Social Security is NOT a Substitute for Annuities[*]

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Abstract

Common wisdom suggests that a fully-funded actuarially fair social security system must increase welfare when households face longevity risk and annuity markets are missing. This wisdom is based on the observation that social security pays benefits as life annuities and therefore appears to complete the market. However, we argue that common wisdom is based on a benefit-only analysis that ignores a fundamental cost—social security crowds out the bequests that households leave (and receive) in general equilibrium. We conduct a general equilibrium cost-benefit analysis of the insurance role of social security, and we show that under very general and empirically relevant conditions, this decline in bequest income offsets any possible gains from access to a public annuity pool. We abstract from distortions to national income and factor prices to show that the equilibrium bequest channel is all that is needed to reach this conclusion.

Key words: Annuities, Uninsurable Longevity Risk, Social Security, General Equilibrium, Bequest Income.
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1 Introduction

A normative rationale for the existence of social security is the provision of lifetime annuity insurance.[1] The private annuity market in the U.S. is very thin, which prevents households from sharing their longevity risk in the market.[2] Given this fact, the prevailing and unchallenged view is that a fully-funded actuarially fair social security system must increase welfare since social security pays benefits as life annuities that provide a higher implicit return than can be found in the market.

In this paper we claim that common wisdom about the insurance role of social security is not correct. We argue that common wisdom is based on a *benefit-only* analysis that ignores a fundamental cost—social security crowds out the bequests that households leave (and receive) in general equilibrium. We show that under very general and empirically relevant conditions, this crowding out of bequest income is in fact large enough to cancel out any welfare gains from participating in a public annuity pool with an above-market return. We abstract from distortions to national income and factor prices to show that the equilibrium bequest channel is all that is needed to reach this conclusion. This is because social security payments, unlike annuities, are independent of saving decisions and therefore do not affect intertemporal trade-offs (Euler equation). Thus social security only affects the lifetime income of the household. But if bequest income is carefully accounted for in general equilibrium, then any increase in lifetime income from participating in a mandatory annuity pool must be offset by a reduction in bequest income or else the aggregate resource constraint is violated.

We first use a stylized two-period OLG model in Section 2 to make our intuition clear. Rational households face survival uncertainty but lack access to annuity markets. They can only save in a zero-interest storage technology. The zero-interest assumption helps us to derive analytical results in a neat and clean way, though later we will show our results only grow stronger with positive interest. Households may die with assets (accidental bequests), which are distributed anonymously to survivors in the population without regard to specific linkages between parents and children (we relax this assumption later). This assumption is motivated by a vast literature in quantitative macroeconomics that uses life-cycle OLG models (see for example Hubbard and Judd (1987), İmrohoroğlu et al. (1995), Conesa and Krueger (1999), Huggett and Ventura (1999), Hong and Ríos-Rull (2007), and Nishiyama and Smetters (2007) among many others). To focus on the longevity insurance aspect of social security and to abstract from its effect on work incentives and retirement, we assume

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[1] See for example Diamond (1977, 2004) and Feldstein (2005).

[2] See Pashchenko (2010) for a survey and quantitative analysis of the leading explanations for why the annuity market is so thin.

inelastic labor supply and exogenous retirement. In this setting we prove analytically that, for any strictly concave utility function, a fully-funded actuarially fair social security system improves welfare when bequest income is fixed but has no effect at all on consumption or welfare when bequest income is endogenous.

Next, in Section 3 we extend the two-period model to a continuous-time OLG model that otherwise shares the same basic assumptions. The continuous-time setting allows us to consider all possible equilibrium age-distributions of bequest income. We again prove analytically that, for any strictly concave utility function, social security has no effect on consumption or welfare when bequest income is endogenous.

Then in Section 4 we conduct numerical experiments to show that our result continues to hold and actually grows stronger in a more general environment with positive returns to saving. We make a direct comparison to Hubbard and Judd (1987). By holding bequest income fixed, we replicate the results found in Hubbard and Judd (1987): a fully-funded social security system can generate large welfare gains when liquidity constraints are absent but these gains are reduced or even reversed when liquidity constraints are present. However, by allowing bequest income to adjust in general equilibrium, we confirm our analytical findings and we uncover a set of new findings not found in Hubbard and Judd (1987): now a fully-funded social security system reduces welfare for all parameterizations and for any assumption about liquidity constraints. Reductions in equilibrium bequest income more than crowd out any gains from mandatory annuitization.

Finally, in Section 5 we relax the anonymous-bequest assumption and adopt an alternative that is less popular (mainly because of its analytical and computational complexity) but perhaps more realistic. We assume bequests are distributed through direct linkages between deceased parents and surviving children (as in Eckstein et al. (1985a) and Abel (1985)). We show that while such an assumption can lead to a welfare role for social security, the welfare gains are due to reducing the variance of the distribution of wealth rather than a better provision of longevity insurance. In this alternative case, there is heterogeneity in wealth across households due to heterogeneity in the timing of the deaths of their entire ancestry. We show that introducing social security can improve social welfare by reducing heterogeneity, but it is not a Pareto move: only those whose parents live the longest (the very poorest) gain, and everyone else loses.

Related Literature

The usefulness of social security as a provider of longevity insurance has been quantitatively explored by Hubbard and Judd (1987), İmrohoroğlu et al. (1995), Conesa and Krueger

(1999), Hong and Ríos-Rull (2007) among many others. These papers study the welfare implications of various social security arrangements in dynamic OLG models with multiple layers of uncertainty, market incompleteness and (possibly) multiple social insurance programs. We complement their quantitative analysis by proving analytically that under very general conditions social security, *in isolation*, fails to improve welfare when annuity markets are missing. To our knowledge, this fundamental result is new to the literature.

It has long been known (at least since Eckstein et al. (1985a) and Abel (1988)) that social security does not improve welfare when households already have access to perfect annuity markets. But our contribution is to show that social security does not improve welfare even when annuity markets are totally missing.

Abel (1986) and Eckstein et al. (1985b) point out that there is a welfare enhancing role for social security in the presence of adverse selection in the annuity market. Social security forces households with different mortality types to pool their survival risk and hence provides a better implicit rate of return than annuity contracts traded in the market. Our assumption that private annuity markets are completely missing biases our model in favor of social security.[3] Yet, social security fails to deliver any welfare gains due to the general equilibrium effect of bequest income.

This paper is also related to the large literature measuring the insurance value of annuitization for representative life-cycle consumers (e.g., Kotlikoff and Spivak (1981), Mitchell et al. (1999), Brown (2001), Davidoff et al. (2005), Lockwood (2012)). The exercise in these papers is to determine how much incremental, nonannuitized wealth would be equivalent to providing access to actuarially fair annuity markets.[4] A robust finding of this exercise is that a 65-year-old adult, with population average mortality, gains up to 30 to 50 percent of his/her retirement wealth from having access to actuarially fair insurance. A key feature of all these studies is the static comparison between full insurance and no insurance at all. A large part of this welfare gain comes from the fact that when people die their assets evaporate from the economy. On the other hand, under full insurance these assets are annuitized and hence the assets of the deceased are transferred to those who survive (the assets do not leave the economy). The conventional interpretation of this vast literature is that any public provision of annuities (e.g., social security) can have very large welfare benefits.[5] Our contribution to this literature is to point out that when the assets of the deceased are carefully account for

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[3]As Hosseini (2010) shows, even in the presence of adverse selection, mandatory annuitization does come with a cost. Social security causes those with the shortest longevity to withdraw from the private annuity market, which pushes annuity prices up and reduces the welfare gains from social security. He shows that this negative welfare effect is quantitatively large.

[4]Lockwood (2012) is an exception in that he considers the comparison between no annuities and annuities that are available at actuarially unfair market rates.

[5]Once again Lockwood (2012) is an exception.

these results go away. This has deep and important implications for what we think about the value of annuitization through social security.

2 A Two-Period Model

Our goal in this section is to provide intuition for our results by using a simple two-period model. In the next section we will generalize our results to a multi-period (continuous time) setting. We use the cleanest possible environment to capture the equilibrium response of bequest income to social security. This prevents us from confounding all other roles of social security with its pure insurance role. Also, to be able to prove results analytically we assume zero interest and no population growth. In Section 4 we verify numerically that our result hold in a more general environment with positive interest and population growth.

We first demonstrate how social security can be welfare improving in a static (non-OLG) setting in which bequests are exogenous (Section 2.1). Then in Section 2.2 we demonstrate how the conclusions of the static analysis can change in an OLG environment in which bequests are endogenously determined in equilibrium.

2.1 Static Model with Exogenous Bequests

Consider a household that lives for a maximum of two periods. We denote the probability of survival to the second period by S. Households have two sources of income in the first period. They inelastically supply one unit of labor and received wage w. They also receive bequest income B. In the first period of life the household can hold an asset k that pays return q. We assume there is a zero-interest storage technology. Therefore, if there is a perfect annuity market the equilibrium return is $q = \frac{1}{S}$. However, if there is no annuity market the equilibrium return is $q = 1$. When annuity markets exist, the return on assets is higher since it pays only to those who are alive. On the other hand, when annuity markets do not exist, households effectively face a higher price for future consumption.

There is a fully-funded actuarially fair social security system that collects on income in the first period at rate τ and pays benefit b to those who are alive in the second period. A balanced budget implies that $Sb = \tau w$.

Let the household flow utility function satisfy $u' > 0$, $u'' < 0$, and the usual Inada conditions. The household problem is to choose consumption c to

$$\max u\left(c_1\right) + Su\left(c_2\right), \tag{1}$$

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subject to

$$c_1 + k = w(1 - \tau) + B, \tag{2}$$

$$c_2 = qk + b. \tag{3}$$

Note that, at an interior solution, the following Euler equation must hold

$$u'(c_1) = qSu'(c_2). \tag{4}$$

If perfect annuity markets exist, $q = \frac{1}{S}$ and the Euler becomes

$$u'(c_1) = u'(c_2), \tag{5}$$

and if annuity markets do not exist, then $q = 1$ and

$$u'(c_1) = Su'(c_2). \tag{6}$$

In the case when there is a perfect annuity market, the household lifetime budget constraint becomes

$$c_1 + Sc_2 = w(1 - \tau) + Sb = w. \tag{7}$$

From the intertemporal budget (7) and Euler condition (5), the consumption allocation is

$$c_1 = c_2 = \frac{w}{1 + S} \equiv c_{FB}, \tag{8}$$

where c_{FB} is the first-best allocation of the social planner's problem. From (7) and (8), it is obvious that social security does not affect consumption and welfare. This result is well known.

In the more relevant case when annuity markets are missing, we show that the effect of social security depends critically on whether bequest income is exogenous or endogenous in the model. In both cases, the Euler equation (6) holds and is not affected by social security. But the lifetime budget will be different in these two cases. When bequest income is exogenous, the lifetime budget of the household is

$$c_1 + c_2 = w(1 - \tau) + b + B = w - Sb + b + B = w + (1 - S)b + B. \tag{9}$$

Taking the derivative with respect to the social security benefit,

$$\frac{\partial(c_1 + c_2)}{\partial b} = 1 - S > 0, \tag{10}$$

since we assume that bequest income B is exogenous and thus $\frac{\partial B}{\partial b} = 0$.

Note that in this static model with exogenous bequests, $(1 - S)k$ units of consumption is wasted and not consumed by anyone. By taking resources from households and carrying them to the next period in the public pool of annuities, social security in effect increases household lifetime income by keeping part of the otherwise wasted assets of the deceased in the economy and paying them to survivors. Hence, social security enhances the lifetime budget and improves welfare under the assumption of exogenous bequest income. Therefore, in a static environment in which bequests are fixed, social security can improve welfare only through an income effect.

However, in a dynamic OLG environment, bequest income is determined endogenously by those who die at the end of the first period. If the introduction of a fully-funded actuarially fair social security system leads to crowding out of asset holdings, it will lead to a lower bequest level. Thus, it is possible that the negative income effect of lower bequests crowds out the positive income effect of social security. In the rest of this paper we show that under very general conditions, this crowding out is in fact large enough to cancel out any welfare gains from participating in a public annuity pool.

2.2   Dynamic Model with Endogenous Bequests

Here we extend the static model to a dynamic OLG model. In the first period of life, the household collects wage income w and bequest income B from the deceased. Social security taxes are levied on wages at rate τ in return for social security benefits b which are paid to those who survive to the second period of life. The probability of surviving to the second period is S. Households lack access to annuities and must instead save in a zero-interest technology. There is no population growth.

The household takes bequest income B and government policy as given and chooses consumption in period one c_1 and claims over consumption in period two k (and hence consumption in the second period c_2)

$$\max u\left(c_1\right) + Su\left(c_2\right), \tag{11}$$

subject to

$$c_1 + k = w(1 - \tau) + B, \tag{12}$$

$$c_2 = k + b. \tag{13}$$

There is a fully-funded actuarially fair social security system. At the aggregate level, social security is self financed

$$Sb = \tau w, \tag{14}$$

and the bequests of the young must equal the assets of those who die early after just one period

$$(1 - S) k = B. \tag{15}$$

A stationary equilibrium in this economy is characterized by household allocations (c_1^*, c_2^*, k^*) and aggregate bequest B^* such that given government policy and bequest B^*, allocation (c_1^*, c_2^*, k^*) solves the household problem (11)-(13) and equations (14) and (15) hold.

In what follows we assume away liquidity constraints. At the end of this section we discuss how the results change if we do not allow borrowing.[6]

In the following proposition we show that a fully-funded actuarially fair social security system crowds out bequest income one for one and hence consumption and welfare in the steady state are unchanged.

Proposition 1 *If annuity markets do not exist, introducing a fully-funded actuarially fair social security system does not affect consumption allocations and welfare.*

Proof. If $k > 0$ (or is not constrained), the following Euler equation describes household intertemporal trade-offs

$$u'(w - \tau w + B - k) = Su'(k + b). \tag{16}$$

After imposing the social security budget constraint (14) and feasibility (15) we get the following equation

$$u'(w - Sb - Sk) = Su'(k + b). \tag{17}$$

The solution to this equation is the equilibrium allocation for a given level of social security benefit. We call this solution $k(b)$.

Consider the case when there is no social security

$$u'(w - Sk(0)) = Su'(k(0)). \tag{18}$$

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[6]And we do the same for the general model using numerical experiments in Section 4.

We claim that $k(b) = k(0) - b$. To verify, replace this into (17)

$$u'(w - Sb - S(k(0) - b)) = Su'(k(0) - b + b),\qquad(19)$$

and simplify

$$u'(w - Sk(0)) = Su'(k(0)),\qquad(20)$$

which holds by definition. This implies that equilibrium consumption must be

$$c_2(b) = k(b) + b = k(0) = c_2(0),\qquad(21)$$

and therefore

$$c_1(b) = c_1(0).\qquad(22)$$

■

While social security is traditionally believed to improve welfare because it shares some of the properties of annuities (both make survival-contingent payments at the expense of the deceased), it cannot complete the market because payments are lump sum and do not affect intertemporal trade-offs. Instead, social security will only have a crowding out effect on saving, which in turn reduces the amount of bequest income households leave (and receive) in general equilibrium, leaving consumption allocations unchanged.

The result presented above depends on the assumption that consumers are not liquidity constrained. For the sake of completeness we show below that if households cannot borrow against their future social security benefit and if social security taxes and benefits are large enough, social security can implement the first best.

Proposition 2 *In the two-period model, if annuity markets do not exist and households cannot borrow against their social security benefits, there is a social security tax and benefit that can implement the first best.*

Proof. Let τ^* and b^* be such that

$$u'(w - \tau^* w) = u'(b^*),\qquad(23)$$

and replacing from the social security budget

$$u'(w - \tau^* w) = u'\left(\frac{\tau^* w}{S}\right).\qquad(24)$$

Note that the Inada conditions imply that there is a $0 < \tau^* < 1$ that satisfies this equation. Let $c_1 = w - \tau^* w$ and $c_2 = \frac{\tau^* w}{S}$. Note that by construction c_1 and c_2 satisfy the household budget constraint and

$$u'(c_1) = u'(c_2) > Su'(c_2), \tag{25}$$

which means under this allocation the household does not wish to save. Hence, c_1 and c_2 are the equilibrium consumption allocation and the household chooses to purchase zero claims on the second period consumption good ($k = 0$). Note that c_1 and c_2 are the first-best allocation in this economy. ∎

Hence, the only way to improve welfare in this economy is by shutting down all intertemporal trade. We next move on to prove our result in a more general multi-period setting.

3   A Continuous-Time OLG Model

In this section we extend the two-period OLG model to continuous time. Age is continuous and indexed by t. Households start work at $t = 0$, retire exogenously at $t = T$, and pass away and exit the model no later than $t = \bar{T}$. At each moment an infinitely divisible cohort is born. The probability of surviving to age t, from the perspective of age 0, is $S(t)$. Thus, $S(0) = 1$ and $S(\bar{T}) = 0$. As above, households lack access to annuities and invest in a saving technology that has a net return 0. We denote household assets by $k(t)$. The assets of the deceased are bequeathed to the surviving population according to the age-distribution $B(t)$. We continue to assume the flow utility function $u(c)$ satisfies $u' > 0$, $u'' < 0$ and the usual Inada conditions. Utility is discounted at rate ρ.

Rational households take bequest income $B(t)$ and government policy as given and solve the following problem

$$\max \int_0^{\bar{T}} e^{-\rho t} S(t) u(c(t)) dt, \tag{26}$$

subject to

$$\dot{k}(t) = y(t) - c(t), \tag{27}$$

$$y(t) = (1 - \tau)w + B(t), \text{ for } t \in [0, T], \tag{28}$$

$$y(t) = b + B(t), \text{ for } t \in [T, \bar{T}], \tag{29}$$

$$k(0) = k(\bar{T}) = 0. \tag{30}$$

The fully-funded actuarially fair social security system is self financed

$$b \int_{T}^{\bar{T}} S(t)dt = \tau w \int_{0}^{T} S(t)dt, \tag{31}$$

and the bequests that are distributed must equal the total assets of the deceased[7]

$$\int_{0}^{\bar{T}} S(t)B(t)dt = \int_{0}^{\bar{T}} \left(-\dot{S}(t)\right) k(t)dt. \tag{32}$$

A stationary equilibrium in this economy is household allocations $(c^*(t), k^*(t))_{t\in[0,\bar{T}]}$ and aggregate bequest distribution $(B^*(t))_{t\in[0,\bar{T}]}$ such that given government policy and bequest $B^*(t)$, household allocations solve the household problem (26)-(30) and equations (31) and (32) hold.

As before, we continue to assume away liquidity constraints. We will quantitatively explore the effect of this assumption on our result in the next section. The following proposition establishes the same result as we proved in the two-period model: social security crowds out private saving (and hence aggregate bequests) one for one and therefore consumption allocations are invariant to the introduction of social security.

Proposition 3 *If annuity markets do not exist, introducing a fully-funded actuarially fair social security system does not affect consumption allocations and welfare.*

Proof. Let $\lambda = u_c(c(0))$ be the marginal utility of consumption at time zero. Intertemporal optimality implies (and note that we ignore liquidity constraints for now)

$$e^{-\rho t} S(t) u_c(c(t)) = \lambda. \tag{33}$$

Therefore

$$c(t) = u_c^{-1}[\lambda e^{\rho t}/S(t)], \tag{34}$$

where u_c^{-1} is the inverse of the marginal utility of consumption. Equilibrium consumption allocations must simultaneously satisfy the household optimization condition (34), household budget constraints (27)-(30), and the aggregate constraints (31) and (32). We begin by combining all constraints (household plus aggregate constraints) into a single equation. To do this, first integrate by parts the aggregate bequest condition (32) using the boundary

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[7]Note that we intentionally consider the entire family of $B(t)$ paths that are feasible. This includes the traditional assumption in macroeconomics that bequests are spread uniformly across the surviving population as well as all other possibilities (such as only the young receive bequests, etc.)

conditions $k(0) = k(\bar{T}) = 0$

$$\int_0^{\bar{T}} S(t)B(t)dt = \int_0^{\bar{T}} S(t)\dot{k}(t)dt. \tag{35}$$

Insert $\dot{k}(t)$ from the household budget constraint (27)

$$\int_0^{\bar{T}} S(t)B(t)dt = \int_0^{\bar{T}} S(t)\left[y(t) - c(t)\right]dt. \tag{36}$$

Insert $y(t)$ from (28) and (29) into (36)

$$\int_0^{\bar{T}} S(t)B(t)dt = \int_0^T S(t)\left[(1-\tau)w + B(t) - c(t)\right]dt + \int_T^{\bar{T}} S(t)\left[b + B(t) - c(t)\right]dt, \tag{37}$$

and then cancel the $B(t)$ terms

$$0 = \int_0^T S(t)\left[(1-\tau)w - c(t)\right]dt + \int_T^{\bar{T}} S(t)\left[b - c(t)\right]dt. \tag{38}$$

Now impose the self-financing assumption of social security

$$0 = \int_0^T S(t)\left[(1-\tau)w - c(t)\right]dt + \int_T^{\bar{T}} S(t)\left[\tau w\frac{\int_0^T S(t)dt}{\int_T^{\bar{T}} S(t)dt} - c(t)\right]dt, \tag{39}$$

and simplify to obtain a single condition that reflects all other constraints

$$\int_0^{\bar{T}} S(t)c(t)dt = \int_0^T S(t)wdt. \tag{40}$$

Any equilibrium consumption allocation must simultaneously satisfy household optimization (34) and the aggregate resource constraint (40). Combine these two

$$\int_0^{\bar{T}} S(t)u_c^{-1}[\lambda e^{\rho t}/S(t)]dt = \int_0^T S(t)wdt. \tag{41}$$

Equation (41) provides the equilibrium determination of λ such that household optimization and budget constraints are satisfied, social security is self financed, and aggregate bequest income equals the assets of the deceased. Because the right side of (41) is independent of τ, it must be the case that λ (and hence consumption at time zero) is also independent of τ. Therefore, equilibrium $c(t)$ for all t must be independent of τ. Consumption and welfare are invariant to social security in equilibrium, despite the fact that households face uninsurable

longevity risk. ∎

Although the case we considered above is very special (zero interest and zero population growth), it demonstrates the main mechanism at play in a neat and clean way. Any gain from the annuitization aspect of social security comes from a positive income effect (and not a better intertemporal trade-off). All of that gain is washed out by a negative income effect on equilibrium bequest income due to crowding out.

We conjecture that this result continues to hold in a more general environment. Since we cannot prove this analytically we resort to numerical exercises to validate this conjecture.

4   Robustness

In the previous section we presented the main result of the paper. In order to derive clean analytical results we made two simplifying assumptions. First we assumed that households can borrow without any restriction. This assumption implies the Euler equation of the household always holds with equality. The other assumption was that the net return on capital is exactly equal to the rate of population growth (and they both equal zero). This assumption implies that under any social security system, households face the same budget constraint in equilibrium. Therefore, these two assumptions imply the same consumption allocation in equilibrium under any social security policy.

In this section we relax both assumptions and we show that the main results still hold true, and, in a sense become stronger. In order to establish this we resort to numerical experiments in line with Hubbard and Judd (1987). We first show that when there is positive population growth and positive net returns on capital, a fully-funded social security system leads to lower consumption and therefore lower welfare when bequest income is endogenous. We then go further and add liquidity constraints and verify that for an empirically relevant level of social security tax and transfer the same result holds.

We have to caution readers that we are not conducting a serious calibration or quantitative exercise. Our paper is making a theoretical point. The purpose of the numerical exercises that we present here is to show that our theoretical results are more general than is implied by the assumptions in Proposition 3.

Our sensitivity analysis follows Hubbard and Judd (1987) and uses the same environment and parameterization. We assume households start working at age 20, retire at age 65 and live up to a maximum age of 110. We use a CRRA utility function $u(c) = c^{1-\sigma}/(1-\sigma)$ and report results for different values of σ. We assume population growth $n = 1\%$ and construct average population survival using male and female mortality data for the cohort of

1920 (Bell and Miller (2005), Table 7).[8] Following Hubbard and Judd (1987) the household age-earning profile is taken from Davies (1981) which is approximated by the following forth-order polynomial for $t \in [20, 65]$,[9]

$$w(t) = -36999.4 + 3520.22t - 101.878t^2 + 1.34816t^3 - 0.00706233t^4. \tag{42}$$

Finally, we assume an annual discount rate of $\rho = 0.015$. To isolate our findings from the factor price effects of social security we assume a small open economy with constant returns on capital, r. We report results for $r = 0.02$, 0.04, and 0.06.[10]

As we demonstrated in the previous section our analytical results do not depend on the timing of the distribution of bequests. We follow Hubbard and Judd (1987) and assume they are received when households are at age 40.

We show the capital-output ratio and aggregate bequest (as a percentage of average earnings) in an economy without social security in Table 1. The top panel shows the quantities for the economy without liquidity constraints and the bottom panel shows numbers for the economy with liquidity constraints.

We now introduce a fully-funded actuarially fair social security system. We choose social security benefits to be 60 percent of average earnings and choose social security taxes to balance the budget,

$$\tau \int_0^T e^{-rt} S(t) w(t) \, dt = b \int_T^{\bar{T}} e^{-rt} S(t) \, dt. \tag{43}$$

Note that by doing this we hold the annuity payment of social security unchanged across different parameter values, however, the tax rates that are required to balance the budget differ for different values of the interest rate, r.[11]

We first present the results with exogenous bequests in Table 2. That is, we hold the household bequest endowment fixed at the level presented in Table 1 and solve the household problem with fully-funded social security. In all cases the capital-output ratio and aggregate

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[8]Hubbard and Judd (1987) only report the source of mortality data, but they do not report which year they are using to construct their survival probabilities. For this reason our numerical results do not match theirs exactly. But they are consistent.

[9]As Hubbard and Judd (1987) claim, and we confirm, the results are not sensitive to the use of alternative age-earning profiles.

[10]We only consider cases in which the return on capital exceeds the population growth rate, $r > n$. If $r < n$, the crowding out of saving is less than one for one (whether social security is pay as you go or fully funded) and the introduction of social security can be welfare improving in the steady state. We don't report these results here because they have less relevance empirically.

[11]Hubbard and Judd (1987) choose a 6% social security tax rate, consistent with the tax on self-employed workers in the early 1980's. We choose a level of benefit that is more in line with today's system in the US and other OECD countries. However, the results are not qualitatively sensitive to the exact level of taxes and transfers.

bequest that households leave after death declines after the introduction of social security. When there is no liquidity constraint, welfare increases for all combinations of the interest rate and risk aversion parameter. However, when households are not allowed to borrow, welfare increases only for low risk aversion. The importance of liquidity constraints is in line with Hubbard and Judd (1987). Next, we show that this pattern is completely reversed when we let bequest income adjust to a new steady state.

Table 3 shows the results in the steady state after the introduction of social security when aggregate bequests are determined endogenously in equilibrium. Social security causes a lower capital-output ratio and bequests, and significantly lower welfare (relative to a steady state with no social security). Note that welfare is significantly lower for all combinations of the interest rate and risk aversion parameter regardless of whether borrowing is allowed or not. Also, note that the only inefficiency in this environment is the absence of annuity insurance. The economy is dynamically efficient, and even though annuity markets are absent, the introduction of a publicly provided annuity through a fully-funded social security system does not improve welfare. This system does not affect the intertemporal trade-offs of the household. It only crowds out saving more than one for one (if the interest rate is larger than the population growth rate) which in equilibrium will reduce bequests that are received by each household and therefore lowers their lifetime income.

To gain better intuition about the effect of social security with and without endogenous bequest income we plot the consumption allocations and asset holdings for the case $\sigma = 2$ and $r = 0.04$ in Figure 1. The solid black lines are allocations under steady states with no social security. The green dashed line shows allocations after the introduction of social security while holding bequest income fixed at its steady state level without social security. Note that consumption is higher at almost all ages. This is because the annuity through social security pays a better implicit return than non-survival-contingent assets that households hold. Also, since bequest income is fixed we don't see any crowding out effect. Social security improves welfare because it provides better longevity risk sharing.

However, once we allow for bequests to adjust to the new steady state, all the positive effect of social security is more than offset by the crowding out of bequest income. The dashed-dotted red line shows allocations in steady states with social security and endogenous bequest income. We see that consumption is lower than consumption without social security in all ages when there are no liquidity constraints and almost all ages when liquidity constraints are present. That is why we see a large decline in welfare. Social security fails to improve welfare, even though it provides actuarially fair annuities. The reason is that social security does not affect intertemporal trade-offs that households face (unless it crowds out all trade).

5 Example with an Alternative Bequest Arrangement

Our result depends on the assumption that the assets of the deceased are distributed anonymously to all survivors in the economy. This assumption is motivated by common practice in the quantitative macroeconomics literature that uses life-cycle OLG models (see for example Hubbard and Judd (1987), İmrohoroğlu et al. (1995), Conesa and Krueger (1999), Huggett and Ventura (1999), Hong and Ríos-Rull (2007) and Nishiyama and Smetters (2007) among many others). It is a convenient assumption that makes models simple and tractable. But it is not innocuous. An alternative assumption that is less popular (mainly for analytical and computational complexity) is that bequests are distributed through direct linkages between deceased parents and surviving children (as in Eckstein et al. (1985a) and Abel (1985)).

In this section we show that while such an assumption can lead to a welfare role for social security, the welfare gains are due to reducing the variance of the distribution of wealth rather than a better provision of longevity insurance. We illustrate this point with a two-period OLG model with logarithmic utility, zero interest and no population growth.[12] We show that introducing social security is not a Pareto move: only those whose parents live the longest (the very poorest) gain, and everyone else loses.

The household problem is

$$\max \ln(c_1) + S \ln(c_2), \tag{44}$$

subject to,

$$c_1 + k = w(1 - \tau) + B, \tag{45}$$

$$c_2 = k + b. \tag{46}$$

And the first-order condition is

$$\frac{1}{w(1-\tau) + B - k} = \frac{S}{k+b}, \tag{47}$$

solving for k (and imposing $b = \frac{\tau w}{S}$)

$$(1 + S)k = S(1 - \tau)w + SB - b, \tag{48}$$

$$k = \frac{S}{1+S}w + \frac{S}{1+S}B - \frac{(1+S^2)\tau w}{S(1+S)}. \tag{49}$$

In this environment, although households are ex ante identical, ex post they differ by the amount of bequest income they receive. Whether a household receives a bequest or not

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[12]For a more general analysis we refer readers to Eckstein et al. (1985a) and Abel (1985).

depends on whether his/her parent has died in the first or second period.

Let ψ be the distribution of the bequest that is received by each new born. Next we will characterize this distribution. Note that each parent survives with probability S. Therefore, independent of parents' assets, a fraction S of newborns receive zero bequest. Let $B_0 = 0$, then

$$\psi\left(B_0\right) = S. \tag{50}$$

These households have savings

$$k_0 = \frac{S}{1+S}w - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)}, \tag{51}$$

and if they die early, they leave all their assets as a bequest. So their children start the economy with

$$B_1 = \frac{S}{1+S}w - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)}, \tag{52}$$

and will have saving

$$
\begin{aligned}
k_1 &= \frac{S}{1+S}w + \frac{S}{1+S}\left(\frac{S}{1+S}w - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)}\right) - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)} \\
&= \frac{S}{1+S}w + \left(\frac{S}{1+S}\right)^2 w - \frac{S}{1+S}\left(\frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)}\right) - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)} \\
&= \left(1 + \frac{S}{1+S}\right)\left(\frac{S}{1+S}w - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)}\right),
\end{aligned} \tag{53}
$$

and if they die early their children receive

$$B_2 = \left(1 + \frac{S}{1+S}\right)\left(\frac{S}{1+S}w - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)}\right), \tag{54}$$

and will have saving

$$
\begin{aligned}
k_2 &= \frac{S}{1+S}w + \frac{S}{1+S}B_2 - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)} \\
&= \left(1 + \frac{S}{1+S} + \left(\frac{S}{1+S}\right)^2\right)\left(\frac{S}{1+S}w - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)}\right).
\end{aligned} \tag{55}
$$

Therefore

$$k_t = \left(\frac{S}{1+S}w - \frac{\left(1+S^2\right)\tau w}{S\left(1+S\right)}\right)\sum_{i=0}^{t}\left(\frac{S}{1+S}\right)^i, \tag{56}$$

and

$$B_0 = 0, \tag{57}$$
$$B_t = k_{t-1} \text{ for } t > 0, \tag{58}$$

also we can find the distribution ψ as

$$\psi(B_t) = S + S(1-S) + \cdots + S(1-S)^t, \tag{59}$$

where t denotes the number of periods since the last ancestor has survived to the second period.

We can find consumption allocations.

$$
\begin{aligned}
c_{1t} &= w(1-\tau) + B_t - k_t \\
&= w(1-\tau) + k_{t-1} - k_t \\
&= w(1-\tau) - \left(\frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}\right)\left(\frac{S}{1+S}\right)^t
\end{aligned}
\tag{60}
$$

and

$$c_{2t} = Sc_{1t} = Sw(1-\tau) - S\left(\frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}\right)\left(\frac{S}{1+S}\right)^t. \tag{61}$$

It is obvious that consumption allocations are not invariant to the social security tax. And, notice that social security only increases consumption for those whose immediate ancestor survived to old age (i.e., for the poorest). We show this in the following proposition.

Proposition 4 *Social security reduces welfare for all households for whom $t > 0$.*

Proof. It is enough to show that c_{1t} decreases with τ if $t > 0$. Note that we can write c_{1t} as

$$c_{1t} = \left(1 - \left(\frac{S}{1+S}\right)^{t+1}\right)w + \left(\frac{(1+S^2)}{S(1+S)}\left(\frac{S}{1+S}\right)^t - 1\right)w\tau. \tag{62}$$

It is enough to show that $\frac{(1+S^2)}{S(1+S)}\left(\frac{S}{1+S}\right)^t < 1$ for $t > 0$. Note that

$$\frac{\ln\left(\frac{(1+S^2)}{S(1+S)}\right)}{\ln\left(\frac{1+S}{S}\right)} < 1 \le t, \quad t = 1, 2, \ldots \tag{63}$$

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for any $S < 1$. Therefore,

$$t \ln \left(\frac{1+S}{S} \right) > \ln \left(\frac{(1+S^2)}{S(1+S)} \right) \tag{64}$$

and

$$\left(\frac{1+S}{S} \right)^t > \left(\frac{(1+S^2)}{S(1+S)} \right), \tag{65}$$

which implies

$$\frac{(1+S^2)}{S(1+S)} \left(\frac{S}{1+S} \right)^t < 1. \tag{66}$$

∎

Note that for any level of social security, there is always a fraction of people who receive zero bequest. Introduction of social security does not change the bequest income for this group in the steady state. Therefore, even though there is crowding out of asset holdings and bequests in the aggregate, the bequest that these households received does not change at all. Therefore, they benefit from the positive income effect from social security and are not harmed by the crowding out.

For those who do receive bequests in equilibrium, there will be a crowding out effect. The larger their bequest income is, the larger is the effect of crowding out on them (i.e., the effect is larger for higher t). And eventually it is so high to cancel out all the positive income effect of social security. In the case of the example presented above the crowding out effect is large enough to cancel out all the positive gains from social security for all those who receive any positive bequest in equilibrium.

6 Conclusion

The conventional view in public economics is that social security improves welfare through the provision of longevity insurance when private annuity markets are missing. However, we show that in a general equilibrium model with closed annuity markets, social security fails to improve welfare under very general conditions.

Our paper highlights the importance of including the general equilibrium effect of bequests when studying the insurance role of social security. The conventional wisdom holds only if we ignore this effect, which violates the aggregate resource constraint. Then social security is (trivially) welfare improving because of a free lunch: the welfare gains from mandatory annuitization come without the cost of lower bequest income. A proper, general

equilibrium cost-benefit analysis eliminates this free lunch and eliminates any insurance role for social security.

References

Abel, Andrew B., "Precautionary Saving and Accidental Bequests," *American Economic Review*, September 1985, *75* (4), 777–91.

_ , "Capital Accumulation and Uncertain Lifetimes with Adverse Selection," *Econometrica*, September 1986, *54* (5), 1079–97.

_ , "Aggregate Savings in the Presence of Private and Social Insurance," *NBER Working Paper*, 1988, (1873).

Bell, Felicitie C. and Michael L. Miller, "Life Tables for the United States Social Security Area 1900–2100," Actuarial Study 120, Social Security Administration August 2005.

Brown, Jeffrey R., "Private Pensions, Mortality Risk, and the Decision to Annuitize," *Journal of Public Economics*, October 2001, *82* (1), 29–62.

Conesa, Juan C. and Dirk Krueger, "Social Security Reform with Heterogeneous Agents," *Review of Economic Dynamics*, October 1999, *2* (4), 757–95.

Davidoff, Thomas, Jeffrey R. Brown, and Peter A. Diamond, "Annuities and Individual Welfare," *American Economic Review*, December 2005, *95* (5), 1573–90.

Davies, James B., "Uncertain Lifetime, Consumption, and Dissaving in Retirement," *Journal of Political Economy*, June 1981, *89* (3), 561–77.

Diamond, Peter A., "A Framework for Social Security Analysis," *Journal of Public Economics*, December 1977, *8* (3), 275–98.

_ , "Social Security," *American Economic Review*, March 2004, *94* (1), 1–24.

Eckstein, Zvi, Martin Eichenbaum, and Dan Peled, "The Distribution of Wealth and Welfare in the Presence of Incomplete Annuity Markets," *Quarterly Journal of Economics*, August 1985, *100* (3), 789–806.

_ , _ , **and** _ , "Uncertain Lifetimes and the Welfare Enhancing Properties of Annuity Markets and Social Security," *Journal of Public Economics*, April 1985, *26* (3), 303–26.

Feldstein, Martin S., "Rethinking Social Insurance," *American Economic Review*, March 2005, *95* (1), 1–24.

Hong, Jay H. and José-Víctor Ríos-Rull, "Social Security, Life Insurance and Annuities for Families," *Journal of Monetary Economics*, January 2007, *54* (1), 118–40.

Hosseini, Roozbeh, "Adverse Selection in the Annuity Market and the Role for Social Security," Working Paper, Arizona State University 2010.

Hubbard, R. Glenn and Kenneth L. Judd, "Social Security and Individual Welfare: Precautionary Saving, Borrowing Constraints, and the Payroll Tax," *American Economic Review*, September 1987, *77* (4), 630–46.

Huggett, Mark and Gustavo Ventura, "On the Distributional Effects of Social Security Reform," *Review of Economic Dynamics*, July 1999, *2* (3), 498–531.

İmrohoroğlu, Ayşe, Selahattin İmrohoroğlu, and Douglas H. Joines, "A Life Cycle Analysis of Social Security," *Economic Theory*, June 1995, *6* (1), 83–114.

Kotlikoff, Laurence J. and Avia Spivak, "The Family as an Incomplete Annuities Market," *Journal of Political Economy*, April 1981, *89* (2), 372–91.

Lockwood, Lee M., "Bequest Motives and the Annuity Puzzle," *Review of Economic Dynamics*, April 2012, *15* (2), 226–43.

Mitchell, Olivia S., James M. Poterba, Mark J. Warshawsky, and Jeffrey R. Brown, "New Evidence on the Money's Worth of Individual Annuities," *American Economic Review*, December 1999, *89* (5), 1299–1318.

Nishiyama, Shinichi and Kent Smetters, "Does Social Security Privatization Produce Efficiency Gains?," *Quarterly Journal of Economics*, November 2007, *122* (4), 1677–1719.

Pashchenko, Svetlana, "Accounting for Non-Annuitization," Working Paper Series WP-2010-03, Federal Reserve Bank of Chicago 2010.

(a) Economy without Liquidity Constraints

	Capital-Output Ratio				Aggregate Bequest (% of avg earnings)			
	σ				σ			
	0.9	2	4	5	0.9	2	4	5
$r = 0.02$	5.55	7.1	8.48	8.88	8	13	17	19
$r = 0.04$	9.42	7.57	7.01	7.78	19	18	20	20
$r = 0.06$	11.37	7.78	5.99	5.62	55	28	22	21

(b) Economy with Liquidity Constraints

	Capital-Output Ratio				Aggregate Bequest (% of avg earnings)			
	σ				σ			
	0.9	2	4	5	0.9	2	4	5
$r = 0.02$	6.81	8.51	10.01	10.44	9	14	19	20
$r = 0.04$	9.78	8.71	8.82	8.95	20	20	23	25
$r = 0.06$	11.5	9.01	8.55	8.5	57	34	33	33

TABLE 1: Capital-output ratio and aggregate bequest income in steady state without social security.

(a) Economy without Liquidity Constraints

	Capital-Output Ratio				Aggregate Bequest (% of avg earnings)				Welfare Change (% of consumption)			
	σ				σ				σ			
	0.9	2	4	5	0.9	2	4	5	0.9	2	4	5
$r = 0.02$	-1.27	0.93	2.84	3.39	-6	-1	4	6	9.44	9.07	8.71	8.6
$r = 0.04$	7.33	4.83	4.04	3.94	9	8	9	10	3.16	3.19	3.16	3.14
$r = 0.06$	11.02	6.68	4.33	3.83	47	19	13	13	0.94	1.13	1.18	1.19

(b) Economy with Liquidity Constraints

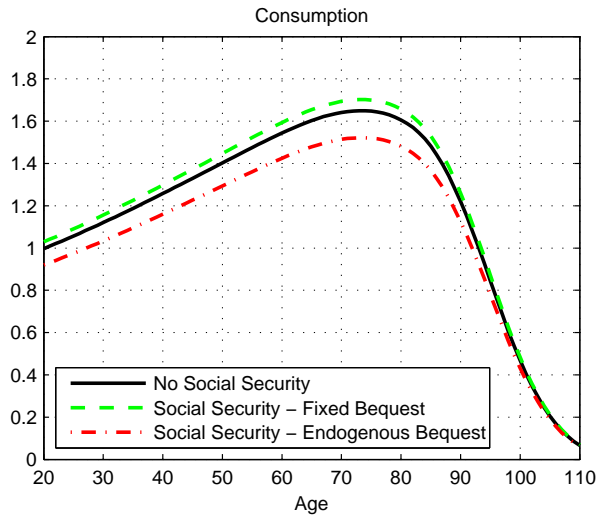|	Capital-Output Ratio				Aggregate Bequest (% of avg earnings)				Welfare Change (% of consumption)			
	σ				σ				σ			
	0.9	2	4	5	0.9	2	4	5	0.9	2	4	5
$r = 0.02$	3.81	5.28	6.78	7.28	3	5	8	9	4.68	3.57	-3.34	-6.36
$r = 0.04$	8.73	7.71	8.55	8.92	13	12	15	16	2.36	1.25	-2.82	-3.87
$r = 0.06$	11.59	10.29	11.09	11.36	50	27	25	26	0.82	0.02	-1.83	-2.06

TABLE 2: Capital-output ratio, aggregate bequest income and welfare change after introduction of social security while holding bequest income fixed.

(a) Economy without Liquidity Constraints

	Capital-Output Ratio				Aggregate Bequest (% of avg earnings)				Welfare Change (% of consumption)			
	σ				σ				σ			
	0.9	2	4	5	0.9	2	4	5	0.9	2	4	5
$r = 0.02$	1.23	3	4.58	5.04	-7	-3	2	3	-4.69	-4.69	-4.69	-4.69
$r = 0.04$	7.81	5.73	5.09	5.01	7	6	7	7	-7.76	-7.76	-7.76	-7.76
$r = 0.06$	10.91	6.98	5.01	4.6	42	17	11	11	-7.44	-7.44	-7.44	-7.44

(b) Economy with Liquidity Constraints

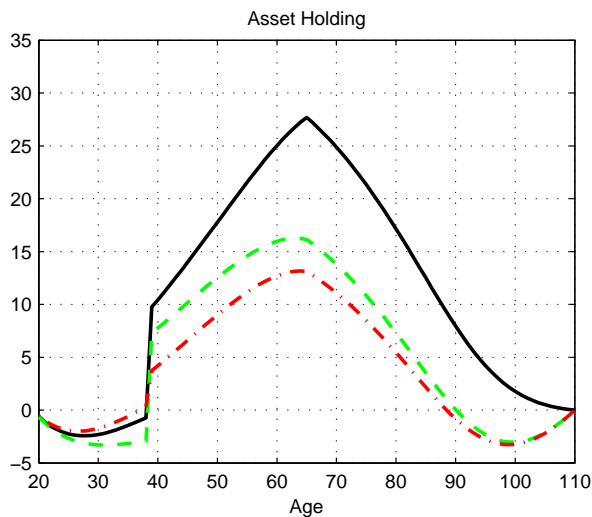|	Capital-Output Ratio				Aggregate Bequest (% of avg earnings)				Welfare Change (% of consumption)			
	σ				σ				σ			
	0.9	2	4	5	0.9	2	4	5	0.9	2	4	5
$r = 0.02$	5.74	6.44	7.16	7.41	2	4	6	6	-1.07	-3.31	-6.71	-7.87
$r = 0.04$	8.84	7.29	7	7.04	12	10	10	11	-4.62	-5.9	-5.3	-4.76
$r = 0.06$	11.12	7.95	7.07	7	45	21	18	18	-6.1	-6.97	-3.24	-2.43

TABLE 3: Capital-output ratio, aggregate bequest income and welfare change at the steady state with social security and endogenous bequests.
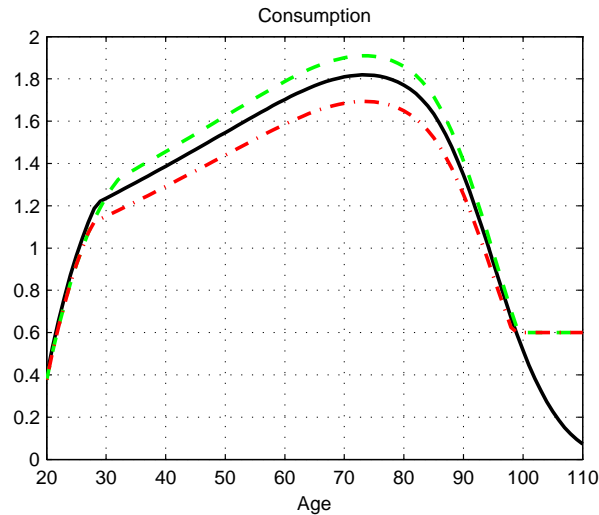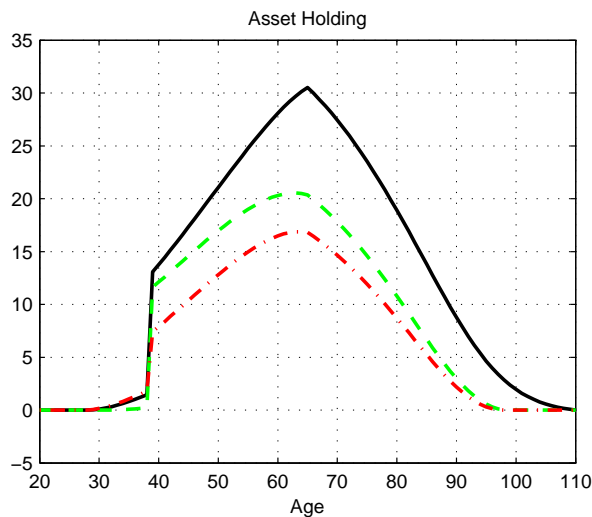
FIGURE 1: Consumption and asset holdings for parameters $\sigma = 2$ and $r = 0.04$. Figures on the left show consumption (top) and asset holding (bottom) for the economy without liquidity constraints. Figures on the right show consumption (top) and asset holding (bottom) for the economy with liquidity constraints. The black solid line is the steady state allocation without social security. The green dashed line is the allocation after the introduction of social security with bequests held fixed (at the steady state level without social security). The red dashed-dotted line is the steady state allocation with social security (and endogenous bequest income).