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THE PRICE PUZZLE AND VAR IDENTIFICATION

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In structural VARs, unexpected monetary tightening often leads to the price puzzle, a counterintuitive increase in inflation in the impulse response function. The identification of impulse responses requires at least a minimal set of structural assumptions, and models exhibiting the price puzzle typically use standard assumptions focusing mainly on relationships among contemporaneous disturbances. This note uses a well-established stylized fact, the long lags of monetary policy, to motivate a simple additional identifying assumption. The assumption eliminates a single term in one equation of the reduced form, and with it the price puzzle.

Keywords: Vector Autoregression, Monetary Policy, Policy Lag

1. INTRODUCTION

More than twenty-five years after the recognition of the price puzzle by Sims (1986), the current literature still contains many examples of papers that find the puzzle in empirical vector autoregression (VARs) and sometimes propose new solutions. Specifically, the puzzle is that unexpected monetary tightening leads to an increase in inflation in the model, instead of a decline as intuition, theory, and empirical evidence would suggest.

Virtually all of the existing proposals for dealing with the price puzzle introduce additional variables into the VAR to mitigate the empirical magnitude of the counterintuitive effects. This article takes the opposite approach of making the model simpler by incorporating a single theoretically motivated exclusion constraint.

The present approach is based on the well-documented tenet that there are substantial lags between monetary policy actions and their observable effects on real economic activity and, particularly, on inflation. Structural VAR identification usually focuses on contemporaneous or long-term restrictions. In contrast, we look here to the monetary lags literature for a simple dynamic restriction. When a standard empirical VAR is reestimated to reflect this one constraint, the price puzzle disappears.

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2. THE PRICE PUZZLE AND SOME PROPOSED SOLUTIONS

We begin with an illustrative structural VAR that exhibits the price puzzle when estimated using U.S. data. The VAR contains three variables: inflation, real GDP growth, and the nominal short-term interest rate. These series are calculated here, respectively, as annualized quarterly log changes in the chain-weighted price index P_t and real GDP Y_t, and as the quarterly average of the federal funds rate.

Let $X_t = (\pi_t, y_t, r_t)'$ be the vector of endogenous variables in the VAR, where $\pi_t = 400 \log(P_t/P_{t-1})$ and $y_t = 400 \log(Y_t/Y_{t-1})$. The VAR includes four quarterly lags of each of the variables and may be represented as

$$X_t = b + \sum_{i=1}^{4} B_i X_{t-i} + u_t, \qquad (1)$$

where b is a constant vector and the B_i are 3×3 matrices of unrestricted constant coefficients. The model is estimated using quarterly U.S. data from Q3 1955 (start of fed funds data) to Q2 2013. Impulse responses are first derived by identifying the shock structure using Choleski factorization of the variance of u_t, with a standard ordering as listed in X_t.

Figure 1 presents impulse responses for the model over 20 quarters, derived by Monte Carlo methods as suggested by Sims and Zha (1999). The solid line is the median of the simulated distribution and the dashed lines represent 0.68 error bands. With one exception, the results in all the panels seem intuitively plausible. For instance, we see along the diagonal panels that inflation and funds rate shocks are very persistent, but that output shocks are much less so. The response of inflation to output is positive, as in a short-run Phillips curve relationship, and the short-term response of output to a positive shock in the funds rate is negative, as generally expected. The responses of the interest rate to both output and inflation shocks are positive, consistent with a policy rule interpretation.

The only exceptional case in Figure 1 is the positive response of inflation to the funds rate: in other words, the price puzzle. This response remains positive for several quarters and is statistically significant after one quarter even at a .95 confidence level. This empirical example provides a clear illustration of the prize puzzle, which is also found in larger VARs and in estimates using data for other time periods.

Various types of solutions to the puzzle have been put forth over the years. Sims (1992) conjectures that central banks have internal information about inflation expectations that is not captured in the simple VAR structure and proposes including a measure of commodity price increases in the VAR as a proxy for unobserved inflation expectations. Other researchers have followed this lead and include various measures of commodity prices in structural VARs—for instance, Kim (1999), Kim and Roubini (2000), Barth and Ramey (2001), Hanson (2004), and Sims and Zha (2006).

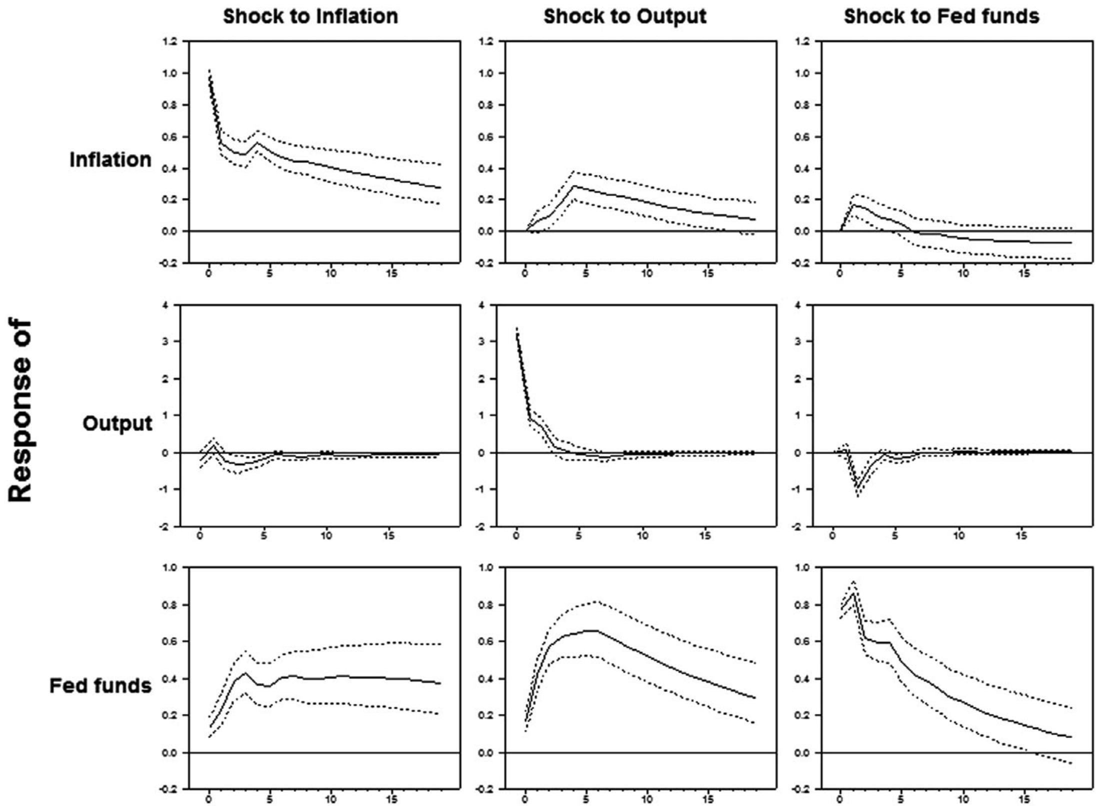

FIGURE 1. Impulse responses with full VAR lag structure. Model (1) is estimated with U.S. data from 1955 Q3 to 2013 Q2. Impulse responses are given by the solid lines. Dotted lines represent 0.68 error bands as suggested by Sims and Zha (1999).

Other authors have added a number of other variables to the VAR in search of the missing link that would absorb the counterintuitive correlations. These include Giordani (2004), Bernanke et al. (2005), Francis and Owyang (2005), Kozicki and Tinsley (2005), Lagana and Mountford (2005), and Herrera and Pesavento (2009). Bierens (2000) extracts common nonlinear trends from the variables, Scholl and Uhlig (2008) constrain the questionable impulse responses to be negative, and Krusec (2010) proposes long-run restrictions as a solution. Finally, other recent explanations invoke the cost channel, as for example in Rabanal (2007), Henzel et al. (2009), and Castelnuovo (2012); deep habits, as in Ravn et al. (2010); nonlinearities, as in Bachmeier et al. (2007); and asymmetric information, as in Tas (2011). The common thread in all of these approaches is that the solution is more complex than the original model.

3. THE LONG LAGS OF MONETARY POLICY

The basic modeling goal of this article is to find a minimal identification structure for a VAR that does not exhibit the price puzzle. In a seminal set of articles, Friedman (1958, 1960, 1961) argued that the lags between monetary policy actions and their influence on prices and economic activity are "long and variable." The lag between policy and inflation is seen to be anywhere from a few quarters to several years. Friedman's results have been confirmed empirically by Bernanke and Gertler (1995) and Batini and Nelson (2001) using more recent data and methodologies.

On the theoretical side, Svensson (1997) proposed a model that exhibits monetary policy lags consistent with observed facts. A version of that model is composed of three equations representing aggregate supply, aggregate demand, and the monetary policy rule:

$$\pi_t = a_1\pi_{t-1} + a_2 y_{t-1} + \varepsilon_t, \tag{2}$$

$$y_t = b_1 y_{t-1} + b_2 (r_{t-1} - \pi_t) + \nu_t, \tag{3}$$

$$r_t = c_1 r_{t-1} + c_2\pi_t + c_3 y_t + \mu_t, \tag{4}$$

where $a_1 > 0$, $a_2 > 0$, ε_t is a supply shock, $b_1 > 0$, $b_2 < 0$, ν_t is a demand shock, $c_i > 0$, $i = 1, 2, 3$, and μ_t is a monetary policy shock. Assume for simplicity that the equilibrium values of all the variables are zero and that the shocks are independent and identically distributed. For our purposes, the key feature of the model is the absence of a direct channel from r_{t-1} to π_t in equation (2), reflecting the monetary policy lag.

If we define $X_t = (\pi_t, y_t, r_t)'$ as before, the model may be written as

$$A_0 X_t = A_1 X_{t-1} + e_t, \tag{5}$$

where A_0 and A_1 are 3×3 matrices, A_0 is lower triangular, and A_1 contains some zero values. The reduced form is

$$X_t = A_0^{-1} A_1 X_{t-1} + A_0^{-1} e_t \equiv B_1 X_{t-1} + u_t \tag{6}$$

with A_0^{-1} lower triangular. The coefficient matrix B_1 has a general form except for the restriction that the (1,3) element is zero, as a consequence of the monetary policy lag.

This exclusion restriction is also seen in the dual moving average (MA) representation, which corresponds directly to the impulse responses. Referring to equation (5), the MA representation of the model is

$$X_t = (A_0 - A_1 L)^{-1} e_t = C_0 e_t + C_1 e_{t-1} + C_2 e_{t-2} + \cdots , \tag{7}$$

where L is the lag operator and $C_j = (A_0^{-1} A_1)^j A_0^{-1}$. The properties of A_0 and A_1 imply that the (1,3) elements of both C_0 and C_1 are zero. Hence the first nonzero effect of the monetary policy shock on inflation occurs with a two-period lag and is negative, as indicated here by the (1,3) element of C_2, which is $a_2 b_2 < 0$.

4. IMPLICATIONS FOR SOLVING THE PRICE PUZZLE

To illustrate the effects on the price puzzle of imposing the zero restriction from Section 3, we repeat the estimation and computation of impulse responses in the empirical VAR of Section 2, but this time with the single additional restriction on B_1. The resulting numerical impulse responses are shown in Figure 2.

Most of the impulse responses in Figure 2 are virtually the same as their counterparts in Figure 1. The only panel that differs appreciably is the response of inflation to the funds rate, which in Figure 2 is never significantly positive and becomes negative within a few quarters. After four quarters, the full error band lies below zero and the reduction in inflation resulting from the shock persists over the rest of the five-year horizon. The price puzzle is thus absent from these results.

This particular solution to the puzzle stems solely from the exclusion of the first lag of the funds rate in equation (2) and consequently in the reduced-form equation for inflation. Impulse responses depend critically on structural identification and we should not underestimate the importance of even seemingly modest identification assumptions on the economic properties of the model. In this case, recognition of the well-documented structural lag in the effect of monetary policy on inflation leads to sensible results and elimination of the price puzzle.

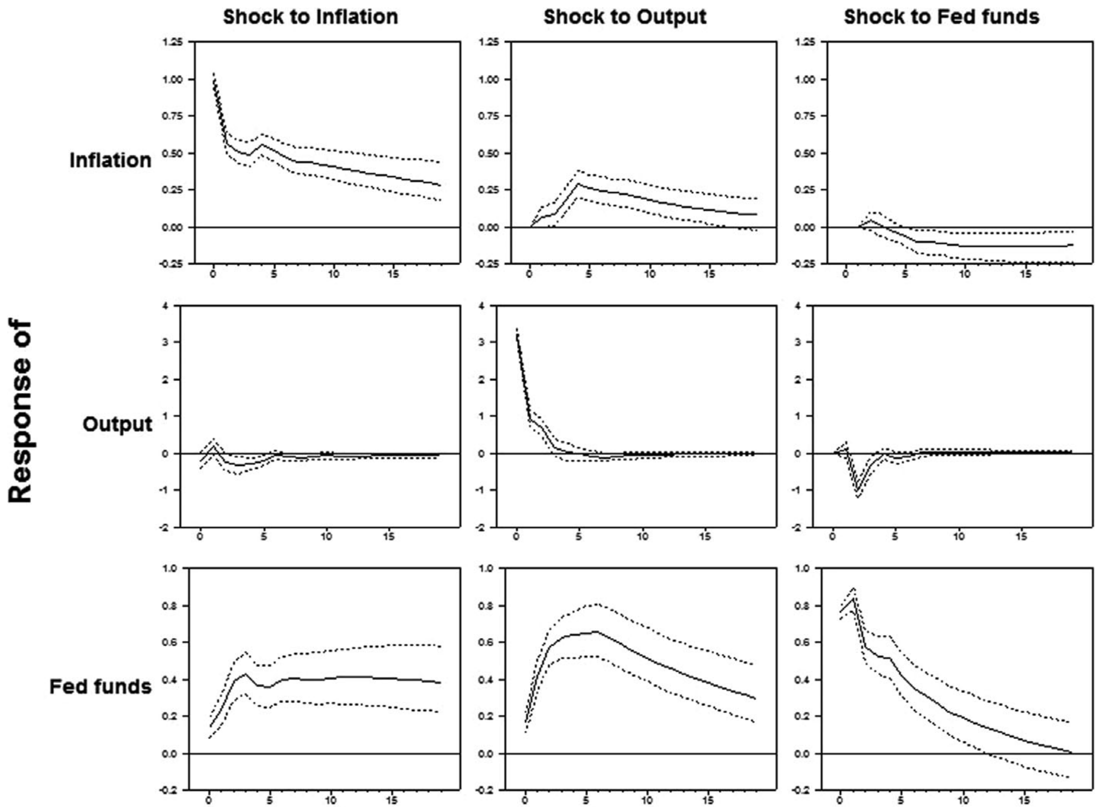

FIGURE 2. Impulse responses with near VAR lag structure: excludes first lag of the fed funds rate in inflation equation. Model (1) is estimated with U.S. data from 1955 Q3 to 2013 Q2 and the restriction $B_{113} = 0$. Dotted lines represent 0.68 error bands as suggested by Sims and Zha (1999).

5. CONCLUDING REMARKS

The construction of impulse responses requires a minimal set of identification restrictions in order to produce sensible structural implications. When the goal is to interpret one of the VAR disturbances as a monetary policy shock, the clear stylized fact of monetary policy lags should be reflected in that minimal set of identification assumptions. The analysis here supports imposing a single zero restriction in the matrix of coefficients of the first lag of the reduced form of our illustrative structural VAR, an approach supported by both theory and empirical work.

When the restriction is applied to an otherwise standard structural VAR that exhibits the price puzzle, the puzzle disappears. Additional variables or model structure are not required in the context of this model.

Note, however, that these findings do not contradict other proposed solutions to the price puzzle and that the zero restriction and those other solutions are not in practice mutually exclusive. The present analysis simply demonstrates that it is possible to obtain an interpretable set of impulse responses in a representative VAR through minimal structural assumptions.

REFERENCES

Bachmeier, Lance, Sittisak Leelahanon, and Qi Li (2007) Money growth and inflation in the United States. *Macroeconomic Dynamics* 11, 113–127.

Barth, Marvin J., III and Valerie A. Ramey (2001) The cost channel of monetary transmission. *NBER Macroeconomics Annual* 16, 199–240.

Batini, Nicoletta and Edward Nelson (2001) The lag from monetary policy actions to inflation: Friedman revisited. *International Finance* 4, 381–400.

Bernanke, Ben S., Jean Boivin, and Piotr Eliasz (2005) Measuring the effects of monetary policy: A factor-augmented vector autoregressive (FAVAR) approach. *Quarterly Journal of Economics* 120, 387–422.

Bernanke, Ben S. and Mark Gertler (1995) Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic Perspectives* 9, 27–48.

Bierens, Herman J. (2000) Nonparametric nonlinear cotrending analysis, with an application to interest and inflation in the United States. *Journal of Business and Economic Statistics* 18, 323–337.

Castelnuovo, Efrem (2012) Testing the structural interpretation of the price puzzle with a cost-channel model. *Oxford Bulletin of Economics and Statistics* 74, 425–452.

Francis, Neville and Michael T. Owyang (2005) Monetary policy in a Markov-switching vector error-correction model: Implications for the cost of disinflation and the price puzzle. *Journal of Business and Economic Statistics* 23, 305–313.

Friedman, Milton (1958) The supply of money and changes in prices and output. In U.S. Congress, Joint Economic Committee, *The Relationship of Prices to Economic Stability and Growth: Compendium,* pp. 241–256. Washington, DC: Government Printing Office.

Friedman, Milton (1960) *A Program for Monetary Stability*. New York: Fordham University Press.

Friedman, Milton (1961) The lag in effect of monetary policy. *Journal of Political Economy* 69, 447–466.

Giordani, Paolo (2004) An alternative explanation of the price puzzle. *Journal of Monetary Economics* 51, 1271–1296.

Hanson, Michael S. (2004) The "price puzzle" reconsidered. *Journal of Monetary Economics* 51, 1385–1413.

Henzel, Steffen, Oliver Hülsewig, Eric Mayer, and Timo Wollmershäuser (2009) The price puzzle revisited: Can the cost channel explain a rise in inflation after a monetary policy shock? *Journal of Macroeconomics* 31, 268–289.

Herrera, Ana María and Elena Pesavento (2009) Oil price shocks, systematic monetary policy, and the "Great Moderation." *Macroeconomic Dynamics* 13, 107–137.

Kim, Soyoung (1999) Do monetary policy shocks matter in the G-7 countries? Using common identifying assumptions about monetary policy across countries. *Journal of International Economics* 48, 387–412.

Kim, Soyoung and Nouriel Roubini (2000) Exchange rate anomalies in the industrial countries: A solution with a structural VAR approach. *Journal of Monetary Economics* 45, 561–586.

Kozicki, Sharon and Peter A. Tinsley (2005) Permanent and transitory policy shocks in an empirical model with asymmetric information. *Journal of Economic Dynamics and Control* 29, 1985–2015.

Krusec, Dejan (2010) The "price puzzle" in the monetary transmission VARs with long-run restrictions. *Economics Letters* 106, 147–150.

Lagana, Gianluca and Andrew Mountford (2005) Measuring monetary policy in the UK: A factor-augmented vector autoregression model approach. *Manchester School, Supplement* 73, 77–98.

Rabanal, Pau (2007) Does inflation increase after a monetary policy tightening? Answers based on an estimated DSGE model. *Journal of Economic Dynamics and Control* 31, 906–937.

Ravn, Morten O., Stephanie Schmitt-Grohe, Martín Uribe, and Lenno Uuskula (2010) Deep habits and the dynamic effects of monetary policy shocks. *Journal of the Japanese and International Economies* 24, 236–258.

Scholl, Almut and Harald Uhlig (2008) New evidence on the puzzles: Results from agnostic identification on monetary policy and exchange rates. *Journal of International Economics* 76, 1–13.

Sims, Christopher A. (1986) Are forecasting models usable for policy analysis? *Federal Reserve Bank of Minneapolis Quarterly Review* 10, 2–16.

Sims, Christopher A. (1992) Interpreting the macroeconomic time series facts: The effects of monetary policy. *European Economic Review* 36, 975–1000.

Sims, Christopher A. and Tao Zha (1999) Error bands for impulse responses. *Econometrica* 67, 1113–1155.

Sims, Christopher A. and Tao Zha (2006) Does monetary policy generate recessions? *Macroeconomic Dynamics* 10, 231–272.

Svensson, Lars E.O. (1997) Inflation forecast targeting: Implementing and monitoring inflation targets. *European Economic Review* 41, 1111–1146.

Tas, Bedri K.O. (2011) An explanation for the price puzzle: Asymmetric information and expectation dynamics. *Journal of Macroeconomics* 33, 259–275.