Inflation Targeting with Sovereign Default Risk*

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Abstract

We analyze the interplay of monetary policy with sovereign debt and risk, motivated by the adoption of inflation targeting by emerging markets since the early 2000s. We build a framework that integrates a workhorse small open economy New Keynesian monetary environment within a canonical sovereign default model. We show that this framework replicates the positive co-movements of sovereign interest rate spreads with domestic nominal rates and inflation, a salient feature of emerging markets data. Our framework rationalizes the experience of Brazil during the 2015 downturn, which featured high inflation, high nominal rates, and high sovereign spreads. Our counterfactual experiment suggests that by raising the domestic rate the Brazilian central bank not only reduced inflation but also alleviated the debt crisis. Finally, we caution that implementing inflation targeting in economies subject to default risk will call for a more volatile domestic policy rate, in response to higher underlying volatility in consumption and inflation.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. Contact: arellano.cristina@gmail.com; yanbai06@gmail.com; mihalache@gmail.com

1 Introduction

Since the early 2000s, following the steps of advanced economies, many central banks in emerging markets have achieved independence from the central government and have adopted an inflation targeting framework for monetary policy. These emerging markets have conquered their historical inflationary episodes, recently experiencing low and stable inflation. Emerging markets have historically also faced recurrent foreign debt crises, with high and volatile interest rate spreads as well as outright default. The theoretical literature studying inflation target and sovereign debt crises, however, has largely studied each of these topics in isolation.[1] In this paper, we analyze the interplay of inflation target for monetary policy and sovereign debt crises. We find that raising nominal interest rates not only brings down inflation but also lowers sovereign interest rate spreads.

We provide a framework that combines the workhorse New Keynesian monetary model of Gali and Monacelli (2005) with the canonical sovereign default model in Arellano (2008). We show that this framework delivers a positive co-movement between sovereign interest rate spreads with domestic nominal interest rates and inflation, which we document as a salient feature in emerging markets data including post inflation targeting era. We apply our framework to the recent Brazilian downturn of 2015, which featured increases in sovereign spreads together with high inflation and tight monetary policy. The Brazilian central bank increased nominal interest rates in response to high inflation relative to its target. We show that our framework can rationalize these dynamics and then use it to evaluate a counterfactual looser monetary policy. In this counterfactual exercise, we find that if Brazil had kept nominal interest rates low inflation would had been higher and the debt crisis would had been exacerbated. Hence we conclude that by adhering to inflation target, Brazil's central bank was able not only to bring inflation down but also to alleviate the debt crisis.

The small open economy model we consider consists of households, firms, a monetary authority, and a fiscal government that borrows internationally. Households value consumption of domestic goods and imported foreign goods. They work in intermediate goods firms that produce domestic varieties. The intermediate goods firms face frictions in setting their prices and are subject to productivity shocks. Firms have to pay an adjustment cost whenever they change their prices, in the tradition of Rotemberg (1982). Final goods firms are competitive and use intermediate goods varieties to produce domestic output,

[1] For example, the influential paper by Gali and Monacelli (2005) analyzes monetary policy in the context of perfect financial markets and the work of Aguiar and Gopinath (2006) and Arellano (2008) study sovereign default in real models.

which is both consumed by domestic households and exported to the rest of the world. The monetary authority sets policy by following a nominal interest rate rule, that depends on the gap between inflation and its target. Monetary policy is subject to shocks, so that the interest rate can deviate from the prescriptions of the rule.

The government borrows from the rest of the world by issuing discount bonds denominated in foreign currency. It is benevolent and uses international borrowing for transfers to households to smooth fluctuations. The government, however, lacks commitment over repaying the debt and can choose to default. The risk of it doing so is reflected in the borrowing rates that it faces on world markets. Default induces costs in terms of utility and productivity.

We consider a Markov problem for the fiscal government. The government internalizes that its borrowing and default decisions induce an allocation and prices for the private economy and monetary authority. It also takes as given the decisions for borrowing and default of future governments, but internalizes that its choice of borrowing today impacts their state variables in the future. We show that our model contains three endogenous functions that depend on the government's borrowing and default choices. The first function is the bond price function that encodes future default decisions: increasing borrowing depresses the bond prices. The second function is the expected marginal utility for households relative to future inflation. This function is increasing in borrowing because future domestic consumption declines with debt. The third function is the expected inflation for firms which is increasing in borrowing. Within this structure the borrowing choices for the government interact with nominal side of the model and with monetary policy.

We show that the optimal borrowing decision for the government interacts with the monetary authority interest rate rule due to the time inconsistent nature of the government. The marginal benefit of borrowings is not only increasing in current consumption but depends on whether it relaxes the intertemporal Euler equation and the New Keynesian Phillips Curve. When nominal interest rates are low, foreign borrowing is beneficial as a way to induce lower expected consumption growth through its impact on future consumption. Such a low rate of consumption growth is useful as it relaxes the domestic inter-temporal Euler equation. Larger borrowing, however, is costly because it induces higher future inflation and tightens the New Keynesian Phillips Curve leading to higher inflation today. In contrast, when nominal interest rates are high, foreign borrowing is costly since it tightens the domestic Euler equation, yet borrowing is also beneficial because it relaxes the Phillips Curve.

We parameterize our model to Brazilian data and perform a quantitative evaluation of our model. We present impulse response functions to monetary and productivity shocks and compare them to a reference model without default risk. This reference model is a variant of the Gali and Monacelli (2005) model with uncontingent debt but otherwise frictionless financial markets. In response to a contractionary i.i.d. monetary shock inflation, output, domestic consumption, and borrowing fall on impact. Unlike the reference model, these effects in our model are persistent because debt takes time to recovery. Nominal rates in our model rise by more than in the reference model because inflation fall by less. Government spreads fall in our model in response to contractionary shock and remain low for several periods while debt is below its initial level. The reference model exhibits no spreads.

These impulse responses show that with default risk, contractionary monetary policy is less powerful in bringing down inflation as inflation falls less in the benchmark than in the reference despite nominal interest rates increasing by more in the benchmark. Tight monetary policy, nevertheless, also brings down government spreads because it reduces the incentives to borrow.

In response to a low productivity shock, consumption and output fall, while spreads, inflation, and nominal interest rates rise. The increases in inflation and nominal rates are higher in the benchmark than in the reference model. This amplification arises in the benchmark because the bond price schedule is tighter with low productivity leading to an increase in spreads. The tight borrowing conditions discourage imports and stimulate exports, to pay off the debt. Such dynamics lead to an appreciation of the exchange rate which dampens the decline of domestic output. Inflation then increases by more because of a higher unit cost of production. In the reference model without default the economy can smooth the shock by taking on additional debt without reducing imports or increasing exports; the abundant borrowing also leads to a depreciation.

The impulse responses to a productivity shock show that with default risk, nominal interest rates and inflation are more volatile. Inflation target in environments with default risk require more aggressive movements in nominal rates to achieve an inflation target goal.

We compare our model implications with Brazilian data in terms of second moments, correlations, and dynamics in the 2014-2016 downturn. In terms of second moments we show that our model delivers a volatility of inflation and nominal interest rates relative to output close to 1.5% and 3% respectively, as in the data. In terms of correlations, our model delivers strong positive comovement of spreads with inflation and nominal interest rates.

Finally, we perform an event analysis and evaluate a counterfactual monetary policy scenario. We focus on the period from 2014 to 2016. During the event output fell in Brazil about 6% below trend, inflation and nominal interest rates increased about 4%, and spreads increased about 3%. We apply our model to this event by feeding the model a sequence of productivity shocks such that it reproduces the dynamics of output. We then compare the model implications for inflation, nominal rates, and spreads to the data. Our model reproduces sizable increases in inflation, nominal rates and spreads. Nominal interest rates increase as the inflation target rule calls for such tightening. The magnitudes of the increases in the model, however, are somewhat larger than in the data.

We then perform the counterfactual experiment. We feed in the same sequence of productivity shocks but impose that the monetary authority does not increase nominal rates. We then compare the paths of inflation, spreads, and output in the counterfactual scenario to the benchmark. We implement this experiment by feeding in expansionary monetary shocks large enough such that nominal rates remain at the 2014 level throughout. In the counterfactual, inflation increases by more, output decreases by less, and spread increase by more. We conclude that the increase in nominal rates in Brazil, during the event, not only controlled inflation but also dampened the debt crisis.

Related Literature Our project builds on insights from two distinct and hitherto unconnected literatures on emerging market business cycle: the work on New Keynesian monetary policy in small open economies, following Gali and Monacelli (2005), and the literature on endogenous, fundamental sovereign default risk, following Eaton and Gersovitz (1981).

We follow the quantitative approach to default of Aguiar and Gopinath (2006) and Arellano (2008), but abstract from the many extensions and applications developed in the recent literature, including long-term debt and maturity choice (Chatterjee and Eyigungor (2012), Arellano and Ramanarayanan (2012), Hatchondo et al. (2016)), taxation and government spending (Cuadra et al. (2010)), debt restructuring and renegotiation (Yue (2010), Pitchford and Wright (2012)), or the work on contagion and transmission of country risk (Arellano et al. (2017), Bocola (2016)). Similarly, our domestic monetary environment is close to the reference model of Gali and Monacelli (2005) and abstracts from the many extensions considered in the (medium-scale) open economy DSGE literature, e.g. Christiano et al. (2011). One methodological difference from such projects is that we use global methods rather than local approximations around the steady state. Furthermore, we focus on a simple interest rate rule as a model of inflation targeting, and do not address optimal monetary policy, along the lines of Schmitt-Grohé and Uribe (2007) or Corsetti et al. (2010).

The literature on sovereign default recently turned to questions raised by nominal rigidities and the currency denomination of debt. Na et al. (2014) emphasize downward rigidity of nominal wages and the incentives it creates for exchange rate management together with the monetary authority's inability to pursue such policies in the presence of an exchange rate peg. In their setting inflation is desirable in that, in the presence of a rigid nominal wage, adjustments in the price level can return the real wage to its efficient level. Our projects differs in two main ways: first, we consider price-setting frictions as opposed to nominal wage rigidity and, second, we model an inflation targeting monetary authority, via an interest rate rule that calls for monetary tightening in the face of rising inflation. Bianchi et al. (2018) study an environment similar to Na et al. (2014) and focus on the public spending dimension of fiscal policy. There, additional spending stimulates aggregate demand and lowers unemployment, but at the cost of worsened terms of borrowing. We also address the fiscal authority's role in determining demand, not directly through public spending but rather via international borrowing, in particular as a response to domestic monetary policy.

Kondo et al. (2016) and Sunder-Plassmann (2018) study the interaction of inflation with defaultable debt. The former considers exogenous inflation, for given covariance structures with fundamentals, while the latter builds on a cash-and-credit model with a constant money supply. Nuno and Thomas (2018) build a continuous time model with local currency debt and a discretionary choice of inflation. In contrast with these papers, we emphasize the joint dynamics of endogenous inflation and country risk, under rule-based monetary policy, and focus on the structure of domestic economy as constraints facing fiscal policy-making.

A large literature, following Calvo (1988), studies whether monetary policy can alleviate self-fulfilling debt crises. Aguiar et al. (2013) study trade-offs raised by monetary policy credibility in a tractable, continuous time model of self-fulfilling default. Concerning the multiplicity of equilibria and the role inflation can play in selecting among them, Corsetti and Dedola (2016) focus on unconventional monetary policy while Bacchetta et al. (2018) embed New Keynesian features in the "slow moving debt crisis" model of Lorenzoni and Werning (2013) to quantify the role of inflation for crisis prevention.

Finally, our model's implications for the terms of trade, nominal and real exchange rate, and centralized borrowing raise a natural comparison with the work on capital controls and exchange rates in small open economies, such as Farhi and Werning (2012), Devereux et al. (2018) and more recently Fanelli (2017).

2 Model

We consider a small open economy which is composed of households, final good producers, intermediate goods firms, a fiscal authority, and a monetary authority. There are three types of goods: imported, intermediate, and final. The final good is produced using all varieties of differentiated, intermediate goods. Each variety is produced with labor. The final good is demanded by both domestic and foreign consumers.

Foreign demand for domestic goods (export demand) is given by

$$X_t = \left(\frac{P_t^d}{\varepsilon_t P_t^*} \right)^{-\rho} \xi,$$

where P_t^* is the price of foreign goods in foreign currency, ξ is the level of overall foreign demand and ρ is the elasticity of demand. P_t^d is the price of domestic goods in local currency and ε_t is the nominal exchange rate, with an increase in ε_t being a depreciation of the home currency. Because the law of one price holds we can write the price of the foreign good in local currency as

$$P_t^f = \varepsilon_t P_t^*.$$

The terms of trade are defined by

$$e_t = \frac{P_t^f}{P_t^d} = \frac{\varepsilon_t P_t^*}{P_t^d}.$$

The foreign demand for domestic goods is a function of the terms of trade and the foreign shock

$$X_t = e_t^{\rho} \xi. \tag{1}$$

We normalize the foreign price P_t^* and ξ to one.

2.1 Households

Households preferences are defined over consumption of domestic C_t and foreign goods C_t^f and labor N_t. Their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t,\ C_t^f, N_t),$$

where the per-period utility function is given by

$$u(C_t,\ C_t^f, N_t) = \theta \log(C_t) + (1-\theta)\log(C_t^f) - \frac{N_t^{1+1/\zeta}}{1+1/\zeta}.$$

Taking aggregate prices as given, the households choose consumption, labor supply, and domestic bonds B_t^d holdings, denominated in local currency. Households own intermediate goods firms and receive profits Φ_t from them. They also receive labor income and government transfers T_t. The budget constraint is given by

$$P_t^d C_t + (1+\tau_f) P_t^f C_t^f + q_t^d B_{t+1}^d \leq W_t N_t + B_t^d + \Psi_t + T_t.$$

where q_t^d is the nominal discount prices of domestic bonds, and τ_f is a constant consumption tax that households pay on imported consumption. It is convenient to write the budget constraint in real terms, deflating by the domestic price index P_t^d

$$C_t + (1+\tau_f) e_t C_t^f + q_t^d b_{t+1}^d \leq w_t N_t + \frac{b_t^d}{\pi_t} + \psi_t + t_t. \tag{2}$$

where real domestic bonds are $b_{t+1} = B_{t+1}^d / P_t^d$, the real wage is $w_t = W_t / P_t^d$, real profits and transfers are $\psi_t = \Psi_t / P_t^d$, $t_t = T_t / P_t^d$, and the gross domestic inflation is $\pi_t = P_t^d / P_{t-1}^d$. We can characterize the representative consumer's problem with the following optimality conditions

$$-\frac{u_{N,t}}{u_{c,t}} = \frac{W_t}{P_t^d} = w_t, \tag{3}$$

$$\frac{u_{c^f,t}}{u_{c,t}} = (1+\tau_f) e_t, \tag{4}$$

$$q_t^d = \beta \mathbb{E}_t \left[\frac{u_{c,t+1}}{u_{c,t}} \frac{1}{\pi_{t+1}} \right]. \tag{5}$$

The nominal interest rate is defined as the inverse of the discount bond price

$$i_t \equiv \frac{1}{q_t^d}.$$

2.2 Final goods producers

The final good is produced from a variety of differentiated intermediates y_{it}, $i \in [0,1]$ under perfect competition,

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}. \tag{6}$$

where η is the elasticity of substitution between intermediate goods. Let the prices of intermediate goods be $\{p_t(i)\}$. The profit maximization problem of the final good producer is

$$\max P_t^d \left[\int_0^1 y_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} - \int_0^1 p_t(i) y_t(i) di.$$

inducing a demand function and a price aggregator

$$y_t(i) = \left(\frac{p_t(i)}{P_t^d} \right)^{-\eta} Y_t, \tag{7}$$

$$P_t^d = \left[\int_0^1 p_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}. \tag{8}$$

2.3 Intermediate goods producers

Each differentiated intermediate good is produced with labor n_{it}, using a constant returns to scale production function, subject to productivity shocks z_t

$$y_{it} = z_t n_{it}. \tag{9}$$

Intermediate goods firms are monopolistically competitive and set the prices for their products taking as given the demand system (7). These firms, however, face price setting frictions in that they have to pay a quadratic adjustment cost when they change their price relative to the target inflation $\overline{\pi}$, as in Rotemberg (1982). Taking as given the wage rate W_t and the final good price P_t^d, an intermediate firm i chooses labor and its price to maximize the present discounted value of their profits

$$\max_{\{p_{it}, n_{it},\}} \mathbb{E}_0 \sum_t Q_{t,0} \left\{ p_{it} y_{it} - (1-\tau) W_t n_{it} - \frac{\varphi}{2} \left(\frac{p_{it}}{p_{it-1}} - \overline{\pi} \right)^2 P_t^d Y_t \right\}$$

subject to the production function, where $Q_{t,0}$ is the stochastic discount factor of households, denominated in units of domestic goods, and τ is a labor subsidy (This is assumed constant, a fiscal policy designed to alleviate inefficiencies induced by market power.).

Using the households' stochastic discount factor and the production function this problem is

$$\max_{\{p_{it}\}} \mathbb{E}_0 \sum_t \beta^t \frac{u_{c,t}}{u_{c,0}} \frac{P_0^d}{P_t^d} \left\{ p_{it} y_{it} - \Omega_t y_{it} - \frac{\varphi}{2} \left(\frac{p_{i,t}(j)}{p_{i,t-1}(j)} - \overline{\pi} \right)^2 P_t^d Y_t \right\}$$

where $\Omega_t = \frac{(1-\tau)W_t}{z_t}$ which we denote as the unit cost. The first order condition for each firm, after imposing symmetry across all firms $p_{it} = P_t^d$, results in

$$\frac{\Omega_t}{P_t^d} = \frac{\eta - 1}{\eta} + \frac{1}{\eta}\varphi\left(\pi_t - \overline{\pi}\right)\pi_t - \frac{1}{\eta}E_t\left[\beta \frac{u_{c,t+1}}{u_{c,t}} \frac{Y_{t+1}}{Y_t}\varphi\left(\pi_{t+1} - \overline{\pi}\right)\pi_{t+1}\right]. \tag{10}$$

This equation is a standard New Keynesian Philips Curve (NKPC) that relates inflation to a measure of contemporaneous unit cost and expected future inflation.

2.4 Monetary Authority

The monetary authority conducts policy using nominal interest rates rules. Nominal rates i_t are set based on the level of inflation relative to the target $\pi_t/\overline{\pi}$, and also respond to a monetary shock m_t

$$i_t = R\left[\left(\frac{\pi_t}{\overline{\pi}}\right)^{\alpha_P}\right] m_t. \tag{11}$$

In equilibrium, the average value of the domestic nominal rate R must satisfy the usual steady state condition,

$$R = \frac{\overline{\pi}}{\beta}. \tag{12}$$

In the quantitative section, we implement interest rate smoothing, common in the New Keynesian literature, to prevent excess volatility of domestic nominal rates in equilibrium. We delay the introduction of this extension to economize on notation in the exposition of the model.

2.5 Fiscal Government and External Debt

The fiscal government is benevolent and maximizes the utility of households. It borrows short-term B_{t+1} at price q_t in foreign currency from international lenders. As in standard New Keynesian models we also let the fiscal government subsidize employment at a time-invariant rate τ, as to undo the markup in goods markets. The government transfers to households the net receipts from its operations. Letting B_t denote the outstanding foreign currency debt of the government, the budget constraint in local currency is

$$T_t + \tau W_t N_t = P_t^f [q_t B_{t+1} - B_t] + \tau_f P_t^f C_t^f \tag{13}$$

where the net capital inflow from debt operations is multiplied by P_t^f to convert it to domestic currency. The government budget constraint in terms of domestic goods is

$$t_t + \tau w_t N_t = e_t [q_t B_{t+1} - B_t] + \tau_f e_t C_t^f \tag{14}$$

Every period the government chooses B_{t+1} and decides whether to default on its outstanding debt. Default induces costs both in terms of productivity and utility. If the government defaults, the economy experiences a reduction in productivity to $z_t^d \leq z_t$, and a utility cost ν_t. The utility cost ν_t is an i.i.d. shock generated by the process $\nu_t \sim \mathcal{N}(\overline{\nu}, \sigma_\nu)$. Default has the benefit that it reduces the level of debt to $\underline{B}$, an exogenous recovery level. For simplicity, we assume that the government continues to borrow immediately after default.[2]

The government's objective is to maximize the present discounted value of the utility derived from consumption by the representative household, net of any default costs,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(C_t, C_t^f, N_t) - D_t \nu_t),$$

where $D_t = 1$ when the government defaults and zero otherwise. The fiscal government borrows from competitive, international lenders that discount the future at a foreign currency rate r^*. The break-even condition for lenders requires that discount bond prices compensate lenders for the expected loss from the default such that

$$q_t B_{t+1} = \frac{1}{1+r^*} \mathbb{E}_t \left[(1 - D_{t+1}) B_{t+1} + D_{t+1} \underline{B} \right].$$

3 Equilibrium

We now describe the equilibrium of this economy. We consider a Markov equilibrium where the government takes into account that its borrowing and default policies affect

[2]We introduce the iid default cost shock ν_t to ease computation. It smooths out the bond price function and facilitates uniform convergence. In the robustness appendix we document its role in the quantitative results.

the allocations of the private equilibrium and the monetary authority response. In the beginning of the period, the aggregate state of this economy include the productivity, monetary, and enforcement shocks $s = \{z, m, \nu\}$ and the government debt B. The fiscal government chooses its policy to default and borrowing D, B'. The private equilibrium and monetary authority's response depend on both the state $\{s, B\}$ and on the government choices because they affect government transfers $t(S)$. Let $S = \{s, B, D, B'\}$ be end of the period state relevant for the private equilibrium.

Definition 1. *Private and Monetary Equilibrium. Given state $\{S\}$, the government policy functions for borrowing $B''(s', B') = H_B(s', B')$, default $D'(s', B') = H_D(s', B')$, and transfer function $t(S)$ consistent with the government budget constraint, the symmetric private and monetary equilibrium consists of*

- *Households policies for domestic goods consumption $C(S)$, foreign goods consumption $C^f(S)$, labor $N(S)$, and domestic debt $B^d(S)$,*
- *Intermediate and final goods firms policies for labor $n(S)$, prices $p^d(S)$ and final domestic goods output $Y(S)$ and exports $X(S)$,*
- *The wage rate $w(S)$, domestic nominal interest rate $i(S)$, aggregate domestic price $P^d(S)$, domestic inflation $\pi^d(S)$, and the terms of trade $e(S)$*

such that: (i) *the policies for households satisfy their budget constraint (2) and optimality conditions (3), (4), (5);* (ii) *the policies of intermediate and final goods firms satisfy their optimization problem (6), (7), (9), and (10);* (iii) *export demand (1) is satisfied* (iv) *labor, domestic goods, and domestic bond markets clear, and balance-of-payment constraint is satisfied* (v) *nominal interest rate satisfies the monetary authority interest rate rule (11).*

The labor market clearing conditions imply that labor demanded by firms equal labor supplied by households $n = N$. Domestic bonds are in zero net supply in the economy, leading to the market clearing condition for bond markets, $B^d = 0$. The resource constraint for domestic goods requires that domestic final good output equal domestic consumption and exports net of the adjustment costs

$$C(S) + X(S) + \frac{\phi}{2}(\pi - \overline{\pi})^2 Y(S) = Y(S) \tag{15}$$

where aggregate output $Y = zN$.

The balance-of-payments constraint requires that net export equals the net capital outflow which equals to the government transfer net of the labor subsidy

$$X(S) - (1+\tau_f)e(S)C^f(S) = t(S) - \tau w(S)L(S). \tag{16}$$

3.1 Government Recursive Formulation

We now describe the government's recursive problem that borrows in international financial markets and can default. The government is benevolent and chooses its policies internalizing that its choices affect the private and monetary equilibrium.

The government trades one-period foreign denominated discount bonds with international lenders and can default on its debt. The government starts with debt B and decides on default D and new borrowing B' that carries price $q(s, B')$. The bond price is an endogenous function that depends on the amount of borrowing B' and the shocks s to compensate for default risk. Lenders discount the future at the international interest rate r^*. The break even condition for lenders imply that the bond price schedule satisfies

$$q(s, B') = \frac{1}{1+r^*}\mathbb{E}_{s'|s}\left[(1 - H_D(s', B')) + H_D(s', B')\underline{B}/B'\right]. \tag{17}$$

where $\underline{B}$ is the recovery if the government defaults and $H_D(s', B')$ is the default policy function of the government.

The government internalizes that its choice of borrowing and default affect the private equilibrium. As is standard in New Keynesian models we set that labor subsidy $(1-\tau) = \frac{\eta-1}{\eta}$ to offset the market power of firms in the steady state. and set $(1+\tau_f) = \frac{\rho}{\rho-1}$ to be equal to the optimal tariff in steady state for the government. [3]

By combining the equilibrium conditions and the government budget constraint, the private and monetary allocations can be summarized with the decision rules for domestic and foreign consumption $\{C(S), C^f(S)\}$, labor $N(S)$, inflation $\pi(S)$, nominal interest rate $i(S)$, and terms of trade $e(S)$ that satisfy the following system of dynamic equations

$$C(S) + e(S)^\rho \xi = \left[1 - \frac{\varphi}{2}\left(\pi(S) - \overline{\pi}\right)^2\right] zN(S) \tag{18}$$

$$e(S)^\rho \xi = e(S)[C^f(S) + B - q(s, B')B'] \tag{19}$$

[3] By setting this tariff, we neutralize the incentive of the government to use debt to exert market power with respect to the downward-sloping demand for its exports, in steady state.

$$\frac{u_{c^f}(S)}{u_c(S)} = \frac{\rho}{\rho - 1} e(S) \tag{20}$$

$$u_c(S) = \beta i(S) M(s, B') \tag{21}$$

$$\frac{1}{z}\frac{u_n(S)}{u_c(S)} = 1 + \frac{1}{\eta - 1}\varphi\left(\pi(S) - \overline{\pi}\right)\pi(S) - \frac{1}{u_c(S) z N(S)} F(s, B') \tag{22}$$

$$i(S) = R\left[\frac{\pi(S)}{\overline{\pi}}\right]^{\alpha_p} m \quad \text{with} \quad R = \frac{\overline{\pi}}{\beta} \tag{23}$$

where $q(s, B')$ satisfies (17) and the functions $M(s, B')$ and $F(s, B')$ are the expectations in the households' Euler condition and the firms' pricing condition given by

$$M(s, B') = \mathbb{E}_{s'|s}\frac{u_c(s', B')}{\pi(s', B')} \tag{24}$$

$$F(s, B') = \frac{\beta}{\eta - 1}\mathbb{E}_{s'|s}\left[z' N(s', B') u_c(s', B')\varphi\left(\pi(s', B') - \overline{\pi}\right)\pi(s', B')\right]. \tag{25}$$

These equilibrium conditions are analogous to those arising from the standard New Keynesian small open economy, e.g. Gali and Monacelli (2005). What is different in our model is that the price of debt $q(s, B')$ depend on default risk and that the choice of government borrowing is different because it internalizes that its choice for borrowing B' affects the bond price function $q(s, B')$, the firms' expected inflation function $F(s, B')$, and households' expected marginal utility function $M(s, B')$. These functions depend on the choice of government borrowing because future government's policies of borrowing and default $H_B(s', B')$ and $H_D(s', B')$ depend on the debt state tomorrow B'. As we show below the shape of these functions are important determinants for the choice of borrowing B' and for the interaction between inflation and default risk.

We now set up the the recursive problem of the government which follows the quantitative sovereign default literature. The government can choose to default any period. Let $V(s, B)$ be the value of the option to default. After default, the debt B is reduced to R, productivity is reduced to z^d, and the government pays the default cost ν. The value of the option to default is then

$$V(s, B) = \max_{D=\{0,1\}}\left\{(1 - D)W(s, B) + D\left[W(z^d, m, \underline{B}) - \nu\right]\right\} \tag{26}$$

where $D = 1$ in default and 0 otherwise, and $W(s, B)$ is the value of repaying debt

$$W(s, B) = \max_{B'}\left\{u(C, C^f, N) + \beta\mathbb{E}_{s'|s}V(s', B')\right\} \tag{27}$$

subject to the private and monetary equilibrium which is characterized by conditions (18) through (23), and the break even condition for the bond price schedule (17).

It is convenient to write the default decision of the government as a cutoff rule based on the default cost ν. Given that default costs are i.i.d., the default decision $D(s, B)$ can be characterized by a cutoff default cost $\nu^*(z, m, B)$ below which the repayment value is equal to the default value such that

$$\nu^*(z, m, B) = W(z, m, B) - W(z^d, m, \underline{B}) \tag{28}$$

and $D(s, B) = 1$ if $\nu \leq \nu^*(z, m, B)$ and zero otherwise. Using this cutoff we can then write the repayment value function as

$$W(s, B) = \max_{B'} \left\{ u(C, C^f, N) + \beta \mathbb{E}_{s'|s} \left[W(s', B') - \int^{\nu^*(z', m', B')} \nu d\Phi(\nu)) \right] \right\} \tag{29}$$

We now define the recursive equilibrium for the economy.

Definition 2. *Equilibrium. Given the aggregate state $\{s, B\}$ a recursive equilibrium consists of government policies for default $D(s, B)$ and borrowing $B'(s, B)$, and government value functions $V(s, B)$ and $W(s, B)$ such that*

- *Taking as given future policy and value functions $H_D(s', B')$, $H_B(s', B')$, $V(s', B')$, and $W(s', B')$, government policies for default and borrowing $D(s, B)$ and $B'(s, B)$ solve the government's optimization problem.*

- *Government policies and values are consistent with the future policies and values.*

3.2 Borrowing with Default Risk and Monetary Policy

In this section we characterize the optimal borrowing decision for the government. As described in the recursive equilibrium the government chooses its borrowing taking into account the effect that borrowing has on both the private equilibrium and the bond price. We manipulate the government's problem and derive its optimality condition for borrowing to illustrate the forces at play. In this derivation we have assumed that the functions in the government problem as differentiable. Optimal borrowing satisfies the following Euler equation

$$qeu_c^g - \tau_1(s, B') - \tau_2(s, B') = \beta \mathbb{E}_{s'|s} \left[e(s', B') u_c^g(s', B') \right] - \mathbb{E}_{s'|s} \tau_3(s', B') \tag{30}$$

where the the wedges τ_1, τ_2, and τ_3 are given by following conditions

$$\tau_1(s, B') = -eu_c^g \frac{\partial q(z, B')}{\partial B'} B'$$
$$\tau_2(s, B') = \mu \frac{1}{M(z, B')} \frac{\partial M(z, B')}{\partial B'} + \gamma \frac{1}{u_C Y} \frac{\partial F(z, B')}{\partial B'}$$
$$\tau_3(s, B') = -\beta \Phi(v^*(s', B')) v^*_{B'}(s', B')$$

and the marginal utility for the government is

$$u_c^g = \left(u_c + \lambda_e \frac{\theta}{1-\theta} \right) \frac{\rho}{\rho - 1}. \tag{31}$$

In equation (30), λ_e is the multiplier on the intratemporal condition that relates domestic and imported consumption of (20), μ is the multiplier on the domestic Euler condition (21), and γ is the multiplier on the New Keynesian Phillips Curve (22).

It is useful to compare our model's Euler equation (30) with an Euler equation that arises in the standard Gali and Monacelli (2005) model. In this reference model with complete markets, no default risk, and private international borrowing, the following optimality condition is satisfied

$$q\, e\, u_c = \beta \mathbb{E}_{s'|s} \left[e'\, u_c' \right]. \tag{32}$$

Recall that given a level of borrowing B' and its price q the allocations and prices in our model are exactly the same as the allocations in this reference model as both models satisfy the system of equations (18) through (23).

Optimal borrowing in our model equates the marginal benefit from of borrowing an extra unit of B' which is the left hand side of equation (30) to the marginal cost which is in the right hand side. The marginal benefit contains various terms. The first term qeu_c^g contains the standard marginal benefit of borrowing also present in the reference model. Borrowing an extra unit increases current consumption by the price in terms of domestic goods qe which is valued by marginal utility u_c. Note that a high e (depreciation) boosts the marginal benefit of borrowing. This first term in our model differs from the reference model in that the government also internalizes that B' alters the optimal tariff and its monopoly power of exporting goods which is encoded in the multiplier λ_e

Our model also contains two additional terms for the marginal benefit of borrowing, namely τ_1 and τ_2. The term $\tau_1 > 0$ reduces the marginal benefit because larger borrowing B' reduces the price $\frac{\partial q(z,B')}{\partial B'} < 0$ due to increased default risk. This force is present in standard sovereign debt and default models and it leads to reduced borrowing. It is interesting that in our model the cost arising from the sensitivity of the bond price is lower when the exchange rate falls (appreciates).

The term τ_2 contains the effect that borrowing has on expected marginal utility $M(z, B')$ and inflation $F(z, B')$ for households and firms times the mutipliers on those conditions μ and γ. These terms capture the time-inconsistency problem for the government arising from incentive to manipulate at time t both expected marginal utility and expected inflation in time $t+1$ with its choice of borrowing B'. As we will see below, in response to a tight money shock τ_2 increases because the multiplier on the domestic Euler equation rises. A main result is that high money shocks lower the marginal benefit from borrowing B' as they increases τ_2.

The marginal cost of borrowing is in the right hand side and it has two components. The first component encodes the standard cost from repaying the debt which is also present in the reference model. This first term also is distorted because of the monopoly power of exporting goods. The second term is a wedge $\tau_3 > 0$ which reduces the cost of borrowing and arises because of default: in default states, the government doesn't pay off the debt.

Uncovered Interest Parity with Default We now derive the uncovered interest parity with default (UIPD). It is useful to define a few relations between terms of trade, exchange rate, consumer and producer price indices. We can derive the consumer price index as the price of bundle of domestic and imported consumption,

$$P^{\text{CPI}} = \frac{(P^d)^\theta e^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}}.$$

The CPI inflation is then

$$\pi^{\text{CPI}} = \pi^\theta \left(\frac{e}{e_-}\right)^{1-\theta}.$$

The real exchange rate e^{CPI} equals the nominal exchange rate ε times the ratio of the world price P^* (set to 1) and P^{CPI},

$$e^{\text{CPI}} = \frac{\varepsilon\, P^*}{P^{\text{CPI}}}.$$

Using these relations, the real exchange rate and nominal devaluation depend on terms of trade and domestic prices,

$$e^{\text{CPI}} = \theta^\theta (1-\theta)^{1-\theta} (P^d)^{1-\theta} e^\theta,$$

$$\frac{\varepsilon'}{\varepsilon} = \frac{e'}{e}\pi.$$

We can derive the UIPD through the Euler equation for domestic bond (21) and the one for foreign bond (30). Let $\hat{x}$ denote the percentage deviation of variable x from its steady state. We can write the log-linearized UIPD as[4]

$$\hat{i}_t = \hat{i}_t^f + \mathbb{E}_t \left[\hat{\pi}_{t+1} + \hat{e}_{t+1} - \hat{e}_t\right] + E_t \hat{\tau}_{t+1}^{\text{UIP}}, \tag{33}$$

where $i^f = 1/q$, the expected nominal devaluation equals $E_t[\hat{\pi}_{t+1} + \hat{e}_{t+1} - \hat{e}_t]$, and $\hat{\tau}^{\text{UIP}}$ is the percentage deviation of the composite wedge τ^{UIP} defined as

$$\tau^{\text{UIP}}(s, s', B') = \frac{u_c}{u_c^g} \frac{u_c^g(s', B')}{u_c(s', B')} \left[1 + \frac{\tau_1(s, B') + \tau_2(s, B') + \tau_3(s', B')}{\beta e(s', B') u_c^g(s', B')}\right].$$

Hence, the composite wedge τ^{UIP} includes all the three wedges in the Euler (30) as well as the discrepancy between the government's and the consumer's marginal rate of substitution (MRS) between current and future consumption.

When the wedges are not time-varying $\mathbb{E}_t \hat{\tau}_{t+1}^{\text{UIP}} = 0$, UIPD (33) goes back to the standard uncovered interest parity (UIP) where the change of domestic nominal interest rate equals the sum of the change in the foreign interest rate and the expected nominal devaluation. In our model, in general the UIP does not hold due to the time-varying wedges arising from default risk reflected in τ_1 and τ_3, the government's time-inconsistency problem reflected in τ_2, and the export externality shown in the MRS discrepancy. In particular, under a bad productivity shock, the government is more likely to default and thus the borrowing schedule is tightened. This leads to a higher default wedge of τ_1 and lifts the domestic nominal rate.

4 Quantitative Analysis

We now describe the parameterization of the model, discuss policy rules, impulse responses, and compare the model implications to the data.

4.1 Inflation, Nominal Rates, and Spreads for Inflation Targeters

Starting in the late 1990s, many emerging markets have worked towards central bank independence and have adopted an inflation target for monetary policy.[5] Inflation levels in

[4]We delegate the detailed derivation to Appendix A.

[5]See Roger (2009) for a description of this process and for a list of Emerging Markets that adopted Inflation Targeting.

these countries have been single digits since the 2000s, which is a dramatic change from the 1980s when many of these countries had very high inflation episodes. At the same time, emerging countries continue to face challenges with their public debt, although debt crises have been dampened as well for many countries.

	Brazil	Mexico	Colombia
Means (%)			
Inflation	6.6	4.3	5.2
Nominal Rates	14.3	6.2	7.9
Spreads	4.5	2.3	3.2
Standard Deviations (%)			
Output	1.6	1.7	1.3
Inflation	2.6	2.9	4.0
Nominal Rates	4.6	2.6	3.6
Spreads	3.5	0.8	1.9
Correlations with Spreads			
Output	-0.16	-0.39	-0.17
Inflation	0.52	0.22	0.34
Nominal Rates	0.79	0.27	0.76

Table 1: Emerging Markets in Inflation Target Era

In this section we present the data of three emerging markets that are inflation targeters: Brazil, Mexico, and Colombia. We focus on data since 2000. We are interested in the connection between of inflation, interest rates, and sovereign spreads. Data are quarterly series. Inflation is computed as a annual change of the CPI. Nominal interest rates are the 3 month rate on government bonds denominated in local currency. Spreads are the EMBI+ spread which are measures of the difference in yields for government bonds denominated in U.S. dollars relative to a U.S. treasury bond. Output is real GDP filtered with an HP filter.

Table 1 presents means and standard deviations for inflation, nominal interest rates, and spreads as well as correlations for these variables with spreads. Inflation in these three countries have been low with a mean close to 5%. Nominal interest rates and spreads have also been fairly low ranging from 6% to 14% and 2% to 5% respectively. The volatility for inflation, nominal rates, and spreads have also been quite modest. In terms of correlations, inflation and nominal rates are positively correlated with spreads. The co-movement between nominal rates and spreads is strong, specially for Brazil and Colombia and equal close to 0.8. As is typical in emerging market, output is negatively correlated with spreads, see Neumeyer and Perri (2004), although the magnitude of these correlations are weaker than the correlation of spreads with nominal rates.

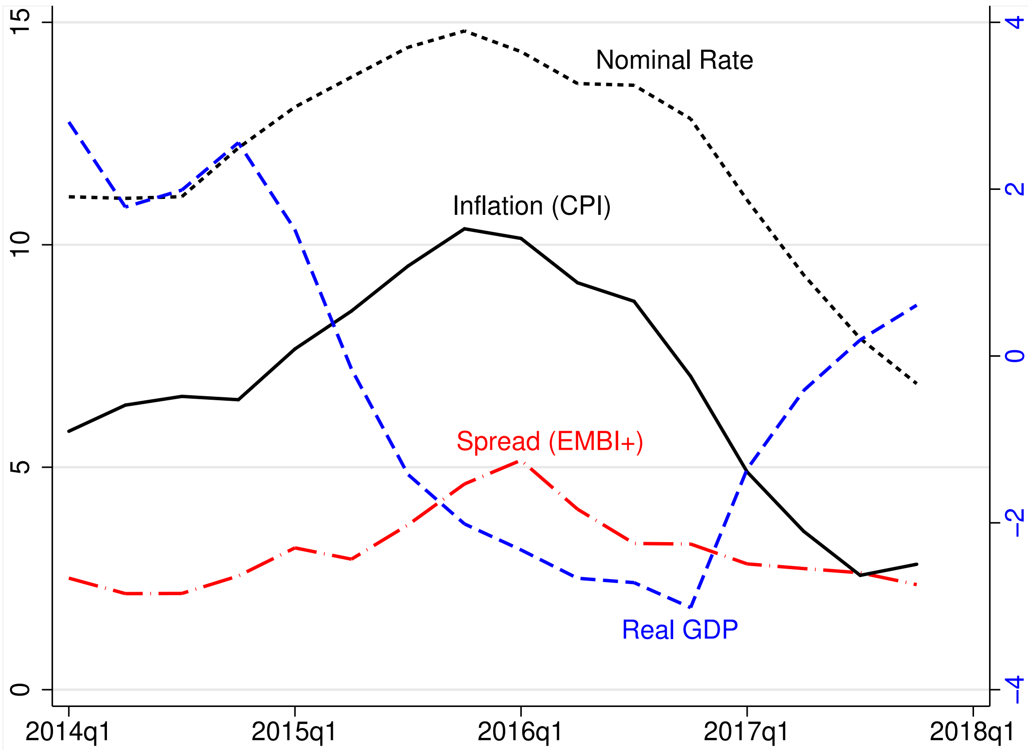


Figure 1: The 2015 Recession in Brazil

We focus our quantitative analysis on Brazil because this is the country that has experienced the largest recession. During 2016, Brazil experienced a severe recession that originated with the decline in commodity prices. As output fell, inflation rose and the central bank tightened policy by raising nominal rates following their inflation target goal. In Figure 1 we plot the dynamics of output, nominal rates, inflation, and spreads from 2014 to 2018. From the beginning of 2015 to mid 2016 output fell about 5%. Inflation increased from about 6% to about 10% and nominal rates increase from about 11% to about 15% in response to the high inflation. Spreads also increased during this period, from about 2% to 5%. At the end of 2016, the economy recovered with output increasing and inflation, nominal rates, and spreads falling.

The dynamics surrounding the Brazilian event illustrate the correlations observed in the longer time series for Brazil, Mexico, and Colombia. Emerging markets that are inflation targeters raise nominal rates in response to high inflation, just as developed countries do. In emerging markets, however, government spreads rise reflecting elevated default risk during these events.

4.2 Interest Rate Smoothing

For our quantitative results, we extend the model by incorporating interest rate smoothing by replacing the interest rate rule (11) with

$$i_t = \left[R \left(\frac{\pi_t}{\overline{\pi}} \right)^{\alpha_P} \right]^{1-\kappa} [i_{t-1}]^{\kappa} \, m_t. \tag{34}$$

Such a specification is standard in the New Keynesian literature and is aimed at preventing excess interest rate volatility (cf. Clarida et al. (2000)). This extension requires us to include the lag of the domestic nominal rate (i_-) as an additional state variables in S but otherwise leaves all other model equations unaltered.

4.3 Parameterization

The model includes three shocks: productivity z, money m, and utility cost ν. We assume that productivity shocks follow an AR(1) process

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_t$$

with ε_t following standard normal distribution. Money shocks are log-normal distributed with mean zero and standard deviation σ_m. Utility cost of default are normally distributed with mean $\bar{\nu}$ and standard deviation σ_ν. Following Chatterjee and Eyigungor (2012), we assume that the productivity after default takes the following form

$$z^d(z) = z - \max\left\{0, \lambda_0 z + \lambda_1 z^2\right\}.$$

We calibrate our model to quarterly Brazilian data from 2004 to 2017. Table 2 presents all the parameters values with the source or targeted moments. There are two sets of parameters. For the first set, we assign their values directly, by relying on one-to-one mappings with the data or reference values in the literature. The second set is unique to Brazil and we choose them jointly to match a set of data moments. The first set of parameters include the Frisch elasticity ζ, the share of domestic consumption θ, export demand elasticity ρ, international interest rate r^*, the goods elasticity η, and the persistence of TFP shock ρ_z. For the Frisch elasticity, we choose a value of 1.0, within the broad range of values used in the IRBC/Open Macro literature, and document robustness in Appendix C. The import share of Brazil is 15 percent, we therefore set θ as 0.85. Export demand elasticity ρ is chosen to be 5 following Devereux et al. (2018). This number is also within the plausible range of estimates in the trade elasticity literature. The international risk-free rate is 2%, consistent with US Treasury yields. The elasticity of substitution between goods varieties η is chosen to be 6, as is standard in the literature, inducing a 20% markup. Given that we are considering a short horizon of the data and that the employment data of Brazil has many missing values, it is difficult to estimate precisely the TFP process. Instead, we set ρ_z as 0.95 and estimate the volatility of the standard error, σ_z, to match the volatility of Brazilian output.

The second group includes the discount factor β, inflation target $\overline{\pi}$, Taylor rule parameter α_p, the Rotemberg adjustment cost φ, and the shock parameters. We choose them jointly so that the model generates the observed moments. Roughly speaking, the volatility of TFP shock is closely related to the output volatility. Under the Taylor rule and the Rotemberg adjustment cost, it is costly to deviate from the inflation target $\overline{\pi}$, and thus the average inflation rate helps pin down $\overline{\pi}$. Both α_p and the Rotemberg adjustment cost affects the inflation volatility, but α_p matters more for the relative volatility of nominal rate to inflation. The standard deviation of utility cost σ_ν and the productivity loss parameters λ_0 and λ_1 all matter for the mean spread and volatility of spread. We therefore target the mean and volatility of EMBI in the data. Financial frictions affect risk sharing and we therefore target the comovement of consumption and export with output. Domestic interest rate target is chosen to be 1.05 to match the average nominal rate of 14.3% in the data.

4.4 Policy Rules

Before describing the model time series, we illustrate the model mechanisms by describing policy rules and the equilibrium functions.

We start by describing the spread function which is an important force driving allocations. We define government spreads as the inverse of the bond price relative to the risk free rate

$$\text{spread}(s, B') = \frac{1}{q(s, B')} - (1 + r^*). \tag{35}$$

In our model, and in the standard sovereign default models, the government's default incentives increase with the level of debt B and with low productivity. The spread function

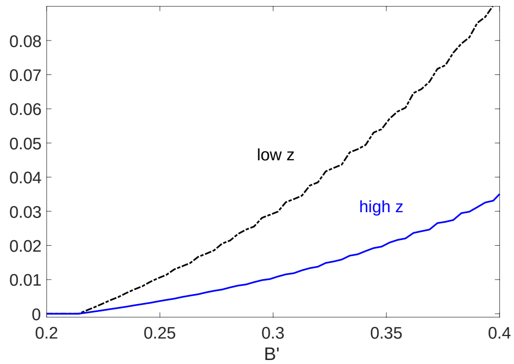
Assigned Parameters		
Share of domestic goods	$\theta = 0.85$	Import share of Brazil
Export demand elasticity	$\rho = 5$	Literature: Devereux et al. (2018)
International rate	$r^* = 2\%$	US Treasury yields
Goods elasticity	$\eta = 6$	Literature: 20% markup
Frisch elasticity	$\zeta = 0.72^{-1}$	Literature, Appendix C
Persistence of TFP shock	$\rho_z = 0.95$	Literature, Appendix C
Export demand level	$\xi = 1$	Normalization
Interest rate smoothing	$\kappa = 0.6$	Literature
Natural rate	$R = \overline{\pi}/\beta$	Equilibrium condition

Parameters from Moment Matching		*Brazilian Data, 2004–2017*
Discount factor	$\beta = 0.99$	Debt service to GDP = 6%
Inflation target	$\overline{\pi} = 1.013$	Mean inflation = 5.9%
Inflation weight in rule	$\alpha = 1.125$	Inflation volatility = 1.8%
Rotemberg adjustment	$\varphi = 12$	Mean nominal rate = 11.2%
Std of TFP shock	$\sigma_z = 0.55\%$	GDP volatility = 1.9%
Std. of money shock	$\sigma_m = 0.31\%$	Nominal rate volatility = 2.2%
Productivity in default	$\lambda_0 = -0.275$	Mean govt spread= 2.6%
	$\lambda_1 = 0.3$	Govt spread volatility = 1%
Std. of default cost	$\sigma_\nu = 0.675\%$	corr(GDP, spread) = −0.40
Recovery level	$\underline{B} = 0.2$	Brazil recovery rate = 86%

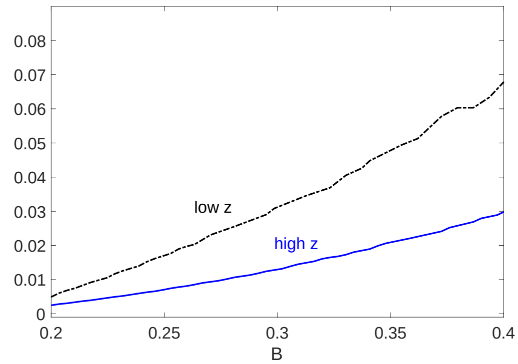
Table 2: Parameter Values

reflects default risk and hence increases with borrowing B' and decreases with productivity. Panel (a) in Figure (2) plots the spread function, as a function of B' relative to average output, for two z levels, a high level z_H and a low level z_L. These functions are independent of the money shock m and default cost ν because these shocks are i.i.d. Panel (a) shows that spreads increase with borrowing. When productivity is low, spreads are higher and increase faster with borrowing. Panel (b) plots the equilibrium government spread as a function of the state of debt B. The equilibrium spread is the spread taking into account the optimal borrowing rule spread$(s, B'(s, B))$. As the plot shows, equilibrium spreads are increasing in the level of debt but the optimal borrowing dampens the dependency of spreads on debt.

We now present the policy rules for the other allocations and prices in our model. Recall that the allocations in our model depend on in equilibrium on the government's state



(a) Spread Function



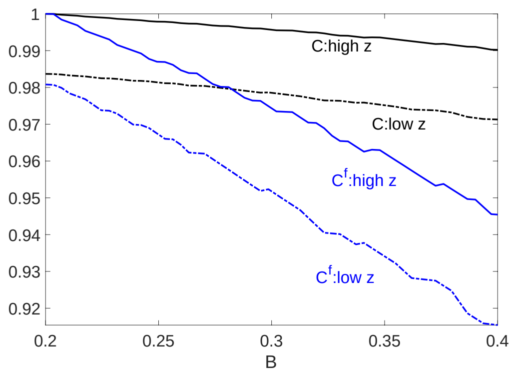
(b) Equilibrium Spread

Figure 2: Spread Function an Equilibrium Spread

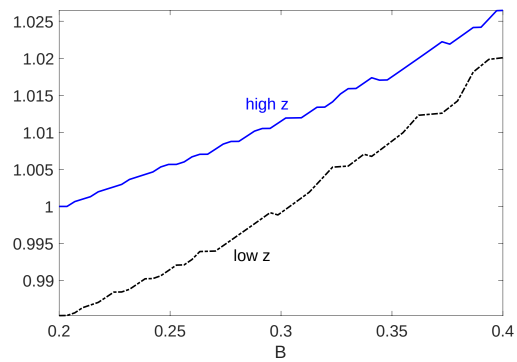
$s = \{z, m, \nu, B\}$ which contain the shocks to productivity, money, and default costs, as well as the endogenous level of debt. In Figure 3 we plot the policy rules for consumption, output, inflation, nominal rates, and terms of trade as a function of the debt level (relative to average output) for two levels of the productivity shock, a high level z_H and a low level z_L holding constant the money and enforcement shock at their median levels.

Panel (a) plots domestic consumption and imported consumption as a function of debt and shows that consumption falls with debt. In our economy with financial restrictions arising from default risk high levels of debt are associated with larger net debt repayment $B - qB'$, which requires larger net exports to pay the debt $X - eC^f$. In equilibrium exports increase and imports decrease to pay the larger debt. The decline in domestic consumption with larger debt occurs precisely because such decline boosts exports. Domestic and imported consumption are lower with low productivity and they decline faster with debt with low productivity. The faster decline of consumption with debt arises because the bond price function is tighter with low productivity due to higher default risk. Panel (b) shows that that in contrast to consumption, output increase with debt. Output increase with debt for similar reasons: high debt coupled with restricted borrowing creates the need for larger exports to pay the debt which leads to an increase in labor supply through a wealth effect. Output lower with low productivity and increases at a faster rate with debt.

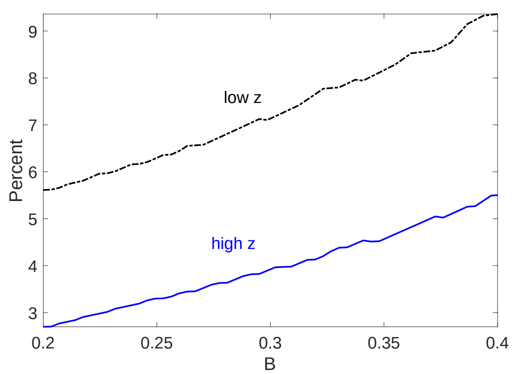
In Panel (c) we plot inflation as a function of debt. Inflation increases with debt because unit costs for firms are higher. Higher unit costs reflect the larger labor used to produce exports to pay off the debt. In Panel (d) we see that nominal interest rates also increase with debt. In response to high inflation, interest rates increase because of the inflation target rule. Low productivity is associated with higher inflation and nominal interest rates. Unit



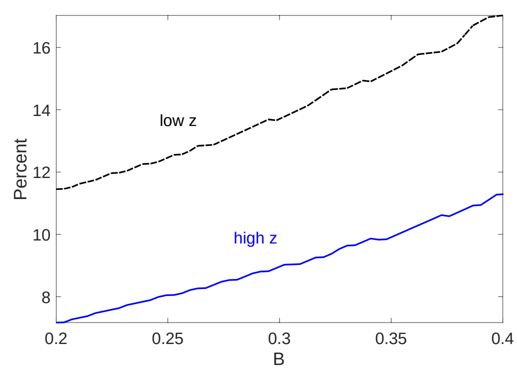
(a) Consumption



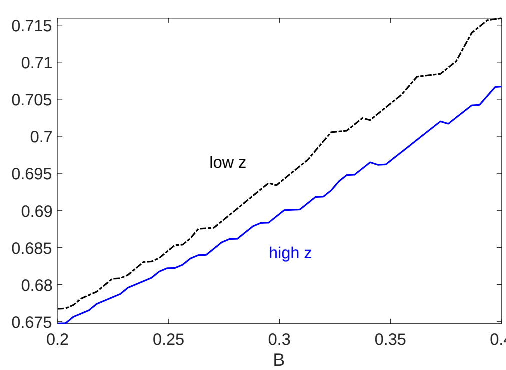
(b) Output



(c) Inflation

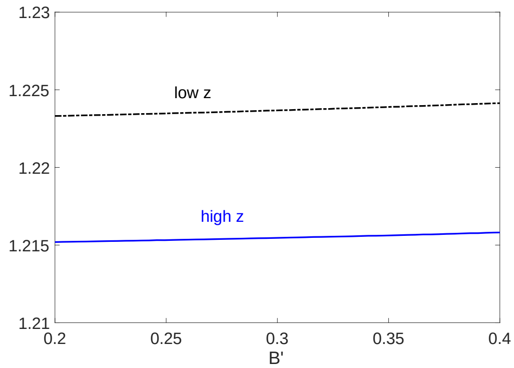


(d) Nominal Interest Rates

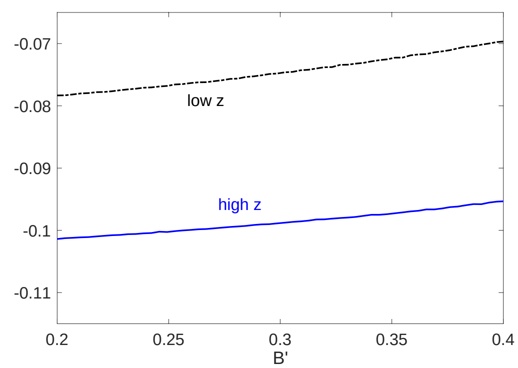


(e) Terms of Trade

Figure 3: Consumption, Output, Inflation, Nominal Rates, Terms of Trade



(a) Expected Marginal Utility (M)



(b) Expected Inflation (F)

Figure 4: Expected Marginal Utility Functions and Expected Inflation Function

costs are higher with low productivity leading to an increase in inflation.

Panel (e) shows that the terms of trade increase (depreciate) with debt. Higher debt is associated with smaller imported consumption and larger exports, both of which increase the price of imports relative to exports. Terms of trade are also high with low productivity.

In our model with lack of commitment, the government not only takes as given the bond price function $q(s, B')$ but also takes as given two additional functions that shape allocations, namely, the households expected marginal utility function $M(s, B')$ and the expected inflation function $F(s, B')$ in equations (21) and (22). In particular, the government understand that its choice of borrowing today can influence the the consumption and inflation that households and firms will choose tomorrow. Figure 4 plot the expected marginal utility and inflation functions as a function of the borrowing choice B' for the two levels of productivity. Expected marginal utility relative to inflation $M(s, B')$ is increasing in borrowing B' because the decrease in domestic consumption with debt is larger than the increase in inflation. Expected inflation times marginal utility is increasing with B' because inflation increases with debt and consumption decreases with debt. Expected marginal utility and expected inflation are decreasing in productivity.

4.5 Impulse Response Functions

We present the impulse response functions of the variables of interest to a contractionary money shock m and to a low productivity realization z.

To highlight the mechanism of default risk and time-consistency problem of the government, we contrast our model with a reference model akin to Gali and Monacelli (2005) but with state-uncontingent private debt and an exogenous, loose borrowing schedule. The reference model differs from the benchmark in two ways. First, there is no fiscal policy (other than the time-invariant taxes/subsidies that correct market power) and international borrowing is conducted by private agents. Hence, unlike the government in the benchmark model, here private agents do not internalize the effect of their borrowing on future inflation and nominal rates. Second, the international bonds are enforceable. We close the reference model and ensure stationarity with a debt-elastic interest rate as in Schmitt-Grohé and Uribe (2003). The bond price function depends the borrowing choice B' as follows:

$$\frac{1}{q(B')} = R + \phi_B \left[\exp(B' - \overline{B}) - 1 \right]. \tag{36}$$

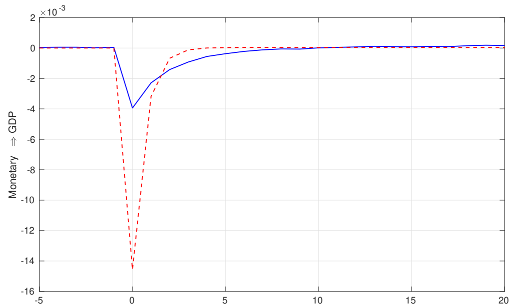
where ϕ_B, and $\overline{B}$ are constants governing the speed of return to steady state and the average level of debt, respectively.

The equilibrium in the reference model includes domestic and foreign consumption $\{C(S), C^f(S)\}$, labor $N(S)$, inflation $\pi(S)$, the nominal rate $i(S)$, terms of trade $e(S)$, private debt $b'(S)$, and equilibrium bond price $q^*(S)$ that satisfies equation (18) to (23), (32), and (36). To parameterize the reference model, we set the coefficient ϕ_B to a small value in the range of 10^{-5}, essentially allowing for a near-constant interest rate. We also choose $\overline{\pi}$ and $\overline{B}$ to generate the same average inflation and debt-to-GDP as in the benchmark. All other parameters are the same as in the benchmark model.[6]

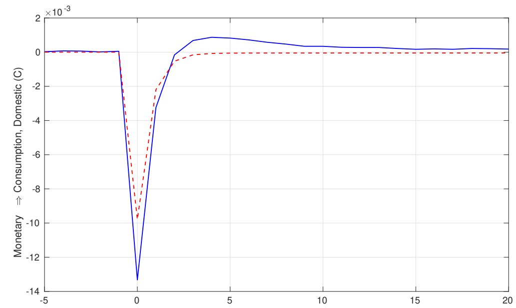
As we discussed in section 3.2, our model creates three wedges relative to the reference model in the optimal choice of foreign debt. These wedges shape the responses of the model to the exogenous monetary and productivity shocks.

We construct the impulse response functions in our nonlinear model following Koop et al. (1996). We simulate a panel of $50,000$ units for 5000 periods. For the first 4950 periods, the shocks follow their underlying Markov chains so that the cross-sectional distribution converges to the ergodic distribution of the model. In period 4951, the *impact period*, normalized to 0 in the plots, we alter shocks for all units by the same amount. From period 4952 onwards, the money or productivity shocks follow the conditional Markov chain. The impulse responses plot the average, across the units, of the variables conditional on not defaulting (We discard defaulting units from the cross-section average. Their inclusion does not alter the properties of the IRFs.).

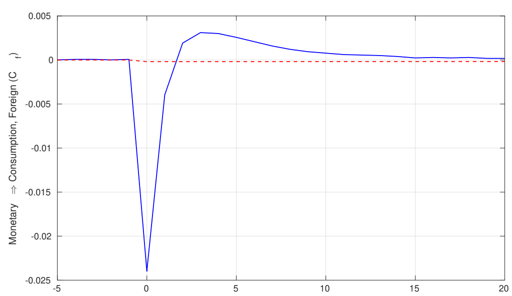
[6]We solve the reference model using Dynare 4.5 for MATLAB, with a first-order log-linear approximation of the equilibrium conditions.



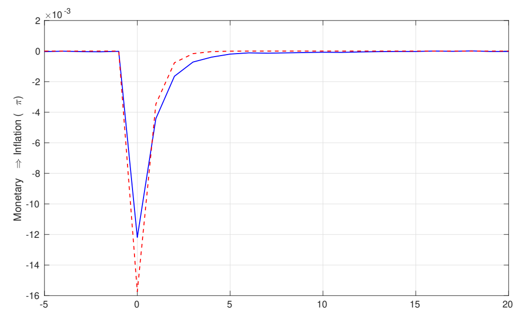
(a) Output



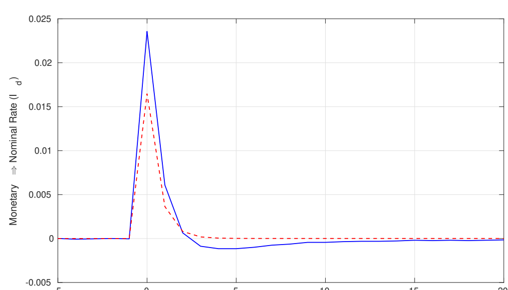
(b) Domestic Consumption (C)



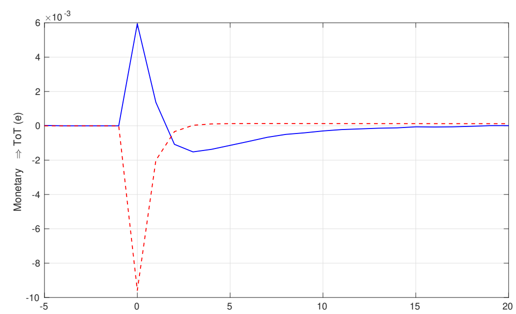
(c) Imported Consumption (C^f)



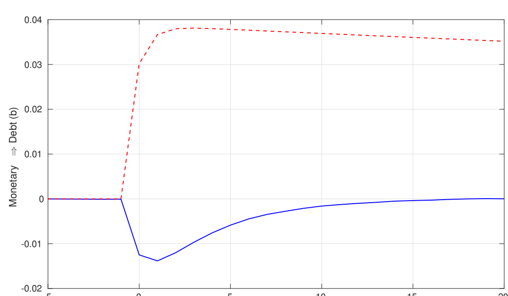
(d) Inflation (π)



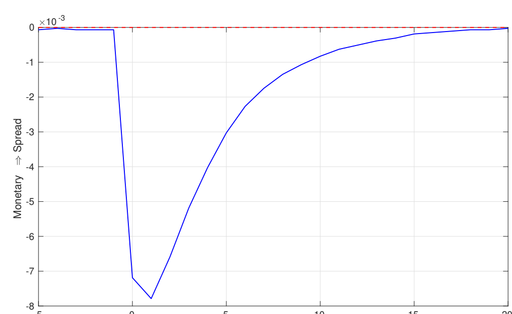
(e) Nominal Interest Rate (i)



(f) Terms of Trade (e)



(g) Debt (B)



(h) Spread

Figure 5: Impulse Response to Monetary Rule Shock. Benchmark model in solid blue, Reference in dashed red.

Money Shocks Figure 5 plot the impulse responses to an increase in the money shock m of aggregate output, domestic consumption, imports, spreads, inflation, nominal interest rates, terms of trade, and debt. The solid lines are for the benchmark model and the dashed lines are for the reference model.

In the benchmark model, we increase the money shock by 4% (annualized) so that the nominal interest rate increases by roughly 2% on impact as shown in Panel (e). On the impact of this tightening of monetary policy, aggregate output in Panel (a) declines by 0.2%, domestic consumption drops by 1.2%, and the imports shrinks about 2.5%. The increase of nominal interest rates leads to a reduction in inflation of about 1.2%. Terms of trade depreciates by about 0.6% and the spread declines 0.7% due to lower government borrowing.

These responses of output, domestic consumption, inflation, and nominal interest rate are similar to those in the standard New Keynesian small open economy models with sticky prices, e.g. Gali and Monacelli (2005), as seen from the dashed lines of the reference model in Figure 5. In both models, a high money shock increases nominal interest rate, which depresses domestic consumption. The low domestic consumption further reduces unit cost of producing and leads to a low inflation and output in this demand-driven economy.

The behavior of imports, terms of trade, borrowing, and sovereign spread are different in the reference and benchmark model. The reference model provides almost perfect risk sharing with incomplete market but loose borrowing limit. With a negative shock, the government increases its borrowing dramatically to smooth out imports. As shown in Panel (c), imports in the reference model are essentially flat. Higher inflows of foreign goods due to borrowing lowers the relative price of foreign goods and leads to an appreciation of the terms of trade.

In contrast, our model features an endogenous spread schedule arising from sovereign default risk. Moreover, the government internalizes the impact of its borrowing choices on future consumption and inflation. In Panel (g), we show that government's borrowings and resulting spreads fall on impact, in response to a contractionary money shock. Here is the reason. In addition to the standard forces in the reference model, higher monetary shock in our model tightens the domestic Euler condition, increases the wedge and reduces the borrowing incentives of the government. The fiscal authority would like to borrow less and increase future consumption to alleviate the pressure on reduced current consumption. Less debt leads to a lower spread and a reduction in imports. Additionally, lower borrowing generates a depreciation of terms of trade.

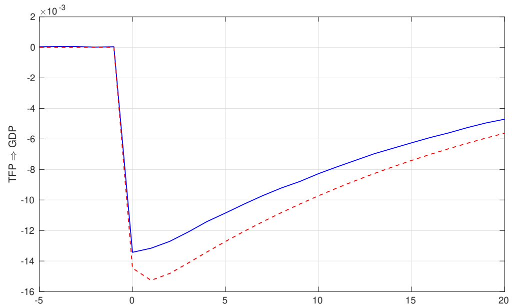
After the impact period, domestic consumption and imported consumption recover and overshoot their final levels while spreads, debt, and inflation slowly recover. The terms of trade appreciate on impact and then slowly revert back to mean. The dynamics in these variables arise in our model because of the persistence in debt and the interest rate smoothing, which give long lasting effects from an iid money shock. These dynamics are not present in standard New Keynesian models that generally feature perfect financial markets.

Productivity Shocks Figure 6 plots the impulse responses for the same variables to declines productivity shock z for both the benchmark and reference model (by about 1.5%). To help understand, we first discuss the responses in the reference model. We then present the IRFs and point out the unique forces in the benchmark model.

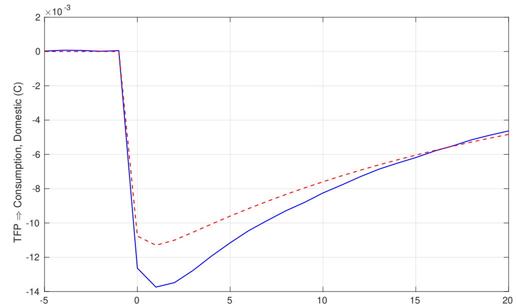
The red dashed lines in Figure 6 plots the IRFs in the reference model. Panel (a), (b), and (c) plot aggregate output, domestic consumption, and imports respectively. As is standard, negative productivity shock reduces output and consumptions. In Panel (d) we see that, as is typical in New Keynesian models, inflation rises with low productivity about 0.4% because firms face a higher unit cost. The response of the domestic nominal interest is in Panel (e). It rises about 0.8% due to the response called for by the interest rate rule to high inflation. Facing a loose borrowing schedule, unaltered by the negative shock, private agents increase their foreign borrowing (see Panel (g)), to smooth out imports. The inflow of foreign goods leads to an appreciation of the terms of trade, i.e. e decreases.

Qualitatively, the benchmark model has similar responses in output, consumption, domestic imports, inflation and nominal interest rate. The magnitudes, however, are very different. In the benchmark, GDP falls less, 1.2% versus 1.4% in the reference. Imports falls by 0.6% more than in the reference model. Inflation and nominal rates have a much large increase on impact, an extra 0.15% increase for inflation and an extra 0.2% increase in nominal rate. Moreover, in contrast to the reference model, the terms of trade appreciation is much more modest in the benchmark.

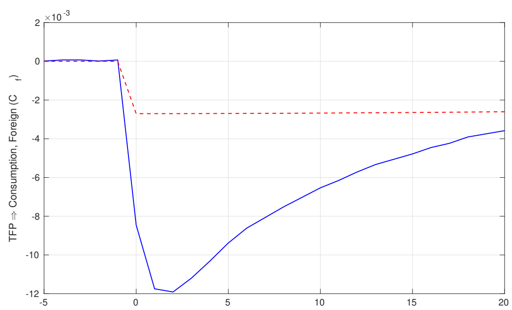
The reason is because the presence of the default risk in the benchmark model. Low productivity shock tightens the borrowing schedule and leads to a higher spread even when the government reduces the debt as seen in the solid blue lines of Panel (g) and (h). This generates a wealth effect and private agents does not decrease their labor as much as in the reference model, which in turn leads to a smaller decline in output. A relative high labor usage in the benchmark drives up the unit cost of firms resulting in higher inflation. In response to the increased inflation in the benchmark, the monetary authority raises the nominal rate even higher than in the reference model.



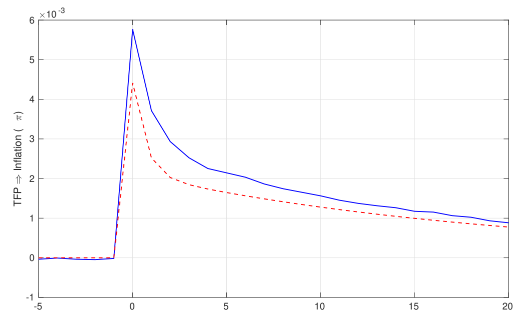
(a) Output



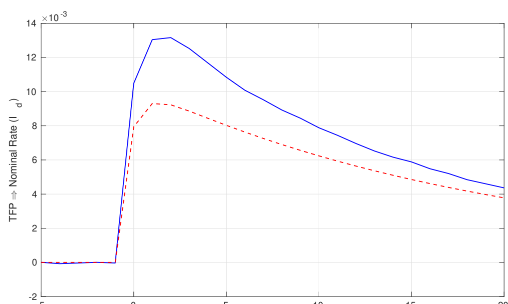
(b) Domestic Consumption (C)



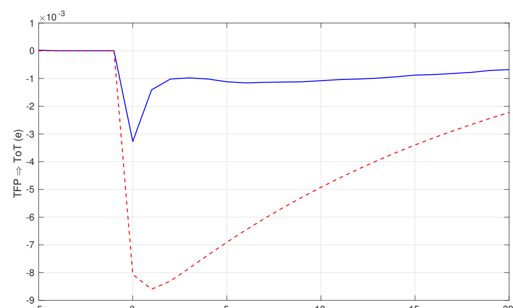
(c) Imported Consumption (C^f)



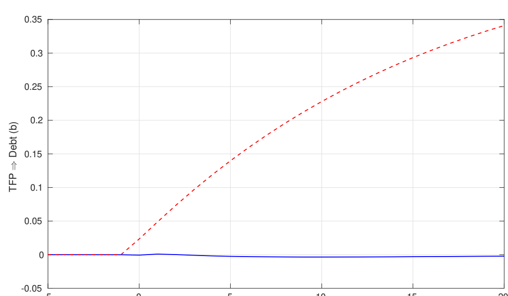
(d) Inflation (π)



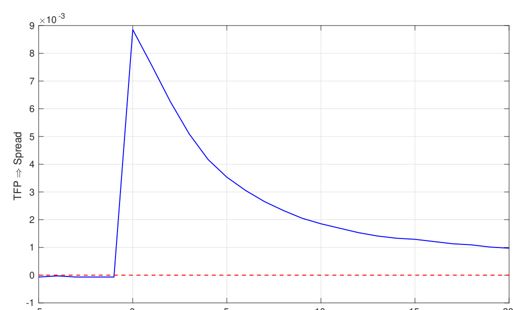
(e) Nominal Interest Rate (i)



(f) Terms of Trade (e)



(g) Debt (B)



(h) Spread

Figure 6: Impulse Response to Productivity Shock. Benchmark model in solid blue, Reference in dashed red.

The dynamics of the terms of trade in Panel (e) are in contrast to the reference model, being shaped by default risk and tight financial conditions as well. With the increase in spread, the government cuts its debt holding and realizes that the country has to export more to pay back its debt. An alternative way to understand is that fewer foreign goods flow into the country due to default risk, and thus domestic goods are relatively abundant.

In summary, low productivity leads to a decline in output and consumption, both domestic and imported. Productivity shocks generate positive co-movement between inflation, nominal interest rates and yields, while sovereign spreads are countercyclical. Moreover, the comparison of the two models highlights the transmission from sovereign spread to monetary policy. Sovereign default risk induces additional volatility in consumption and inflation, so that the interest rate rule calls for more aggressive monetary tightening, i.e. larger responses of the nominal rate to shocks.

4.6 Second Moments

Table 3 reports the mean and standard deviation of key variables as well as their correlations with sovereign spread for the data, the benchmark model, and the reference model.

	Brazil	Benchmark	Reference
Means (%)			
Inflation	5.9	6.1	6.1
Nominal Rate	11.2	11.6	11.6
Spread	2.6	2.7	—
Standard Deviations (%)			
Output	1.9	2.0	1.9
Inflation	1.8	2.1	2.0
Nominal Rate	2.2	**2.1**	**1.4**
Spread	0.9	0.9	—
Correlations with Spread			
Output	-0.4	-0.5	—
Inflation	0.5	**0.7**	—
Nominal Rate	0.8	**0.4**	—

Table 3: Moments

Overall, our moment matches well the Brazilian data. The mean inflation, nominal interest rate and spread are close to the data. The volatility of inflation and the nominal rate are comparable but somewhat higher in the model. The model generates relatively smoother spreads than the data though, 0.7 versus 0.9. Both the model and the data exhibit inflation that is positively correlated with spread, the coefficient is 0.5 for both. The correlation between spread and nominal rate is 0.8 in the data and 0.3 in the model. In the model, the value reflects the relative mix of productivity and monetary shocks, with productivity shocks inducing a higher correlation and monetary ones the opposite. As is usual in the sovereign default literature, spread is countercyclical.

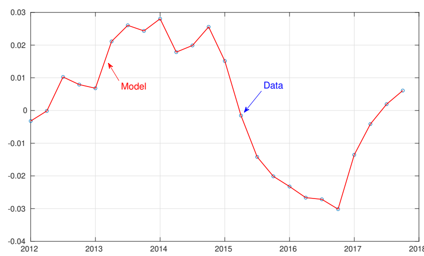
Table 3 also reports the moments from the reference model. Here without default risk, spreads are trivially zero. By construction, the reference model has the same average inflation and nominal rate as the benchmark. The volatility of nominal interest rate is, however, only about half of that in the benchmark. Inflation is also less volatile in the reference model than that in the benchmark. (Note that all parameters are kept the same between the two models.)

This comparison shows that in an environment with default risk, a central bank engaged in inflation targeting must apply much more active policy. This is because inflation responds more to productivity shock with default risk, requiring a larger change in the nominal interest rate to keep inflation close to the target, as seen in the IRFs to productivity or money shocks.

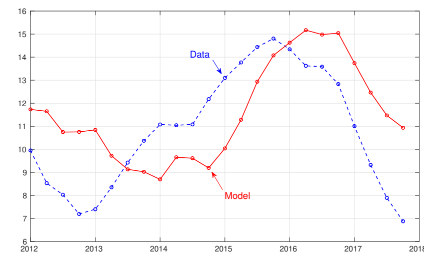
5 Brazil Event

We now compare the quantitative implications of the model to the Brazil data. We consider the 2012 to 2018 subsample, with a focus on the 2015 recession. To highlight the impact of monetary policy on sovereign default risk, we also conduct a counterfactual experiment where the nominal rate is fixed at its 2014 level. The experiment shows that if the central bank would have been more dovish during the recession, Brazil would face much higher inflation and spread.

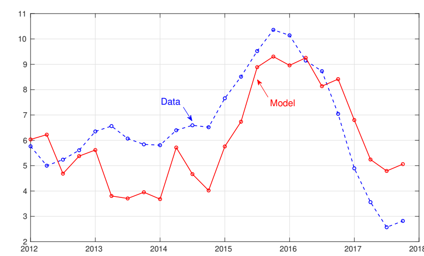
To replicate the Brazilian event in the model, we feed in productivity shocks such that the model reproduces the time path of output observed in the data. For this benchmark event, we choose as initial level of debt given by the mean of the limiting distribution and hold the money shock m at its mean level. We then compare the predictions of the model for inflation, the nominal interest rate, government spreads, and nominal exchange rates to the data.



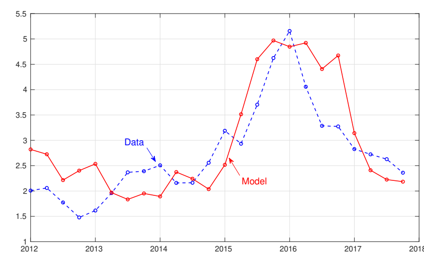
(a) Output



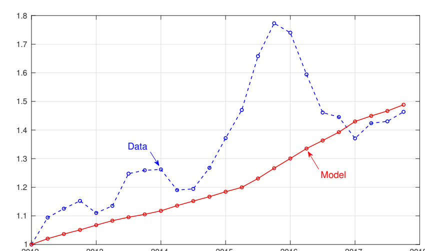
(b) Nominal Interest Rate



(c) Inflation

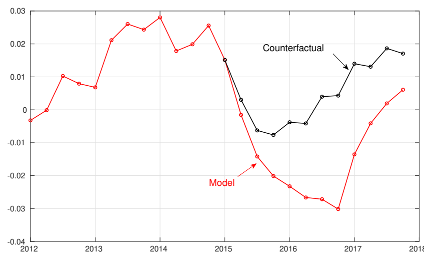


(d) Sovereign Spread

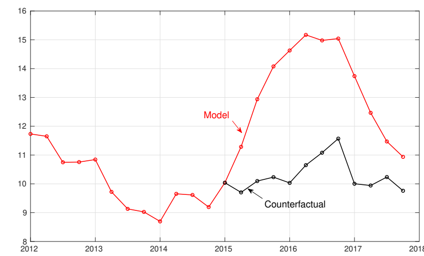


(e) Nominal Exchange Rate

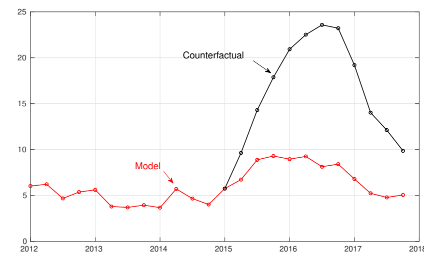
Figure 7: Event: Brazil



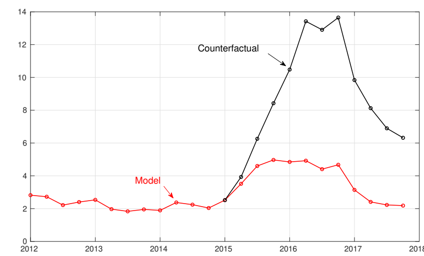
(a) Output



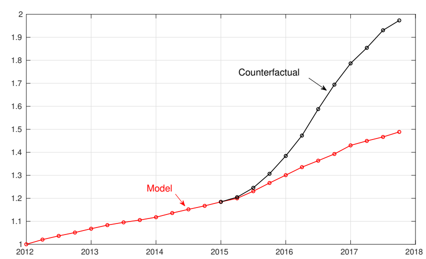
(b) Nominal Interest Rate



(c) Inflation



(d) Sovereign Spread



(e) Nominal Exchange Rate

Figure 8: Event: Counterfactual Experiment

The blue lines with circles in Figure 7 plot the data series. Brazil experienced a recession from 2014 to mid 2016, and GDP declines from 3% above trend to 3% below trending, a 6% decline in total. It then recovers after 2016Q3. During this period, inflation increases by 4%, the nominal rate increases by 2%, and spreads rise from about 2% to 5%. When GDP recovers after 2017, inflation, nominal rate, and spread all fall. During the recession, the nominal exchange rate depreciates 40% and then appreciates during the recovery.

The red lines in Figure 7 plot the corresponding series in the model. To match the dynamics of GDP in the data, the model requires that the underlying productivity shock first decreases from 2014 to 2016Q3 and then increases afterwards. This implies that during the recession, the unit cost of producing increases, which leads to an increase in inflation. Monetary policy responds to a high inflation with a high nominal rate. Low productivity also drives up the sovereign default risk. Quantitatively, the model matches well the peak of inflation and spread during recession, around 10% of inflation and 5% of spread. The model also produces the observed recoveries from 2016Q3 on, inflation decreases about 6% and spread lowers about 2%.

There are two forces driving the nominal exchange rate. On the one hand, lower productivity makes the domestic good scarem leaning towards appreciation. On the other hand, tightening of the bond price schedule reduces external debt and generates a depreciation. Overall, the second effect dominates, nominal exchange rate depreciates. However, due to the first effect, the depreciation is not as high as in the data during the recession. Moreover, the Armington structure of trade embeded in the model has difficulties in replicating the substantial volatility in the exchange rate from the data.

To understand the impact of monetary policy on the fiscal policy and sovereign default risk, we conduct a counterfactual experiment with a dovish central bank. In this experiment, instead of following the policy called for by its inflation target, the central bank keeps a low nominal interest rate, similar to its 2014 level. We feed in a sequence of monetary shocks so keep resulting rates near this initial value. The counterfactual series are plotted with the black color in Figure 8. As we show in the IRF, a lower money shock induces a lower nominal interest rate and thus higher current consumption. This raises the unit cost of production, which in turn generates high inflation. In 2017, the inflation rate in the counterfactual scenario would be more than 7 percent higher than the benchmark case. Lower nominal interest rate from the dovish central bank also leads to a higher spread, since government increases its borrowing to shift future consumption to the current period. The exchange rate therefore depreciated even further.

In summary, our model matches well Brazilian downturn around 2016 and 2017 in both real and nominal terms. Our model also provides an good laboratory to understand the interplay between monetary and fiscal policy. Had Brazil's central bank deviated from its pursue of price stability, the recession would have been milder but at a cost of much higher inflation and a deeper debt crisis.

6 Conclusion

[TODO]

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A Deriving Uncovered Interest Parity with Default

We can derive the uncovered interest parity under default (UIPD) through the Euler equation for domestic bond (21) and the one for foreign bond (30). We can rewrite foreign bond Euler (30) as

$$1 = \frac{1}{q}\beta E_{s'}\left[\frac{e(s',B')u_c(s',B')}{eu_c}\tau^{\text{UIP}}(s',b')\right] \tag{37}$$

where the composite wedge τ^{UIP} includes all the three wedges in the Euler as well as the discrepancy between the government's and the consumer's marginal rate of substitution between current and future consumption:

$$\tau^{\text{UIP}}(s,s',B') = \frac{u_c}{u_c^g}\frac{u_c^g(s',B')}{u_c(s',B')}\left[1 + \frac{\tau_1(s,B') + \tau_2(s,B') + \tau_3(s',B')}{\beta e(s',B')u_c^g(s',B')}\right].$$

We can also rewrite the domestic bond Euler (21) as

$$1 = \beta i(S)E\left[\frac{u_c(S')}{u_c(S)\pi(S')}\right]. \tag{38}$$

Equating equation (37) and (38), we have

$$\beta i(S)E\frac{u_c(S')}{u_c(S)\pi(S')} = \frac{1}{q}\beta E_{s'}\left[\frac{e(s',B')u_c(s',B')}{eu_c}\tau^{\text{UIP}}(s',b')\right].$$

Let $i^f = 1/q$ and $\hat{x}$ be the percentage deviation of x from a steady state. We log-linearize the above equation and get

$$\hat{i}_t = \hat{i}_t^f + E_t[\hat{\pi}_{t+1} + \hat{e}_{t+1} - \hat{e}_t] + E_t\hat{\tau}_{t+1}^{\text{UIP}}$$

B Numerical Implementation

We solve the model with discrete value function iteration methods. The state variables are the two shocks (z, to TFP, and m, to the Taylor rule), the nominal rate in the previous period i_-, and the level of debt B. The two shocks constitute a VAR(1) process that we discretize into a Markov chain with 21 and 11 support points respectively (covering ± 3 and ± 4 standard deviations). The grid for the nominal rate consists of 11 points set to cover the ergodic distribution of the nominal rate in equilibrium. The grid for debt has 144 points, equally spaced over $[\underline{B}, \underline{B} + 0.1]$. The iid shock to the cost of default ν is discretized over a grid with 301 points, spanning ± 3 standard deviations, so as to evaluate fast its CDF and the conditional expectation at the end of (29).

The algorithm proceeds as follows

1. We start with initial guesses for the value function W_0 and the bond price schedule q_0, together with guesses for the F_0 and M_0 functions.

2. For each point in the state space, (z, m, i_-, B), we consider all potential choices $B' \in [\underline{B}, \overline{B}(z)]$ where $\overline{B}(z)$ is the peak of the bond sale proceeds schedule, in states with TFP z, i.e.
$$\overline{B}(z) = \arg\max_{B'} \left\{ \arg\min_{i} q_0(z, i, B')B' \right\}$$
(Note that the bond price schedule is not a function of the interest rate rule shock m since the shock is iid and therefore its current value provides no information about default behavior in the future.)

3. For each B' choice considered in state (z, m, i_-, B), we solve for the behavior of the private sector, characterized by the system of equations (15)-(22). For each $< C^f, i >$ guess
$$\mathcal{B} \equiv B - q_0^{\text{interp}}(z, \mathbf{i}, B')B'$$
$$\pi = \overline{\pi} \left[\frac{\mathbf{i}}{mR^{1-\kappa} i_-^{\kappa}} \right]^{1/(1-\kappa)\alpha_p}$$
consumption of domestic goods (C),
$$C = \frac{1}{\beta \, \mathbf{i} \, M_0^{\text{interp}}(z, \mathbf{i}, B')}$$

the terms of trade (e), and labor supply (N).

$$e = \left(\frac{\mathbf{C^f} + \mathcal{B}}{\xi}\right)^{1/(\rho-1)}$$

$$\text{exports } = e^{\rho}\xi = \left(\frac{\mathbf{C^f} + \mathcal{B}}{\xi}\right)^{\rho/(\rho-1)} \xi = \left(\mathbf{C^f} + \mathcal{B}\right)^{\rho/(\rho-1)} \xi^{1/(\rho-1)}$$

$$N = \frac{C + \text{ exports}}{z\left[1 - \frac{\varphi}{2}\left(\pi - \overline{\pi}\right)^2\right]}$$

We then look for the pair $< C^f, i >$ associate with private sector choices that are consistent with the Phillips Curve (22) under F_0, and the relative demand for foreign goods, using the Powell hybrid method.

$$\begin{cases} \frac{\chi}{\theta} N^{1+\mu} + F^{\text{interp}}(s, \mathbf{i}, B') = \frac{zN}{C}\left[1 + \frac{\varphi}{\eta - 1}\left(\pi - \overline{\pi}\right)\pi\right] \\ \frac{C}{\mathbf{C^f}} = \frac{\rho}{\rho - 1}\frac{\theta}{1 - \theta} e \end{cases}$$

Finally, using this private sector responses, we compute the value associated with each potential B'. The government maximizes over these and the maximum is stored in W_1, at the appropriate coordinates.

4. Using the new function W_1 we compute a new default threshold for the ν shock, $\nu^*(z, m, i_-, B)$, according to equation (28), and the associated default probabilities for each state.

5. We construct a new bond price schedule, q_1, by iterating on equation (17); then update the F_1 and M_1 functions according to equations (25) and (24) respectively. Dampening the update of the q schedule is never necessary but possible, yet seldom yields a faster convergence, for dampening factors in the 0.01–0.3 range.

6. If $||W_1 - W_0||_\infty < 10^{-6}$, $||q_1 - q_0||_\infty < 10^{-5}$, $||F_1 - F_0||_\infty < 10^{-5}$, and $||M_1 - M_0||_\infty < 10^{-5}$ we stop; otherwise we iterate ($W_0 \leftarrow W_1$, $q_0 \leftarrow q_1$, $F_0 \leftarrow F_1$, $M_0 \leftarrow M_1$) and move to step 2 above. Convergence, as defined here, is achieved within 700–1000 iterations, largely due to the relatively high value of β used in the calibration.

Whenever a function is superscripted with the "interp" label, we use linear interpolation (and extrapolation, whenever necessary) over the i_- dimension of the state space. In equilibrium, extrapolation is never necessary.

C Parameter Robustness

This appendix documents the robustness of the main results to alternative assumptions about the Frisch elasticity of labor supply, the persistence of the TFP shock, and the volatility of the iid shock to the value of default. Finally, we explore the role of the interest rate smoothing parameter κ for equilibrium dynamics.

[TODO]