

Misallocation Effects of Labor Market Frictions

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VERY PRELIMINARY AND INCOMPLETE

Abstract

We ask whether, and in what respect, labor market frictions lead to misallocation of workers across firms. We answer this question in a heterogeneous-firm model with imperfectly directed search. Some workers can direct their search, while others are uninformed about the location of wage offers ex ante and are assigned to job openings randomly. In equilibrium, high-productivity firms attract both types of workers, whereas low-productivity firms forego attracting directed searchers so as to extract surplus from the random searchers. Relative to the social optimum, too many firms take advantage of their market power and attract only random searchers, inducing all the directed searchers to concentrate at high-productivity firms. This results in a misallocation of labor away from the middle and toward the top of the productivity distribution. An appropriately chosen minimum wage raises employment and welfare.

Keywords: Directed search, random search, labor markets, minimum wage, misallocation.

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1 Introduction

Deviations of wages from the marginal product of labor are pervasive. Recent evidence on rising labor market concentration, as well as the ongoing debates about the desirability of the minimum wage, have focused attention on the the efficiency-reducing effects of labor market power and the potential role of policy interventions.¹ However, while it is well recognized that imperfect labor market competition distorts aggregate employment, its effects on the allocation of workers across firms have received far less scrutiny. Indeed, both the textbook monopsony model and the search and matching literature have emphasized the effects of labor market power on aggregate employment; the minimum wage, in turn, affects welfare in these models by either exacerbating or dampening the employment distortion.

This focus on aggregates, however, potentially misses important distributional effects. In the presence of heterogeneous productivity, not all firms may take advantage equally of their market power. As a result, employment would be distorted more in some firms than in others. Policy interventions, in turn, will not affect all firms equally either. In the most rudimentary example, the minimum wage will lower the profits of firms for which it binds, hence possibly driving the low-wage firms out of the market in favor of high-wage firms. Understanding such allocative effects requires combining heterogeneity with labor market frictions.

Importantly, failure to explicitly model the frictions giving rise to market power may lead to misleading policy prescriptions, because these very same frictions also affect the constrained efficient allocation. This insight, though often missing in policy debates, is in fact well known in search theory. The constrained efficient allocation in a search model involves non-zero unemployment. A policymaker observing unemployment in such a world would not be able to improve welfare, thus giving drastically different prescriptions from someone observing unemployment from a perfectly competitive frictionless standpoint. To give a more sophisticated example, consider a setting in which low-productivity firms partially crowd-out high productivity firms due to, e.g. search or information frictions. A minimum wage may improve efficiency by driving some of these low-productivity firms out of the market. But suppose instead that some workers are employed at low-productivity firms because it is prohibitively costly to move to high-productivity firms. In this case, the minimum wage can only hurt, because driving out the low-productivity firms would not reallocate the workers to high-productivity firms anyway.² In other words, the very same frictions that distort the decentralized equilibrium outcome may also constrain the social planner. This is in sharp contrast to the standard textbook treatment, which typically involves comparing the

¹See e.g. De Loecker and Eeckhout (2017) for evidence of rising markups, or Azar et al. (2017) for evidence of rising labor market concentration.

²This insight is closely related to the literature on endogenously incomplete insurance markets. If some insurance markets are exogenously missing, providing public insurance typically improves welfare. However, if insurance markets are endogenously constrained by, e.g. enforcement frictions, the effects of providing public insurance are far less obvious since it may exacerbate the very same enforcement frictions. We make an analogous point for labor markets.

competitive equilibrium and monopsony to *the same* social planner’s allocation.

Motivated by these considerations, we consider a model of the labor market with heterogeneous firms and labor market frictions that constrain the allocation of workers across them. There are three key model ingredients. First, the labor market is frictional in the tradition of the competitive search literature: firms post wages, understanding that higher-wage vacancies will be filled more quickly in equilibrium. Second, firms are heterogeneous in productivity. Third, similarly to Lester (2011), some, but not all, workers can direct their search. A fraction of workers are directed searchers, who choose which job openings to apply for, understanding that higher posted wages attract more competing applicants. The remaining fraction are random searchers, who are assigned to vacancies randomly; hence their only decision is whether to accept or reject the posted wage.³ Firms will thus face a choice between attracting only random searchers or attracting both kinds of searchers. This information friction will lead to firm market power and potential misallocation of workers across firms of different productivity levels.

We show that, in equilibrium, too many workers are employed at high-productivity firms. This contrasts with the intuitive prediction that lack of information results in too many workers at low-productivity firms. Both the constrained efficient allocation and the decentralized equilibrium in this environment are characterized by a threshold rule: firms above some productivity threshold attract both random and directed searchers, whereas firms with productivity below this threshold attract only random searchers. However, the equilibrium productivity threshold is always higher than the socially efficient one. In other words, too many firms take advantage of their market power in equilibrium. These firms forego attracting directed searchers so as to extract more surplus from the random searchers. This induces too many directed searchers to queue up at the high-productivity firms. As a result, there is a misallocation of labor away from the middle and toward the top of the productivity distribution, a “flight to quality” type phenomenon.

We then use the model to study the effect of a minimum wage. A suitably chosen minimum wage does not bind for the high-productivity firms that attract directed searchers, but binds for the low-productivity firms attracting random searchers only. The latter firms are then induced to pay higher wages, thereby attracting workers away from the high-productivity firms. This reallocation of workers is efficiency-improving since it alleviates the congestion at high-productivity firms. While the previous literature has focused overwhelmingly on the aggregate employment effects of the minimum wage, we emphasize its allocative effects. Notably, the minimum wage *does* affect employment in our framework, even in the absence of an extensive entry margin; in fact, the minimum wage just described raises employment. However, the reasons for this are distinct from the conventional narrative. In the standard monopsony model, mandating higher wages results in more workers willing to work because of an upward-sloping labor supply curve. In our model, mandat-

³This modeling assumption can be interpreted literally as differences in information about available job openings. However, an alternative interpretation is differences in mobility, whereby some workers have lower costs of moving for a high-wage job than others. This will similarly generate a “captive” group of workers.

ing higher-wages results in a reallocation of workers from firms with many applicants to firms with few applicants; this raises employment in spite of the total measure of workers remaining fixed.

This paper builds on, and contributes to, the existing literature on imperfectly directed search, such as Lester (2011), Lentz and Moen (2017), or Shi (2018). Our model environment follows Lester (2011), in which a fraction of workers are directed searchers, while the remaining workers are random searchers.⁴ In this setting, like ours, the fraction of firms that choose to attract only random searchers differs from what would be socially efficient. However, in these papers all the firms are identical, and hence the planner would set the measure of firms attracting only random searchers to zero. Moreover, because firms are homogeneous, the fraction of random searchers among workers does not affect the constrained-efficient allocation. We contribute to this literature by introducing heterogeneity among firms. As a result of this modification, the information friction constrains the social planner as well, and in general there is a non-zero measure of firms exclusively targeting random searchers even at the social optimum. Constrained inefficiency is now driven by an incorrect allocation of workers across firms of differing productivities.

Our paper is directly relevant for work on labor misallocation induced by search frictions. This literature has recognized that frictions can lead to an inefficient composition of jobs or inefficient allocation of workers among them. For example, in Bertola and Caballero (1994), Acemoglu (2001), and Davis (2001), too many workers are allocated to low-productivity firms: there, search is random, and the key friction leading to inefficiency is akin to an investment holdup problem in the labor market. In Acemoglu and Shimer (1999) and Golosov et al. (2013), search is directed, but workers are risk-averse; these papers show that there will likewise be misallocation towards low-productivity firms if workers cannot insure against the risk of not finding a job. In Galenianos et al. (2011), as in our work, it is market power, driven by a finite number of firms, that leads to misallocation of workers, namely too many workers being employed at low-productivity firms. In our paper, where market power is instead driven by imperfectly directed search, we show that there is misallocation toward *high-productivity* firms, contrary to the previous literature. Notably, such a result would not arise when search is either fully random or fully directed. We also share with Galenianos et al. (2011) the prediction that a binding minimum wage reallocates workers towards low-productivity firms; however, in our setting this reallocation can be welfare-improving.

2 Environment

We consider a static model. There is a measure 1 of firms, each with one job opening. Firms are heterogeneous in productivity $y \in [0, \bar{y}]$, which is drawn from a distribution F with

⁴See Bethune et al. (2016) for an application to a monetary economy. Both Lester (2011) and Bethune et al. (2016) are set in the context of a product market rather than a labor market, and hence agents are buyers and sellers rather than workers and firms, but this distinction is inconsequential except for the interpretation.

density f . There is a measure 1 of workers, all initially unemployed. A fraction $\psi \in [0, 1]$ of workers are random searchers, who will be assigned randomly across all the vacancies. The remaining fraction $1 - \psi$ are directed searchers, who can choose which vacancy to target. Matching works as follows. If the searcher-vacancy ratio at a particular vacancy is λ , the vacancy gets filled with probability λ , and each worker applying to that vacancy has a probability $m(\lambda)/\lambda$ of being matched. A worker matched with a firm of productivity y produces y ; unmatched workers and firms produce 0. The matching function $m(\lambda)$ satisfies the standard assumptions $m' > 0, m'' < 0$. For future reference, we also define the function $g(\lambda) = m(\lambda) - \lambda m'(\lambda)$. By the assumptions on m , we see that g satisfies $g'(\lambda) > 0$ and $g(\lambda) < m(\lambda)$.

Note that if $\psi = 0$, this is a standard competitive search environment. However, with $\psi > 0$, there will be at least ψ workers at each vacancy. Hence the queue length at each vacancy satisfies $\lambda \geq \psi$. At the other extreme, $\psi = 1$, we have a standard random search environment in which $\lambda = 1$ always.

3 Planner's problem

The planner chooses the distribution of workers across posted vacancies. Thus, the planner's problem can be written as choosing $\lambda(y)$ for every $y \in [0, \bar{y}]$ so as to maximize

$$\int_0^{\bar{y}} m(\lambda(y)) y f(y) dy. \tag{1}$$

The planner maximizes (1) subject to two constraints. First, there is a resource constraint, which says that the total measure of workers at all the vacancies must add up to 1:

$$\int_0^{\bar{y}} \lambda(y) f(y) dy = 1. \tag{2}$$

Second, and crucially, the planner must respect the randomness of search for some workers. This means that the planner must assign the ψ random searchers randomly across all the posted vacancies, thereby assigning at least ψ workers to each vacancy. Hence, the constraint states

$$\lambda(y) \geq \psi \quad \forall y \in [0, \bar{y}]. \tag{3}$$

Let η be the Lagrange multiplier on (2), and let $f(y) \mu(y) dy$ be the Lagrange multiplier on (3) for each y . We can write the Lagrangian

$$\begin{aligned} \mathcal{L} = & \int_0^{\bar{y}} m(\lambda(y)) y f(y) dy \\ & + \eta \left(1 - \int_0^{\bar{y}} \lambda(y) f(y) dy \right) \\ & + \int_0^{\bar{y}} \mu(y) [\lambda(y) - \psi] f(y) dy. \end{aligned} \quad (4)$$

The first-order condition for $\lambda(y)$ can be written as

$$\mu(y) = \eta - m'(\lambda(y)) y. \quad (5)$$

Whenever the constraint (3) binds, we have $\lambda = \psi$ and therefore $\mu(y) = \eta - m'(\psi) y$, which is strictly decreasing in y . Therefore, the constraint binds for all y below some threshold and does not bind for y above it. Denoting this threshold by \tilde{y}_p (where the subscript p denotes the planner's allocation throughout), we immediately see that it satisfies

$$m'(\psi) \tilde{y}_p = \eta. \quad (6)$$

Hence (5) can be rewritten as

$$\mu(y) = m'(\psi) \max\{0, \tilde{y}_p - y\}. \quad (7)$$

Thus, firms with $y < \tilde{y}_p$ face a binding constraint (3) and hence have queue length $\lambda_p(y) = \psi$. Firms with $y \geq \tilde{y}_p$ have queue length $\lambda_p(y) = (m')^{-1}(\eta/y)$. We can therefore write

$$\lambda_p(y) = \max\left\{\psi, (m')^{-1}(\eta/y)\right\}. \quad (8)$$

Note that $\lambda_p(y)$ thus defined is non-increasing in η for each y , and continuous and non-decreasing in y . The multiplier η is then pinned down as the unique value for which the resource constraint holds with equality, i.e.

$$\int_0^{\bar{y}} \max\left\{\psi, (m')^{-1}(\eta/y)\right\} f(y) dy = 1. \quad (9)$$

Uniqueness of the solution follows from the fact that the integral is strictly decreasing in η . This can be summarized as

Lemma 1 *The constrained-efficient allocation is characterized by a number η and a function $\lambda_p(y)$ satisfying (8) and (9). There exists a solution to this system, and it is unique.*

Next, we consider what happens when there is an increase in ψ , i.e. fewer workers are

informed. A key lesson is that ψ affects the constrained-efficient allocation.

Lemma 2 *An increase in ψ raises η and \tilde{y}_p .*

This is seen most clearly from (9). An increase in ψ raises the mass of workers in low-productivity firms. For the total measure of workers to stay equal to 1, the mass of workers in high-productivity firms must fall. This is accomplished efficiently by both a rise in the cutoff productivity above which directed searchers are located, and a decrease in the number directed searchers at each productivity level above the cutoff. The shadow benefit of a worker rises since workers are now more scarce at high-productivity firms.

4 Equilibrium

We now analyze the decentralized equilibrium and show how and in what respects it differs from the planner's allocation. Each firm decides what wage to post. The $1 - \psi$ directed searchers observe all the posted wages and decide to which firm to apply. The ψ random searchers are assigned to vacancies randomly. The combination of these choices determines the queue length $\lambda(y)$ at each productivity type y . If a firm posts a wage w' and attracts a queue length λ' , the utility of a directed searcher applying to that firm is $\frac{m(\lambda')}{\lambda'}w'$. Define the market utility \mathcal{U} to be the maximum utility across all submarkets that a directed searcher can obtain:

$$\mathcal{U} \equiv \max_{w', \lambda'} \frac{m(\lambda')}{\lambda'} w'. \quad (10)$$

Then a firm will not attract directed searchers unless it offers utility of at least \mathcal{U} . A firm of productivity y offering less than that utility will attract random searchers only, therefore receiving profits $m(\psi)(y - w)$. From this it is easy to conclude that a firm that chooses not to attract directed searchers will offer a wage of zero, and its profits are therefore

$$\pi^R(y) = m(\psi)y. \quad (11)$$

On the other hand, a firm of productivity y that would like to attract some directed searchers solves the problem

$$\pi^D(y) = \max_{w, \lambda} m(\lambda)(y - w) \quad (12)$$

subject to

$$\frac{m(\lambda)}{\lambda} w \geq \mathcal{U}. \quad (13)$$

It is easy to see that (13) will bind. Solving for w using the binding constraint (13) and substituting into (12), we get the maximization problem

$$\pi^D(y) = \max_{\lambda} m(\lambda)y - \lambda\mathcal{U}; \quad (14)$$

the solution satisfies

$$m'(\lambda)y = \mathcal{U}, \quad (15)$$

and the maximized profit is therefore

$$\pi^D(y) = g\left((m')^{-1}(\mathcal{U}/y)\right)y, \quad (16)$$

recalling that $g(\lambda) = m(\lambda) - \lambda m'(\lambda)$. Comparing (16) to (11), we conclude that a firm will choose to attract directed searchers if and only if

$$g\left((m')^{-1}(\mathcal{U}/y)\right) \geq m(\psi).$$

Note that $g\left((m')^{-1}(\mathcal{U}/y)\right)$ is increasing in y , by concavity of m . As a result, there is a threshold \tilde{y}_d such that a firm chooses to attract directed searchers if and only if $y \geq \tilde{y}_d$. This threshold is the unique \tilde{y}_d satisfying

$$g\left((m')^{-1}(\mathcal{U}/\tilde{y}_d)\right) = m(\psi). \quad (17)$$

The queue length, denoted by $\lambda_d(y)$, therefore satisfies

$$\lambda_d(y) = \begin{cases} \psi, & \text{if } y < \tilde{y}_d, \\ (m')^{-1}(\mathcal{U}/y), & \text{if } y \geq \tilde{y}_d. \end{cases} \quad (18)$$

Observe that $\lambda_d(y)$ is non-increasing in \mathcal{U} for each y , and non-decreasing in y . However, unlike the planner's $\lambda_p(y)$ defined by (8), $\lambda_d(y)$ is discontinuous at the threshold \tilde{y}_d . This is because $g(\lambda) < m(\lambda)$ for all λ , and so

$$g\left((m')^{-1}(\mathcal{U}/\tilde{y}_d)\right) = m(\psi) > g(\psi),$$

implying $(m')^{-1}(\mathcal{U}/\tilde{y}_d) > \psi$. In words, the firm with productivity \tilde{y}_d is indifferent between posting a wage of 0 (hence attracting a queue length of ψ) and posting a strictly positive wage high enough to attract directed searchers. Since the latter entails a discrete increase in the wage, indifference requires a discrete increase in the queue length.

Finally, the market utility \mathcal{U} of directed searchers is pinned down by the market clearing condition

$$\begin{aligned} 1 &= \int_0^{\bar{y}} \lambda_d(y) f(y) dy \\ &= \psi F(\tilde{y}_d) + \int_{\tilde{y}_d}^{\bar{y}} (m')^{-1}(\mathcal{U}/y) f(y) dy \end{aligned} \quad (19)$$

To summarize, we have

Lemma 3 *The decentralized equilibrium is characterized by \mathcal{U} , \tilde{y}_d , and a function $\lambda_d(y)$ satisfying (17), (18) and (19). There exists a decentralized equilibrium, and it is unique.*

Uniqueness easily follows from the observation that (17) defines \tilde{y}_d as an increasing function of \mathcal{U} , and (19) defines \tilde{y}_d as a decreasing function of \mathcal{U} .

We next compare the equilibrium to the constrained efficient allocation. Unlike the constrained efficient allocation, the equilibrium queue length $\lambda_d(y)$ features a discrete increase at the threshold \tilde{y}_d . For market clearing to hold, i.e. for the average queue length to still equal 1 despite the jump at the threshold, it must be that the threshold is strictly greater than the planner's threshold \tilde{y}_p . This yields the main inefficiency result.

Proposition 1 *Suppose that $\psi \in (0, 1)$. Let $\eta, \lambda_p(\cdot)$ be the constrained efficient allocation, with the corresponding threshold \tilde{y}_p defined by (6). Let $\mathcal{U}, \lambda_d(\cdot)$ be the decentralized equilibrium allocation, with the corresponding threshold \tilde{y}_d defined by (17). Then the following holds: (i) $\mathcal{U} < \eta$, (ii) $\tilde{y}_d > \tilde{y}_p$, and (iii) $\lambda_d(y) > \lambda_p(y)$ for all $y \geq \tilde{y}_d$.*

Proof. See Appendix. ■

The results are illustrated in Figure 1, which shows the queue length as a function of productivity under both the constrained efficient allocation and the decentralized equilibrium allocation. Under the equilibrium allocation, $\lambda_d(y)$ jumps at \tilde{y}_d from ψ to $\tilde{\lambda}_d = (m')^{-1}(\mathcal{U}/y)$, and stays strictly above $\lambda_p(y)$ thereafter. Intuitively, in equilibrium, too many firms choose to attract only random searchers in order to extract the full surplus from those random searchers. In doing so, they do not internalize the congestion this causes at higher-productivity firms, since all the directed searchers now queue up for those high-productivity firms.

5 Effects of the minimum wage

In this section, we consider the effect of a minimum wage w_{\min} , which binds only for random searchers. We show the following. First, an increase in w_{\min} lowers \tilde{y}_d and raises \mathcal{U} . Second, we show that welfare is monotonically decreasing in \tilde{y}_d for $\tilde{y}_d > \tilde{y}_p$, implying that such a minimum wage improves welfare. Third, a properly chosen minimum wage implements the planner's allocation. This optimal minimum wage is increasing in ψ .

A minimum wage w_{\min} binds for directed searchers if there exists a y in equilibrium such that $\lambda(y) > \psi$ and $w(y) = w_{\min}$. If this is the case, there are no firms that attract only random searchers, since such firms would have to be posting a wage strictly below w_{\min} . But this implies that $\tilde{y}_d < \tilde{y}_p$, so the minimum wage “overshoots” the social planner's level. Therefore, we consider minimum wages that bind only for random searchers.

The first question is whether the equilibrium still has the threshold property. Note that the profit of a D firm looks the same, except for the general equilibrium effect on \mathcal{U} . Therefore, we have

$$\pi^D(y) = g\left((m')^{-1}(\mathcal{U}/y)\right)y \quad (20)$$

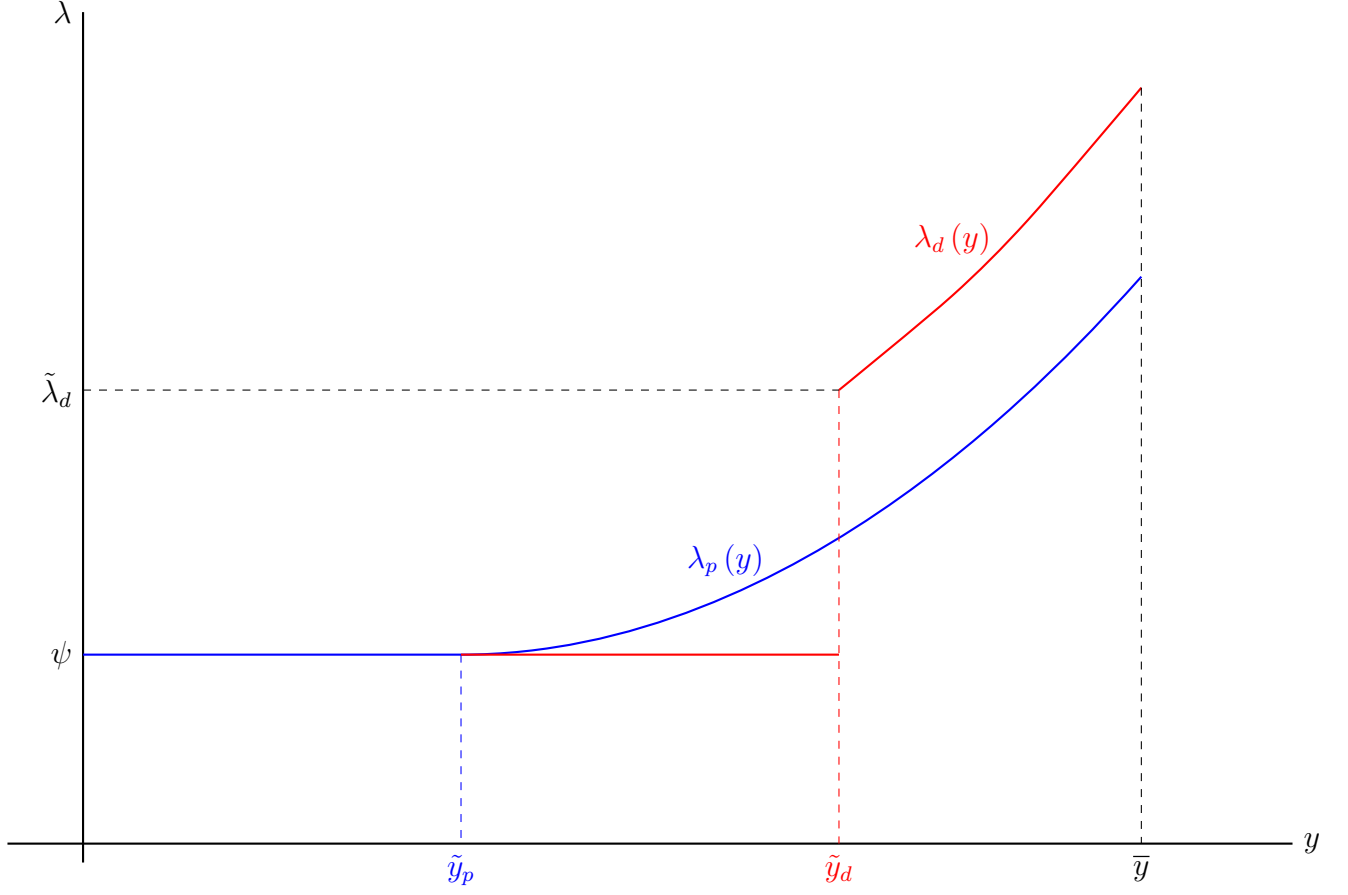


Figure 1: Equilibrium ($\lambda_d(y)$) vs. efficient allocation ($\lambda_p(y)$).

The profit of an R firm, however, is

$$\pi^R(y) = m(\psi)(y - w_{\min}) \quad (21)$$

Applying the envelope theorem to $\pi^D(y)$,

$$\frac{d}{dy} \{ \pi^D(y) - \pi^R(y) \} = m \left((m')^{-1}(\mathcal{U}/y) \right) - m(\psi) > 0, \quad (22)$$

This means that $\pi^D(y) \geq \pi^R(y)$ if and only if $y \geq \tilde{y}_d$, and there is a unique \tilde{y}_d satisfying

$$g \left((m')^{-1}(\mathcal{U}/\tilde{y}_d) \right) \tilde{y}_d = m(\psi)(\tilde{y}_d - w_{\min}) \quad (23)$$

As before, the queue length is determined by

$$\lambda_d(y) = \begin{cases} \psi, & y \leq \tilde{y}_d \\ (m')^{-1}(\mathcal{U}/y), & y > \tilde{y}_d \end{cases} \quad (24)$$

The equilibrium now satisfies (23), (24), and the market-clearing condition. Defining $\epsilon(\lambda) = \lambda m'(\lambda) / m(\lambda)$, we have

Proposition 2 *Suppose there is a minimum wage $w_{\min} < \tilde{y}_p \epsilon(\psi)$. The equilibrium exists and it is unique. In this equilibrium, the minimum wage binds only for R firms. Furthermore, as long as it does not surpass $\tilde{y}_p \epsilon(\psi)$, an increase in w_{\min} (i) lowers \tilde{y}_d , (ii) raises \mathcal{U} , (iii) raises employment, and (iv) raises total welfare.*

Proof. See Appendix. ■

The same reasoning establishes that the planner's allocation can be restored by setting the minimum wage at the highest level at which it does not bind for directed searchers.

Corollary 1 *The constrained-efficient outcome is achieved by setting $w_{\min} = \tilde{y}_p \epsilon(\psi)$.*

The proof follows from the fact that

$$\tilde{y}_p \epsilon(\psi) = \tilde{y}_p \frac{\psi m'(\psi)}{m(\psi)} = \eta \frac{\psi}{m(\psi)},$$

where η is the shadow value of a worker in the planner's problem. We then note that $\frac{\psi}{m(\psi)}$ is increasing in ψ by the properties of m , and η is increasing in ψ by Lemma 2.

6 Discussion

Our analysis has established that imperfect information about terms of trade offered by jobs leads a misallocation of workers across jobs - in particular, a flight-to-quality type phenomenon whereby too many informed workers are concentrated at high-productivity firms. As a result, policy interventions can improve welfare by reallocating workers. We have illustrated this by showing that a suitably chosen minimum wage can increase welfare and employment. While we have focused on imperfect information as a source of distortion, largely similar conclusions would arise if workers faced mobility frictions, i.e. some workers are unable to reallocate to better-paying firms. Our analysis is thus of relevance to the recent debates concerning the consequences of employer market power in the US and the desirability of labor market regulation, as well as to labor markets in developing countries where information frictions are rampant.

There are several natural directions for future analysis. First, there are potentially interesting implications for the effects of information technology, as captured by the fraction of search that is directed rather than random. Our model implies that a reduction in ψ leads to a reallocation of workers across firms. Consider the thought experiment of reducing ψ while keeping fixed the threshold \tilde{y}_d . This will result in fewer workers employed at low-productivity firms, and more at high-productivity firms. However, the reduction in the fraction of random searchers will endogenously lower \tilde{y}_d , inducing the marginal firms to now attract some directed searchers. This counteracting response implies that the effect on employment at the

top-productivity firms - and therefore on the utility of the workers - is ambiguous. Similarly, an improvement in information has non-trivial effects on welfare. This is most easily seen by noticing that the equilibrium outcome is constrained-efficient for either $\psi = 0$ or $\psi = 1$ (either perfectly directed or perfectly random search) but not for intermediate values of ψ . Thus, there are ambiguous output and welfare effects of an improvement in information.

Second, an extension of our model allows for endogenous vacancy posting. With such an entry margin, the social planner would face a tradeoff between shutting down some firms and letting them attract workers away from higher productivity firms. In equilibrium, firms' entry decisions entail an inefficiency since they do not internalize the crowding out effect they have on higher productivity firms. This standard congestion externality is combined with the misallocation effect highlighted in the baseline model, leading to too many workers at the top and bottom of the productivity distribution, and too few in the middle. This adds an additional dimension to the policy analysis above, since, e.g. a binding minimum wage now affects both which firms choose to post a vacancy and the allocation of workers across active firms.

Third, potential strategic complementarities can arise when information is an endogenous choice. Suppose that a worker can decide, at a cost, to be a directed searcher. This can be interpreted as a time and effort cost of learning about available job openings, or even the choice of an occupation that allows for greater geographic mobility. The fraction of search that is directed is now an endogenous variable that depends on the market utility of directed searchers, which, in turn, depends itself on this fraction. In particular, as discussed above, an increase in the fraction of random searchers can potentially lead to more congestion of directed searchers at the very high productivity firms, potentially reducing the payoff from being a directed searcher. This may result in multiple equilibria that differ in proportion of random searchers, worker allocation, and welfare. Moreover, labor market policies discussed earlier now affect employment, output and welfare not only directly, but also by changing the incentives to acquire information. This is the subject of ongoing research.

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A Proofs

Proof of Proposition 1. We first show that $\mathcal{U} < \eta$. By market clearing,

$$\begin{aligned} \int_0^{\bar{y}} \max \left\{ \psi, (m')^{-1}(\eta/y) \right\} f(y) dy &= 1 = \int_0^{\tilde{y}_d} \psi f(y) dy + \int_{\tilde{y}_d}^{\bar{y}} (m')^{-1}(\mathcal{U}/y) f(y) dy \\ &< \int_0^{\bar{y}} \max \left\{ \psi, (m')^{-1}(\mathcal{U}/y) \right\} f(y) dy. \end{aligned}$$

Since $\int_0^{\bar{y}} \max \left\{ \psi, (m')^{-1}(x/y) \right\} f(y) dy$ is strictly decreasing in x , it follows that $\mathcal{U} < \eta$. Next, suppose that $\tilde{y}_d \leq \tilde{y}_p$. Then $(m')^{-1}(\mathcal{U}/y) > \psi$ for all $y \in (\tilde{y}_d, \tilde{y}_p]$, and so

$$\begin{aligned} 1 &= \int_0^{\tilde{y}_d} \psi f(y) dy + \int_{\tilde{y}_d}^{\bar{y}} (m')^{-1}(\mathcal{U}/y) f(y) dy \\ &> \int_0^{\tilde{y}_d} \psi f(y) dy + \int_{\tilde{y}_d}^{\tilde{y}_p} \psi f(y) dy + \int_{\tilde{y}_p}^{\bar{y}} (m')^{-1}(\mathcal{U}/y) f(y) dy \\ &= \int_0^{\tilde{y}_p} \psi f(y) dy + \int_{\tilde{y}_p}^{\bar{y}} (m')^{-1}(\mathcal{U}/y) f(y) dy \\ &> \int_0^{\tilde{y}_p} \psi f(y) dy + \int_{\tilde{y}_p}^{\bar{y}} (m')^{-1}(\eta/y) f(y) dy, \end{aligned}$$

contradicting market clearing. This proves that $\tilde{y}_d > \tilde{y}_p$. Finally, $\mathcal{U} < \eta$ implies $(m')^{-1}(\mathcal{U}/y) > (m')^{-1}(\eta/y)$ and therefore $\lambda_d(y) > \lambda_p(y)$ for all $y \geq \tilde{y}_d$. ■

Proof of Proposition 2. It is straightforward to see that (23) defines \tilde{y}_d as an increasing function of \mathcal{U} for any w_{\min} , and a decreasing function of w_{\min} for any \mathcal{U} . Next, the market-clearing condition

$$\psi F(\tilde{y}_d) + \int_{\tilde{y}_d}^{\bar{y}} (m')^{-1}(\mathcal{U}/y) f(y) dy = 1 \quad (25)$$

defines \tilde{y}_d as a decreasing function of \mathcal{U} . In particular, differentiating (25) with respect to \mathcal{U} , we get

$$\frac{d\tilde{y}_d}{d\mathcal{U}} = \frac{1}{f(\tilde{y}_d) \left((m')^{-1}(\mathcal{U}/\tilde{y}_d) - \psi \right)} \int_{\tilde{y}_d}^{\bar{y}} \frac{\partial}{\partial \mathcal{U}} (m')^{-1}(\mathcal{U}/y) f(y) dy < 0 \quad (26)$$

Together with (23), this implies that the equilibrium is unique. Furthermore, the implicit function theorem implies that an increase in w_{\min} lowers \tilde{y}_d and raises \mathcal{U} (with \tilde{y}_d implicitly defined as a function of \mathcal{U} via (25)). This also means that, to show that aggregate employment and welfare are increasing in w_{\min} , it is sufficient to show that they are increasing in \mathcal{U} .

Aggregate employment is given by

$$E = \int_0^{\bar{y}} m(\lambda_d(y)) f(y) dy = m(\psi) F(\tilde{y}_d) + \int_{\tilde{y}_d}^{\bar{y}} m\left((m')^{-1}(\mathcal{U}/y)\right) f(y) dy \quad (27)$$

Differentiating (27) with respect to \mathcal{U} and using (26), we obtain

$$\frac{dE}{d\mathcal{U}} = \int_{\tilde{y}_d}^{\bar{y}} \left[m'(\lambda_d(y)) - \frac{m\left((m')^{-1}(\mathcal{U}/\tilde{y}_d)\right) - m(\psi)}{(m')^{-1}(\mathcal{U}/\tilde{y}_d) - \psi} \right] \frac{\partial}{\partial \mathcal{U}} \left((m')^{-1}(\mathcal{U}/y) \right) f(y) dy, \quad (28)$$

and note that the term in brackets is negative:

$$m'(\lambda_d(y)) - \frac{m\left((m')^{-1}(\mathcal{U}/\tilde{y}_d)\right) - m(\psi)}{(m')^{-1}(\mathcal{U}/\tilde{y}_d) - \psi} < \mathcal{U}/\tilde{y}_d - \frac{m\left((m')^{-1}(\mathcal{U}/\tilde{y}_d)\right) - m(\psi)}{(m')^{-1}(\mathcal{U}/\tilde{y}_d) - \psi} < 0 \quad (29)$$

by concavity of m . Since $\frac{\partial}{\partial \mathcal{U}} \left((m')^{-1}(\mathcal{U}/y) \right) < 0$, this establishes that $\frac{dE}{d\mathcal{U}} > 0$, and therefore E is increasing in w_{\min} . We use a similar argument for welfare, which is given by

$$\begin{aligned} \mathcal{W} &= \int_0^{\bar{y}} m(\lambda_d(y)) y f(y) dy \\ &= m(\psi) \int_0^{\tilde{y}_d} y f(y) dy + \int_{\tilde{y}_d}^{\bar{y}} m\left((m')^{-1}(\mathcal{U}/y)\right) y f(y) dy \end{aligned} \quad (30)$$

Differentiating with respect to \mathcal{U} and using (26), we obtain

$$\frac{d\mathcal{W}}{d\mathcal{U}} = \int_{\tilde{y}_d}^{\bar{y}} \left[\mathcal{U} - \frac{m\left((m')^{-1}(\mathcal{U}/\tilde{y}_d)\right) - m(\psi)}{(m')^{-1}(\mathcal{U}/\tilde{y}_d) - \psi} \tilde{y}_d \right] \frac{\partial}{\partial \mathcal{U}} \left((m')^{-1}(\mathcal{U}/y) \right) f(y) dy > 0 \quad (31)$$

■