Faraday’s Law

Chapter
30

HW08 is up on WebAssign, due Thursday 04/04

Last lecture in a nutshell…

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

Biot-Savart

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

Magnetic force between two // conductors

$$\oint \overrightarrow{B} \cdot d\overrightarrow{\ell} = \mu_0 I_{encl}$$

Ampere's law

$$\phi_B = \int \vec{B} \cdot d\vec{A}$$

Magnetic Flux

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Maxwell's 2nd equation
(Gauss' law of magnetism)

Induced Fields

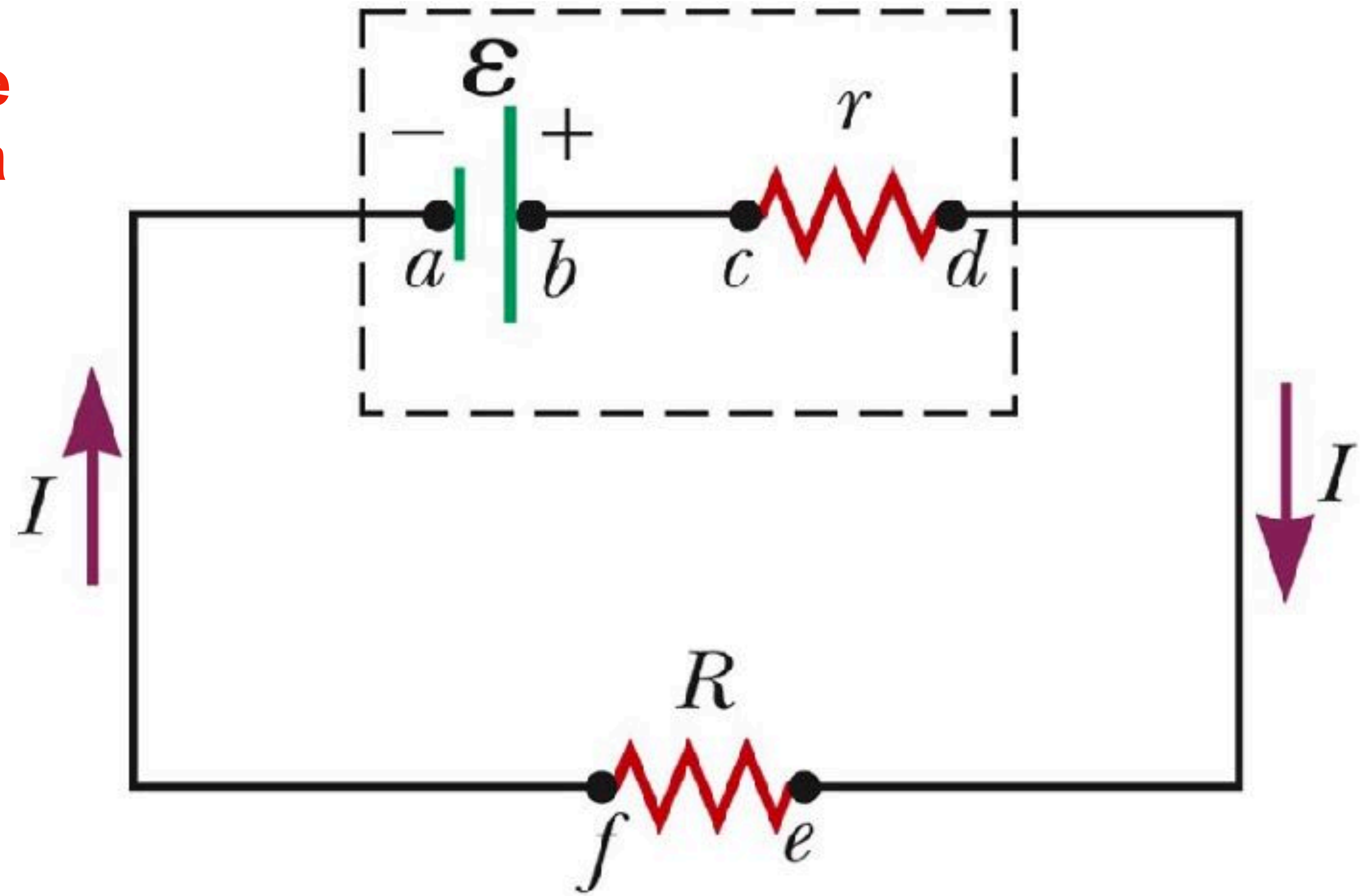
A charge creates an electric field.
A moving charge creates a magnetic field.
Fields can also be induced.
Faraday and Henry in 1831 showed that **an emf can be induced in a circuit by a changing magnetic field.**

In other words: a changing magnetic field induces an electric field.

The results of these experiments led to *Faraday's Law of Induction.*
An *induced current* is produced by a changing magnetic field.
There is an *induced emf* associated with the induced current.
A current can be produced without a battery present in the circuit.
Faraday's law of induction describes the induced emf.

Electromotive Force: Reminder

The electromotive force (emf), ε, of a battery is the maximum possible voltage that the battery can provide between its terminals.

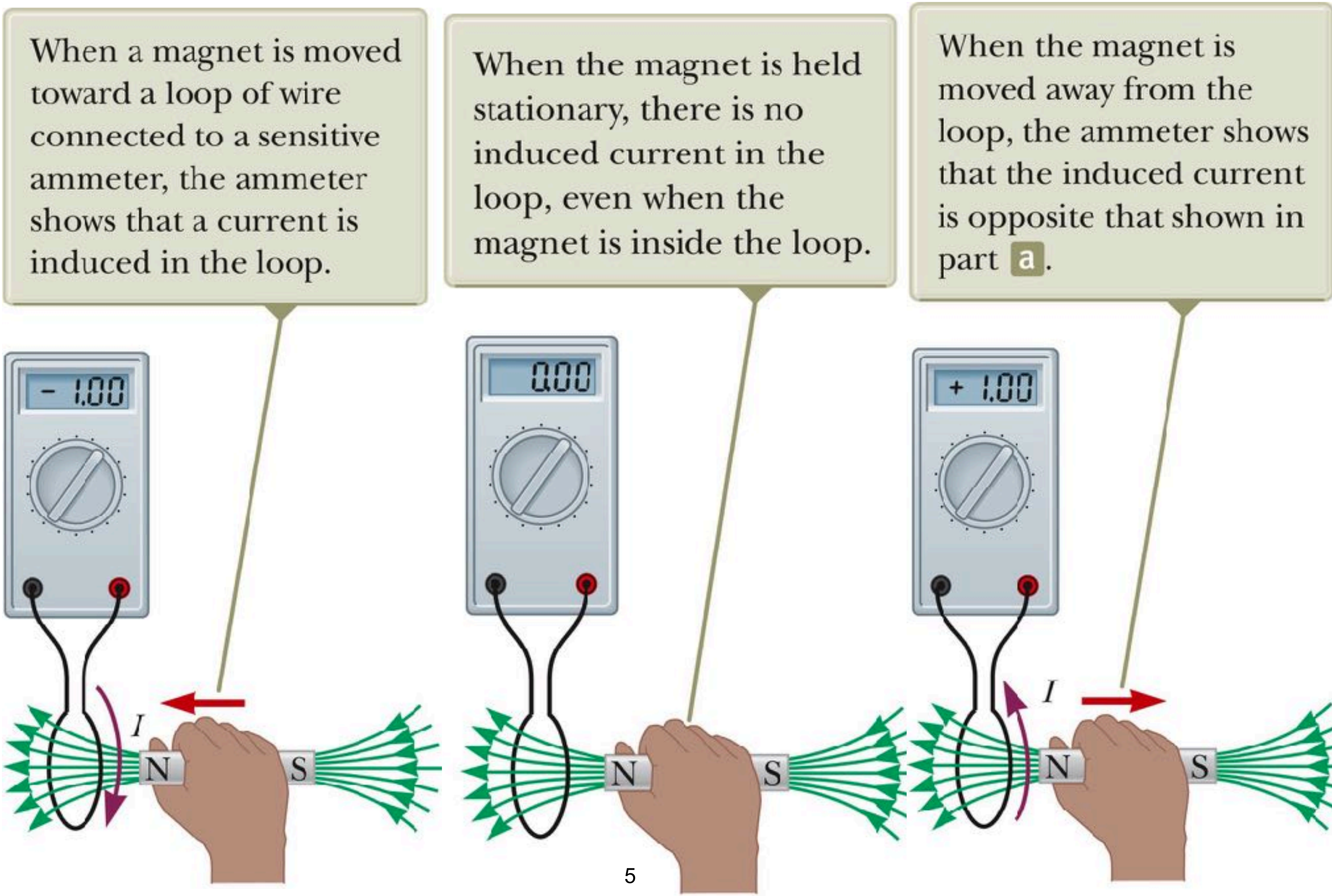


- The emf supplies energy, **it does not apply a force.**

$$\varepsilon = IR + Ir$$

$\boxed{\varepsilon = W/q}$ (work per charge)

EMF Produced by a Changing Magnetic Field



EMF Produced by a Changing Magnetic Field, Summary

The ammeter deflects when the magnet is moving toward or away from the loop.

The ammeter also deflects when the loop is moved toward or away from the magnet.

Therefore, **the loop detects that the magnet is moving relative to it**.

- We relate this detection to a change in the magnetic field.
- This is the induced current that is produced by an induced emf.

Faraday’s Law of Induction

The **emf induced in a circuit** is directly proportional to the time rate of change of the magnetic flux through the circuit. Mathematically,

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}$$

Faraday’s law

Remember Φ_B is the magnetic flux through the circuit and is found by

$$\phi_B = \int \vec{B} \cdot d\vec{A}$$

If the circuit consists of N loops, all of the same area, and if Φ_B is the flux through one loop, an emf is induced in every loop and Faraday’s law becomes

$$\boxed{\varepsilon = -N\frac{d\Phi_B}{dt}}$$

Faraday's Law – Example

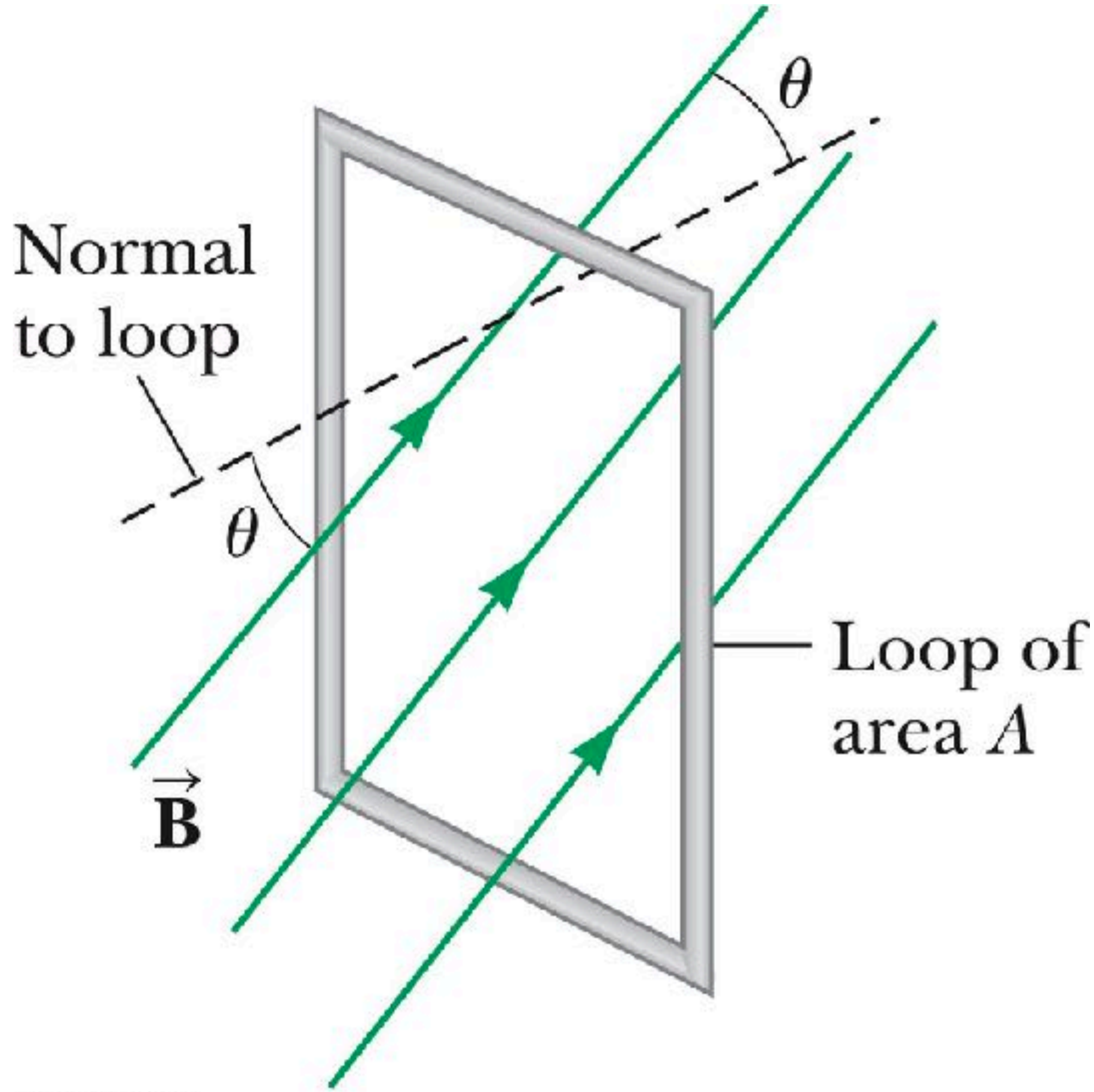
Assume a loop enclosing an area A lies in a uniform magnetic field.

The magnetic flux through the loop is:

$$\Phi_B = BA \cos \theta.$$

The induced emf is :

$$\epsilon = -\frac{d(BAcos\theta)}{dt}$$



Ways of Inducing an emf

$$\epsilon = -\frac{d(BAcos\theta)}{dt}$$

The magnitude of the magnetic field can change with time.

The area enclosed by the loop can change with time.

The angle between the magnetic field and the normal to the loop can change with time.

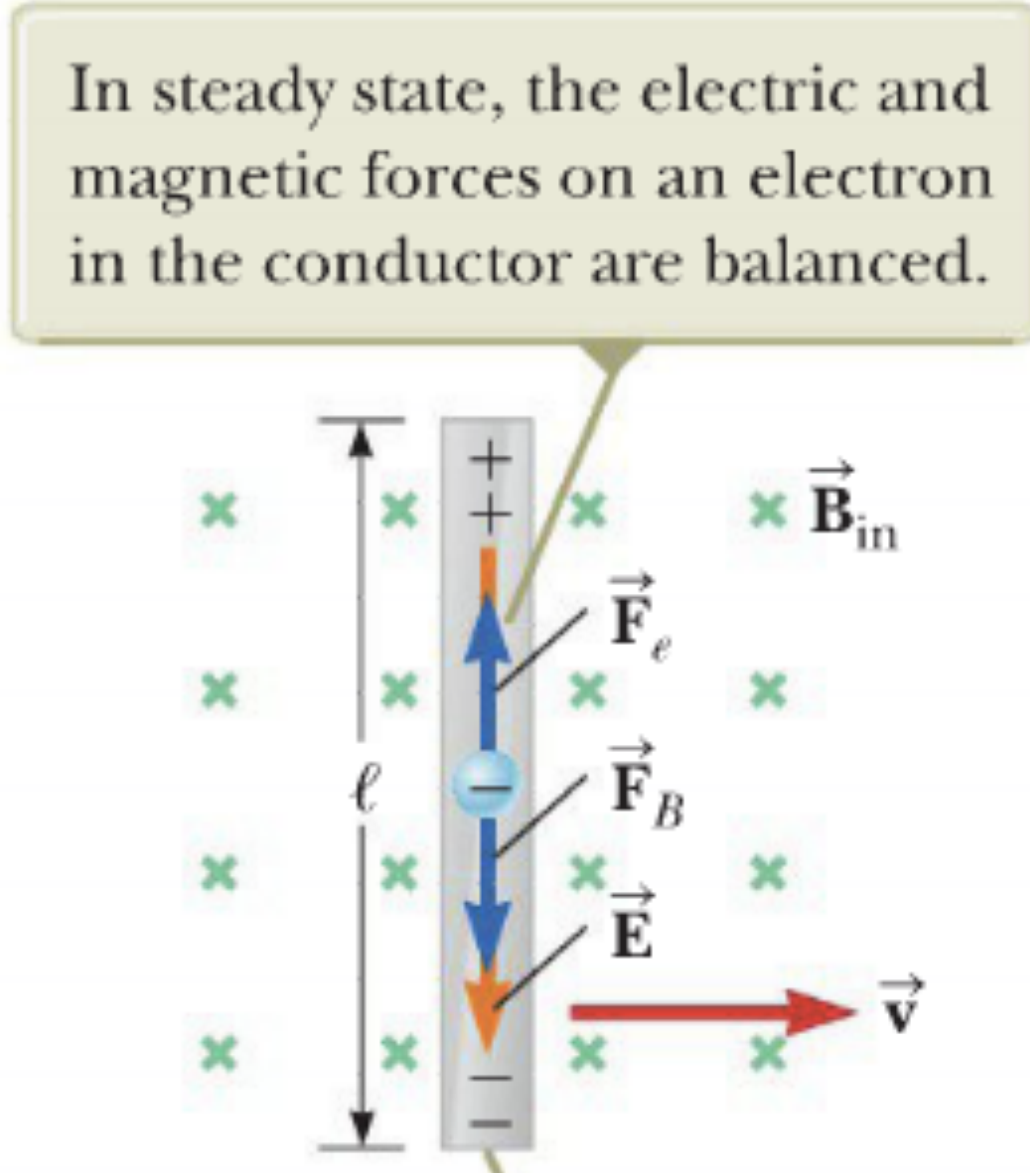
Any combination of the above can occur.

Motional emf

A motional emf is the **emf induced in a conductor moving through a constant magnetic field.**

The electrons in the conductor experience a magnetic force, that is directed along ℓ .

$$\vec{F} = q\vec{v} \times \vec{B}$$



Initially + and - charges are uniformly distributed

Motional emf

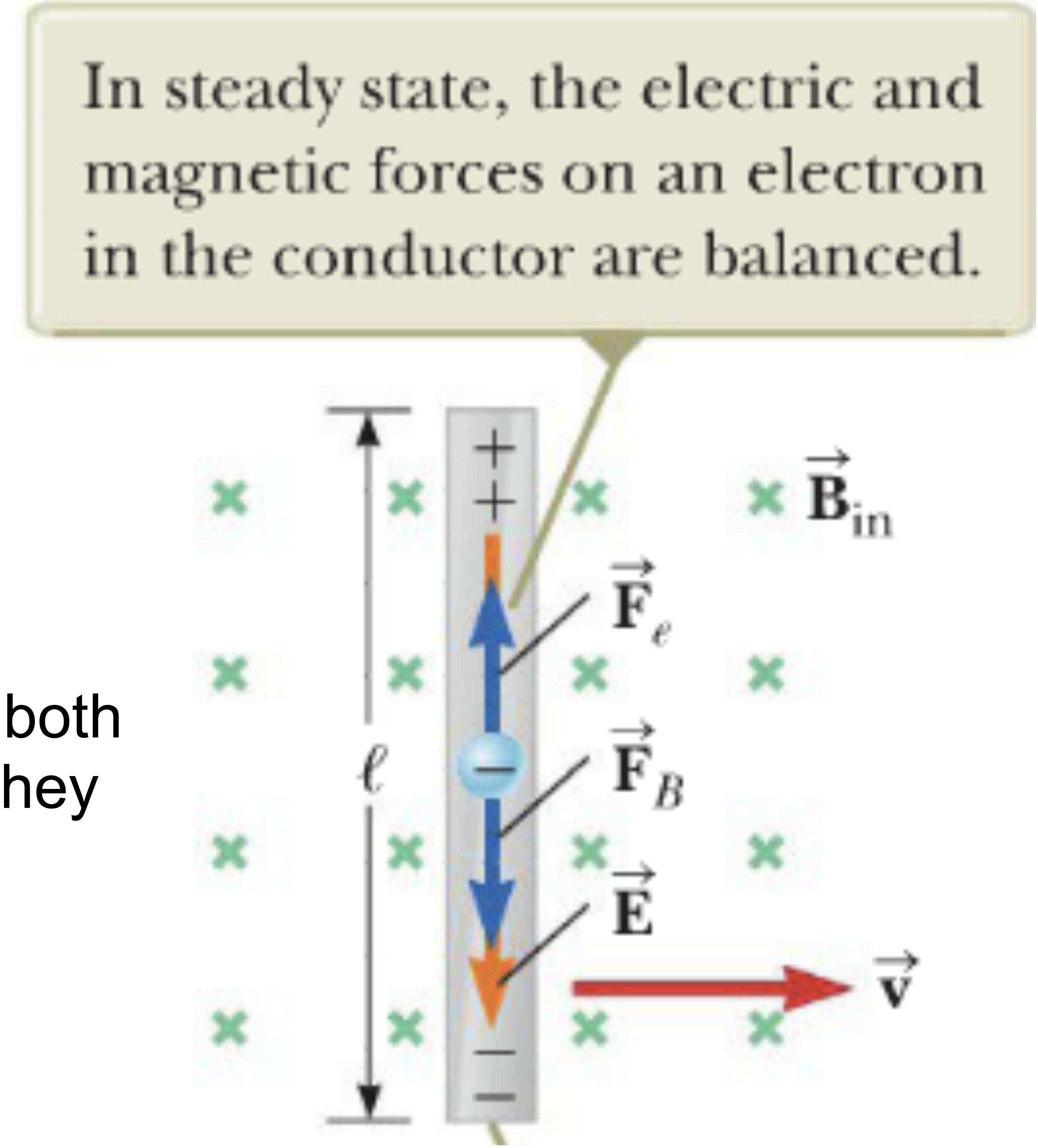
The force makes the electron move downwards. The charges are separate (+ on top, - on bottom).

The charge separation creates an electric field.

The charges accumulate at both ends of the conductor until they reach equilibrium:

$$F_E = F_B$$
$$qE = qvB$$

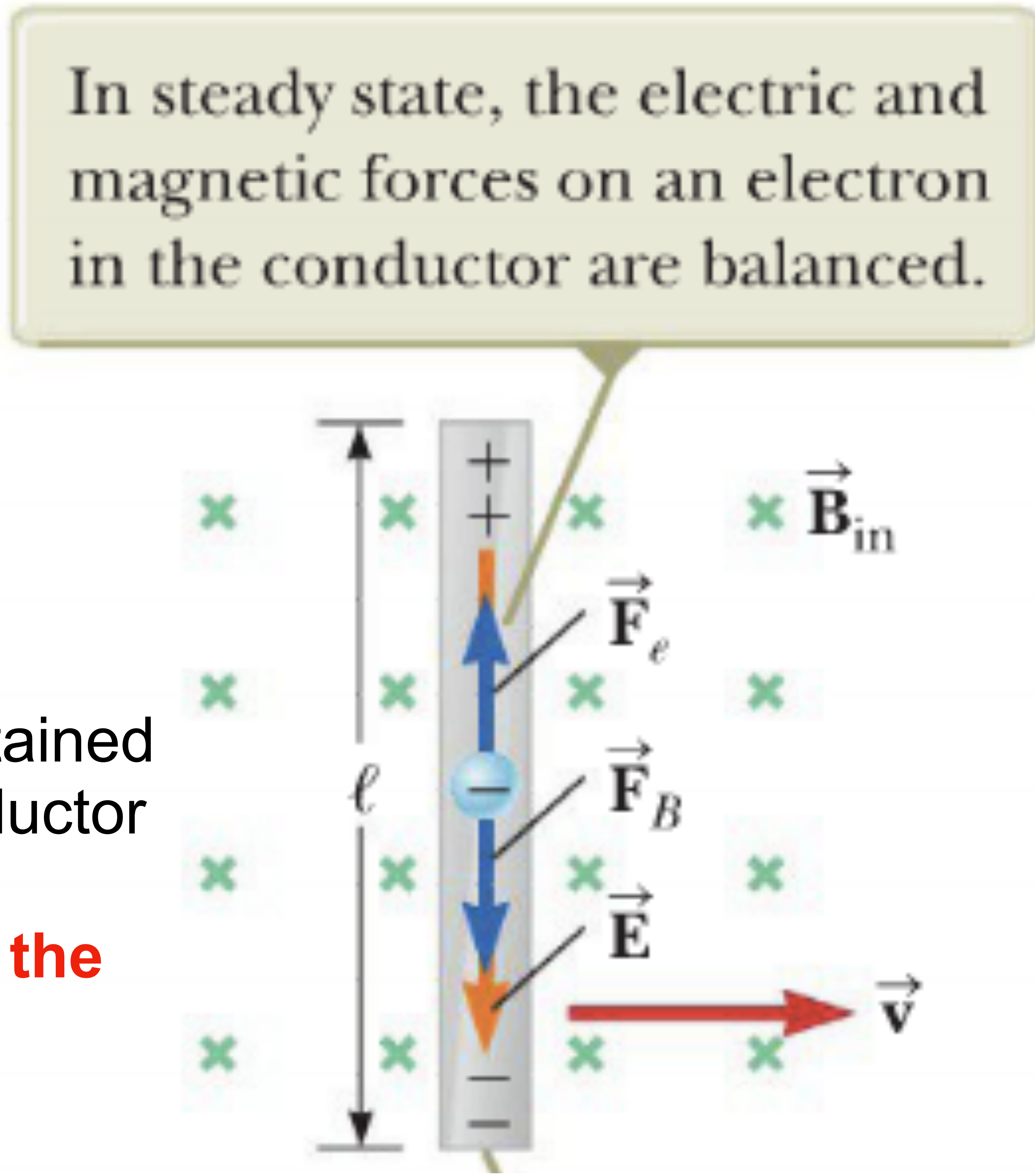
$$\boxed{E = vB}$$



Motional emf

The electric field is related to the potential difference across the ends of the conductor ($E = -dV/dx$):

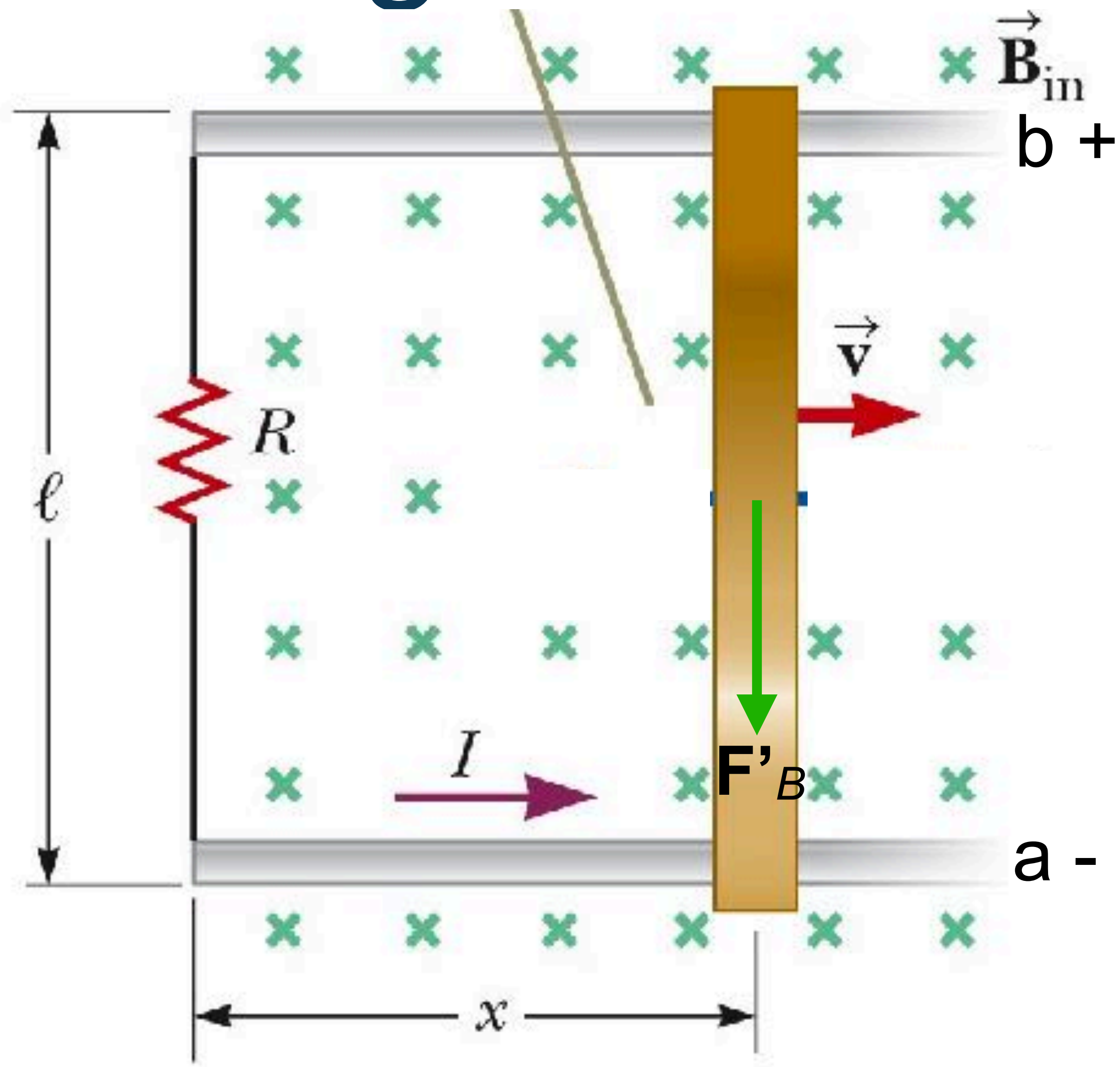
$$\boxed{\Delta V = E\,\ell = B\,\ell\,v}$$



A potential difference is maintained between the ends of the conductor **as long as the conductor continues to move through the uniform magnetic field.**

If the direction of the motion is reversed, the polarity of the potential difference is also reversed.

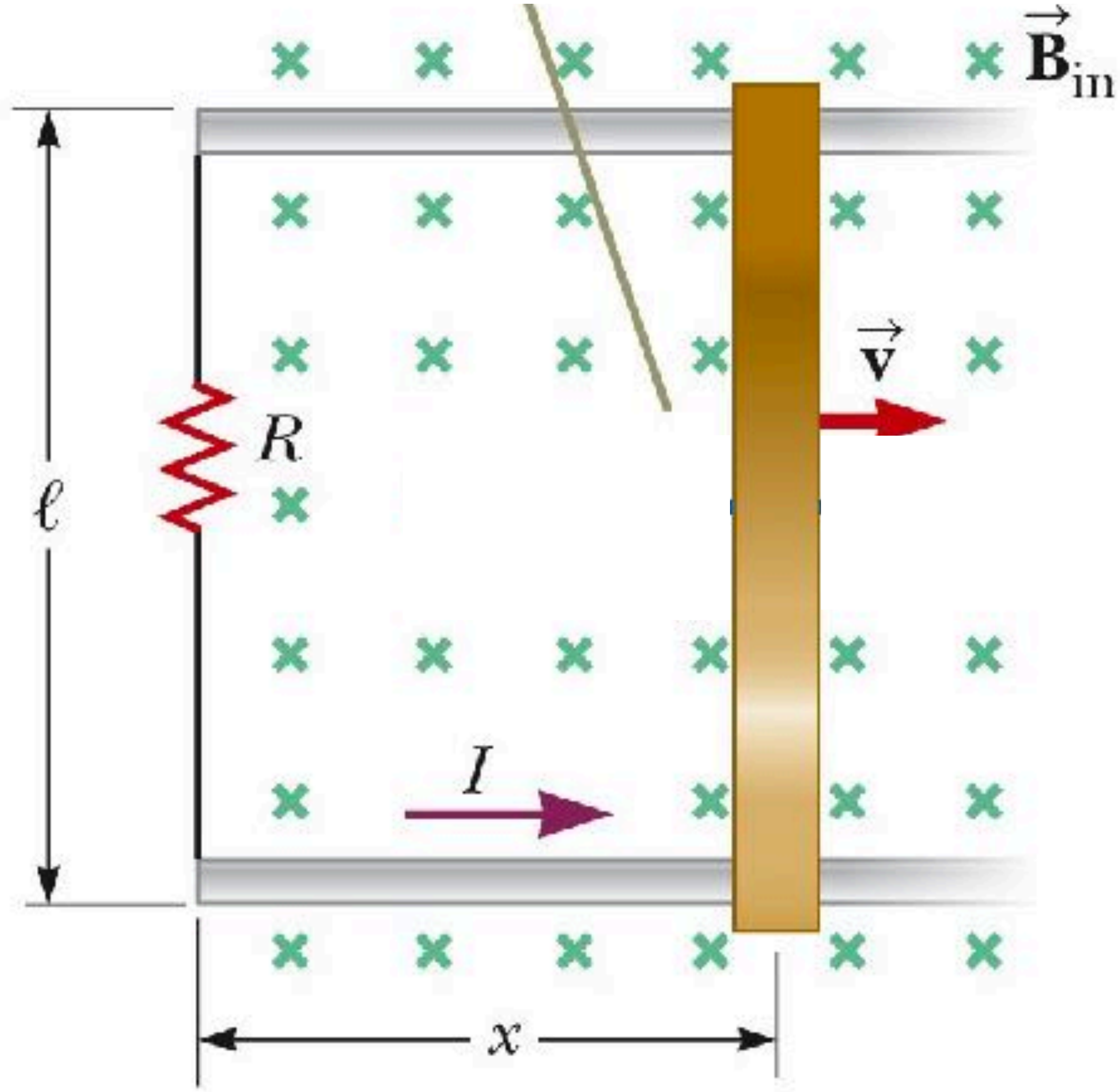
Sliding Conducting Bar

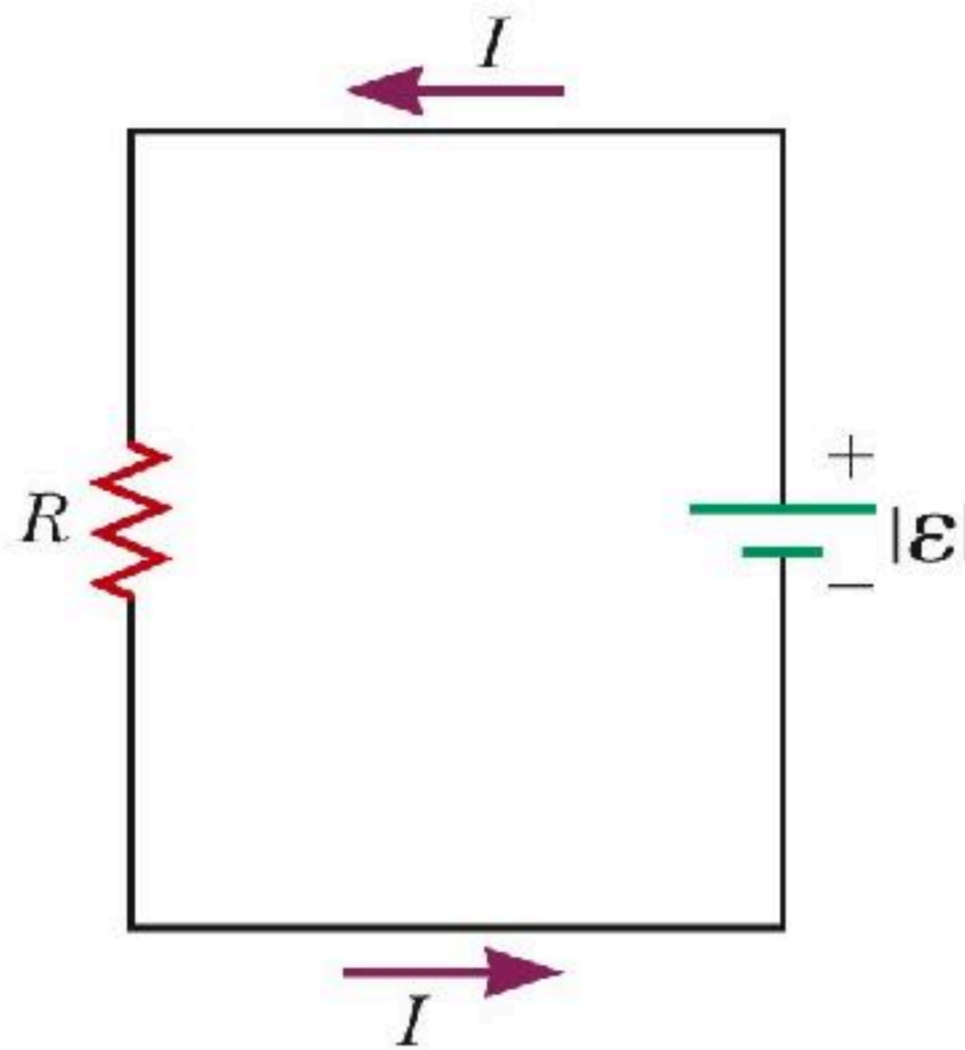


A conducting bar is moving to the right through a uniform B field into the page.
This creates a magnetic force **F'**$_B$ along the direction of the bar (see previous slides).
This separates the charges in the bar and creates an E field, which creates a potential difference between point a and b.

Assume the bar has zero resistance (emf = V) and the stationary part of the circuit has a resistance R (the load resistance).
Because you have a potential, and you have a resistance you have a current.
Remember, current is the flow of positive charges, so the current goes from + to -, so counterclockwise.

Sliding Conducting Bar





This situation can be **represented by a circuit** (it is a circuit).
Just like in all the other circuits we have seen, the question is:
- Find the potential difference (in this case the emf, because the bar itself has zero resistance)
- Find the current

Sliding Conducting Bar, cont.

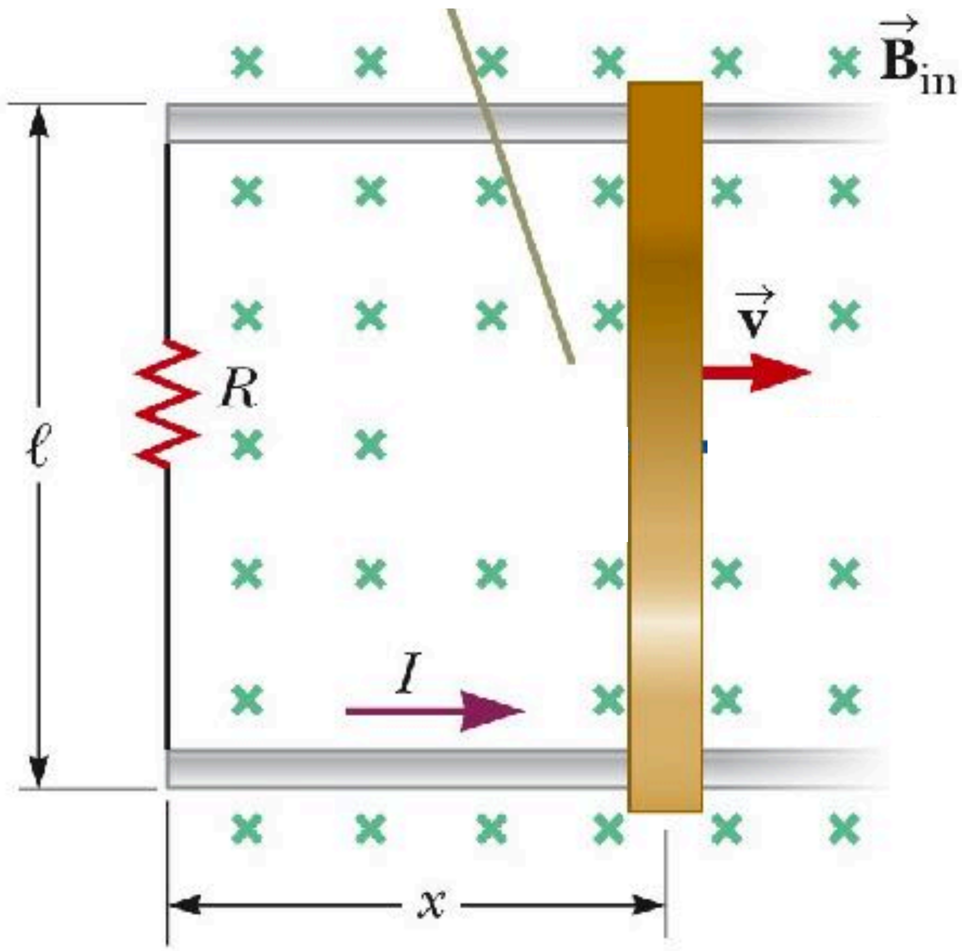
The magnetic flux through the loop is :

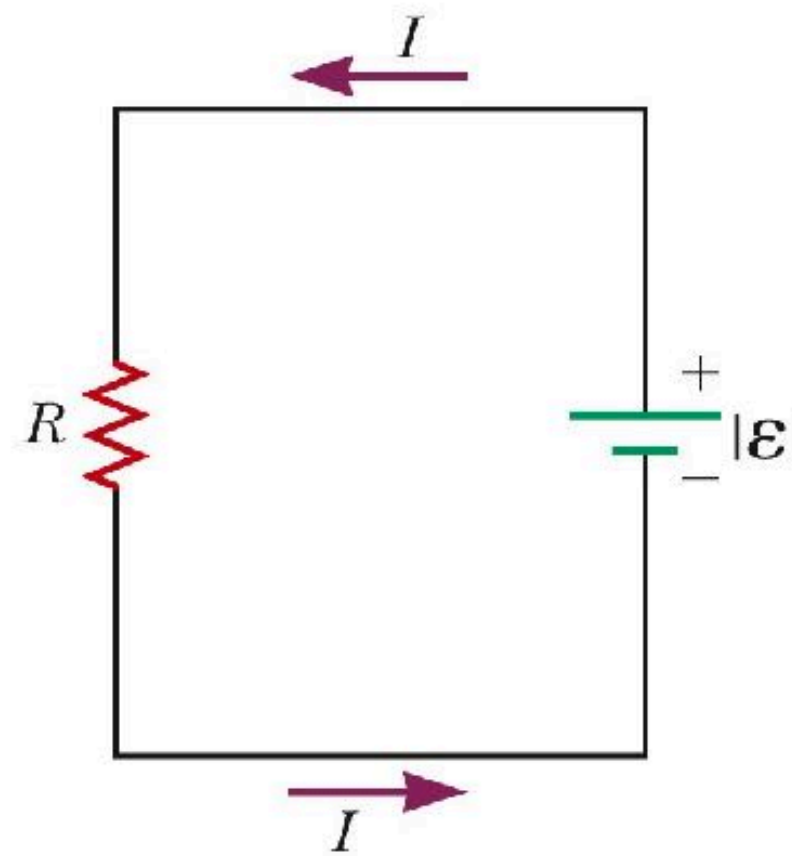
$$\phi_B = \int \vec{B} \cdot d\vec{A} \qquad \phi_B = B\ell x$$

The induced emf is:

$$\begin{aligned} \epsilon &= -\frac{d\phi_B}{dt} \\ &= -\frac{d(B\ell x)}{dt} \\ &= -B\ell\frac{dx}{dt} \end{aligned}$$

$$\boxed{\epsilon = -B\ell v}$$





The resistance in the circuit is *R*, so the current is $\boxed{I = \frac{|\epsilon|}{R} = \frac{B\ell v}{R}}$

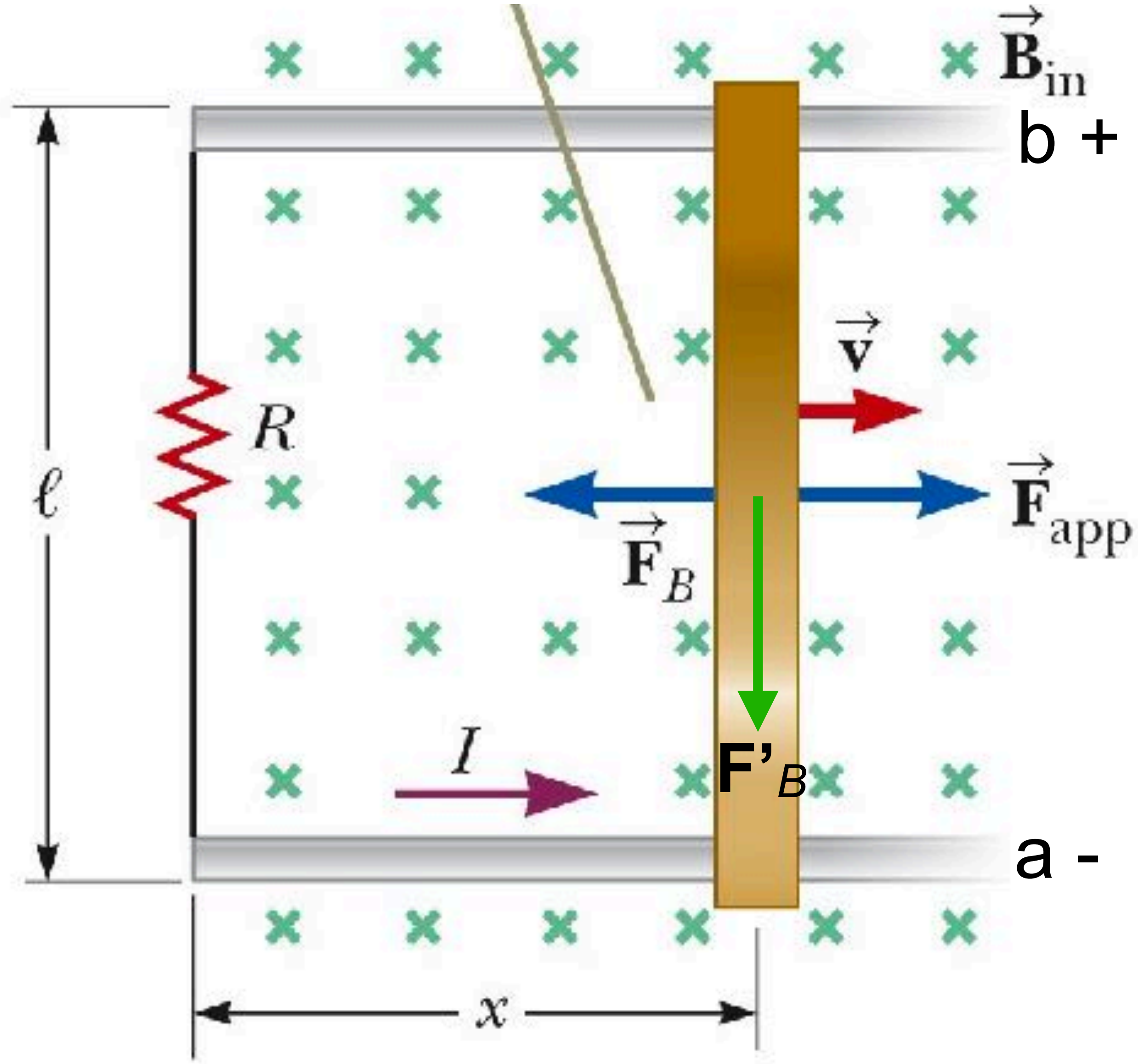
Sliding Conducting Bar

Moving the bar in the B field creates a B force along the bar, which creates separation of charges, which creates an electric field which induces an emf which induces a current.

Not the end of the story.

A current in a B field in turns creates a B force **F**$_B$: (this one is directed to the left)

$$\vec{F} = I\vec{L} \times \vec{B}$$



The bar moves at constant velocity so there is no acceleration so

$$\Sigma \mathbf{F} = 0$$

$$F_B = F_{app}$$

$$I\ell B = F_{app}$$

F'$_B$ doesn't count because it's internal.

Sliding Conducting Bar, Energy

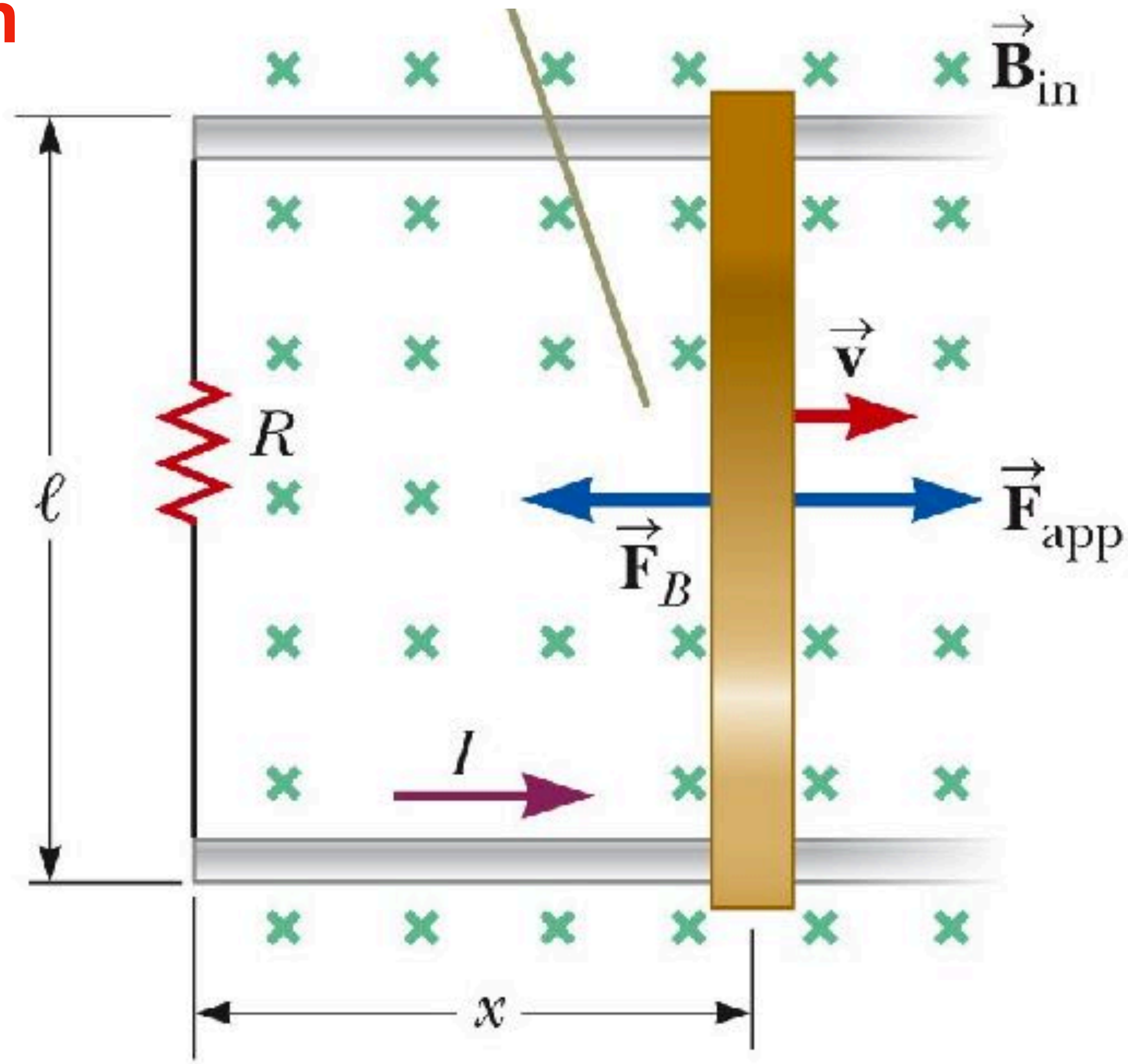
The **applied force does work on the conducting bar**: W = Fx

The change in energy of the system during some time interval must be equal to the transfer of energy into the system by work:

E = W = Fx

The power input is equal to the rate at which energy is delivered to the resistor.

P = E/t = W/t = Fx/t = Fv



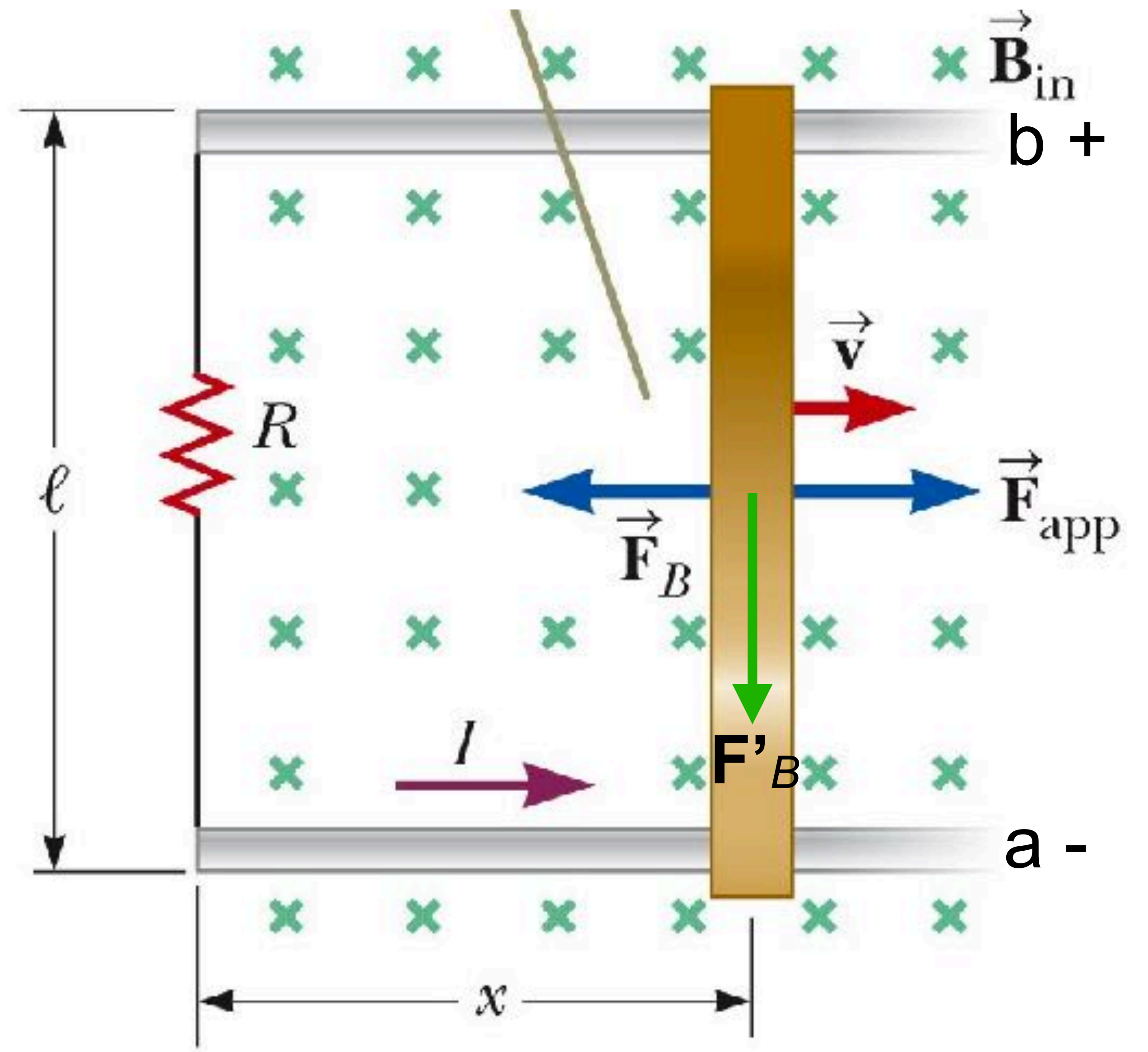
$$I = \frac{|\epsilon|}{R} = \frac{B\ell v}{R}$$

$$\boxed{P = F_{app}v = I\ell Bv = \frac{B^2\ell^2v^2}{R} = \frac{\epsilon^2}{R}}$$

Sliding Conducting Bar, Lenz' law

Moving the bar in the B field creates a B force along the bar, which creates separation of charges, which creates an electric field which induces an emf which induces a current. The current creates a magnetic force.

The current also creates another B field (directed out of the page by the RHR)



As the bar slides to the right, the area of the loop becomes bigger and bigger. This means that the magnetic flux increases with time. **The created current creates a field that opposes this change.**

—> Lenz' law

Lenz's Law

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

stop here?

Faraday's law indicates that the induced emf and the change in flux have **opposite algebraic signs.**

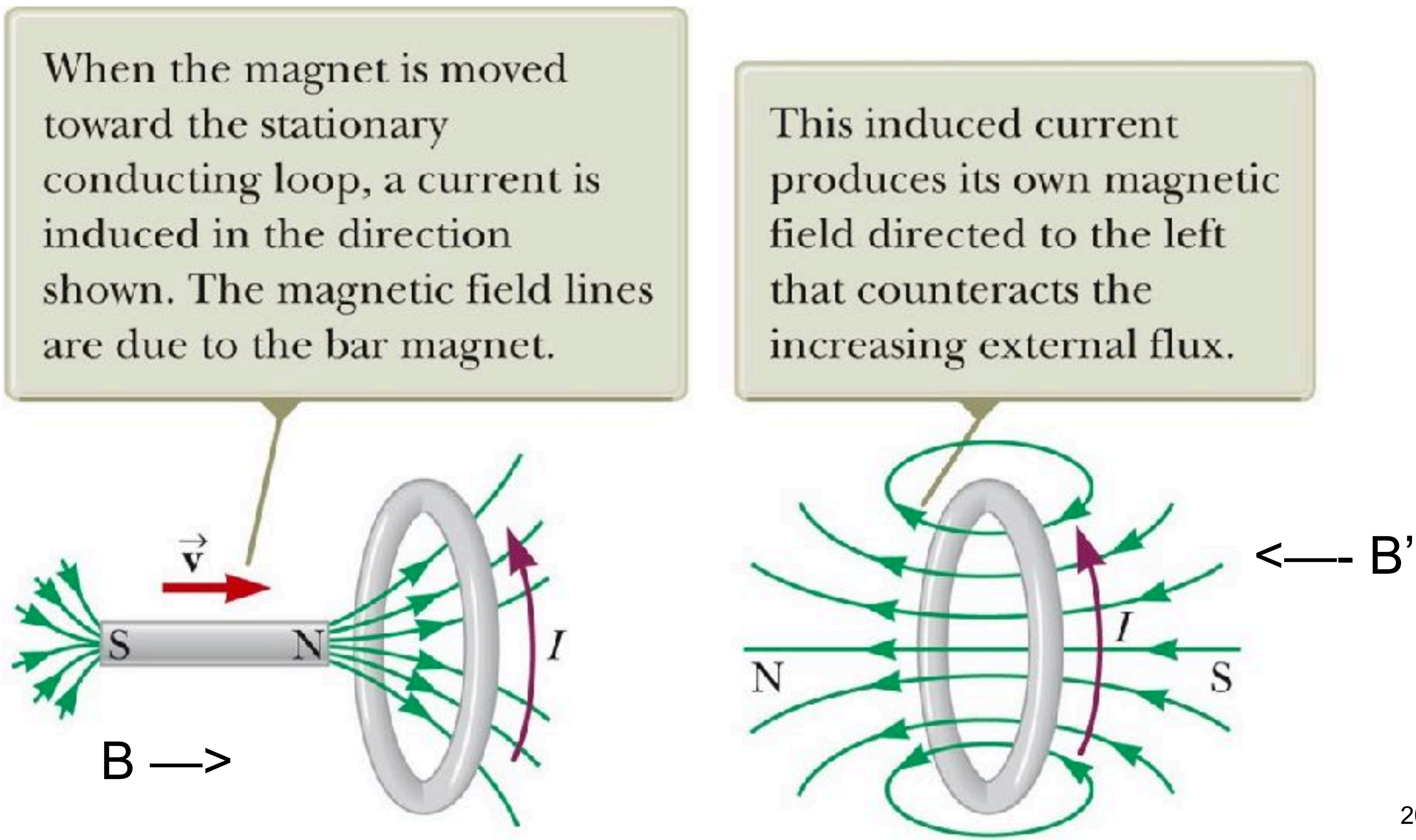
This has a physical interpretation that has come to be known as Lenz's law (after German physicist Heinrich Lenz)

Lenz's law: ***the induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.***

The induced current tends to keep the original magnetic flux through the circuit from changing.

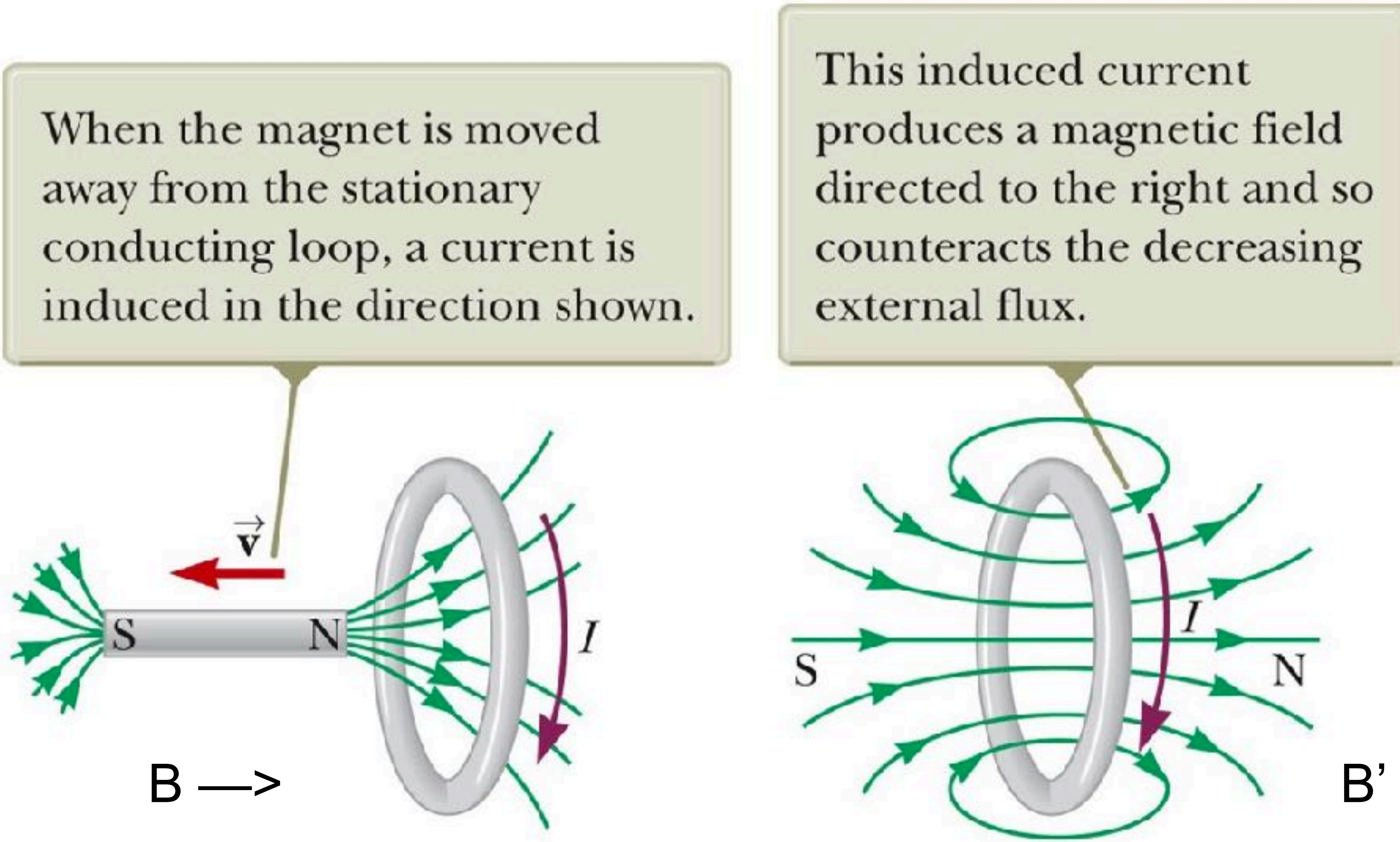
Induced Current Directions – Example

A magnet is placed near a metal loop.
Find the direction of the induced current in the loop **when the magnet is pushed toward the loop.**



Induced Current Directions – Example

A magnet is placed near a metal loop.
Find the direction of the induced current in the loop **when the magnet is pulled away from the loop .**



B —>

B’ —>

Induce emf and electric fields

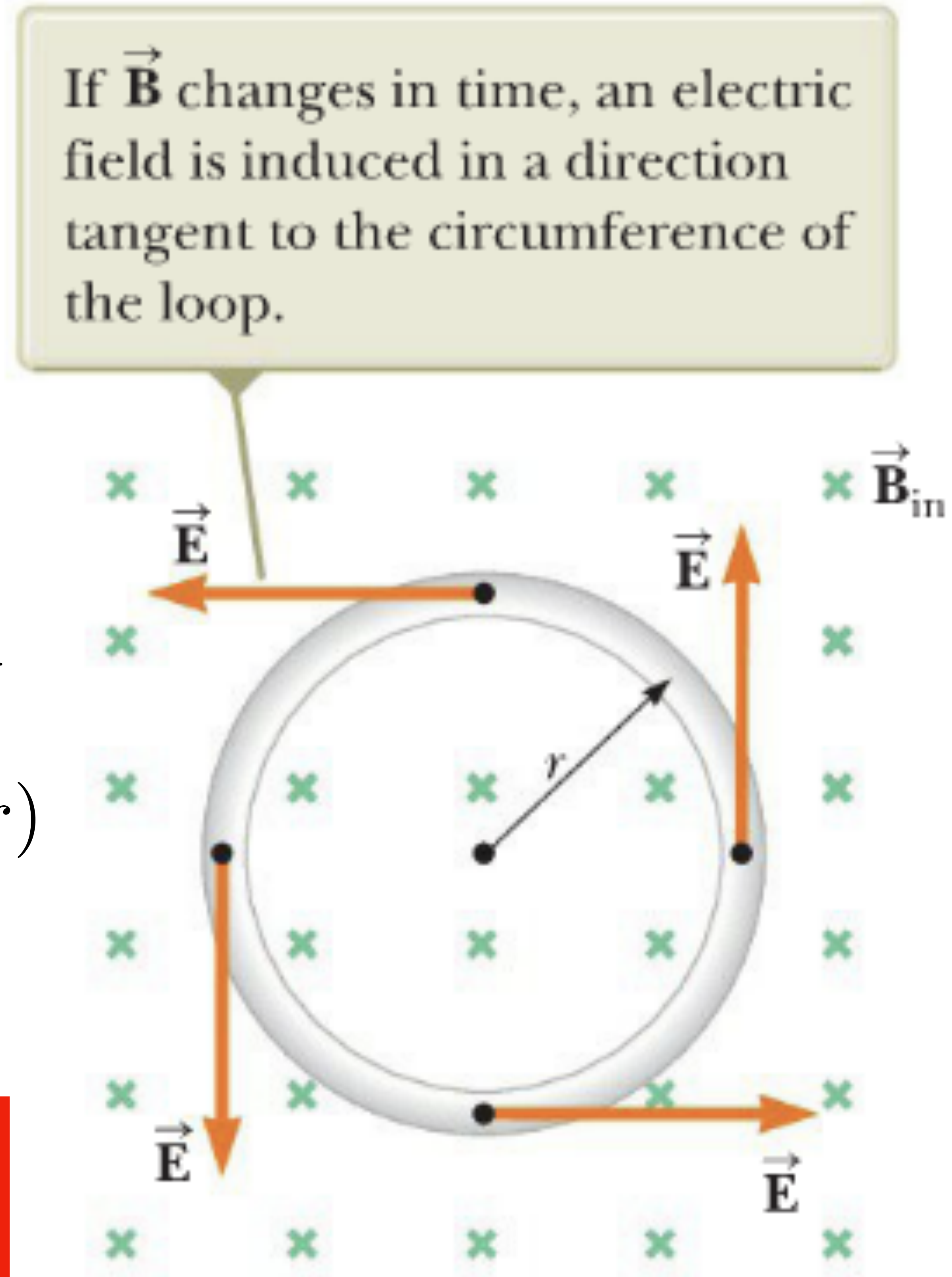
Magnetic field changes with time. Induces an emf, which induces a current, which implies that an electric field is also induced.
The E field must be tangent to the loop because that's the direction the charges move.

$$\epsilon = \frac{W}{q} \qquad W = \epsilon q \qquad \varepsilon = -\frac{d\Phi_B}{dt}$$

$$W = Fd \qquad F_e = qE \qquad W = F_e(2\pi r)$$

$$\epsilon q = qE(2\pi r) \qquad E = \frac{\epsilon}{2\pi r}$$

$$\boxed{E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt}}$$



This is for a loop!

Faraday's Law - General Form

An electric field is created in the conductor as a result of the changing magnetic flux.

Even in the absence of a conducting loop, a changing magnetic field will generate an electric field in empty space.

This induced electric field is **nonconservative** (unlike the electric field produced by stationary charges)

The emf for any closed path can be expressed as the line integral of the electric field over the path:

$$\boxed{\epsilon = \oint \vec{E} \cdot d\vec{s} = -\frac{d\phi_B}{dt}}$$

Maxwell's 3rd equation

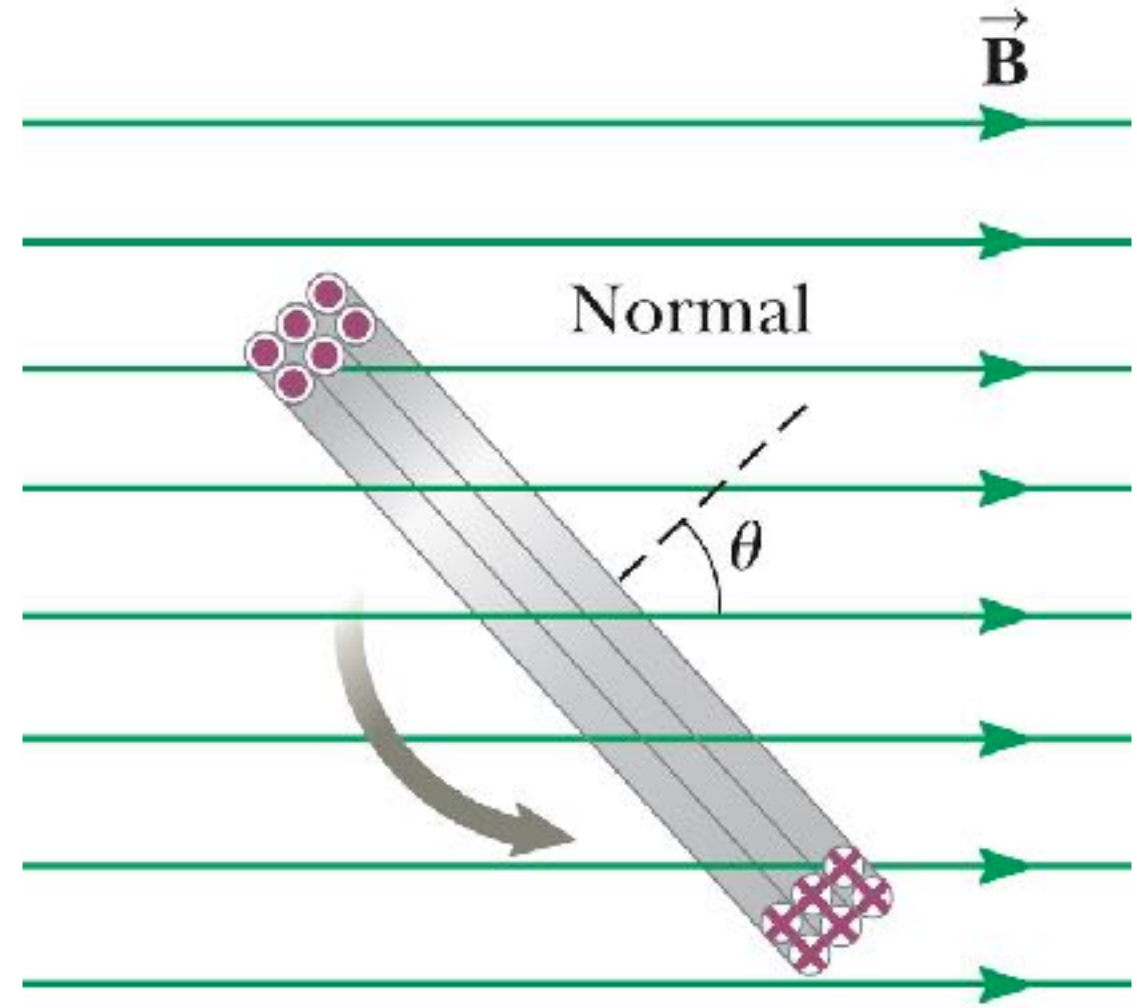
$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$\phi_B = \int \vec{B} \cdot d\vec{A}$$

The field cannot be an electrostatic field because if the field were electrostatic, and hence conservative, the line integral of $\overrightarrow{E} \cdot d\vec{s}$ over a closed loop would be zero and it isn't.

Rotating Loop

Assume a loop with N turns, all of the same area rotating in a magnetic field with angular speed ω.

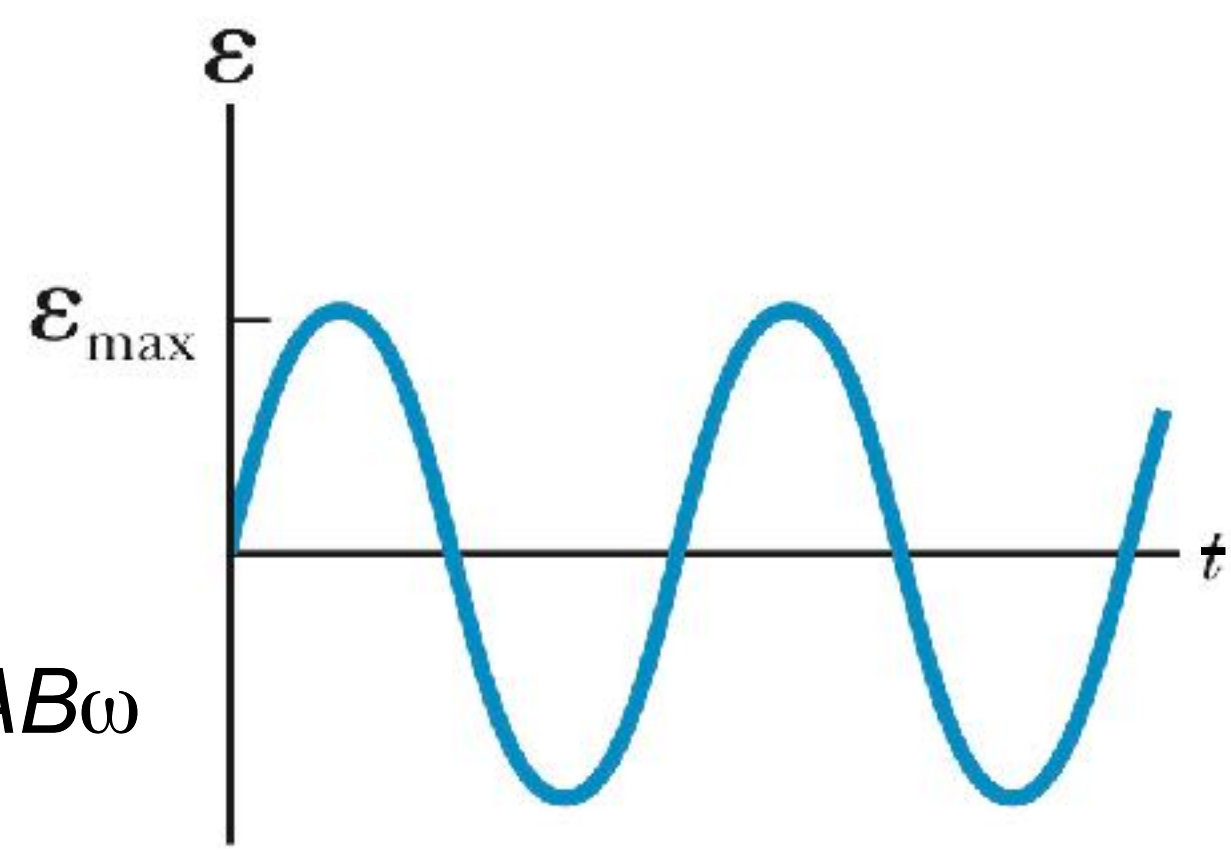


The flux through the loop at any time t is:

$\Phi_B = BA \cos \theta = BA \cos \omega t$

The induced emf in the loop is:

$$\begin{aligned}\varepsilon &= -N\frac{d\Phi_B}{dt} \\ &= -NBA\frac{d}{dt}(\cos \omega t) \\ &= NBA\omega \sin \omega t\end{aligned}$$



This is sinusoidal, with $\varepsilon_{max} = NAB\omega$

Conceptual Question

Why does Lenz' Law go the direction that it goes? (opposite)

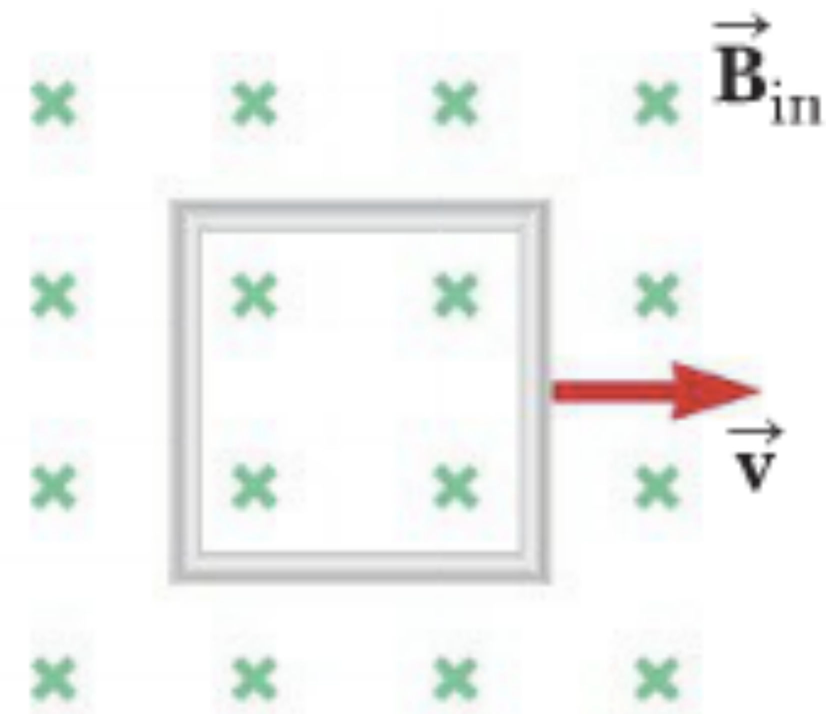
Conceptual Question : solution

Otherwise +ive feedback loop. Perpetual motion, energy from nowhere

More conceptual Questions

1) If I decreases in time, what direction is the current induced in the loop?

2) You pull a positively charged loop at constant v in a uniform B, do you have charge separation? Do you have an induced current, if so in which direction?



3) I increases with time.
This induces a current in the inner loop.

- What is the direction of this current?
- How is this current affected by the dimensions of the loops?

More Conceptual Questions: Solution

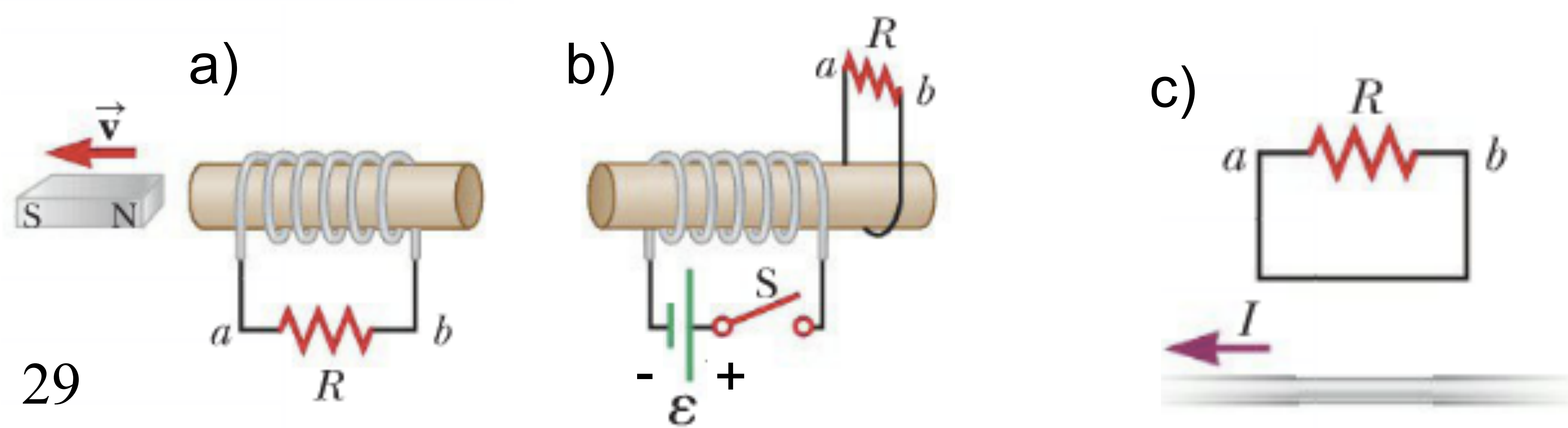
1) clockwise: I decreases in time, the flux through the loop decreases. So Lenz's law says the other current will compensate for that.

2) no current induced (B flux is constant). charge separation: top is +: q**v** x **B** the force along the line is still there (F')

3) clockwise and magnitude depends on dimension of loops
$I = Blv/R$

Practice Problem Example 2

22. Use Lenz's law to answer the following questions concerning the direction of induced currents. Express your answers in terms of the letter labels *a* and *b* in each part of Figure P31.22. (a) What is the direction of the induced current in the resistor *R* in Figure P31.22a when the bar magnet is moved to the left? (b) What is the direction of the current induced in the resistor *R* immediately after the switch S in Figure P31.22b is closed? (c) What is the direction of the induced current in the resistor *R* when the current *I* in Figure P31.22c decreases rapidly to zero?



Practice Problem Example 2: solution

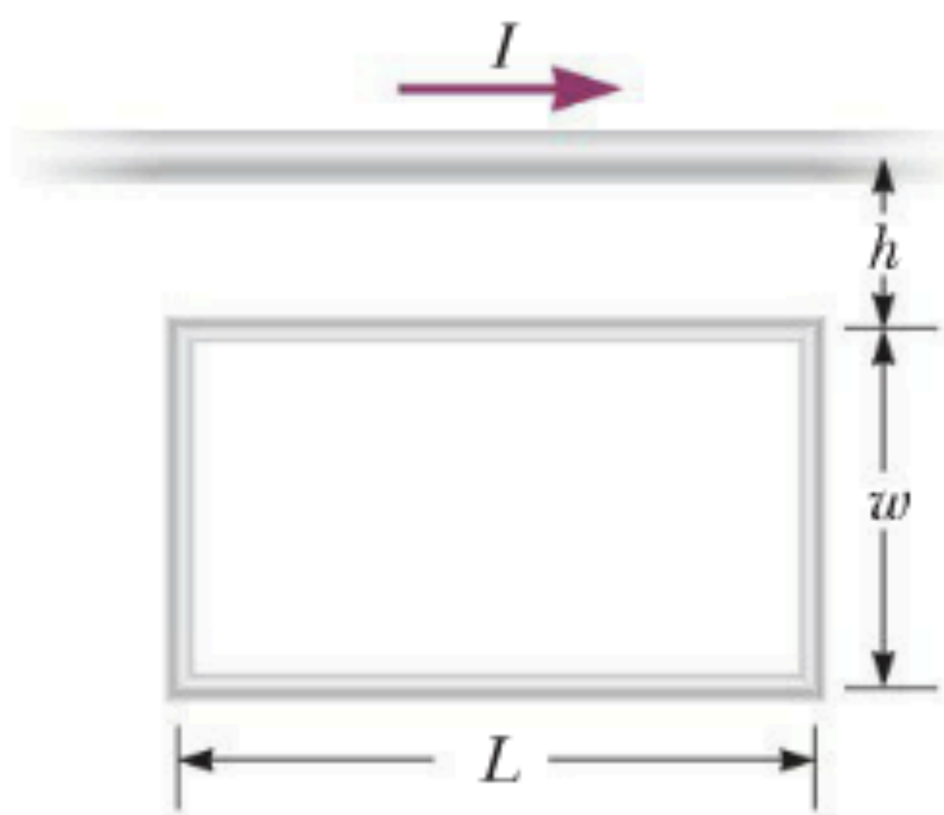
(a) $\vec{\mathbf{B}}_{\text{ext}} = B_{\text{ext}}\hat{\mathbf{i}}$ and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_{\text{induced}} = B_{\text{induced}}\hat{\mathbf{i}}$ (to the right) and the current in the resistor is directed from a to b, $\boxed{\text{to the right}}$.

(b) $\vec{\mathbf{B}}_{\text{ext}} = B_{\text{ext}}\left(-\hat{\mathbf{i}}\right)$ increases; therefore, the induced field $\vec{\mathbf{B}}_{\text{induced}} = B_{\text{induced}}\left(+\hat{\mathbf{i}}\right)$ is to the right, and the current in the resistor is directed from a to b, $\boxed{\text{out of the page}}$ in the textbook picture.

(c) $\vec{\mathbf{B}}_{\text{ext}} = B_{\text{ext}}\left(-\hat{\mathbf{k}}\right)$ into the paper and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_{\text{induced}} = B_{\text{induced}}\left(-\hat{\mathbf{k}}\right)$ into the paper, and the current in the resistor is directed from a to b, $\boxed{\text{to the right}}$.

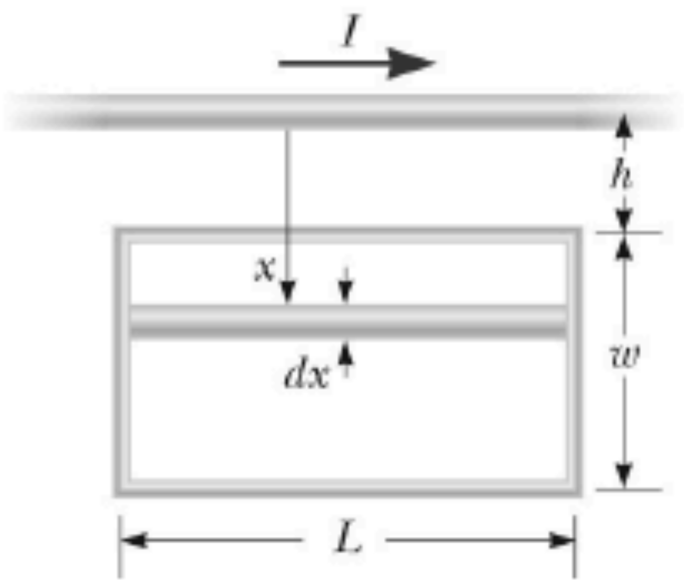
Practice Problem Example 1

13. W A loop of wire in the shape of a rectangle of width w and length L and a long, straight wire carrying a current I lie on a tabletop as shown in Figure P31.13. (a) Determine the magnetic flux through the loop due to the current I. (b) Suppose the current is changing with time according to $I = a + bt$, where a and b are constants. Determine the emf that is induced in the loop if $b = 10.0$ A/s, $h = 1.00$ cm, $w = 10.0$ cm, and $L = 1.00$ m. (c) What is the direction of the induced current in the rectangle?



Practice Problem Example 1: Solution

(a) At a distance x from the long, straight wire, the magnetic field is $B = \dfrac{\mu_0 I}{2\pi x}$. The flux through a small rectangular element of length L and width dx within the loop is



ANS. FIG. P31.13

slide 9
Biot Savart

$$d\Phi_B = \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \frac{\mu_0 I}{2\pi x} L dx:$$

$$\Phi_B = \int_h^{h+w} \frac{\mu_0 IL}{2\pi} \frac{dx}{x} = \boxed{\frac{\mu_0 IL}{2\pi} \ln\left(\frac{h+w}{h}\right)}$$

(b) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi} \ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi} \ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$$

where $\dfrac{dI}{dt} = \dfrac{d}{dt}(a+bt) = b$:

Practice Problem Example 1: Solution

$$\mathcal{E} = -\frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)(1.00\ \mathrm{m})}{2\pi}$$

$$\times \ln\left(\frac{0.010\ 0\ \mathrm{m} + 0.100\ \mathrm{m}}{0.010\ 0\ \mathrm{m}}\right)(10.0\ \mathrm{A/s})$$

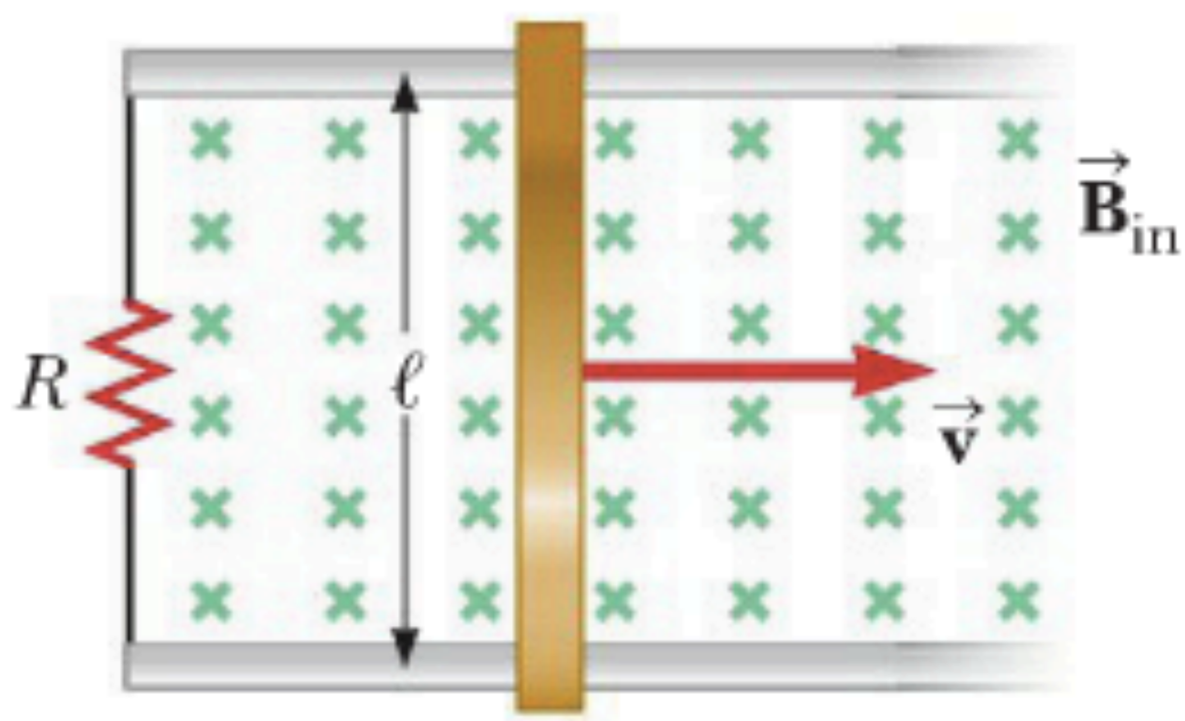
$$= -4.80 \times 10^{-6}\ \mathrm{V}$$

Therefore, the emf induced in the loop is $\boxed{4.80\ \mu\mathrm{V}}$.

(c) The long, straight wire produces magnetic flux into the page through the rectangle, shown in ANS. FIG. P31.13. As the magnetic flux increases, the rectangle produces its own magnetic field out of the page to oppose the increase in flux. The induced current creates this opposing field by traveling $\boxed{\text{counterclockwise}}$ around the loop.

Practice Problem Example 3

34. A conducting bar of length ℓ moves to the right on two frictionless rails as shown in Figure P31.34. A uniform magnetic field directed into the page has a magnitude of 0.300 T. Assume $R = 9.00\ \Omega$ and $\ell = 0.350$ m. (a) At what constant speed should the bar move to produce an 8.50-mA current in the resistor? (b) What is the direction of the induced current? (c) At what rate is energy delivered to the resistor? (d) Explain the origin of the energy being delivered to the resistor.



Practice Problem Example 3: Solution

(a) The motional emf induced in the bar must be $\mathcal{E} = IR$, where I is the current in this series circuit. Since $\mathcal{E} = B\ell v$, the speed of the moving bar must be

$$v = \frac{\mathcal{E}}{B\ell} = \frac{IR}{B\ell} = \frac{(8.50 \times 10^{-3}\ \text{A})(9.00\ \Omega)}{(0.300\ \text{T})(0.350\ \text{m})} = \boxed{0.729\ \text{m/s}}$$

(b) The flux through the closed loop formed by the rails, the bar, and the resistor is directed into the page and is increasing in magnitude. To oppose this change in flux, the current must flow in a manner so as to produce flux out of the page through the area enclosed by the loop. This means the current will flow $\boxed{\text{counterclockwise}}$.

Practice Problem Example 3: Solution

(c) The rate at which energy is delivered to the resistor is

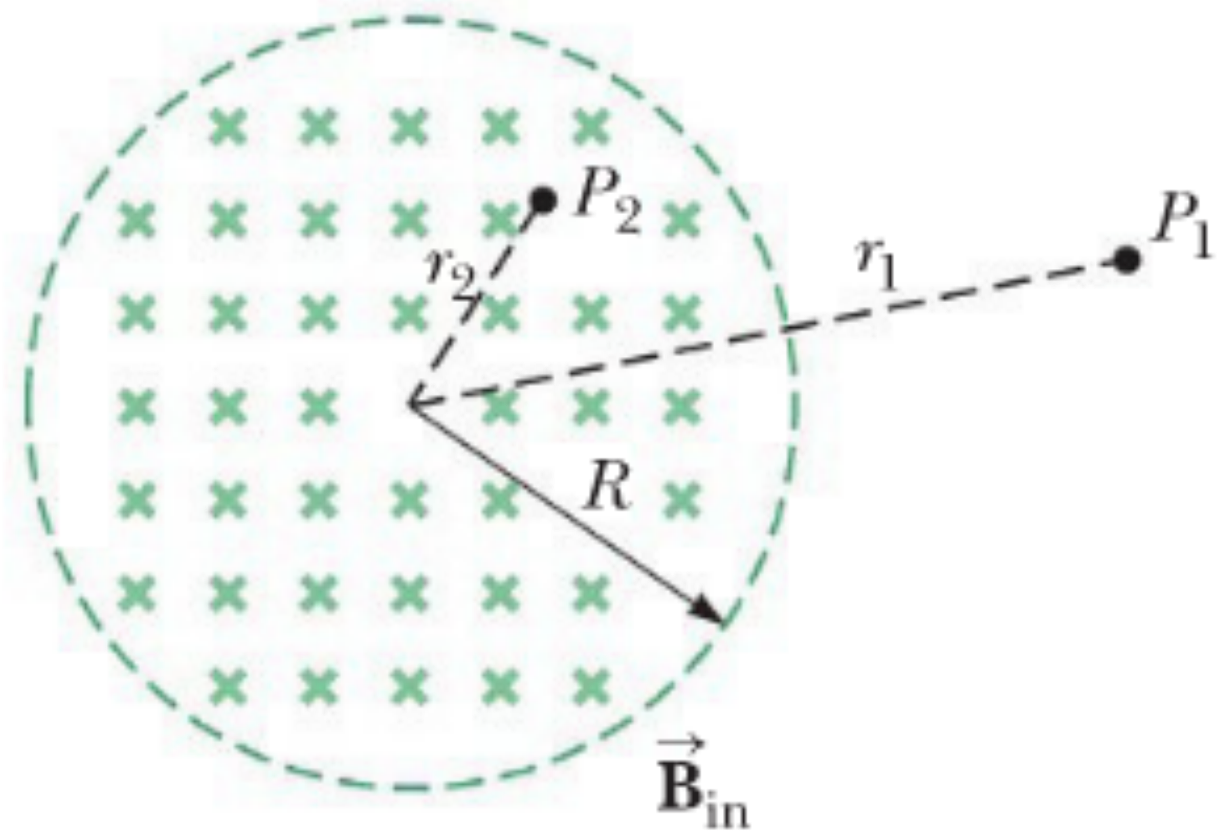
$$P = I^2R = \left(8.50 \times 10^{-3}\ \text{A}\right)^2 (9.00\ \Omega)$$

$$= 6.50 \times 10^{-4}\ \text{W} = \boxed{0.650\ \text{mW}}$$

(d) Work is being done by the external force, which is transformed into internal energy in the resistor.

Practice Problem Example 4

39. Within the green dashed circle shown in Figure P31.39, the magnetic field changes with time according to the expression $B = 2.00t^3 - 4.00t^2 + 0.800$, where B is in teslas, t is in seconds, and $R = 2.50$ cm. When $t = 2.00$ s, calculate (a) the magnitude and (b) the direction of the force exerted on an electron located at point P_1, which is at a distance $r_1 = 5.00$ cm from the center of the circular field region. (c) At what instant is this force equal to zero?



Practice Problem Example 4: solution

Point P_1 lies outside the region of the uniform magnetic field. The rate of change of the field, in teslas per second, is

$$\frac{dB}{dt} = \frac{d}{dt}\left(2.00t^3 - 4.00t^2 + 0.800\right) = 6.00t^2 - 8.00t$$

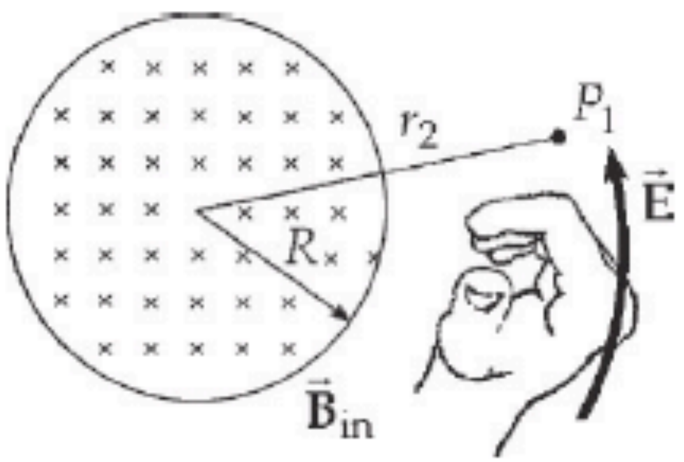
where t is in seconds. At $t = 2.00$ s, we see that the field is increasing:

$$\frac{dB}{dt} = 6.00(2.00)^2 - 8.00(2.00) = 8.00\ \mathrm{T/s}$$

(a) The magnitude of the electric field is (refer to Section 31.4 and Equation 31.8)

$$|E| = \frac{r}{2}\frac{dB}{dt} = \frac{r}{2}\left(6.00t^2 - 8.00t\right)$$

$$= \frac{0.050\,0}{2}\left[6.00(2.00)^2 - 8.00(2.00)\right] = 0.200\ \mathrm{N/C}$$



What happened to r1?

The magnitude of the force on the electron is

$$F = qE = eE = \left(1.60 \times 10^{-19}\ \mathrm{C}\right)\left(0.200\ \mathrm{N/C}\right) = \boxed{3.20 \times 10^{-20}\ \mathrm{N}}$$

Practice Problem Example 4: solution

(b) Because the electron holds a negative charge, the direction of the force is opposite to the field direction. The force is tangent to the electric field line passing through at point P_1 and clockwise.

(c) The force is zero when the rate of change of the magnetic field is zero:

$$\frac{dB}{dt} = 6.00t^2 - 8.00t = 0 \rightarrow t = \boxed{0} \text{ or } t = \frac{8.00}{6.00} = \boxed{1.33\text{ s}}$$