$. This leads to

$$V^b_{(s)} = e^{-x^s}(1 - f - c) \tag{12}$$

The second order condition is satisfied. Substituting it back yields the optimal retail price of sellers

$$p^s = 1 - f - \frac{x^s e^{-x^s}}{1 - e^{-x^s}}(1 - f - c), \tag{13}$$

and the equilibrium value of sellers

$$V^s = \left(1 - e^{-x^s} - x^s e^{-x^s}\right)(1 - f - c).$$ (14)

We now turn to p^m. First, it is easy to see that x^m can't be larger than B as there are simply no more than B buyers. Also, any $k > B$ will not help to improve the matching probability, but only incur extra inventory costs. Without loss of generality, we impose $x^m, k \in [0, B]$. The optimal retail price p^m is characterized by maximizing the intermediary's expected profit taking k and f as given, subject to the condition that buyers are indifferent between visiting independent sellers and the middleman,

$$V^b = \eta^m (x^m) (1 - p^m).$$

Critically, the intermediary holds a continuum of inventories so that it can influence V^b and x^s purposely. To reflect all these effects, we should take into account that $x^s (x^m)$ and $V^b (x^m)$ are functions of x^m satisfying (11) and (12). Thus, the buyers' indifference condition $V^b_{(s)} = V^b_{(m)}$ becomes

$$p^m (x^m) = 1 - \frac{1}{\eta^m (x^m)} e^{-\frac{B - x^m}{S}} (1 - f - c).$$ (15)

This expression clearly states that p^m is positively correlated with the transaction fee f. In particular, when f is not bounded above, that is $f = 1 - c$ is feasible, the intermediary sets a monopoly price for its inventory $p^m = 1$, which makes the middleman mode more profitable; when f is bounded above due to outside competition, the monopoly price is not possible, and the marketmaking mode might become more attractive. Whether f will be restricted depends on the market structure and the configuration on exclusivity. As we shall see later, the restriction on f is the key factor for the intermediary's optimal structure.

Substituting p^m from (15), the problem can be regarded as if the intermediary is choosing an expected queue x^m instead of the price,

$$\max_{x^m} \Pi (x^m; k, F) = Bg^b + Sg^s + Tf + x^m \eta^m (x^m) p^m (x^m) - kc.$$ (16)

This problem can be easily solved as shown in Lemma 2.

Lemma 2 *Given $B > 0, S > 0, f \in [0, 1 - c]$ and $k \in [0, B]$, the optimal expected queue length of the middleman satisfies $x^m = \min\{\tilde{x}^m (f), k\}$, where $\tilde{x}^m (f) \in (0, B]$ satisfies*

$$1 - e^{-\frac{B - x^m}{S}} - \frac{x^m}{S} e^{-\frac{B - x^m}{S}} (1 - f - c) = 0.$$

The intuition behind the lemma is as follows. Given the coordination frictions in the intermediated platform, a larger middleman sector creates more transactions that potentially gives higher profits. However, to attract more buyers to the middleman (a higher x^m), the intermediary has to lower the price p^m and give buyers a higher surplus V^b. This trade off determines the unique $\tilde{x}^m(f)$. Moreover, having more buyers than the inventory $x^m > k$ does not increase transactions since it is bounded above by k. This suggests any $x^m > k$ can't be optimal. That is, $x^m = \min\{\tilde{x}^m(f), k\}$.

As shown in the lemma, inventory k only plays a role as a turning point of the piecewise profit function, and thus any interior solution of x^m is not affected by the value of k. This feature will help to further clarify the optimal k in the first stage of the game. We thus finish the characterization of the directed search equilibrium, which is summarized in the following lemma.

Lemma 3 *Given an announced set of fees, $F = \{f, g^b, g^s\}$, inventory k, a mass of $B > 0$ buyers and a mass of $S > 0$ sellers have decided to participate in the C market, a directed search equilibrium in the marketplace exists and is unique. It is described by queues of buyers at each seller, $x^s \in (0, B/S)$, and at the middleman, $x^m \in (0, B)$, equilibrium prices $p^s, p^m \in (0, 1)$, and the value of buyers $V^b \in (0, 1)$, of sellers $V^s \in (0, 1)$, and of the intermediary $\Pi \in (0, B)$ satisfying (11), (12), (13), (15), (14), (16) and lemma 2.*

4.2   Intermediation structure and fees

At the first stage of the game, the intermediary chooses inventory k and the fee structure F, taking into account of its effect on $x^m(k, F)$. The divide and conquer conditions (5), (6), (7) and (8) of section 3 continue to hold. Substituting them into (16) leads to profits for each strategy. The DbCs strategy divides buyers by $g^b = -\lambda^b \beta (1 - c)$ and extracts $g^s = V^s$ from sellers by registration fees,

$$
\begin{aligned}
\Pi_{DbCs} &= -B\lambda^b \beta (1 - c) + S \left(1 - e^{-x^s}\right) (p^s + f - c) \\
&\quad + x^m \eta^m (x^m) p^m - kc.
\end{aligned} \tag{17}
$$

The DsCb strategy divides sellers by $g^s = -\lambda^s (1 - \beta)(1 - c)$ and extracts $g^b = V^b$ from buyers,

$$
\begin{aligned}
\Pi_{DsCb} &= -S\lambda^s (1 - \beta)(1 - c) + S \left(1 - e^{-x^s}\right)(1 - p^s) \\
&\quad + x^m \eta^m (x^m) p^m - kc.
\end{aligned} \tag{18}
$$

The profits of the marketmaker sector are the same as marketmaker's profits (9) and (10) in section 3, except $x^s = \frac{B-x^m}{S} < \frac{B}{S}$.

Next, we show that the optimal inventory is such that the intermediary in the second stage will always choose exactly the same amount of buyers, $x^m = k^*$. Suppose this is not the case, then according to lemma 2, $x^m = \tilde{x}^m(f) < k$. The intermediary can do better by saving the unnecessary inventory $k - \tilde{x}^m(f)$. Thus, we shall focus on the candidate values such that $k \leq \tilde{x}^m(f)$. This implies that once k is determined, the optimal x^m is equal to it and is independent of f.

Given this argument, the intermediary can choose f ignoring its effect on x^m. Following the same intuition as in section 2, setting the largest $f = 1 - c$ is profit maximizing. For the marketmaker sector, levying the highest f is necessary to extract all users' surplus. The middleman sector also benefits from imposing a higher price since p^m is increasing in f according to (15).

When $f = 1 - c$, according to (13) and (15), equilibrium prices $p^s = c$ and $p^m = 1$. The profit functions (17) and (18) become

$$\Pi_{DbCs} = -B\lambda^b\beta(1-c) + S\left(1 - e^{-x^s}\right)(1-c) + k(1-c);$$
$$\Pi_{DsCb} = -S\lambda^s(1-\beta)(1-c) + S\left(1 - e^{-x^s}\right)(1-c) + k(1-c).$$

In both expressions, the first items regarding the dividing step are constant. Only the rest part $S\left(1 - e^{-x^s}\right)(1-c) + k(1-c)$ matters for the trade-off on k. It is a sum of the marketmaker profit $S\left(1 - e^{-x^s}\right)(1-c)$, which is decreasing in k, and the middleman profit $k(1-c)$, which is increasing in k. Therefore, increasing inventory level k is equivalent to increasing the middleman sector and compressing the marketmaker sector, and vice versa. A marginal increase in k increases trades of the middleman mode by 1, and decreases trades in the marketmaker mode by less than 1. This can be verified by taking the derivative of the marketmaker transactions $S\left(1 - e^{-x^s}\right)$ with respect to k, $\frac{\partial S\left(1 - e^{-x^s}\right)}{\partial k} = e^{-x^s} < 1$. Thus, the middleman mode would create more transactions. Since the per-transaction profit is always $1 - c$ for a trade in either sector, a higher k certainly renders a higher reward for the intermediary. This optimal strategy is characterized in the following proposition.

Proposition 2 *Given $k \geq 0$, the profit-maximizing transaction fee and inventory set by a monopolist marketmaking middleman is*

$$f^* = 1 - c, \ k^* = B.$$

17

The corresponding profits of DbCs and DsCb strategies are

$$\Pi_{DbCs} = -B\lambda^b \beta (1-c) + B(1-c),$$

$$\Pi_{DsCb} = -S\lambda^s (1-\beta)(1-c) + B(1-c).$$

Proof. Substituting p^s and p^m from (13) and (15) into the profits (17) and (18) gives

$$\begin{aligned}
\Pi_{DbCs} &= -B\lambda^b \beta (1-c) + S\left(1 - e^{-x^s}\right)(1-c) \\
&\quad + k(1-c) - Be^{-x^s}(1-f-c), \tag{19} \\
\Pi_{DsCb} &= -S\lambda^s (1-\beta)(1-c) + S\left(1 - e^{-x^s}\right) f \\
&\quad + k(1-c) + (Sx^s - x^m) e^{-x^s} (1-f-c). \tag{20}
\end{aligned}$$

The optimal f follows from the first order conditions, since the first order derivatives of (19) and (20) with respect to f are positive,

$$\begin{aligned}
\frac{\partial \Pi_{DbCs}}{\partial f} &= Be^{-x^s} > 0, \\
\frac{\partial \Pi_{DsCb}}{\partial f} &= S\left(1 - e^{-x^s} - x^s e^{-x^s}\right) + x^m e^{-x^s} > 0.
\end{aligned}$$

The optimal registration fee is a corner solution, $f^* = 1 - c$.

Evaluating the first order derivatives of (19) and (20) with respect to k at $f^* = 1 - c$ gives

$$\begin{aligned}
\frac{\partial \Pi_{DbCs}}{\partial k}\big|_{f=1-c} &= \left(1 - e^{-x^s}\right)(1-c) > 0, \\
\frac{\partial \Pi_{DsCb}}{\partial k}\big|_{f=1-c} &= \left(1 - e^{-x^s}\right)(1-c) > 0,
\end{aligned}$$

for any $0 \le k < B$. Therefore, the optimal $k^* = B$.

Substituting $f^* = 1 - c$ and $k^* = B$ back into (19) and (20) yields the optimal profits. ∎

According to proposition 2, the intermediary uses the same optimal k^* and f^* for DbCs and DsCb strategies, the profits only differ in the dividing step. Given $B\lambda^b = S\lambda^s$, when buyers' bargaining power β in the D market is lower, DbCs is adopted; when sellers' bargaining power $1 - \beta$ is lower, DsCb is favored.

5 Multi-market search

Exclusivity of intermediation services is quite a restrictive assumption in many circumstances, in particular when focusing on intermediation on the Internet. It is not hard to imagine a consumer

looking for some specific good will usually visit his favorite intermediation service providers first, and if he is not satisfied, he might continue to search in his living city. This section is therefore devoted to the analysis on how an intermediary works with non-exclusive intermediation being possible. Under non-exclusive intermediation, buyers and sellers use the intermediary's service as well as search by themselves. We start with an adjustment on timing, then analyze two cases, one with the C market opens first, and the other with the D market opens first.

Timing Non-exclusivity makes the operating time of the intermediary relevant. If the intermediation operates before the D market, then buyers and sellers still hold their outside options when participating in the C market. Therefore, if they are not matched at the intermediary, there are still chances in the D market. In this way, there will be a substantial difference compared to the exclusivity case, as will be shown in sections 5.2 and 5.3. In contrast, if the intermediation operates after the D market closes, then by the time that the C market opens, all participating buyers and sellers are those who are not matched in the D market. We essentially go back to a exclusivity situation as will be shown in section 5.1. Instead of any specific sequence between the C and D market, we allow the intermediary to choose its business hours. A comparison of the two operating time shows that the intermediary would like to take the first moving advantage. This partly explains why many online retailers are enthusiastic in making their websites fast and easy to use, providing a wide range of information on the commodity, offering personalized service such as special offer emails tailored to a customer's interest, etc. These are to enhance customer experiences, create loyalty and thus make the intermediary a first-mover.

Formally, the timing of events is as follows.

1. A competitive wholesale market opens where the intermediary stocks its inventory as a middleman, denoted by $k \in [0, B]$. The wholesale market then closes.

2. The C and D retail markets open. The intermediary chooses the opening time, either before or after the D market closes.

3. The intermediary announces a set of fees $F = \{f, g^b, g^s\}$ for the marketmaker sector and the inventory k of the middleman sector, where the transaction fee $f \in [0, 1-c]$, the registration fees $g^b, g^s \in [-1, 1]$.

4. Observing the fees and the inventory, buyers and sellers independently decide which mar-

ket(s) to participate in. If the C market opens first, individuals could attend the C market and then the D market (if they are not matched in the C market), or ignore the C market and only join the D market later. If the D market opens first, individuals could attend the D market and then the C market (if they are not matched in the D market). Since the D market is of free access, there is no reason to ignore the D market and only join the C market later. Registration fees need to be paid in advance before entering the C market, whereas entering the D market is free.

5. Matched pairs are formed in each market, and transactions are implemented.

5.1 The case where the D market opens first

Given the D market is of free access, all individuals attend it. After the matches in the D market, there are $B(1 - \lambda^b)$ buyers and $S(1 - \lambda^s)$ sellers left unmatched. Since agents have lost outside options, a slightly negative registration fee is enough to divide one type, $g^i = 0, i = B, S$. According to proposition 2, it is optimally for the intermediary to be a pure middleman, and charge a monopolistic price $p^m = 1$. The corresponding profit is $B\left(1 - \lambda^b\right)(1 - c)$. We will compare this profit with that when the C market opens first and decide which opening time is optimal for the intermediary in proposition 4.

5.2 The case where the C market opens first

In this subsection, we follow backward induction to solve the directed search equilibrium in the second stage of the game, and the intermediary's profit maximization problem in the first stage.

5.2.1 Second stage: the directed search equilibrium

We now consider the directed search equilibrium when the C market opens first. Under pessimistic expectations, a profitable strategy for the intermediary must be a divide-and-conquer strategy. This amounts to all buyers and sellers who participate in the C markets. Among a mass of B buyers, Sx^s will visit some independent sellers, and the rest x^m will visit the middleman,

$$Sx^s + x^m = B. \tag{21}$$

The equilibrium value of a buyer visiting an independent seller is

$$V^b_{(s)} = \eta^s \left(x^s\right)\left(1 - p^s - f\right) + \left(1 - \eta^s \left(x^s\right)\right)\lambda^b e^{-x^s}\beta\left(1 - c\right). \tag{22}$$

where $\eta^s(x^s) \equiv \frac{1-e^{-x^s}}{x^s}$ as defined before. This equation can be interpreted as follows. With a chance of $\eta^s(x^s)$, the buyer is served and gets the value of the commodity after paying the price p^s and the transaction fee f. With a probability $1 - \eta^s(x^s)$, the buyer will not be served. Still he gets $\beta(1-c)$ if he is matched in the D market with another seller who can't trade in the C market. This happens with a probability of $\lambda^b(1 - x^s\eta^s(x^s)) = \lambda^b e^{-x^s}$.

Similarly, the equilibrium value of buyers visiting the middleman is

$$V_{(m)}^b = \eta^m(x^m)(1 - p^m) + (1 - \eta^m(x^m))\lambda^b e^{-x^s}\beta(1-c). \tag{23}$$

where $\eta^m(x^m) \equiv \frac{\min\{k,x^m\}}{x^m}$ is the serving probability at the middleman. With probability $\eta^m(x^m)$, the buyer will be served and receives $1 - p^m$. In the equilibrium, a buyer should be indifferent between visiting an independent seller or the middleman in the centralized market. Thus, $V_{(s)}^b = V_{(m)}^b$.

Consider the value of a seller. With probability $x^s\eta^s(x^s)$, the seller will be matched in the C market and receives $p^s - c$. With probability of $(1 - x^s\eta^s(x^s))\lambda^s$, the seller will not be matched in the C market but be matched in the D market with another buyer. To eventually conclude a transaction in the D market, the matched buyer must fail to trade in the C market, which happens with probability $\xi(x^m, k)$, defined as

$$\xi(x^m, k) \equiv 1 - \left(\frac{x^m}{B}\eta^m(x^m, k) + \frac{B - x^m}{B}\eta^s(x^s)\right).$$

The expression in the brackets is the expected chance of a buyer to trade in the C market. In particular, x^m out of the B buyers are matched at the middleman, and the rest $B - x^m$ are matched in the marketmaker sector. Summarizing all the elements yields the seller's equilibrium value

$$V^s = x^s\eta^s(x^s)(p^s - c) + (1 - x^s\eta^s(x^s))\lambda^s\xi(x^m, k)(1 - \beta)(1 - c).$$

Finally, the profits of the intermediary consists of two parts, platform fees and inventory profits,

$$\Pi(p^m; k, F) = Bg^b + Sg^s + Tf + x^m\eta^m(x^m)p^m - kc, \tag{24}$$

where T is the expected amount of trades on the platform. Registration fees g^b, g^s are taken as given at the moment.

Sellers Consider a deviating seller posting a price $p^d \neq p^s$. This seller is characterized by an expected queue of buyers measured by $x^d \neq x^s$. The optimal price of this seller can be determined

by maximizing the expected profit,

$$V^s = \left(1 - e^{-x^d}\right)\left(p^d - c\right) + e^{-x^d}\lambda^s\xi\left(x^m, k\right)\left(1 - \beta\right)\left(1 - c\right),$$

subject to a market utility condition indicating that buyers are indifferent between visiting this deviating seller and other sellers,

$$V^b = \frac{1 - e^{-x^d}}{x^d}\left(1 - p^d - f\right) + \left(1 - \frac{1 - e^{-x^d}}{x^d}\right)\lambda^b e^{-x^s}\beta\left(1 - c\right).$$

Each seller is infinitely small, and therefore can take V^b as given.

To solve this problem, first substitute the price from the constraint into the objective to get the expected profit of the deviating seller in terms of the arrival rate x^d. Taking V^b as given, the optimal x^{d*} satisfies the first order condition $\frac{\partial V^s\left(x^{d*}\right)}{\partial x^d} = 0$. The second order condition is satisfied. In a symmetric equilibrium, all sellers have the same arrival rate $x^{d*} = x^s$. The equilibrium price p^s satisfies

$$p^s - c = \left(1 - \beta_C\right)\left(v - f\right) + \lambda^s\xi\left(1 - \beta\right)\left(1 - c\right), \tag{25}$$

where $\beta_C \equiv \frac{x^s e^{-x^s}}{1 - e^{-x^s}}$ is the bargaining power for buyers induced by directed search equilibrium, and $v \equiv \left[1 - \lambda^b e^{-x^s}\beta - \lambda^s\xi\left(1 - \beta\right)\right]\left(1 - c\right)$ can be regarded as the total payoff that can be divided by a buyer and a seller in the C market, taking into account their outside market values $\lambda^b e^{-x^s}\beta$ and $\lambda^s\xi\left(1 - \beta\right)$, respectively.

Inserting the equilibrium price p^s (25) yields the equilibrium values of buyers and sellers

$$V^b_{(s)} = e^{-x^s}\left(v - f\right) + \lambda^b e^{-x^s}\beta\left(1 - c\right), \tag{26}$$

$$V^s = \left(1 - e^{-x^s} - x^s e^{-x^s}\right)\left(v - f\right) + \lambda^s\xi\left(1 - \beta\right)\left(1 - c\right). \tag{27}$$

Finally, using equilibrium values $V^b_{(m)}$ in (23), $V^b_{(s)}$ in (26) and $V^b_{(s)} = V^b_{(m)}$, the equilibrium price p^m can be written as

$$p^m = 1 - \lambda^b e^{-x^s}\beta\left(1 - c\right) - \frac{x^m e^{-x^s}}{\min\left\{k, x^m\right\}}\left(v - f\right). \tag{28}$$

Incentive constraints Because the intermediary opens first, agents know whether they are matched in the I market before the D market opens. But even an agent is matched in the I market, he might forgo this trading opportunity and continue to search in the coming D market. For example, a matched seller will receive $p^s - c$ by trading on the I market platform, and his expected value in the coming D market is $\lambda^s\xi\left(x^m, k\right)\left(1 - \beta\right)\left(1 - c\right)$, thus the seller would indeed

to trade in the I market if and only if

$$p^s - c \geq \lambda^s \xi(x^m, k)(1 - \beta)(1 - c).$$

Similarly, to have buyers trade in the I market ex post, prices must satisfy

$$1 - p^m \geq \lambda^b (1 - x^s \eta^s(x^s)) \beta (1 - c),$$

$$1 - p^s - f \geq \lambda^b (1 - x^s \eta^s(x^s)) \beta (1 - c),$$

where the first and the second condition correspond to a buyer visiting the middleman and the platform, respectively.

Using expression (25) for p^s and (28) for p^m, the three incentive constraints can be simplified to one restriction on f

$$f \leq v \equiv \left[1 - \lambda^b e^{-x^s} \beta - \lambda^s \xi(x^m, k)(1 - \beta) \right] (1 - c). \tag{29}$$

Basically, inequality (29) imposes an upper bound on the transaction fee that depends on the outside option values of buyers $(\lambda^b e^{-x^s} \beta)$ and sellers $(\lambda^s \xi(x^m, k)(1 - \beta))$. It differs from the natural upper bound of $1 - c$ as in the exclusivity case.

Intermediary   We now turn to the intermediary's pricing p^m. With the one-to-one relationship (28) between p^m and x^m, and taking intermediation fees and inventory as given, the intermediary's problem in the second stage is to choose the optimal x^m, subject to the incentive constraint,

$$\max_{x^m \in [0,B]} \Pi(x^m; k, F) = S\left(1 - e^{-x^s}\right) f + \min\{k, x^m\} p^m(x^m) - kc$$

$$s.t. \ f \leq v. \tag{30}$$

Problem (30) can be solved using the standard Kuhn-Tucker method as shown in lemma 4. The intuition behind the lemma is exactly the same as behind lemma 2.

Lemma 4 *Given $B > 0, S > 0, f \in [0, 1 - c]$ and $k \in [0, B]$, the optimal expected queue of the middleman satisfies $x^m = \min\{\tilde{x}^m(f), k\}$, where $\tilde{x}^m(f) \in (0, B]$ does not depend on k.*

Finally, the directed search equilibrium analyzed so far is summarized in the following lemma.

Lemma 5 *Given an announced set of fees, $F = \{f, g^b, g^s\}$, and inventory k, $B > 0$ buyers and $S > 0$ sellers have decided to participate in the C market, a directed search equilibrium exists*

and is unique. It is described by a queue of buyers at the middleman $x^m \in (0, B)$, a queue of buyers at each seller $x^s \in (0, B/S)$, the equilibrium prices $p^s, p^m \in (0, 1)$, and the value of buyers $V^b \in (0, 1)$, of sellers $V^s \in (0, 1)$, and of the intermediary $\Pi \in (0, B)$ satisfying (21), (24), (25), (26), (27), (28), and lemma (4).

5.2.2 First stage: intermediation structure and fees

In this section, we study the determination on the intermediary's optimal fees and inventory. Concerning the divide and conquer strategy, the difference to exclusive service is that now any negative registration fee ensures that individuals take the C market as the second home to cash the subsidy, since they need not to give up the D market to do so. So attracting one side of the market becomes less costly. By contrast, conquering the other side becomes more difficult, since the conquered side still holds the trading opportunity in the D market.

The DsCb condition under non-exclusive service is

$$\text{Ds: } -g^s \geq 0,$$

$$\text{Cb: } V^b - g^b \geq \lambda^b e^{-x^s} \beta (1 - c).$$

The divide seller condition Ds tells us that the intermediary should provide a slightly negative entrance fee so that all sellers will participate in and cash the fee even when there is no buyer at all. The conquer buyer condition Cb thus extracts the surplus of buyers by giving them merely the value of only attending the D market. Since matching is first implemented in the C market, buyers devoted to the D market can only trade successfully with a probability of $\lambda^b e^{-x^s}$, and thus the value of the D market is $\lambda^b e^{-x^s} \beta (1 - c)$. Similarly, the DbCs condition under non-exclusive service is

$$\text{Db: } -g^b \geq 0,$$

$$\text{Cs: } V^s - g^s \geq \lambda^s \xi (x^s, x^m) (1 - \beta) (1 - c).$$

Let's first consider the relationship between x^m and the optimal inventory. In particular, whether the intermediary would like to hold more inventory than its queue length, $k > x^m$. As in section 3, the optimal behavior of the intermediary must be consistent. That is, the optimal inventory is such that the intermediary can later accommodate exactly the same amount of buyers, $x^m (k, f) = k$. Less buyers than inventory $(\tilde{x}^m (f) < k)$ manifests an unnecessary spending on the

inventory. The intermediary gains a higher profit by reducing the redundant inventory. We thus
have the following lemma.

Lemma 6 *The optimal inventory satisfies $k \leq \tilde{x}^m(f)$.*

Given $x^m = k$, the accounting identity becomes

$$B = k + Sx^s. \tag{31}$$

The matching probability at the middleman becomes $\eta^m = 1$.

The intermediary's problem in the first stage is to choose the optimal inventory k and trans-
action fee f to maximize

$$\Pi(k, F) = Bg^b + Sg^s + S\left(1 - e^{-x^s}\right) f + k\left(p^m(k, f) - c\right)$$

subject to incentive constraint (29) and divide-and-conquer conditions described above.

Applying $V^b_{(s)} = V^b_{(m)}$, we have

$$p^m(k, f) - c = \left(1 - \lambda^b e^{-x^s}\beta\right)(1 - c) - e^{-x^s}(v - f).$$

Substituting the divide-and-conquer conditions yields registration fees under DbCs

$$g^b = 0, \ g^s = \left(1 - e^{-x^s} - x^s e^{-x^s}\right)(v - f),$$

and registration fees under DsCb

$$g^s = 0, \ g^b = e^{-x^s}(v - f).$$

To solve the problem, let's start with the transaction fee. The intuition in previous sections
continues to hold. An increasing in f raises platform fees and price p^m directly, but also reduces
buyers and sellers' values and thereby reduces their registration fees. However, the overall effect is
positive since divide and conquer strategies only extract the value of one type, either that of the
buyers or sellers. The optimal f is ultimately bounded above, imposed by the incentive constraint
$f = v$.

To continue with the choice on k, insert the bounded $f = v$, the registration fees under either
divide and conquer strategy become zero, and the profits of DbCs and DsCb then become the
same,

$$\Pi(k) = \left\{ S\left(1 - e^{-x^s}\right)\left(1 - \lambda^b e^{-x^s}\beta - \lambda^s \xi(1 - \beta)\right) + k\left(1 - \lambda^b e^{-x^s}\beta\right) \right\}(1 - c).$$

It is a sum of profits in two sectors, which again implies that the problem of choosing the optimal inventory is essentially to allocate buyers between the two modes. Increasing inventory is equivalent to increasing the fraction of middleman and decreasing the fraction of the platform, and vice versa.

There are two concerns involved in choosing the relative sizes of the platform and the middleman. First, it follows from previous sections that the middleman mode offers a lower out-of-stock risk, and a higher k creates more transactions. What's new in the current setting is the outside competition, represented by buyers and sellers' decentralized market values. These outside values cap the intermediary's price as stated in the incentive constraints, $f \leq v$. Reflected in the profit function, buyers' outside value $\lambda^b e^{-x^s} \beta (1-c)$ is deprived from intermediary's profit for each transaction, whether it is on the platform or at the middleman, and sellers' outside value $\lambda^s \xi (x^m, k) (1-\beta)(1-c)$ is deprived for each transaction of the platform. Notably, as the intermediary moves towards a larger middleman sector (k increases), less sellers will be matched in the C market due to the shrinking platform, more buyers will be matched in the C market by trading with the middleman. Therefore, more sellers and less buyers are left to be available in the D market. Accordingly, the buyers' outside value increases and sellers' outside value decreases. When k is large, the former one is the main concern. The intermediary would have the incentive to decrease k in order to lower the buyers' outside value. In the following proposition, we show that this competition effect is so strong that it overcomes the effect of the transaction efficiency and yields an optimal inventory $k^* < B$. We thus explained the endogenous choice of a hybrid intermediation mode.

Proposition 3 *With non-exclusivity, when the C market opens first, the intermediary's optimal strategy is to charge zero registration fees for buyers and sellers, the optimal transaction fee $f^* = v(k^*)$, the optimal inventory is set to be strictly smaller than B, $k^* \in [0, B)$. In particular, when*

$$1 - \lambda^b e^{-\frac{B}{S}} \beta - e^{-\frac{B}{S}} v \left(\frac{B}{S} \right) (1-\beta) + \lambda^b \left(1 - e^{-\frac{B}{S}} \right) \left(1 - e^{-\frac{B}{S}} - \beta \right) \leq 0, \qquad (32)$$

the pure marketmaker mode ($k^* = 0$) is optimal.

Proof. Given that

$$\frac{\partial \Pi(k)}{\partial k} = \left\{ \begin{array}{c} -e^{-x^s} v + 1 - \lambda^b e^{-x^s} \beta - \frac{S(1-e^{-x^s})+k}{S} \lambda^b e^{-x^s} \beta \\ + \lambda^b \left(1 - e^{-x^s} \right)^2 (1-\beta) \end{array} \right\} (1-c),$$

we have $\frac{\partial \Pi(k)}{\partial k}|_{k=B} = -\frac{B}{S}\lambda^b \beta (1-c) < 0$. This shows $k^* < B$. When $\frac{\partial \Pi(k)}{\partial k}|_{k=0} \leq 0$ (which is simplified to inequality (32) in the proposition), $k^* = 0$. Since $\Pi(k)$ is concave over the relevant set of k,

$$\frac{\partial^2 \Pi(k)}{\partial k^2} = \left(-\frac{e^{-x^s}}{S}v - \frac{3\lambda^b}{S}e^{-x^s}\left(1 - e^{-x^s}\right) - \frac{k}{S^2}\lambda^b e^{-x^s}\beta \right)(1-c) < 0 .$$

the second order condition holds. ∎

If we set $S = 1 - B$, we can visualize inequality (32) in the space of $\left(\beta, B, \lambda^b\right)$, as shown in figure ??. The shadowed region is the parameter set that yields $k^* = B$. As one can see, this happens when the mass of buyers B is small and the buyer's outside option is large (λ^b and β are of big values).
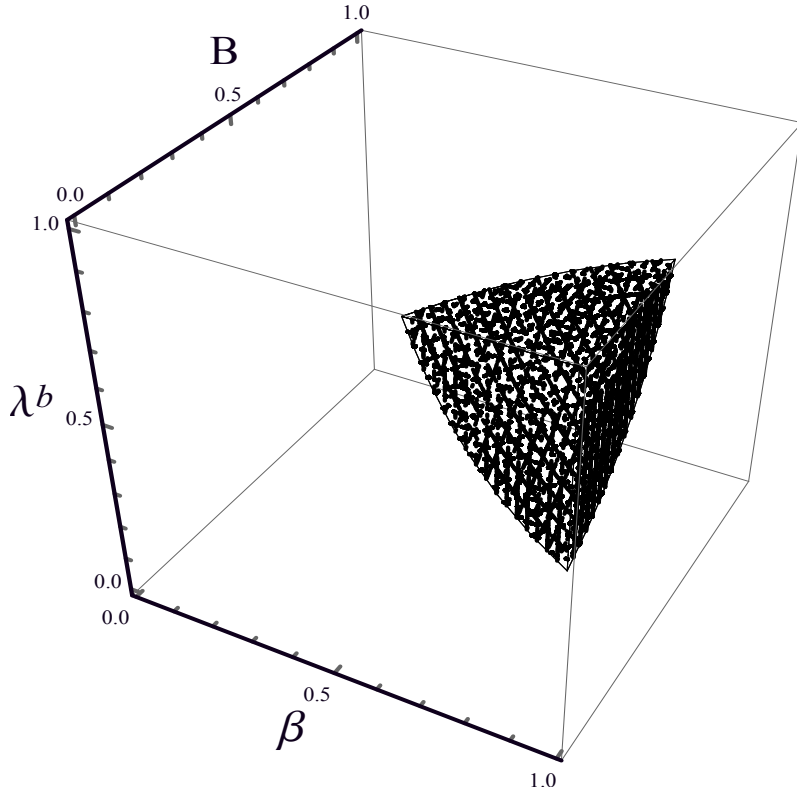

Figure 2: Illustration of the parameter space (shadowed region) where pure marketmaker mode is optimal

The explanation of proposition 3 is consistent with the rise of the online shopping and the decline of brick and mortar stores. If we regard the decentralized market as the enormous traditional high street shops, and the centralized market as the online intermediary such as Amazon,

then we do see that the intermediary undercuts the price in the decentralized market to engage customers in the model, which seems to fit the commonly held view that online intermediaries benefit consumers by lowering prices. However, as is shown, the consumers are actually worse due to the platform strategy. The decline of the high street shops reduces buyers' market value, so that the intermediary can eventually impose a even higher price. If one takes into account the traditional store experiences, such as browsing and lingering in the bricks-and-mortar bookstores or record shops, the utility decrease described by the model would only be a lower bound.

Next, we study the optimal inventory with regards to changes of the buyer's bargaining power β. When β increases, the buyers' outside market value $e^{-x^s}\lambda^b\beta(1-c)$ increases. This indicates the effect of using the marketmaker mode to lower buyers' outside value becomes more important. As a result, the optimal k should decrease.

Corollary 1 (Comparative statics) *An increase in buyer's bargaining power β in the decentralized market leads to a lower optimal inventory k^*.*

To compare whether the intermediary chooses to move first, we evaluate the first-move profit at a suboptimal inventory level of $k = B$. This yields a profit of $B\left(1-\lambda^b\beta\right)(1-c)$, which is strictly larger than that when the intermediary opens after the D market $B\left(1-\lambda^b\right)(1-c)$. We conclude that the intermediary would optimally take the "first move advantage".

Proposition 4 *With non-exclusive intermediation, the intermediary optimally opens the C market before the D market, and uses the optimal strategy characterized in proposition 3.*

6 Discussion and extensions

This part discusses the implications of the previous results and some extensions.

Efficiency and welfare The social optimum is achieved when all buyers are matched with suppliers, either independent sellers or the middleman. Apparently, when the inventory of the middleman is set at $k = B$, all buyers are matched. As long as the marketmaker mode is employed, the directed search friction renders a less efficient allocation. This implies that the optimal strategy of the intermediary is, however, not optimal for society. Given our analysis before, the reason lies in

the competition from the outside market. The incentives for the intermediary to be a marketmaker is to absorb sellers from the outside market and thus reduce buyers' outside value. In other words, it is market competition, rather than market power, that brings efficiency and welfare loss. This conclusion is at odds with the common wisdom.

In terms of welfare of buyers and sellers, we find that under exclusivity, buyers and sellers get their expected values in the decentralized market, the same as when there is no intermediary. Under non-exclusivity, the intermediary attracts transactions using a platform, which lowers the buyer and seller's expected values. They get even less than that without the intermediary.

Mortenson and Pissarides type matching functions in the D market   Our conclusion holds for a very general setting of the D market. Instead of a linear matching function, assume the flow of contacts between sellers and buyers in the D market is given by a matching technology

$$M = M\left(B^D, S^D\right),$$

where B^D and S^D denote the amount of buyers and sellers that actually participate in the D market. It is standard to assume the function M is continuous, nonnegative, with $M\left(0, S^D\right) = M\left(B^D, 0\right) = 0$ for all $\left(B^D, S^D\right)$.

Define the matching probabilities $\lambda^b\left(B^D, S^D\right) = \frac{M\left(B^D, S^D\right)}{B^D}$ and $\lambda^s\left(B^D, S^D\right) = \frac{M\left(B^D, S^D\right)}{S^D}$ for buyers and sellers, respectively. Under exclusive intermediation, we have the same conclusion with new defined matching probabilities $\lambda^b = \lambda^b\left(B, S\right)$, $\lambda^s = \lambda^s\left(B, S\right)$.

With non-exclusive intermediation, under a fairly reasonable condition that $\lambda^b\left(B^D, S^D\right)$ is decreasing in B^D and increasing in S^D, we can show our conclusion holds. If the C market opens first, then the matching in the D market depends on how many buyers and sellers are not matched in the C market. In particular, $\min\{k, x^m\} + S\left(1 - e^{-x^s}\right)$ buyers and $S\left(1 - e^{-x^s}\right)$ sellers are matched in the C market, and thus

$$
\begin{aligned}
B^D &= B - \min\{k, x^m\} - S\left(1 - e^{-x^s}\right), \\
S^D &= S - S\left(1 - e^{-x^s}\right).
\end{aligned}
$$

Following the same derivation as in section 5, we have the intermediary's profits

$$\Pi\left(k\right) = \left\{ \begin{array}{c} S\left(1 - e^{-x^s}\right)\left(1 - \lambda^b\left(B^D, S^D\right)\beta - \lambda^s\left(B^D, S^D\right)\left(1 - \beta\right)\right) \\ + k\left(1 - \lambda^b\left(B^D, S^D\right)\beta\right) \end{array} \right\}\left(1 - c\right),$$

where the first item in the brace is the platform profit, and the second item is the middleman profit. The first order derivative with respect to inventory is

$$\frac{\partial \Pi(k)}{\partial k} = \left\{ \begin{array}{c} -e^{-x^s} \left(1 - \lambda^b \left(B^D, S^D \right) \beta - \lambda^s \left(B^D, S^D \right) (1 - \beta) \right) \\ +S \left(1 - e^{-x^s} \right) \left(1 - \frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial k} \beta - \frac{\partial \lambda^s \left(B^D, S^D \right)}{\partial k} (1 - \beta) \right) \\ + \left(1 - \lambda^b \left(B^D, S^D \right) \beta \right) - k \frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial k} \beta \end{array} \right\} (1 - c).$$

Evaluating this derivative at $k = B$ drops all items in the brace except the last one,

$$\frac{\partial \Pi(k)}{\partial k} |_{k=B} = -B \frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial k} \beta (1 - c).$$

To make a pure middleman model non-optimal ($\frac{\partial \Pi(k)}{\partial k}|_{k=B} < 0$), we need $\frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial k} > 0$, which can be decomposed into two parts,

$$\frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial k} = \frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial B^D} \frac{\partial B^D}{\partial k} + \frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial S^D} \frac{\partial S^D}{\partial k}.$$

Since $\frac{\partial B^D}{\partial k} < 0$ and $\frac{\partial S^D}{\partial k} > 0$, we need $\frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial B^D} \leq 0$, $\frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial S^D} \geq 0$, $\frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial B^D} \times \frac{\partial \lambda^b \left(B^D, S^D \right)}{\partial S^D}$ $\neq 0$.

Heterogenous search costs   In the real world, individuals have different search costs in the outside market. This may be due to many factors, such as shopping experiences, living locations, whether the agent is a local resident or a foreigner, etc. In our model, we can also capture this by assuming agents sample a search cost from a distribution. This gives more flexibility for the intermediary. In particular, besides divide and conquer strategies, there is a new strategy where the intermediary just sets a high price to extract all trade surplus and waits for the high search cost agents to arrive. The flip side is that such a strategy would disappear in the non-exclusivity case. To see why, consider a pure marketmaker. In this scenario, dividing an agent is free by simply giving the agent a small positive value. Thus, the low search cost agents can also be divided. Moreover, if these low search cost agents are not matched in the D market, their outside market value becomes zero, and they would like to trade in the C market. This means, there exists a divide and conquer strategy, where the intermediary serves as the second source for low search cost agents that strictly dominates the proposed new strategy. Therefore, our modeling doesn't lose much in this aspect.

7   Conclusion

This paper presents a theory of a hybrid intermediary using a standard directed search approach within a two-sided markets framework. When the intermediary works as a marketmaker, it organizes a marketplace where price and capacity information spreads efficiently and only coordination frictions exist, i.e. search is directed. When the intermediary works as a middleman, it purchases a large inventory from a competitive wholesale market and resells to buyers. Middleman's inventory can provide buyers with immediate service under market frictions, thereby it increases the number of transactions.

We explored whether and when the mixture of the two modes, the marketmaking middleman, would be optimal, under both single-market and multiple-market search technologies. In the former case, we find the intermediary's optimal mode is to be a pure middleman in order to create as many trades as possible. In the latter case, due to competition from other retailers (the decentralized market in the model), the intermediary has to slash retail price in order to compete with the outside market. Rather than going to a cut-throat price war, the intermediary's best response is to have rivals join its business and become a marketmaking middleman.

This article considers only the monopoly case. For future research, it would be interesting to also study the oligopoly case. In particular, we could check whether our conclusion still holds when another marketmaking middleman is explicitly characterized. Since our model provides a microfoundation for the mixture of middleman mode and marketmaker mode, it can be potentially extended to discuss more specific intermediaries such as marketmakers in financial markets.

8 Appendix

8.1 Proof of lemma 2

The intermediary aims to maximize $\Pi(p^m; k, f)$. Substituting $p^m = 1 - \frac{x^m}{\min\{k, x^m\}} e^{-x^s} (1 - f - c)$ gives

$$\Pi(x^m; k, f) = Bg^b + Sg^s + S\left(1 - e^{-x^s}\right) f + \min\{k, x^m\} - x^m e^{-x^s} (1 - f - c) - kc.$$

To find the optimal x^m, consider two cases: $x^m > k$ or $x^m \leq k$.

When $x^m > k$, the first order derivative is

$$\frac{\partial \Pi(x^m; k, f)}{\partial x^m} = -e^{-x^s} f - e^{-x^s} (1 - f - c) - \frac{x^m e^{-x^s}}{S} (1 - f - c) < 0,$$

thus the optimal $x^m \leq k$.

When $x^m \leq k$, the first order derivatives is

$$\frac{\partial \Pi(x^m; k, f)}{\partial x^m} = 1 - e^{-x^s} (1 - c) - \frac{x^m}{S} e^{-x^s} (1 - f - c).$$

$\frac{\partial \Pi(x^m; k, f)}{\partial x^m} = 0$ gives $\tilde{x}^m(f)$.

Notice $\frac{\partial \Pi(x^m; k, f)}{\partial x^m}|_{x^m = 0} = 1 - e^{-\frac{B}{S}} (1 - c) > 0$, thus $\tilde{x}^m(f) > 0$. But

$$\frac{\partial \Pi(x^m; k, f)}{\partial x^m}|_{x^m = B} = c - \frac{B}{S} (1 - f - c) \gtrless 0,$$

depending on f.

The second order condition is satisfied since $\Pi(x^m; k, f)$ is concave in x^m,

$$\frac{\partial^2 \Pi(x^m; k, f)}{\partial (x^m)^2} = -\frac{1}{S} e^{-x^s} (1 - c) - \frac{S + x^m}{S^2} e^{-x^s} (1 - f - c) < 0.$$

8.2 Proof of lemma 4

Substitute $p^m(x^m)$ into the profit function, we have the problem as

$$\max_{x^m} \Pi(x^m; k, F) = S\left(1 - e^{-x^s}\right) f + \min\{k, x^m\} \left(1 - \lambda^b e^{-x^s} \beta (1 - c)\right) - x^m e^{-x^s} (v - f)$$
$$s.t. \quad f \leq v,$$

where $v \equiv \left(1 - \lambda^b e^{-x^s} \beta - \lambda^s \xi (1 - \beta)\right) (1 - c)$, and $\xi = 1 - \frac{\min\{k, x^m\}}{B} - \frac{S}{B}\left(1 - e^{-x^s}\right)$.

First, we show that any $x^m > k$ can't be optimal.

Suppose $x^m > k$ and the incentive constraint is binding ($f = v$), then set the Lagrangian as

$$\mathcal{L} = S\left(1 - e^{-x^s}\right) f + k\left(1 - \lambda^b e^{-x^s} \beta (1 - c)\right) - x^m e^{-x^s} (v - f) + \mu(v - f),$$

where $\mu > 0$. However,

$$\frac{\partial \mathcal{L}}{\partial x^m} = -e^{-x^s} f - \frac{k}{S} \lambda^b e^{-x^s} \beta (1 - c) - \mu \frac{\lambda^b e^{-x^s}}{B} (1 - c) < 0.$$

Suppose $x^m > k$ and the incentive constraint is not binding ($f < v$). We can derive the first order derivative

$$\frac{\partial \Pi(x^m; k, F)}{\partial x^m} = -e^{-x^s} f - \left(1 + \frac{x^m}{S}\right) e^{-x^s} (v - f)$$
$$- \frac{k}{S} \lambda^b e^{-x^s} \beta (1 - c) + \frac{x^m e^{-x^s}}{B} \lambda^b e^{-x^s} (1 - c).$$

If $\frac{\partial \Pi(x^m; k, F)}{\partial x^m} < 0$ for $\forall x^m \in (k, B]$, then we can immediately conclude any $x^m > k$ can't be optimal.

If $\frac{\partial \Pi(x^m; k, F)}{\partial x^m} > 0$ for $\forall x^m \in (k, B]$, we must have the optimal $x^m = B$. But a marginal increase in k at the first stage of the game raises profits by $\left(1 - \lambda^b \beta\right) (1 - c)$ till $k = x^m$. A contradiction.

If $\exists \breve{x}^m \in (k, B]$ such that $\frac{\partial \Pi(x^m; k, F)}{\partial x^m}|_{\breve{x}^m} = 0$, then optimality can only be achieved at such $\breve{x}^m$. The second order derivative follows that

$$\frac{\partial^2 \Pi(x^m; k, F)}{\partial x^{m2}}|_{\breve{x}^m} = \frac{1}{S} \frac{\partial \Pi(x^m; k, F)}{\partial x^m}|_{\breve{x}^m} + \phi(\breve{x}^m)$$

where

$$\frac{\partial \Pi(x^m; k, F)}{\partial x^m}|_{\breve{x}^m} = \left\{ \begin{array}{c} -e^{-x^s} f - \frac{k}{S} \lambda^b e^{-x^s} \beta (1-c) - e^{-x^s} (v-f) \\ -\frac{\breve{x}^m}{S} e^{-x^s} (v-f) - \breve{x}^m e^{-x^s} \frac{\partial v}{\partial x^m}|_{\breve{x}^m} \end{array} \right\},$$

$$\phi(\breve{x}^m) \equiv \left\{ \begin{array}{c} -e^{-x^s} \frac{\partial v}{\partial x^m}|_{\breve{x}^m} - \frac{1}{S} e^{-x^s} (v-f) - \frac{\breve{x}^m}{S} e^{-x^s} \frac{\partial v}{\partial x^m}|_{\breve{x}^m} \\ -\breve{x}^m e^{-x^s} \left(\frac{\partial v}{\partial x^m}|_{\breve{x}^m} + \frac{\partial^2 v}{\partial x^{m2}}|_{\breve{x}^m} \right) \end{array} \right\},$$

$$\frac{\partial v}{\partial x^m}|_{\breve{x}^m} = -\frac{\lambda^b}{S} e^{-x^s} (1-c), \quad \frac{\partial^2 v}{\partial x^{m2}}|_{\breve{x}^m} = -\frac{\lambda^b}{S^2} e^{-x^s} (1-c).$$

Given $\frac{\partial \Pi(x^m; k, F)}{\partial x^m}|_{\breve{x}^m} = 0$, we have

$$\frac{1}{S} e^{-x^s} (v-f) = \frac{1}{S} \left\{ \begin{array}{c} -e^{-x^s} f - \frac{k}{S} \lambda^b e^{-x^s} \beta (1-c) \\ -\frac{\breve{x}^m}{S} e^{-x^s} (v-f) - \breve{x}^m e^{-x^s} \frac{\partial v}{\partial x^m}|_{\breve{x}^m} \end{array} \right\}$$

$$\leq -\frac{1}{S} \breve{x}^m e^{-x^s} \frac{\partial v}{\partial x^m}|_{\breve{x}^m},$$

thus $\frac{\partial^2 \Pi(x^m; k, F)}{\partial x^{m2}}|_{\breve{x}^m} = \phi(\breve{x}^m) > 0$. Thus the second order condition is not satisfied.

Second, we consider the problem when $x^m \leq k$. The Lagrangian becomes

$$\mathcal{L} = S \left(1 - e^{-x^s} \right) f + x^m \left(1 - \lambda^b e^{-x^s} \beta (1-c) \right) - x^m e^{-x^s} (v-f) + \mu (v-f).$$

Since there is no k in this expression, the solution should only depends on f and other parameters, denoted as $\tilde{x}^m(f)$.

Sum up the analysis, we conclude the optimal queue x^m at the second stage is

$$x^m(k, f) = \min\{\tilde{x}^m(f), k\}.$$

8.3   Proof for Corollary 1

We first deal with the corner solutions. If $k^* = 0$, then increase in β certainly won't decrease the optimal inventory as it is on the lower bound.

Turn to the interior solution $k^* \in (0, B)$. Define $F(k, \beta) = \frac{1}{1-c} \frac{\partial \Pi}{\partial k}(k, \beta) = 0$. Given $\frac{\partial k}{\partial \beta} = -\frac{\frac{\partial F(k, \beta)}{\partial \beta}}{\frac{\partial F(k, \beta)}{\partial k}}$, and $\frac{\partial F(k, \beta)}{\partial k} < 0$, to show $\frac{\partial k}{\partial \beta} < 0$, we only need to prove $\frac{\partial F(k, \beta)}{\partial \beta} < 0$. This is the case,

$$\frac{\partial F(k, \beta)}{\partial \beta} = \frac{1}{1-c} \frac{\partial^2 \Pi}{\partial \beta \partial k}(k, \beta)$$

$$= -e^{-x^s} \lambda^s \xi - \lambda^b \left(1 - e^{-x^s} \right) - \frac{S \left(1 - e^{-x^s} \right) + k}{S} \lambda^b e^{-x^s} - \lambda^b \left(1 - e^{-x^s} \right)^2 < 0.$$

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