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Representation Theory and Schubert Calculus: Combinatorics and Interactions

This research project contains applications of combinatorics to: (1) affine crystals, which are graphs encoding representations of quantum affine algebras; (2) Whittaker functions on p-adic groups, which are a basic tool in the theory of automorphic forms and the construction of Dirichlet series; (3) modern Schubert calculus. The emphasis is on combinatorial methods for computation, new avenues of research, and interesting connections. The main combinatorial model to be used is the alcove model introduced by the PI and A. Postnikov, which was since developed by the PI and other researchers.

An important part of the project is the continuation of the PI's collaboration with S. Naito, D. Sagaki, A. Schilling, and M. Shimozono, on uniform combinatorial models for level 1 Kirillov-Reshetikhin (KR) crystals, in all affine types. Previous work included the development of a generalization of the alcove model, which revealed an important connection between KR crystals and Macdonald polynomials. The PI will investigate the connection of these objects to affine Demazure crystals, and will pursue the generalization of his previous work to level s KR crystals.

Beside crystals, the PI will investigate important related structures, such as the poset defined by the inclusion of Mirkovic-Vilonen (MV) polytopes; the latter are moment map images of MV cycles in the geometric Satake picture. The main goal is to lift the well-known combinatorics of Coxeter groups to crystals and the MV poset.

The PI is pursuing formulas for the so-called metaplectic spherical and non-metaplectic Iwahori Whittaker functions (with B. Brubaker and G. Chinta). These formulas are of two types: in terms of the corresponding irreducible Lie algebra characters (or Demazure characters), cf. the Casselman-Shalika formula, and sums over combinatorial sets. The main tools are the affine Hecke algebra (and a metaplectic generalization of it, to be constructed), and the alcove model.

In Schubert calculus, the PI has projects related to various cohomologies of generalized flag varieties. The main problem is to find combinatorial formulas for multiplying Schubert classes. In the cohomology case, the PI proposes a uniform approach based on previous work of his and of Thomas-Yong, which also integrates a geometric rule combinatorialized by Knutson. A project with A. Buch and L. Mihalcea is to prove the PI's conjectured Chevalley multiplication formula in quantum K-theory. The PI will pursue the extension of Schubert calculus to complex cobordism, and will investigate combinatorial aspects related to the geometric Satake correspondence.

The PI will broadly disseminate his research results, both in the US and abroad. He intends to write a book on the alcove model and its applications. These also extend to particle physics. An important aim of this project is guiding the research through intensive computer exploration. The PI will work on related Sage implementations with the group formed by A. Schilling, M. Shimozono, and N. Thiery, as well as his former student, A. Lubovsky. The PI's research is integrated with teaching through the development of several graduate classes (as part of a new concentration in Lie theory in his department) and the supervision of PhD students. He currently advises two PhD students and intends to start advising another one in the near future. Several problems in this project are suitable for graduates. The PI is involved in the organization of several mathematical events. He will continue to organize the colloquium at Albany, and intends to co-organize a regional combinatorics meeting and a workshop at a mathematical institute with some of his collaborators.