RATES OF CONVERGENCE FOR THE ITERATES OF CESÀRO OPERATORS. A PROBABILISTIC APPROACH

José. A. Adell and A. Lekuona. Universidad de Zaragoza.

We obtain sharp rates of convergence in the usual sup-norm for the nth iterates $D^n f$ and $C^n f$ of continuous and discrete Cesàro operators, respectively. In both cases, the best possible rate of convergence is $n^{-1/2}$, and such a rate is attained under appropriate integrability conditions on f. Otherwise, the rates of convergence could be extremely poor, depending on the behavior of f near the boundary. To show these results, we introduce probabilistic representations of $D^n f$ and $C^n f$ involving standardized sums of independent identically distributed random variables and binomial mixtures, respectively, which allow us to use the classical Berry-Esseen bounds in the context of the central limit theorem.

Bilinear Forms on the Dirichlet Space

Nicola Arcozzi, University of Bologna

Let $\mathcal{D}$ be the classical Dirichlet space, the Hilbert space of holomorphic functions on the disk. Given a holomorphic symbol function b we define the associated Hankel type bilinear form, initially for polynomials f and g, by $T_b(f,g) :=< fg, b >_{\mathcal{D}}$, where we are looking at the inner product in the space $\mathcal{D}$. We let the norm of T_b denotes its norm as a bilinear map from $\mathcal{D}\times\mathcal{D}$ to the complex numbers. We say a function b is in the space $\mathcal{X}$ if the measure $d\mu_b := |b'(z)|^2 dA$ is a Carleson measure for $\mathcal{D}$ and norm $\mathcal{X}$ by

$$\|b\|_{\mathcal{X}} := |b(0)| + \||b'(z)|^2 dA\|_{CM(\mathcal{D})}^{1/2}.$$

Our main result is T_b is bounded if and only if $b \in \mathcal{X}$ and

$$\|T_b\|_{\mathcal{D}\times\mathcal{D}} \approx \|b\|_{\mathcal{X}}.$$

This is a joint work with Richard Rochberg, Eric Sawyer, Brett D. Wick.

On φ-normal functions; Zeros, preimages, sequence characterization,5-point theorem and Blaschke quotients

Rauno Aulaskari and Jouni Rättyä, University of Joensuu

Let $\varphi : [0,1) \to (0,\infty)$ be an increasing function such that $\varphi(r)(1-r) \to \infty$, as $r \to 1^-$. A meromorphic function f in the unit disk belongs to the class $\mathcal{N}^\varphi$ of φ-normal functions if its spherical derivative satisfies $f^\#(z) = \mathcal{O}(\varphi(|z|))$ as $|z| \to 1^-$. This presentation is devoted to study meromorphic φ-normal functions. The zero distribution and the distribution of preimages of distinct points in the image set of φ-normal functions are studied, and a sequence characterization and a version of Lappan's 5-point theorem are established. Moreover, interpolating Blaschke quotients in $\mathcal{N}^\varphi$ are characterized. Little "oh" analogues of all the results are also discussed.

A space of projections on the Bergman space

Oscar Blasco, Universidad de Valencia

We define a set of projections on the Bergman space A^2, which is parameterized by an affine subset of a Banach space of holomorphic functions in the disk and which includes the classical Forelli-Rudin projections. (This is a joint work with Salvador Pérez-Esteva, UNAM, Mexico)

Heisenberg uniqueness pairs in the plane. Three parallel lines.

Daniel Blasi Babot, Universitat Autònoma de Barcelona

A Heisenberg uniqueness pair is a pair (Γ, Λ), where Γ is a curve in the plane and Λ is a set in the plane, with the following property: let μ be any bounded Borel measure in the plane supported on Γ, which is absolutely continuous with respect to arc length, and let $\widehat{\mu}$ be its Fourier transform. Then,

$$\widehat{\mu}_{|\Lambda} = 0 \quad \Rightarrow \quad \mu \equiv 0.$$

We characterize the Heisenberg uniqueness pairs for Γ being three parallel lines $\Gamma = \mathbb{R} \times \{\alpha, \beta, \gamma\}$ with $\alpha < \beta < \gamma$, $(\gamma - \alpha)/(\beta - \alpha) \in \mathbb{N}$.

Dynamics of composition operators on spaces of real analytic functions

José Bonet, Universidad Politécnica de Valencia

Joint work with Pawel Domański (Univ. Poznań, Poland).// The purpose of this talk is to present certain results about the dynamics of composition operators $C_\varphi(f) := f \circ \varphi$ on spaces of real analytic functions defined on an open subset Ω of $\mathbb{R}^d$, φ a real analytic self map on Ω. We characterize when the operator C_φ is power bounded, i.e. when the orbits of all the elements under C_φ are bounded. In the case under investigation, every power bounded operator C_φ is even (uniformly) mean ergodic in the sense that the the sequence of Cesaro means $(\frac{1}{N}\sum_{n=1}^{N} C_\varphi^n)_{N\in\mathbb{N}}$ converges uniformly on the bounded sets to a projection P. In certain cases we determine the projection P explicitly. Several consequences about hypercyclic composition operators on spaces of real analytic functions are also obtained.

Explicit null-solutions of some n-dimensional Dirac-type operators: a unified approach

Isabel Cação, Universidade de Aveiro

The n-dimensional Euclidean Dirac operator is in the center of an extremely rich function theory, the so-called Clifford analysis, generalizing to higher dimensions the classical complex analysis. Using Clifford analytic tools, we provide a unified approach to obtain explicit null-solutions of some Dirac-type operators combined with the radially symmetric n-dimensional Euler operator. The obtained solutions show strong connections between Special Functions and homogeneous polynomials in the kernel of the Dirac operator.

On Nehari-type theorems for truncated Wiener-Hopf operators

Marcus Carlsson, Purdue University

Truncated Wiener-Hopf operators, or unitary equivalent versions, go under various names, e.g. Toeplitz operators on the Paley-Wiener space or truncated Hankel operators on R^+, and they have been studied by various people, most notably R. Rochberg, V. Peller and more recently, A. Baranov, Isabelle Chalendar, Emmanuel Fricain, Javad Mashreghi and Dan Timotin. We provide significantly improved constants for the existing Nehari type theorems, and moreover, we provide a more tractable norm estimate in terms of the discrete BMO(Z). Time allowing, we also discuss connections with approximations of the "symbol" with sparse sums of exponential functions.

Multiplier algebras of holomorphic mean Lipschitz spaces

Hong Rae Cho, Pusan National University

For $1 \leq p < \infty$ and $\alpha > 0$ let Λ^p_α be holomorphic mean Lipschitz spaces on the unit disc. It is shown that if $\alpha > 1/p$ the space Λ^p_α is a multiplicative algebra, and therefore is its own multiplier algebra. In this paper we also prove that if $\alpha > 0$ is non-integer with $\alpha \leq 1/p$, then Λ^p_α are not multiplicative algebras. In these non-regular cases, we give some sufficient condition for a holomorphic function to be a pointwise multiplier of Λ^p_α.

Carleson measures: Old and New

Boo Rim Choe, Korea University

In 1961 L. Carleson initiated the notion of Carleson measures in the course of his celebrated solution of the Corona Theorem. Since then the Carelson measures have been generalized to various settings in great generality by Hömander, Duren, Hastings, Stegenga, ... and have been playing central roles in many areas of complex/harmonic analysis. A variety of characterizations for Carleson measures has been discovered during the last half century. In this lecture we present new characterizations in connection with BMO and dual berezin transforms.

This presentation is based on joint works with Hyungwoon Koo and Michael Stessin.

Compactness of Hankel operators on pseudoconvex domains

Zeljko Cuckovic, University of Toledo

Using the D-bar Neumann operator, we study compactness of Hankel operators whose symbols are smooth up to the boundary. We show that compactness depends on the behavior of the symbol on the analytic structure in the boundary of the domain. We also study compactness of a product of Hankel operators on product domains. This is joint work with Sonmez Sahutoglu.

Cubic Column Relations in Truncated Moment Problems

Raúl Curto, University of Iowa

Inverse problems naturally occur in many branches of science and mathematics. An inverse problem entails finding the values of one or more parameters using the values obtained from observed data. A typical example of an inverse problem is the inversion of the Radon transform. Here a function (for example of two variables) is deduced from its integrals along all possible lines. This problem is intimately connected with image reconstruction for X-ray computerized tomography.

Moment problems are a special class of inverse problems. While the classical theory of moments dates back to the beginning of the 20th century, the systematic study of *truncated* moment problems began only a few years ago. In this talk we will first survey the elementary theory of truncated moment problems, and then focus on moment problems admitting cubic column relations.

For a degree $2n$ real d-dimensional multisequence $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{i \in Z_+^d, |i| \leq 2n}$ to have a representing measure μ, it is necessary for the *associated moment matrix* $M(n)$ to be positive semidefinite, and for the *algebraic variety* associated to β, V_β, to satisfy rank $M(n) \leq$ card V_β as well as the following *consistency condition*: if a polynomial $p(x) \equiv \sum_{|i| \leq 2n} a_i x^i$ vanishes on V_β, then $p(\beta) := \sum_{|i| \leq 2n} a_i \beta_i = 0$. In previous joint work with L. Fialkow and M. Möller, we proved that for the *extremal* case (rank $M(n) =$ card V_β), positivity and consistency are sufficient for the existence of a (unique, rank $M(n)$-atomic) representing measure.

In recent joint work with Seonguk Yoo we consider cubic column relations in $M(3)$ of the form (in complex notation) $Z^3 = itZ + u\bar{Z}$, where u and t are real numbers. For (u, t) in the interior of a real cone, we prove that the algebraic variety V_β consists of exactly 7 points, and we then apply the above mentioned solution of the extremal moment problem to obtain a necessary and sufficient condition for the existence of a representing measure. This requires a new representation theorem for sextic polynomials in Z and $\bar{Z}$ which vanish in the 7-point set V_β.

A NEW CHARACTERIZATION OF THE LORENTZ SPACES $L(p,1)$ FOR $1 < p < \infty$ AND APPLICATIONS

GERALDO SOARES DE SOUZA, Auburn University

In 1950, **G. G. Lorentz** introduced in his paper entitled "**Some New Functional Spaces**" at Annals of Mathematics, the function spaces denoted by $\Lambda(\alpha)$ for $0 < \alpha < 1$, defined as the set of real measurable functions $f(x), 0 < x < 1$ for which

$$||f||_{\Lambda(\alpha)} = \alpha \int_0^1 x^{\alpha-1} f^\star(x) dx,$$

where $f^\star$ is the decreasing rearrangement of f.
In this talk we give two simple characterizations of the special atoms space of $\Lambda\left(\frac{1}{p}\right)$ for $1 < p < \infty$ based on a generalizations of the special atoms spaces, introduced by **Geraldo De Souza** in his earlier works. The space $\Lambda\left(\frac{1}{p}\right)$ is nowadays denoted by $L(p,1)$.
We use these characterizations to give a rather simple proof of **Weiss-Stein** theorem on the extension of operators on $L(p,1)$ and also we take a look at Carleson Theorem on Convergence of Fourier series for this space.

On a theme of Beurling

Ron Douglas, Texas A&M University

Almost sixty years ago Beurling considered two problems for the backward shift operator motivated by spectral synthesis. Not so much these problems but the techniques he used to solve them, have had a continuing and profound effect on operator theory and harmonic analysis and their connections with function theory.

Beurling's results have incisive interpretations in the language of Hilbert modules over the algebra of polynomials in one variable. In this talk we explore some of the connections which result and their extensions to more general classes of Hilbert modules related to reproducing kernel Hilbert spaces. While some results, both recent and not-so-recent will be placed in this context, much of the emphasis will be on open questions motivated by consequences of Beurling's original results. Although much of the focus will be on the one-variable case, issues of multivariate operator will also enter the picture.

Harmonic Bergman kernels and Berezin transforms

Miroslav Englis, Academy of Sciences of the Czech Republic

Fefferman's description of the boundary behaviour of the Bergman kernel has found numerous applications in complex analysis and geometry. Another application concerns the asymptotics of the Berezin transform, which in turn is used in mathematical physics for quantization on Kaehler manifolds. The talk will explore whether and how these facts extend also to the context of Bergman spaces of harmonic, rather than holomorphic, functions, improving, among others, upon some earlier results of Krantz and of Kang and Koo.

Schatten class membership of Hankel operators on the unit sphere

Quanlei Fang, SUNY at Bufallo

Let H_f be a Hankel operator on the Hardy space of the unit sphere in $\mathbf{C}^n$, $n \geq 2$. We determine the membership of H_f in the Schatten class $\mathcal{C}_p$ for all possible symbol functions f in the L^2 of the sphere. In the case $p > 2n$, $H_f \in \mathcal{C}_p$ if and only if H_f maps the constant function 1 into the Besov space $\mathcal{B}_p$. In the case $p \leq 2n$, the membership $H_f \in \mathcal{C}_p$ implies $H_f = 0$. This is a joint work with Jingbo Xia.

Recent advances in elliptic complex geometry

Franc Forstneric, University of Ljubljana

Elliptic complex geometry (named after Mikhail Gromov who introduced the notion of an elliptic compex manifold and of an elliptic submersion in 1989) concerns itself with those complex analytic properties of a complex manifold that imply the Oka principle (the complex analytic version of the homotopy principle) for mappings from Stein manifolds to the given manifold. Furthermore, in the spirit of Grothendieck, one studies ellipticity properties not only of manifolds, but also of holomorphic mappings between them; in the latter case the Oka properties refer to the existence of holomorphic liftings of a holomorphic map to the base manifold, granted that a continuous lifting exists. In this talk I will survey the main recent advances and applications, and indicate some directions of future investigation.

JACOBI MATRICES AND QUADRATURE RULES ON THE UNIT CIRCLE AND THE REAL LINE

Pablo González-Vera, La Laguna University

As it is known, Jacobi matrices play a fundamental role in the efficient computation of Gaussian quadrature formulas. On the other hand, Szegö quadratures represent the analogous on the unit circle with Gaussian formulas so that their computation can be efficiently carried out in terms of Hessenberg matrices. In this talk, we will first show how under certain symmetry conditions on the measure supported on the unit circle, computation of Szegö formulas can be made through the solution of an eigenvalue problem involving Jacobi matrices of dimension E[n/2] instead of Hessenberg matrices of dimension n. Here n denotes the number of nodes of the corresponding quadrature rule, and as usual, E[x] the integer part of x. Secondly, we will also see how Szegö quadratures can be used to deduce the existence of certain Gauss-type formulas with some preassigned nodes on a finite interval and whose computation reduces to numerically solve an eigenvalue problem involving again Jacobi matrices. As an illustration, several numerical experiments will be performed.This is a joint work with R. Cruz-Barroso and F. Perdomo-Pío.

Use of representations of C*-algebras in multivariable operator theory

Palle Jorgensen, University of Iowa

We use representation theory in the study of problems in multivariable operator theory, and orthogonal harmonic analysis. The notion of selfsimilar measures involves representations of Cuntz algebras.

Cuntz algebras are infinite algebras on a finite number of generators, and on certain relations. By their nature, they are selfsimilar and they therefore ideally serve to encode iterated function systems (IFSs) and their measures. At the same time, their representations offer (in a more subtle way) a new harmonic analysis of IFS-fractal measures. These are the measures which arise naturally in multivariable operator theory. Even though the Cuntz algebras initially entered into the study of operator-algebras and physics, in recent yearsthey, and their representation, have found increasing use in pure and applied problems, wavelets, fractals, signals.

Quasi-wandering subspaces in the Bergman space

Kou Hei Izuchi, Korea University

In this talk, we consider quasi-wandering subspaces in the Bergman space L^2_a over the unit disk. Let $\mathcal{B}$ be the Bergman shift on L^2_a and let I be a nontrivial invariant subspace of L^2_a. Let P_I be the orthogonal projection from L^2_a onto I. It is proved that $P_I\mathcal{B}(L^2_a \ominus I)$ is not dense in I if and only if $I \cap \mathcal{D} \neq \{0\}$, where $\mathcal{D}$ is the Dirichlet space.

Von Neumann Inequalities for Weighted Symmetric Fock Spaces

H. Turgay Kaptanoğlu, Bilkent University

The weighted Fock symmetric spaces considered are reproducing kernel Hilbert Spaces of holomorphic functions in the unit ball of C^N. We also call them Dirichlet spaces, and their kernels are $K_q(z,w) = (1-\langle z,w\rangle)^{-(1+N+q)}$ for $q > -(1+N)$. We obtain von Neumann inequalities for row contractions of operators on these spaces for $q \geq -N$. It turns out that the sharpest of these inequalities is the one for $q = -N$, which was discovered earlier by Drury and Arveson for the space named after them. This is joint work with Semra Ö. Kaptanoğlu.

COMMUTING NILPOTENT OPERATORS AND MAXIMAL RANK

Semra Ö. Kaptanoğlu, Middle East Technical University

Let $X, \widetilde{X}$ be commuting nilpotent matrices over k with nilpotency p^t, where k is an algebraically closed field of positive characteristic p. We show that if $X - \widetilde{X}$ is a certain linear combination of products of commuting nilpotent matrices, then X is of maximal rank if and only if $\widetilde{X}$ is of maximal rank.

Holomorphic almost periodic functions on coverings of complex manifolds

Damir Kinzebulatov, University of Toronto (Joint work with Alexander Brudnyi.)

We introduce a class of holomorphic almost periodic functions on regular coverings of Stein manifolds that embraces two classical theories of almost periodic functions: Bohr's holomorphic almost periodic functions on tube domains and von Neumann's almost periodic functions on groups.

In particular, we discuss the following related results: a variant of 'holomorphic Peter-Weyl theorem', extension theorems from holomorphic almost periodic submanifolds, some results on structure of the maximal ideal spaces of certain subalgebras of bounded holomorphic almost-periodic functions and geometry of almost periodic analytic sets. Our approach is based on the following two presentations of holomorphic almost periodic functions: as holomorphic sections of a certain holomorphic Banach vector bundle and as 'holomorphic' functions on the 'Bohr compactification' of the covering, a holomorphic fibre bundle which inherits some properties of the underlying Stein manifold.

Composition Operators on holomorphic Sobolev Spaces in B_n

Hyungwoon Koo, Korea University

We study the composition operator C_Φ on holomorphic Sobolev spaces induced by an analytic self-map Φ of B_n in $\mathbf{C}^n$ that extends to be smooth on $\overline{B_n}$. We characterize the boundedness and the compactness of C_Φ on $A^p_{\alpha,s}$, and prove the jump phenomenon of C_Φ on $A^p_{\alpha,s}$. Moreover, we show an interesting result that the boundedness of C_Φ on $A^p_{\alpha,s}$ is equivalent to the compactness of $C_\Phi : A^p_{\alpha,s} \to A^q_{\beta,t}$ for appropriate $A^q_{\beta,t}$, for example $A^q_{\beta,t} = A^p_{\alpha+1/4,s}$. We provide examples to show that our results are sharp.

Weighted Inequalities for Singular Integrals

Michael T Lacey, Georgia Institute of Technology

The two weight problem for the Hilbert transform H is a central question in Operator Theory and in Harmonic Analysis: For which pairs of weight u, v does H map $L^2(u)$ to $L^2(v)$? We describe recent progress on the general question, and applications of these general results to the case of $u = v \in A_2$. In the latter case, we will establish the sharp result in terms of the A_2 characteristic for a range of Calderón-Zygmund operators, extending prior sharp results on this question due to Petermichl-Volberg and Petermichl, for the Beurling, Hilbert and Riesz transforms. Joint work with Tuomas Hytonen, Stefanie Petermichl, Maria Carmen Reguera, Eric Sawyer, Ignacio Uriate-Tuero, and Armen Vagharshakyan.

On Higher Order Metrics Associated with Berezin's Operator Calculus

Bo Li, University of Toledo

Abstract: We discuss the mth order Bergman metric and the mth order Carathéodory-Reiffen metric of Burbea, and a new higher order metric arised in the study of Berezin's operator calculus on bounded domains in $\mathbb{C}^n$. Some comparison results among them and the corresponding classical intrinsic metrics are established on certain domains.

Inequalities for eigenvalues of sums of self-adjoint operators

Wing Suet Li, Georgia Institute of Technology

Consider self-adjoint operators $A, B, C : \mathcal{H} \to \mathcal{H}$ on a finite dimensional Hilbert space such that $A + B + C = 0$. Let $\{\lambda_j(A)\}$, $\{\lambda_j(B)\}$, and $\{\lambda_j(C)\}$ be sequences of eigenvalues of A, B, and C counting multiplicity, arranged in decreasing order. In 1962, A. Horn conjectured that the relations of $\{\lambda_j(A)\}$, $\{\lambda_j(B)\}$, and $\{\lambda_j(C)\}$ can be characterized by a set of inequalities defined inductively. This problem was eventually solved by A. Klyachko and Knutson-Tao in the late 1990s. In this talk we will show that these inequalities are also valid for selfadjoint elements in a finite factor. The major difficulty in our argument is the proof that certain generalized Schubert cells have nonempty intersection. In the finite dimensional case, it follows from the classical intersection theory. However, there is no readily available intersection theory for von Neumann algebras. Our argument requires a good understanding of the combinatorial structure of honeycombs, and produces an actual element in the intersection algorithmically, and it seems to be new even in finite dimensions.

Moment theory on $H^\infty(\Delta)$

Charlie Micchelli, SUNY at Albany

In this talk we describe the envelope of functions which goes through a prescribed number of point on the interval $(-1, 1)$, real on the real axes and extends to a function analytic in the unit disc which is bounded by one there.

Real-Part Theorems for Solutions of the Riesz System in $\mathbb{R}^3$

J. Morais, Institute of Applied Analysis TU Freiberg

It is truly rare that a paper that has been set aside for more than a century finds its way back to scientific spotlight. Yet this is exactly what the short 1892 paper of Jacques Hadamard has accomplished in the last decade. During the last years, much effort has been done regarding the treatment of multi-dimensional analogues and other generalizations of Hadamard's real part theorem. Excellent contributions to this subject have been made, in particular, by Hadamard, Landau, Wiman, Jensen, Koebe, Borel, Riesz, Littlewood, Titchmarsh, Rajagopal, Elkins, Holland, Hayman, Levin and Kresin and Maz'ya. Being a classical object of analysis it became recently more and more important to perform an analogous study in higher dimensions and/or for other partial differential equations. One way to generalize complex function theory to higher dimensional spaces is offered by following the Riemann approach. In this context the generalization of holomorphic functions is given by the null solutions of generalized Cauchy-Riemann or Dirac systems, known as monogenic functions. This approach is nowadays called Clifford analysis.

In view of many diverse applications in physics and engineering, in this lecture the author generalizes the Hadamard's real part theorem to the three-dimensional Euclidean space in the framework of quaternionic analysis.

Morita Transforms, Tensor Algebras and Analytic Crossed Products

Paul Muhly, University of Iowa

In this talk, I will show how tensor operator algebras of C^*-correspondences are Morita equivalent to analytic crossed products. As a special case, we will see that Popescu's noncommutative disc algebra on d generators is Morita equivalent to the crossed product obtained from the compact operators and the endomorphism induced by a Cuntz d-tuple.

Hadamard Type Extremal Problems and Optimal Recovery of Analytic Functions

K. Yu. Osipenko, Moscow State University

The well-known Hadamard three-circle theorem states that if $f(z)$ is a holomorphic function on the annulus $r_1 \le |z| \le r_2$ and

$$M(r) = \max_{|z|=r} |f(z)|,$$

then

$$M(\rho) \le M(r_1)^{\frac{\log r_2/r}{\log r_2/r_1}} M(r_2)^{\frac{\log r/r_1}{\log r_2/r_1}}$$

for any three concentric circles of radii $r_1 < \rho < r_2$.

For functions f from the Hardy space $H^2(\mathbb{B}^n)$ we consider the analogous extremal problem

$$\|f(\rho z)\|_{H^2(\mathbb{B}^n)} \to \max, \quad \|f(r_1 z)\|_{H^2(\mathbb{B}^n)} \le \delta_1, \quad \|f(r_2 z)\|_{H^2(\mathbb{B}^n)} \le \delta_2.$$

This problem is closely connected with the problem of optimal recovery of f on the sphere of radius ρ from the information about traces on the spheres of radii r_1 and r_2 given with errors. The optimal error of such recovery is defined as follows

$$E_\rho(r_1, r_2, \delta_1, \delta_2) = \inf_m \sup_{\substack{f \in H^2(\mathbb{B}^n),\ y_j \in L_2(\sigma_{r_j}),\ j=1,2 \\ \|f(r_j z) - y_j(r_j z)\|_{L_2(\sigma)} \le \delta_j,\ j=1,2}} \|f(\rho z) - m(y_1, y_2)(\rho z)\|_{L_2(\sigma)},$$

where the lower bound is taken over all maps (methods) $m\colon L_2(\sigma_{r_1}) \times L_2(\sigma_{r_2}) \to L_2(\sigma_\rho)$ and $d\sigma_r(z)$ are the positive normalized rotationally invariant measures on the spheres $r\mathbb{S}^{n-1}$ ($\sigma = \sigma_1$). Any method $\widehat{m}$ for which the lower bound is attained is called an optimal recovery method.

Let

$$(\lambda_1, \lambda_2) = \left(\frac{r_2^2 - \rho^2}{r_2^2 - r_1^2} \left(\frac{\rho}{r_1}\right)^{2s}, \frac{\rho^2 - r_1^2}{r_2^2 - r_1^2} \left(\frac{\rho}{r_2}\right)^{2s} \right),$$

if

$$\left(\frac{r_1}{r_2}\right)^{s+1} \le \frac{\delta_1}{\delta_2} < \left(\frac{r_1}{r_2}\right)^{s}, \quad s \in \mathbb{Z}_+,$$

and $(\lambda_1, \lambda_2) = (0, 1)$, if $\delta_1 \ge \delta_2$.

Theorem 1 ([1])**.** *The error of optimal recovery is given by*

$$E_\rho(r_1, r_2, \delta_1, \delta_2) = \sqrt{\lambda_1 \delta_1^2 + \lambda_2 \delta_2^2}$$

and the method

$$\widehat{m}(y_1, y_2)(z) = \sum_{k=0}^{\infty} \frac{1}{\lambda_1 r_1^{2k} + \lambda_2 r_2^{2k}} \sum_{|\alpha|=k} (\lambda_1 r_1^k c_\alpha^{(1)} + \lambda_2 r_2^k c_\alpha^{(2)}) z^\alpha,$$

where

$$c_\alpha^{(j)} = \frac{(n+|\alpha|-1)!}{n!\alpha!} \int_{\mathbb{S}^{n-1}} y_j(r_j z)\overline{z}^\alpha \, d\sigma(z), \quad j = 1, 2,$$

is optimal.

It appears that it is possible to construct a collection of optimal recovery methods.

Theorem 2. *For all β_k, $k = 0, 1, \ldots$, such that*

$$\lambda_2 \left(\frac{\rho}{r_1}\right)^{2k} |\beta_k|^2 + \lambda_1 \left(\frac{\rho}{r_2}\right)^{2k} |1-\beta_k|^2 \le \lambda_1\lambda_2 \tag{1}$$

all methods

$$\widehat{m}(y_1, y_2)(z) = \sum_{k=0}^{\infty} \sum_{|\alpha|=k} \left(\frac{\beta_k}{r_1^k} c_\alpha^{(1)} + \frac{1-\beta_k}{r_2^k} c_\alpha^{(2)} \right) z^\alpha$$

are optimal.

Assume that $\delta_1 < \delta_2$. Let $K_1 = \max\{\, k \in \mathbb{Z}_+ : \rho^{2k} \le \lambda_1 r_1^{2k} \,\}$, $K_2 = \min\{\, k \in \mathbb{Z}_+ : \rho^{2k} \le \lambda_2 r_2^{2k} \,\}$.

From Theorem 2 we have

Corollary 1. *For all $0 \le k_1 \le K_1$, $k_2 \ge K_2$ and β_k, $k = k_1 + 1, \ldots, k_2 - 1$, such that* (1) *holds all methods*

$$m(y_1, y_2)(z) = \sum_{k=0}^{k_1} \sum_{|\alpha|=k} \frac{c_\alpha^{(1)}}{r_1^k} z^\alpha + \sum_{k=k_1+1}^{k_2-1} \sum_{|\alpha|=k} \left(\frac{\beta_k}{r_1^k} c_\alpha^{(1)} + \frac{1-\beta_k}{r_2^k} c_\alpha^{(2)} \right) z^\alpha + \sum_{k=k_2}^{\infty} \sum_{|\alpha|=k} \frac{c_\alpha^{(2)}}{r_2^k} z^\alpha$$

are optimal.

References

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Toeplitz operators with distributional symbols

Antti Perälä, University of Helsinki

We study Toeplitz operators in analytic function spaces. We focus on the case where the symbol is a distribution. It turns out that the membership of the symbol a to a weighted Sobolev-type space $W_\nu^{-m,\infty}$

is sufficient for the boundedness of T_a. Examples and corresponding results on compactness are also provided.

Irregular behavior of orbits of operators
Gabriel T. Prajitura, SUNY at Brockport

We will discuss the asymptotic behavior of orbits of operators, with special emphasis on irregular orbits of composition operators.

Optimal Approximation by Rational Functions of a Given Degree
Tao Qian, University of Macau

Let $\mathcal{R}_n$ be the set of all rational functions of the form

$$R(z) = \sum_{j=0}^{r-1} c_j z^j + \sum_{l=1}^{L} \sum_{k=1}^{K_l} \frac{d_l^k}{(z-b_l)^k}, \tag{2}$$

where $0 \le r \le n, 1 \le K_l \le n, l = 1, ..., L, c_{r-1} \prod_{l=1}^{L} d_l^{K_l} \neq 0$, and

$$r + \sum_{l=1}^{L} K_l = n.$$

If $R \in \mathcal{R}_n$, then we call R *an n-partial fraction.* We call R_f an *n-critical partial fraction of* $f \in H^2(\mathbb{D})$, if

$$\|f - R_f\|_2 = \min\{\|f - R\|_2 \ : \ R \in \mathcal{R}_n\}.$$

The talk will outline a proof of the existence of an n-critical partial fraction R_f and its algorithm.

Compact and weakly compact composition operators on BMOA
Eero Saksman, University of Helsinki

Let C_ϕ be a analytic composition operator on the unit disc. We answer a question of Bourdon, Cima, Matheson and Tjani by proving that C_ϕ is weakly compact on BMOA (or on VMOA) if and only if it is compact. As a byproduct, the known compactness criteria on BMOA are simplified considerably. The talk is based on a joint work with J. Laitila, P. Nieminen and H.-O. Tylli (University of Helsinki).

Closed ideals of some new analytic area Nevanlinna type classes in the unit disk

R.F.Shamoyan, Bryansk State University

We introduce new area NEvanlinna type spaces in the unit disk and provide characterizations of their zero sets and parametric representations of these classes,which lead us to complete descriptions of closed ideals of mentioned spaces. References.R.Shamoyan,H.Li,Closed ideals of some new analytic area NEvanlinna type classes in the unit disk.Preprint,2009.

Fundamental Solutions of Some Evolution Equations

Erwin Suazo, University of Puerto Rico, Mayagüez

In this talk we discuss applications of operator theory to evolution equations. We find explicit solutions to the Cauchy initial value problem for the linear Schrödinger equation (SE) with a general quadratic time-dependent Hamiltonian in $\mathbb{R}^d$ by first constructing fundamental solutions explicitly. Estimates of the evolution operator relevant in the study of well-posedness for the nonlinear case are also presented. The analogous diffusion equation is studied. In a similar fashion as with the SE, an explicit solution is constructed and in this case uniqueness is an immediate consequence of the maximum principle for parabolic equations on bounded domains and the extension method to unbounded domains introduced by M. Krzyzanski. Finally, we exemplify our results for the case of the SE with the Caldirola-Kanai Hamiltonian, and for the case of a Difusion-type equation we will use the Fokker-Planck equation.

Closed-Range Composition Operators on $\mathbb{A}^2$ and the Bloch space

Maria Tjani, University of Arkansas

For an analytic self-map of the unit disk $\mathbb{D}$ we give necessary and sufficient conditions for the composition operator C_ϕ to be closed-range on the Bloch space $\mathcal{B}$. We establish an extension of the Julia-Caratheodory Theorem and use it to show that if C_ϕ is closed-range on the Bergman space $\mathbb{A}^2$, then it is closed-range on $\mathcal{B}$. The converse of this fails with a vengence: we construct a thin Blaschke product B so that C_B is norm preserving on $\mathcal{B}$ yet compact on $\mathbb{A}^2$. This is joint work with John Akeroyd and Pratibha Ghatage.

On compactness of commutators and semi-commutators of Toeplitz operators on the Bergman space

Nikolai Vasilevski, CINVESTAV del I.P.N.

Given a C*-subalgebra A of algebra $L_\infty(D)$, denote by T(A) the C*-algebra generated by all Toeplitz operators with symbols in A and acting on the Bergman space over the unit disk D. We will discuss the compactness properties of commutators and semi-commutators of Toeplitz operators from T(A) as well as the structural properties of T(A) and other operator algebras related to the above compactness properties.

Open problems in the theory of Toeplitz operators on Bergman spaces

Jani Virtanen, NYU

The talk gives a review of known results and open problems concerning boundedness, compactness, and Fredholm properties of Toeplitz operators acting on Bergman spaces with integrable (matrix) symbols.

The p-Faber-Krahn Inequality Revisited

Jie Xiao, Memorial University

Abstract: When revisiting the Faber-Krahn inequality for the principal p-Laplacian eigenvalue of a bounded open set in Euclidean space with smooth boundary, we report that this inequality may be improved but also characterized through the Maz'ya capacity method, the Euclidean volume, Green's potential, the Sobolev type inequality and Moser-Trudinger's inequality.

The core operator in the Hardy space over the bidisk

Ron Yang, SUNY at Albany

In the classical Hardy space over the unit disk, invariant subspaces are described by inner functions. However, in the Hardy space over the bidisk, inner function no longer plays a central role in the study of invariant subspaces. The central role appears to be played by the so-called core operator. We will take a look at the core operator in this talk.

On product of Toeplitz operators on the Bergman space

Abdel Yousef, University of Toledo

In this talk I will discuss the zero product problem of Toeplitz operators when one of the symbols has certain polar decomposition and the other is a general bounded symbol. Also, for certain class of symbols f, I will describe the Toeplitz operators T_f , which commute with $T_{z+\overline{g}}$, where g is analytic.

Essential norms of composition operators between Bloch type spaces

Ruhan Zhao, SUNY Brockport

For $\alpha > 0$, the α-Bloch space is the space of all analytic functions f on the unit disk D satisfying

$$\|f\|_{B^\alpha} = \sup_{z\in D} |f'(z)|(1-|z|^2)^\alpha < \infty.$$

Let φ be an analytic self-map of D. We show that, for $0 < \alpha, \beta < \infty$, the essential norm of the composition operator C_φ mapping from B^α to B^β can be given by the following formula:

$$\|C_\varphi\|_e = \left(\frac{e}{2\alpha}\right)^\alpha \limsup_{n\to\infty} n^{\alpha-1}\|\varphi^n\|_{B^\beta}.$$