Lecture

on Chapter 29

Biot Savard Law - Ampere's Law

(t)

HW07 is up on WebAssign, due Thursday 03/07

Magnetic Fields

The origin of the magnetic field is moving charges.

The magnetic field due to various current distributions can be calculated.

—> Use Ampere's law and Biot Savart law

Biot-Savart Law

Mathematical expression that gives the magnetic field at some point in space due to a current.

The magnetic field described by the Biot-Savart Law is the field *due to* a given current carrying conductor.

- Do not confuse this field with any external field applied to the conductor from some other source.

$$\boxed{d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s} \times \hat{r}}{r^2}}$$

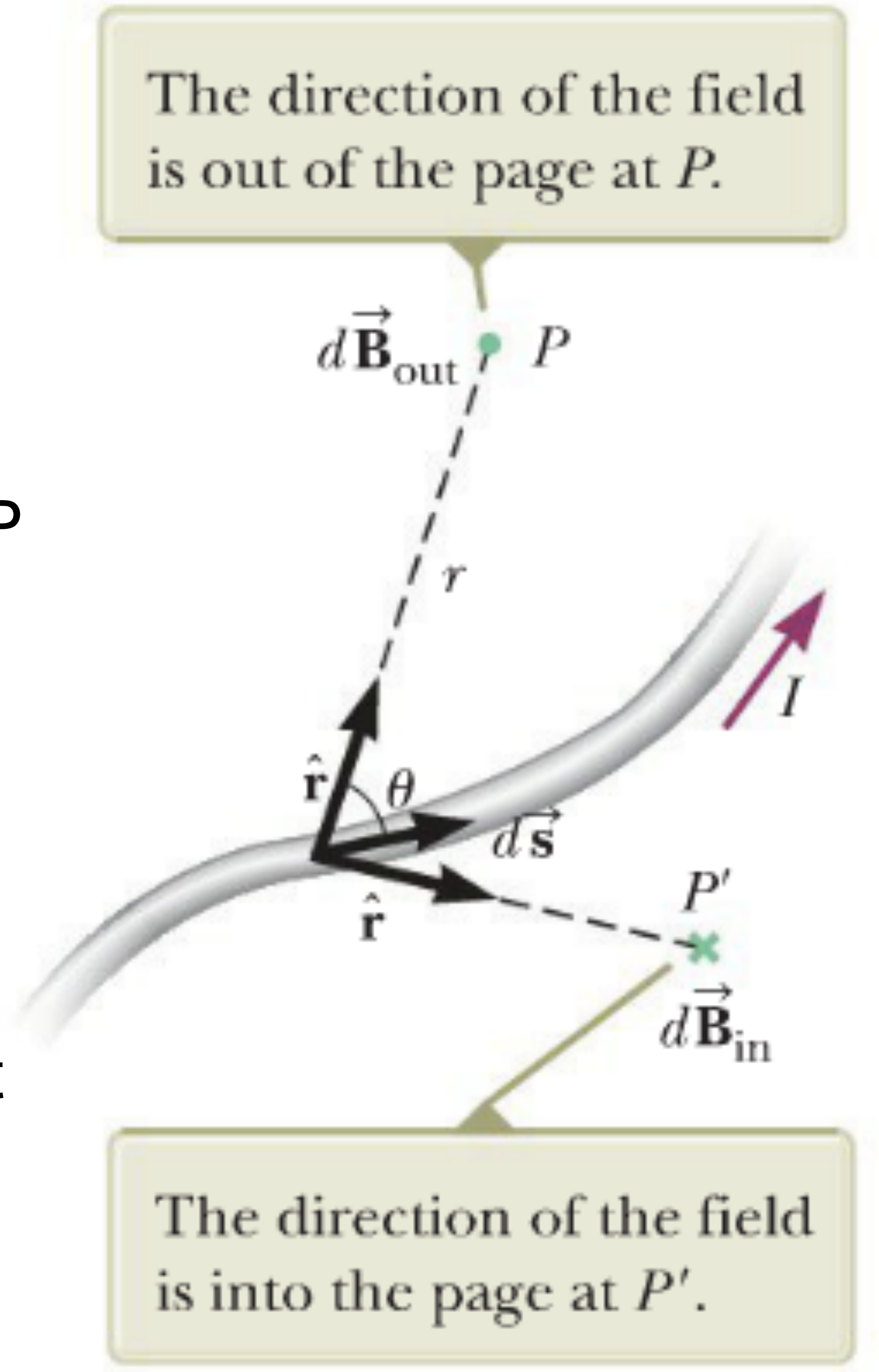
The constant μ_o is called the **permeability of free space.**

$\mu_o = 4\pi \times 10^{-7}$ T· m / A = how well you can get a magnetic field in vacuum.

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s} \times \hat{r}}{r^2}$$

- you want to find the field at point P
- **r** is the unit vector from the small current distribution in the wire pointing in the direction of P
- d**B** is perpendicular to **r** and d**s** (cross product)
- d**B** is proportional to $1/r^2$
- d**B** is still function of the current I and the element of length of the wire d**s**



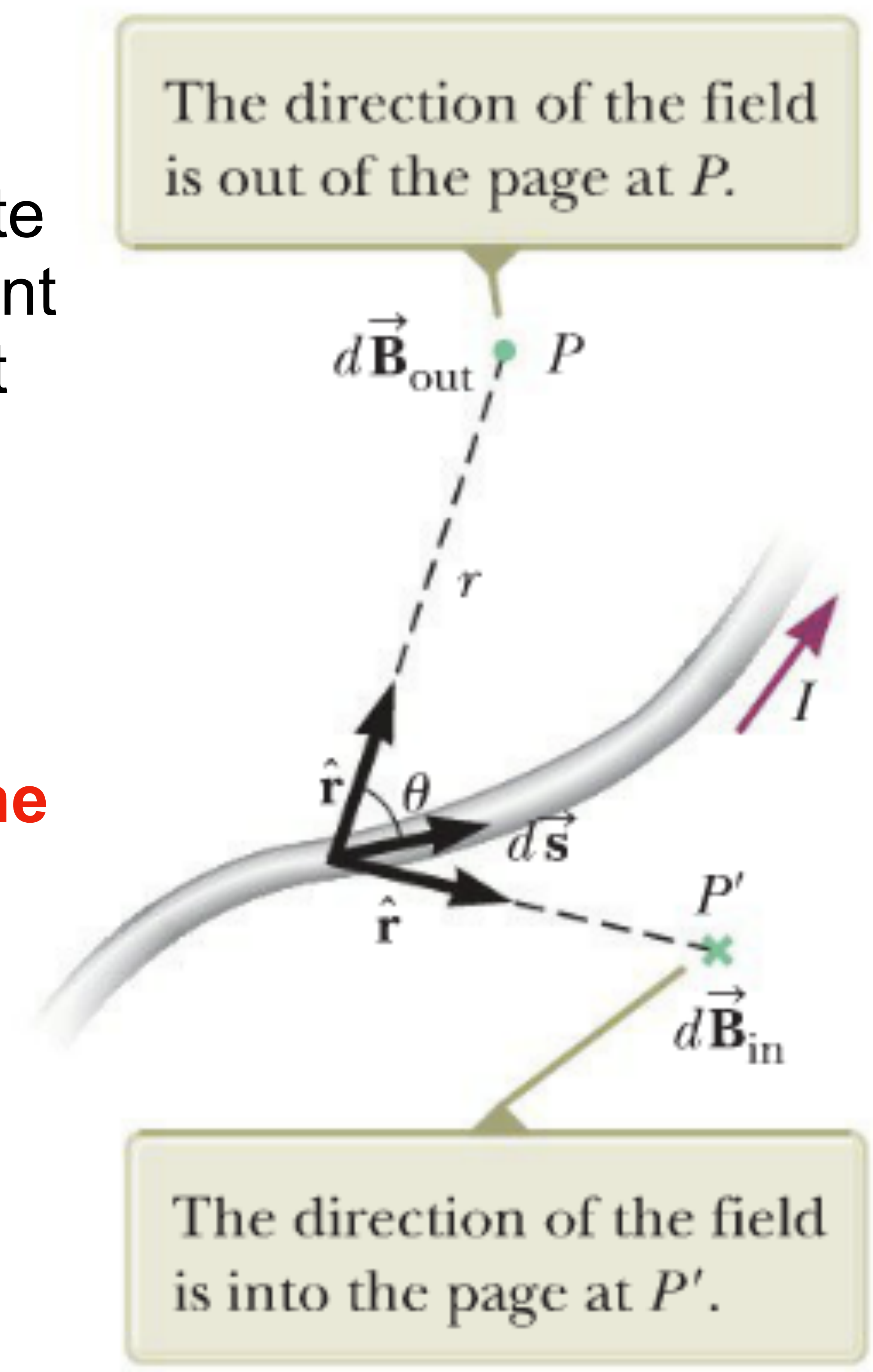
Total Magnetic Field

What we just saw is a law to calculate a magnetic field due to a small current distribution in a small length element ds.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

To find **the magnetic field due to the whole wire,** you must integrate the previous equation:

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}}$$



Magnetic Field Compared to Electric Field

Distance

- The magnitude of the magnetic field varies as $1/r^2$, the inverse square of the distance **from the current source**
- The electric field due to a point charge also varies as $1/r^2$, the inverse square of the distance **from the charge.**

Direction

- The electric field created by a point charge is **radial in direction.**
- The magnetic field created by a current element is **perpendicular to both the length element ds and the unit vector.**

Source

- An electric field is established by an **isolated electric charge**.
- The current element that produces a magnetic field must be **part of an extended current distribution.**
  - **Therefore you must integrate over the entire current distribution.**

How to solve a problem

- Find magnetic field at point P.
- Use Biot Savard law.
- Calculate the field due to a small current distribution
- Integrate to find the total field.

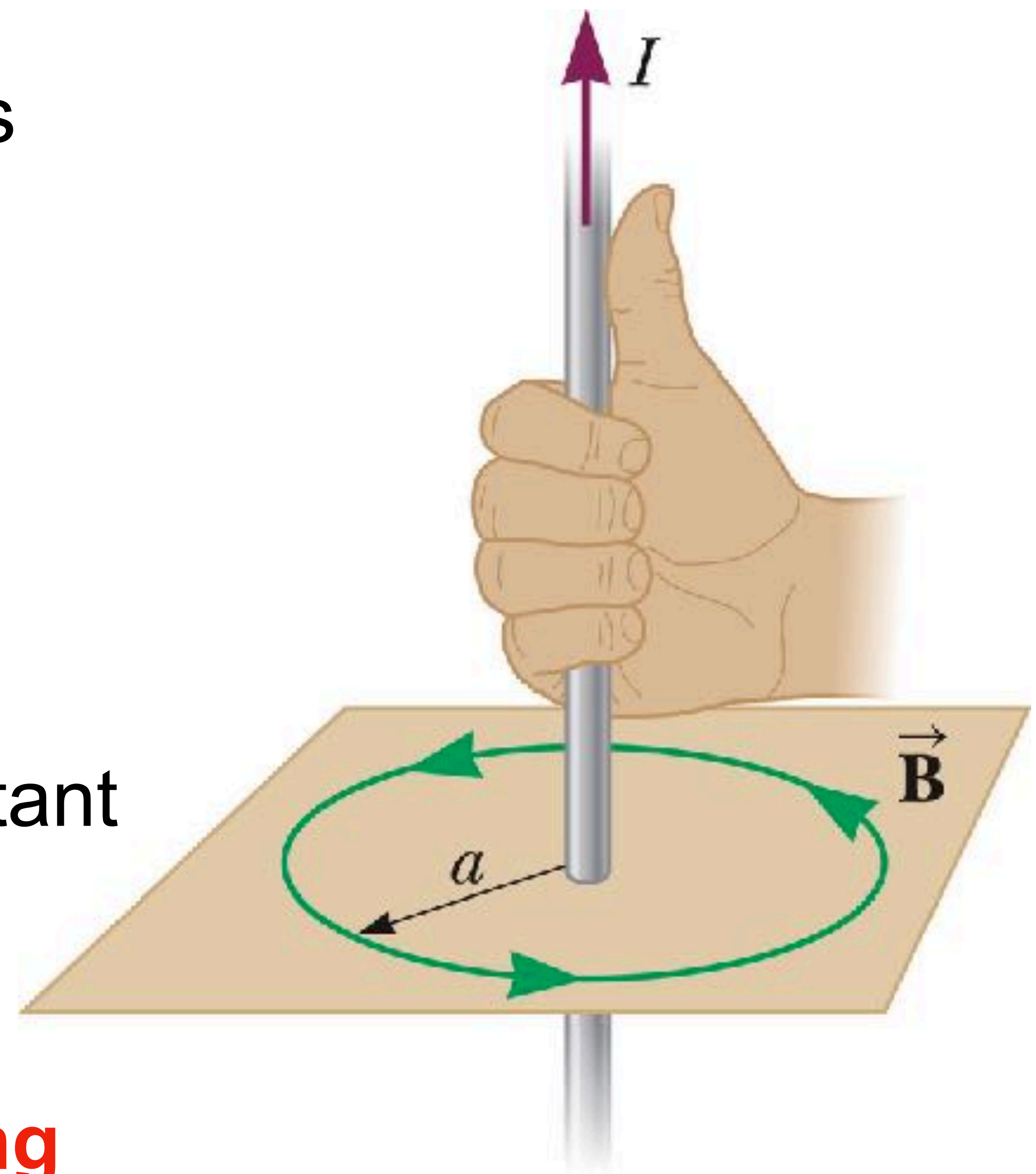
Magnetic Field for a Long, Straight Conductor: Direction

The magnetic field lines are circles concentric with the wire.

The field lines lie in planes perpendicular to the wire.

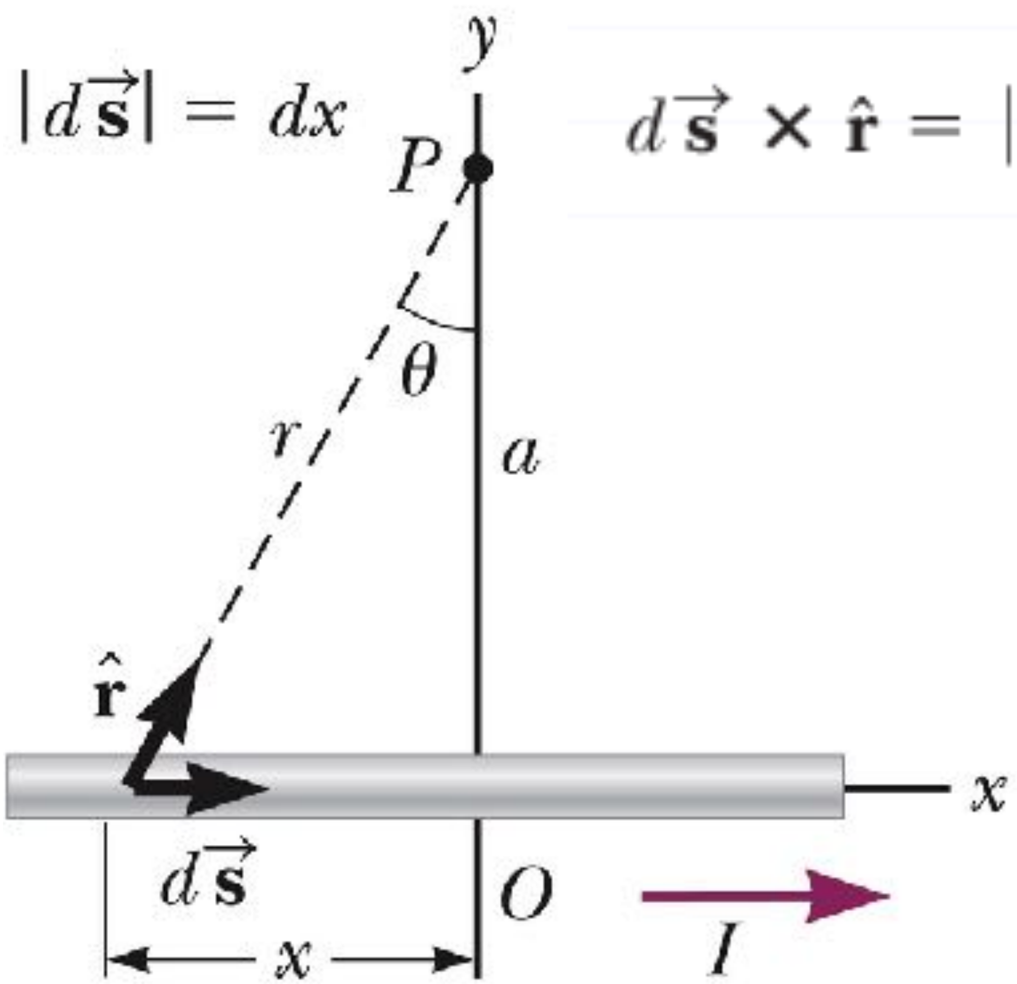
The magnitude of the field is constant on any circle of radius *a.*

The right-hand rule for **determining the direction of the field** is shown.



Magnetic Field for a Long, Straight Conductor

$|d\vec{\mathbf{s}}| = dx$



$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|\,\hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{\mathbf{k}} = (dx\cos\theta)\hat{\mathbf{k}}$$

$$(1) \quad d\vec{\mathbf{B}} = (dB)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\,\hat{\mathbf{k}}$$

$$(2) \quad r = \frac{a}{\cos\theta}$$

sec=1/cos

$$x = -a\tan\theta$$

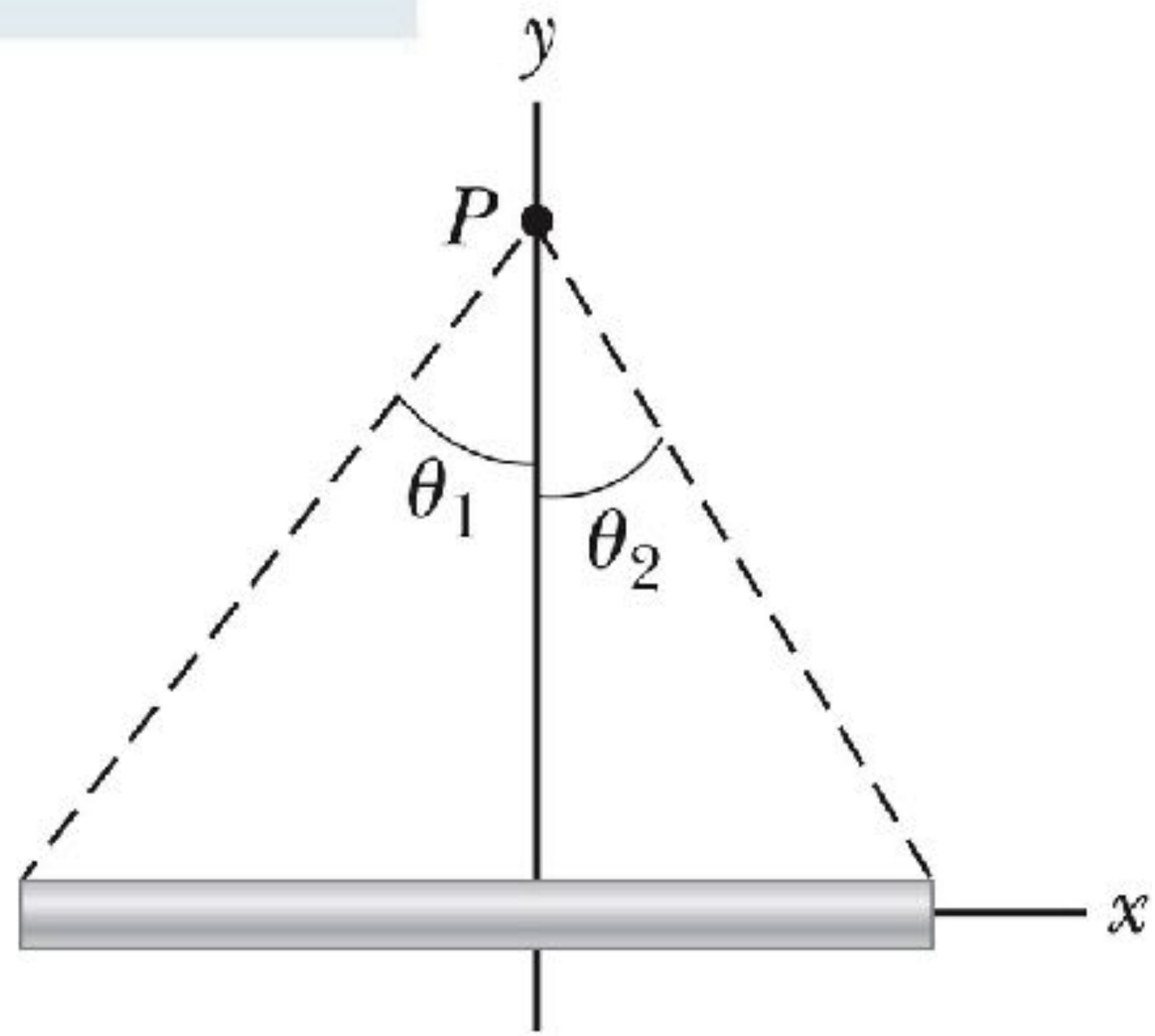
$$(3) \quad dx = -a\sec^2\theta\ d\theta = -\frac{a\,d\theta}{\cos^2\theta}$$

$$(4) \quad dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\,d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta\ d\theta$$

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\ d\theta = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

Magnetic Field for an infinite, Straight Conductor

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$



If the conductor is an infinitely long, straight wire:

$\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$

The field becomes :

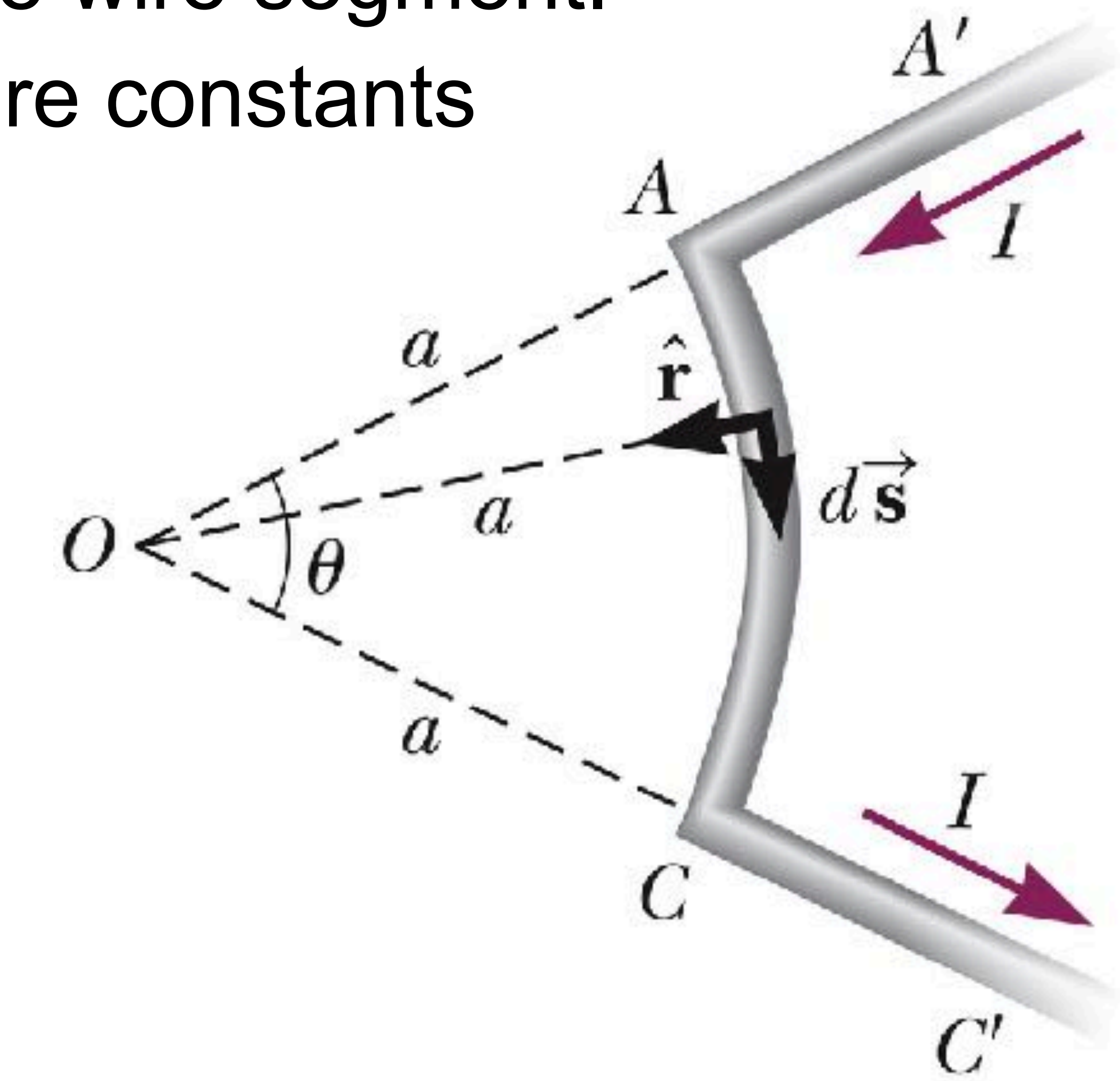
$$\boxed{B = \frac{\mu_o I}{2\pi a}}$$

Magnetic Field for a Curved Wire Segment

Find the field at point O due to the wire segment.

Integrate, remembering I and a are constants

- θ is in radians



$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds}{a^2}$$

$$B = \frac{\mu_0 I}{4\pi a^2}\int ds = \frac{\mu_0 I}{4\pi a^2}s$$

$$\boxed{B = \frac{\mu_0 I}{4\pi a^2}(a\theta) = \frac{\mu_0 I}{4\pi a}\theta}$$

Magnetic Field for a Circular Loop of Wire

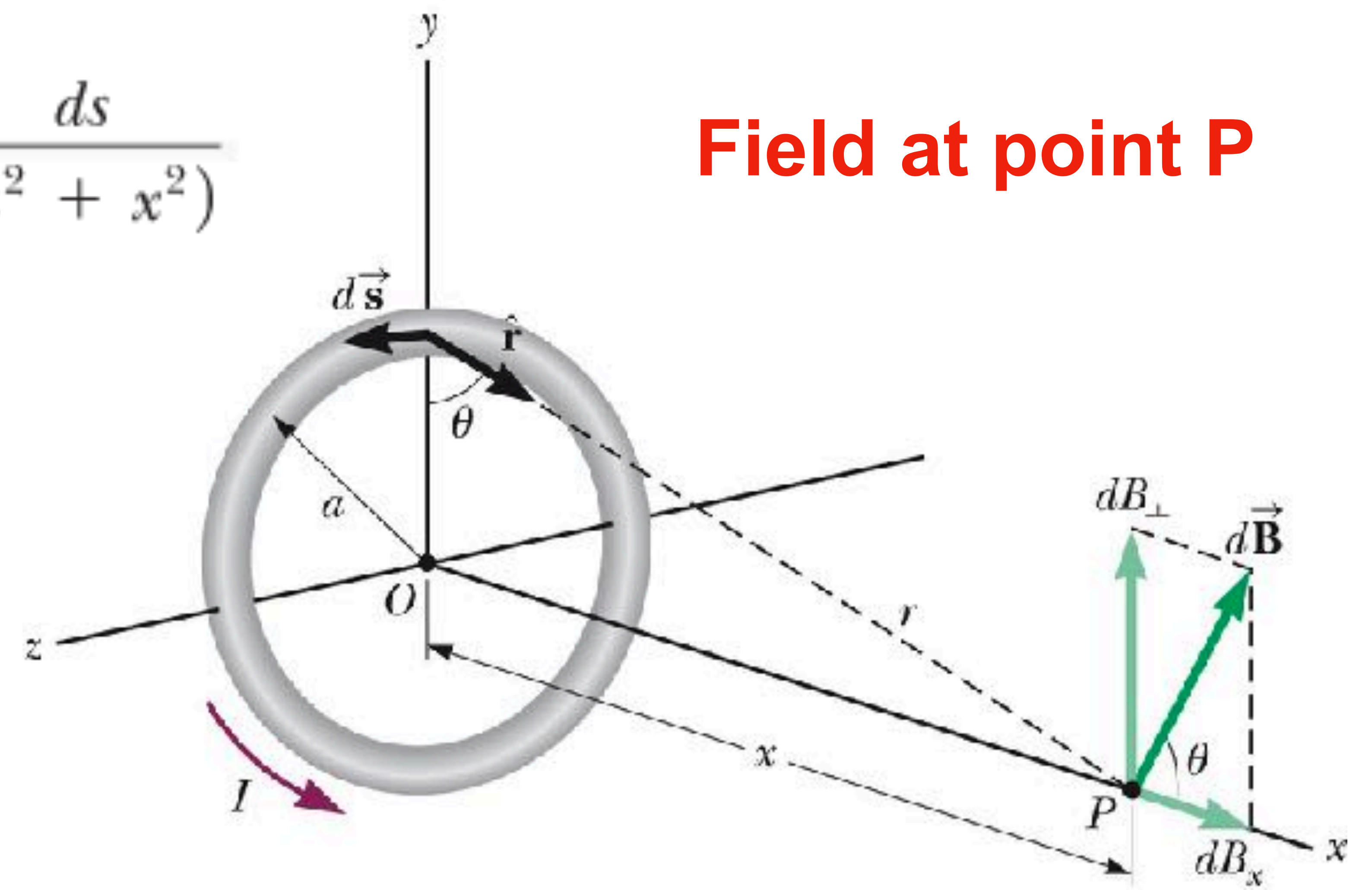
$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2 + x^2)}$$

$$dB_x = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2 + x^2)}\cos\theta$$

$$B_x = \oint dB_x = \frac{\mu_0 I}{4\pi}\oint\frac{ds\cos\theta}{a^2 + x^2}$$

$$\cos\theta = \frac{a}{(a^2 + x^2)^{1/2}}$$

Field at point P



$$B_x = \frac{\mu_0 I}{4\pi}\oint\frac{ds}{a^2 + x^2}\left[\frac{a}{(a^2 + x^2)^{1/2}}\right] = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{3/2}}\oint ds$$

$$\boxed{B_x = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{3/2}}(2\pi a) = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}}$$

Magnetic Field for a Circular Loop of Wire

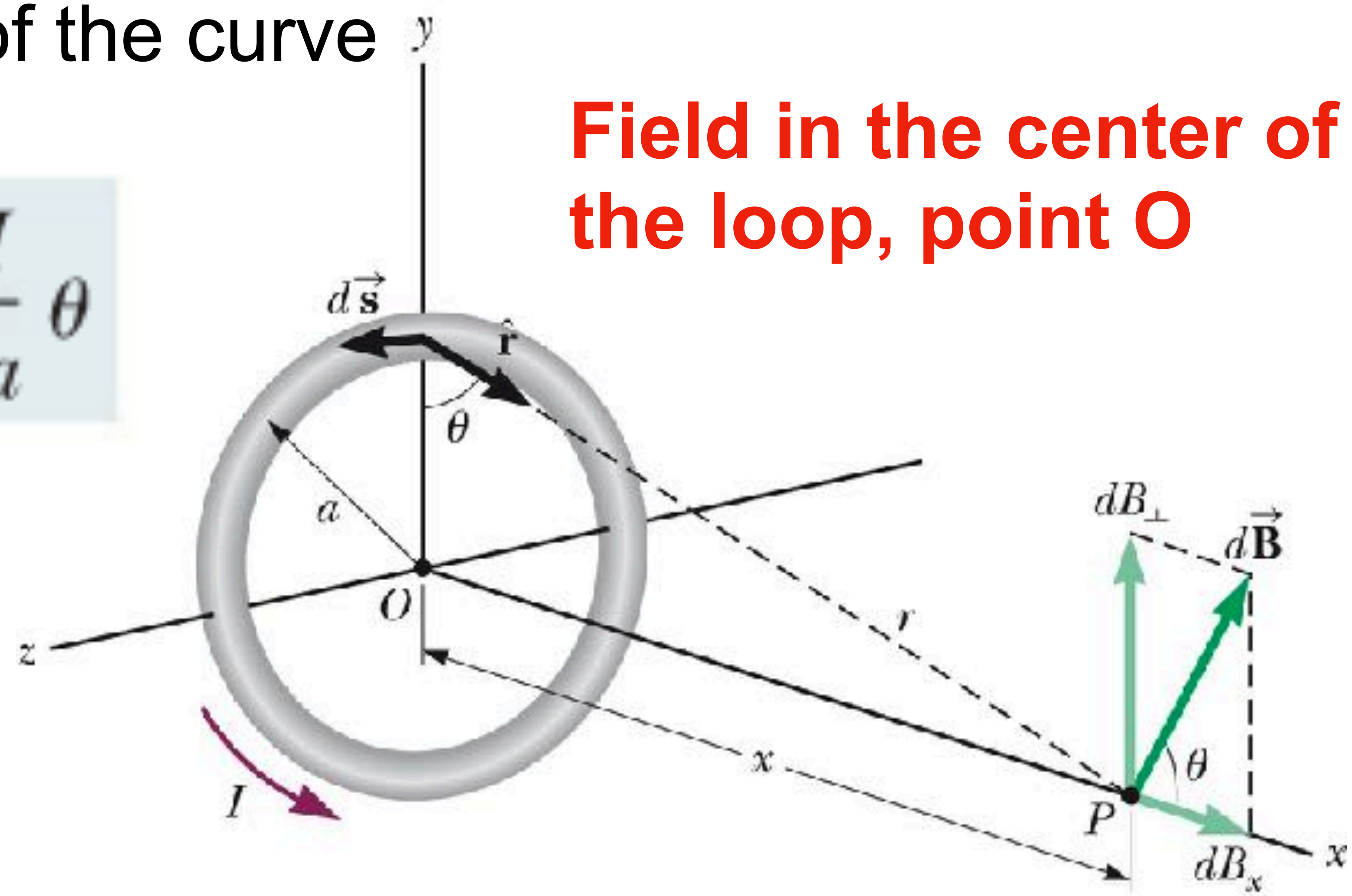
Either go from the results of the curve (slide 11):

$$B = \frac{\mu_0 I}{4\pi a^2}(a\theta) = \boxed{\frac{\mu_0 I}{4\pi a}\theta}$$

- θ = 2π for a full circle

$$B = \frac{\mu_0 I}{4\pi a} 2\pi = \frac{\mu_0 I}{2a}$$

Field in the center of the loop, point O



Or use the result from slide 12 at x = 0:

$$B_x = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2+x^2)^{3/2}}(2\pi a) = \frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}} \boxed{= \frac{\mu_0 I}{2a}}$$

Magnetic Field Lines for a Loop

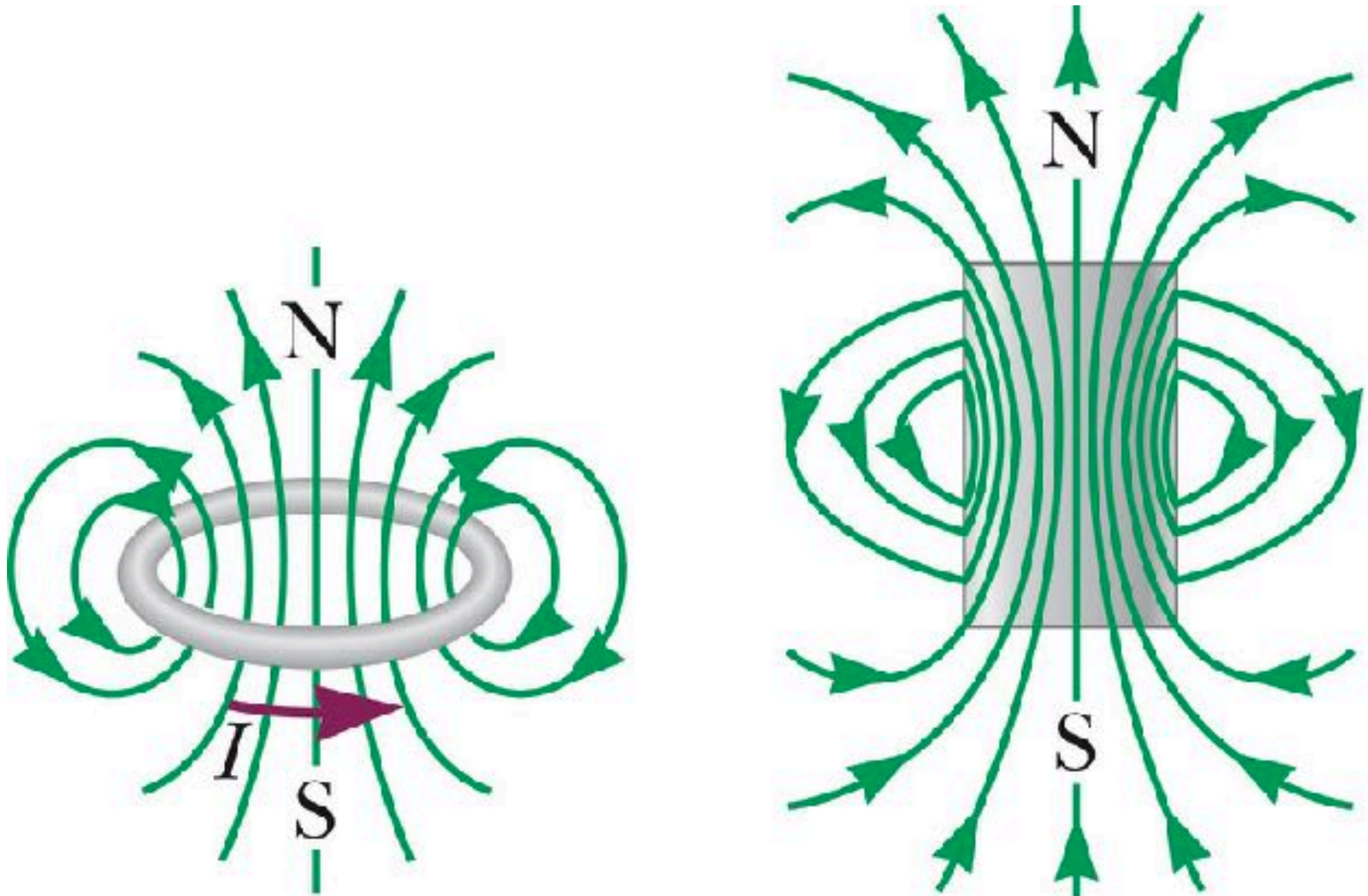


Figure (a) shows the magnetic field lines surrounding a current loop.

Figure (b) compares the field lines to that of a bar magnet.

Notice the similarities in the patterns.

The field created is NOT uniform!

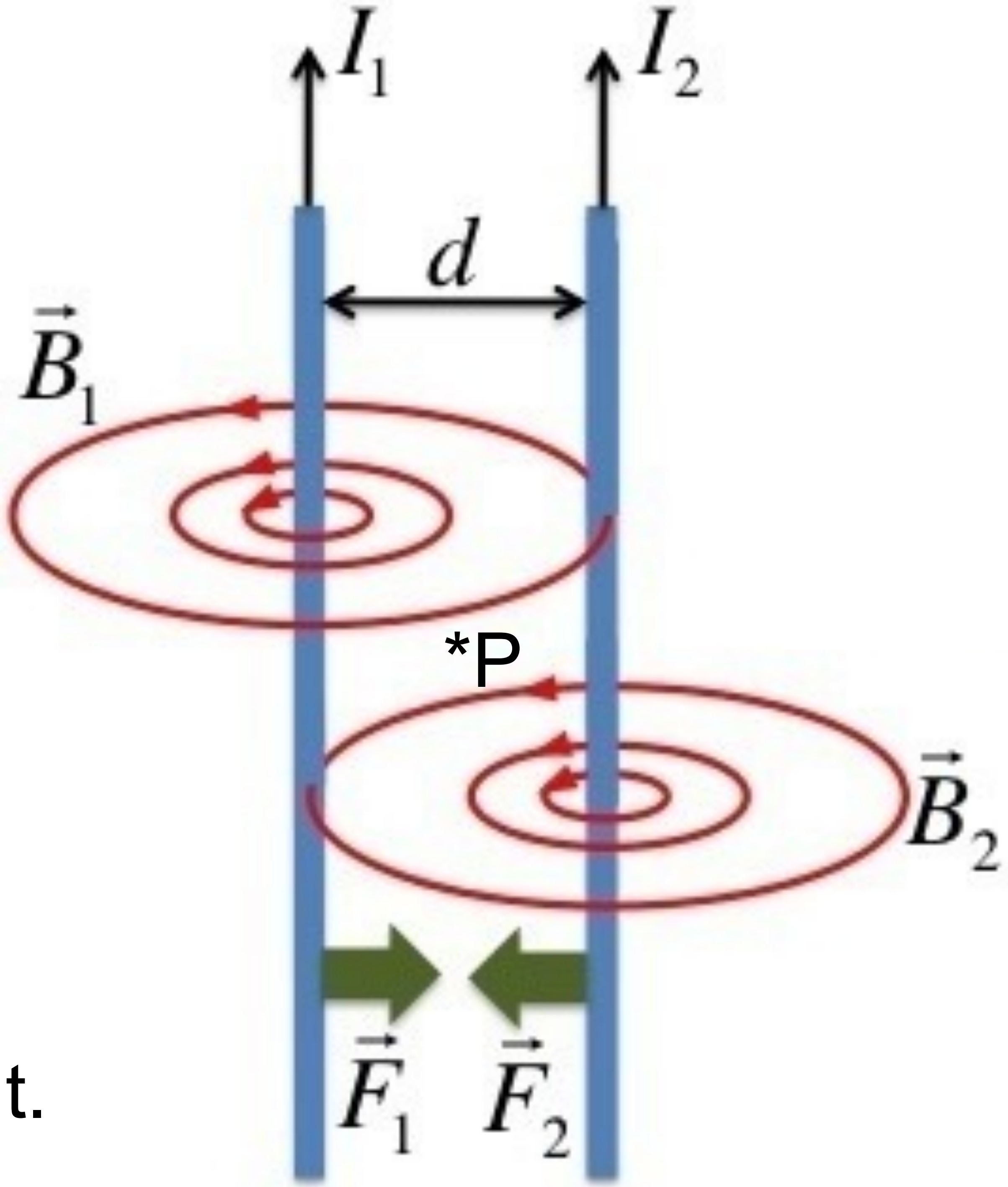
Magnetic Field Between Two Parallel Conductors

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$

Use the right hand rule!

The field at point P midway between the two wires is 0.
The two contributions cancel out.



Magnetic Force Between Two Parallel Conductors

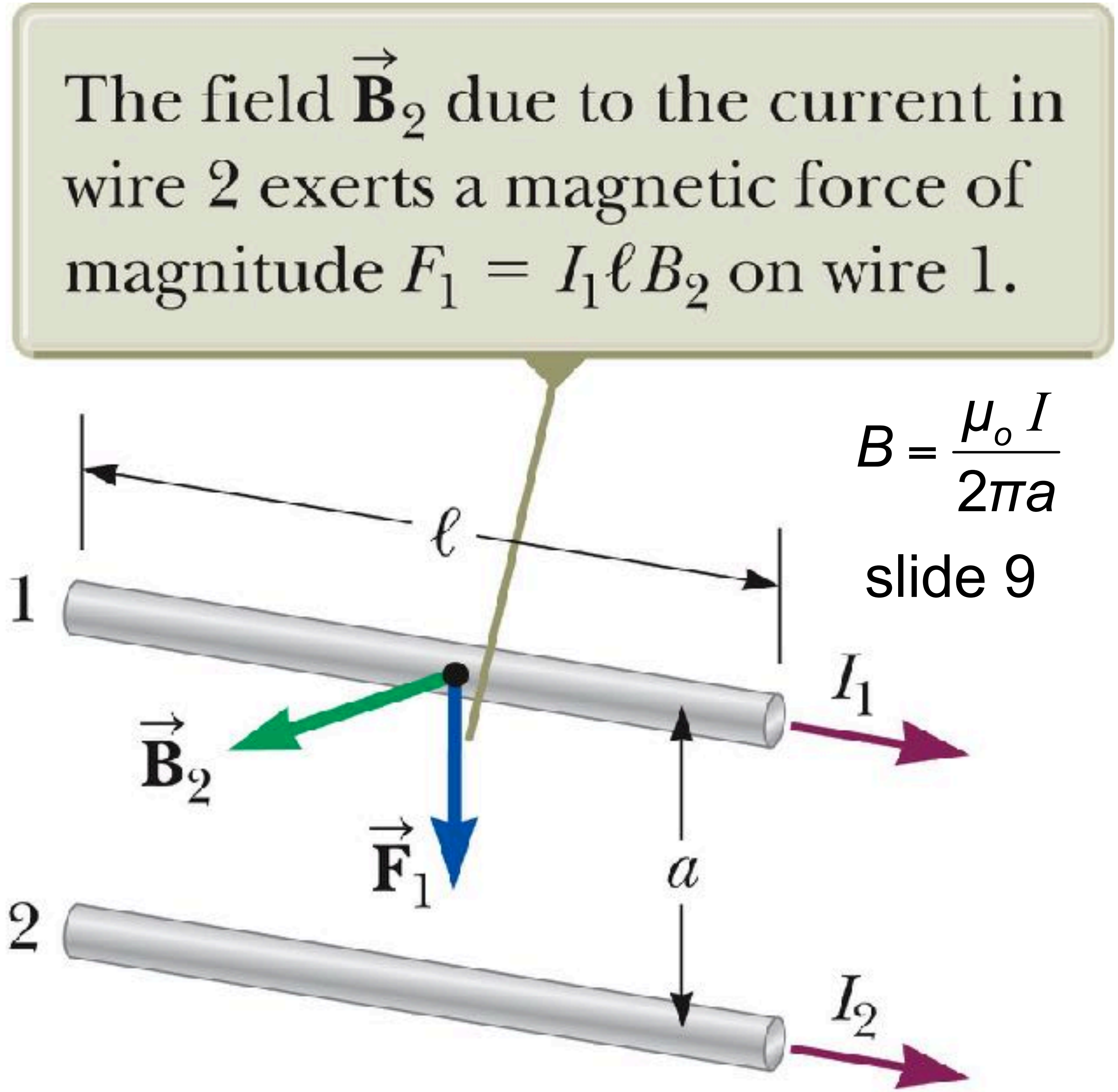
$$\vec{F} = I\vec{L} \times \vec{B}$$

L and B are perpendicular so:

$$F_1 = I_1 \ell B_2$$

$$= I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a} \right)$$

$$\boxed{= \frac{\mu_0 I_1 I_2}{2\pi a} \ell}$$

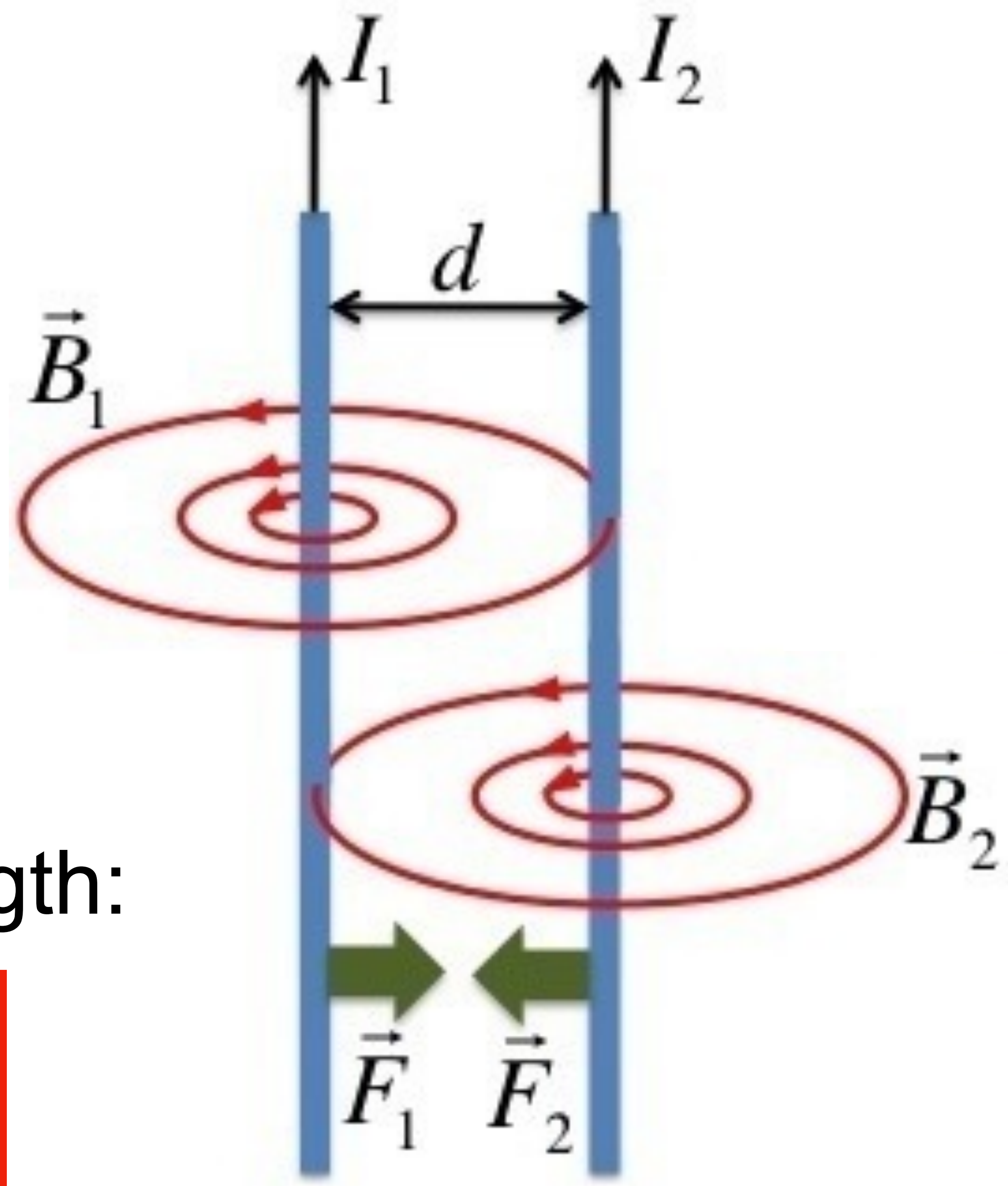


$$B = \frac{\mu_o I}{2\pi a}$$

slide 9

Magnetic Force Between Two Parallel Conductors

- Parallel conductors carrying **currents in the same direction attract** each other.
- Parallel conductors carrying current in opposite directions repel each other.



Often described as the force per unit length:

$$\boxed{\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}}$$

The derivation assumes both wires are long compared with their separation distance.

- Only one wire needs to be long.
- The equations accurately describe the forces exerted on each other by a long wire and a straight, parallel wire of limited length,

Ampere and Coulomb

The force between two parallel wires can be used to define the ampere.

When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A.

The SI unit of charge, the coulomb, is defined in terms of the ampere.

When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C.

Conceptual Questions

1. Can you treat a current as point like, the same way you treat charge?

2. What does Biot-Savart law allow you to do?

3. Is the magnetic field created by a current loop uniform?

Conceptual Questions

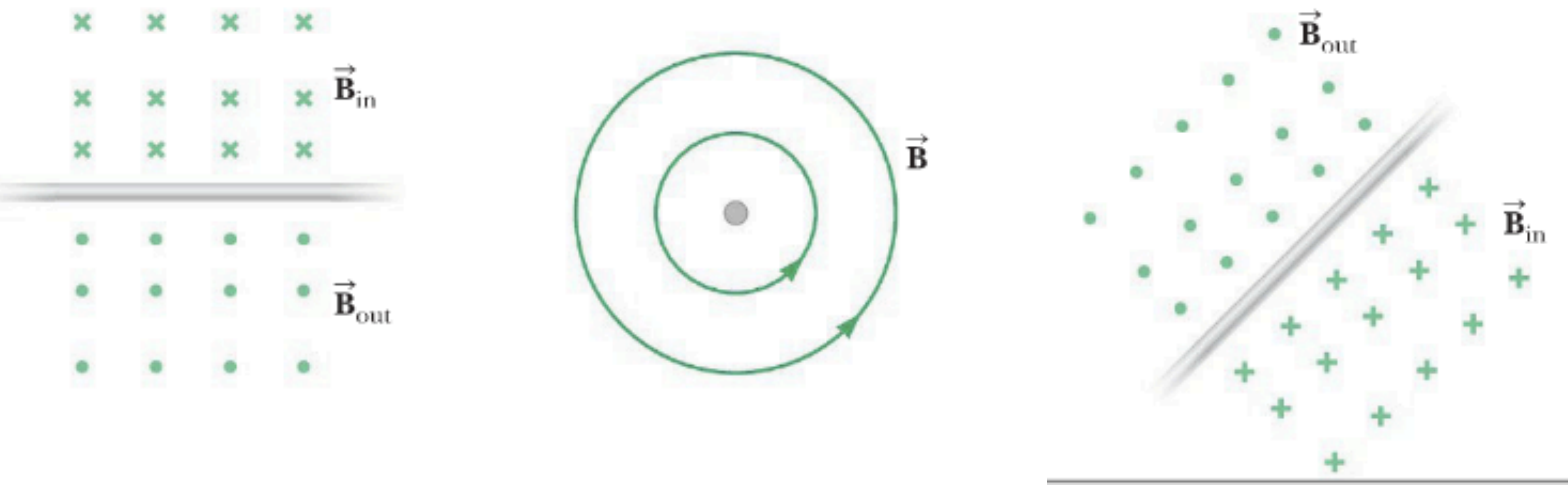
1. Can you treat a current as point like, the same way you treat charge? NO!

2. What does Biot-Savart law allow you to do? Calculate the field due to current distribution

3. Is the magnetic field created by a current loop uniform? No. Slide 14

Example problem #1

2. In each of parts (a) through (c) of Figure P30.2, find the direction of the current in the wire that would produce a magnetic field directed as shown.



Example problem #1: Solution

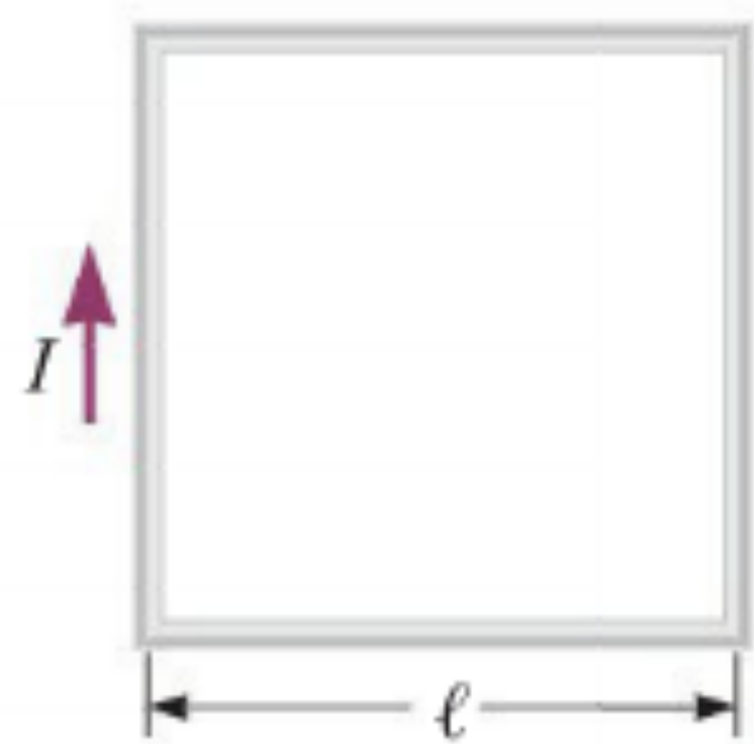
Imagine grasping the conductor with the right hand so the fingers curl around the conductor in the direction of the magnetic field. The thumb then points along the conductor in the direction of the current. The results are

(a) $\boxed{\text{toward the left}}$ (b) $\boxed{\text{out of the page}}$ (c) $\boxed{\text{lower left to upper right}}$

stop here?

Example problem #2

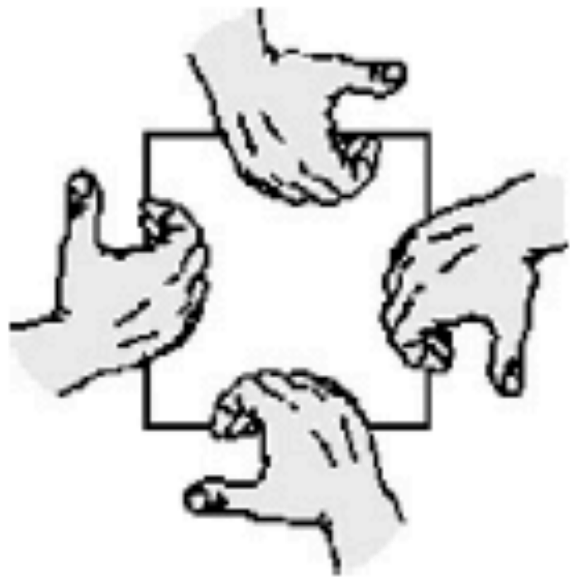
5. M (a) A conducting loop in the shape of a square of edge length $\ell = 0.400$ m carries a current $I = 10.0$ A as shown in Figure P30.5. Calculate the magnitude and direction of the magnetic field at the center of the square. (b) **What If?** If this conductor is reshaped to form a circular loop and carries the same current, what is the value of the magnetic field at the center?



Example problem #2: Solution

(a) Use Equation 30.4 for the field produced by each side of the square.

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$



where $\theta_1 = 45.0°$, $\theta_2 = -45.0°$, and $a = \frac{\ell}{2}$

Each side produces a field into the page. The four sides altogether produce

$$B_{\text{center}} = 4B = 4\frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

$$= \frac{\mu_0 I}{\pi \ell/2}[\sin 45.0° - \sin(-45.0°)]$$

$$= \frac{2\mu_0 I}{\pi \ell}\left[\frac{2}{\sqrt{2}}\right] = \frac{2\sqrt{2}\mu_0 I}{\pi \ell}$$

$$B = \frac{2\sqrt{2}(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(10.0\ \text{A})}{\pi(0.400\ \text{m})}$$

$$= 2\sqrt{2} \times 10^{-5}\ \text{T} = \boxed{28.3\ \mu\text{T into the page}}$$

Example problem #2: Solution

(b) For a single circular turn with $4\ell = 2\pi R$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 \pi I}{4\ell} = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)(10.0\ \mathrm{A})}{4(0.400\ \mathrm{m})}$$

$$= \boxed{24.7\ \mu\mathrm{T}\ \text{into the page}}$$

Example problem #3

There is no homework due on 3/14. No quiz on that day.
When's your midterm exam? (note - *not* OPEN book)

Example problem #3: Solution

Thursday, March 14: in this room, taking up the entire class slot (to 10:20)
* Between 5 and 10 problems to work out (Part I). Max 1-2 on magnetism
* About 10 multiple-choice / TvF / matching problems (Part II)
* ~1-3 essay questions (Part III) by which I mean short answer.

Will include all material we have covered so far EXCEPT FOR no math problems from Ch. 29 (this one)

Bring: 1 calculator (NOT laptop or phone or tablet), 1 pen or pencil, and a formula sheet you write yourself (hint: look at all red boxes from lecture!)
One standard 8.5x11” sheet. Both sides. Any font size, anything you want
Constants will be provided to you, but formulas won’t be provided at all
UNLESS not a part of our class (e.g., geometry: volume of a sphere, etc.)

Example problem #4

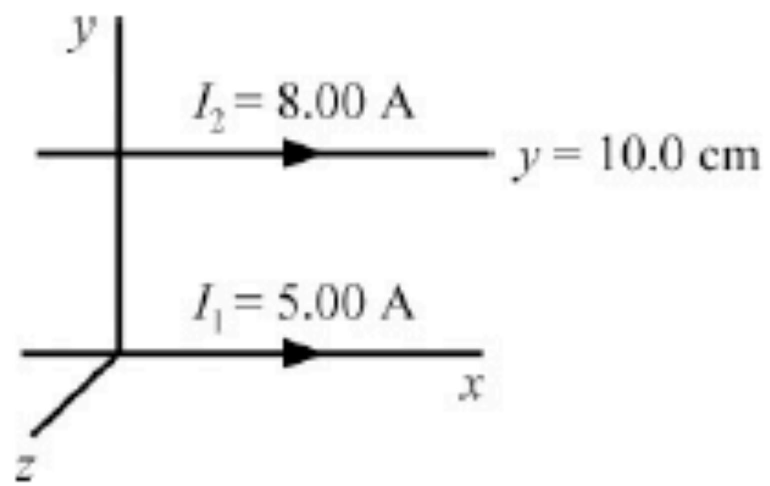
21. Two long, parallel conductors, separated by 10.0 cm, **W** carry currents in the same direction. The first wire carries a current $I_1 = 5.00$ A, and the second carries $I_2 = 8.00$ A. (a) What is the magnitude of the magnetic field created by I_1 at the location of I_2? (b) What is the force per unit length exerted by I_1 on I_2? (c) What is the magnitude of the magnetic field created by I_2 at the location of I_1? (d) What is the force per length exerted by I_2 on I_1?

Example problem #4: Solution

Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

(a) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\hat{\mathbf{k}} = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(5.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \boxed{1.00 \times 10^{-5}\ \mathrm{T\ out\ of\ the\ page}}$$



Example problem #4: Solution

(b) $\vec{\mathbf{F}}_B = I_2\vec{\ell}\times\vec{\mathbf{B}} = (8.00\text{ A})\left[(1.00\text{ m})\hat{\mathbf{i}}\times\left(1.00\times10^{-5}\text{ T}\right)\hat{\mathbf{k}}\right]$

$$= \left(8.00\times10^{-5}\text{ N}\right)\left(-\hat{\mathbf{j}}\right)$$

ANS. FIG. P30.21(b)

$$\vec{\mathbf{F}}_B = \boxed{8.00\times10^{-5}\text{ N toward the first wire}}$$

(c) $\vec{\mathbf{B}} = \dfrac{\mu_0 I}{2\pi r}\left(-\hat{\mathbf{k}}\right) = \dfrac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(8.00\text{ A})}{2\pi(0.100\text{ m})}\left(-\hat{\mathbf{k}}\right)$

$$= \left(1.60\times10^{-5}\text{ T}\right)\left(-\hat{\mathbf{k}}\right)$$

ANS. FIG. P30.21(c)

$$\vec{\mathbf{B}} = \boxed{1.60\times10^{-5}\text{ T into the page}}$$

Example problem #4: Solution

(d) $\vec{\mathbf{F}}_B = I_1 \vec{\ell} \times \vec{\mathbf{B}} = (5.00\text{ A})\left[(1.00\text{ m})\hat{\mathbf{i}} \times (1.60 \times 10^{-5}\text{ T})(-\hat{\mathbf{k}})\right]$

$= (8.00 \times 10^{-5}\text{ N})(+\hat{\mathbf{j}})$

$\vec{\mathbf{F}}_B = \boxed{8.00 \times 10^{-5}\text{ N towards the second wire}}$

Example problem #5

There is V for electric field E. Does magnetic field B have an associated potential?

Example problem #5: Solution

YES, but unlike Voltage it is a VECTOR not a SCALAR. It is called the vector potential, A. (As a 4-vector in GR, V plays the role of time from space-time)

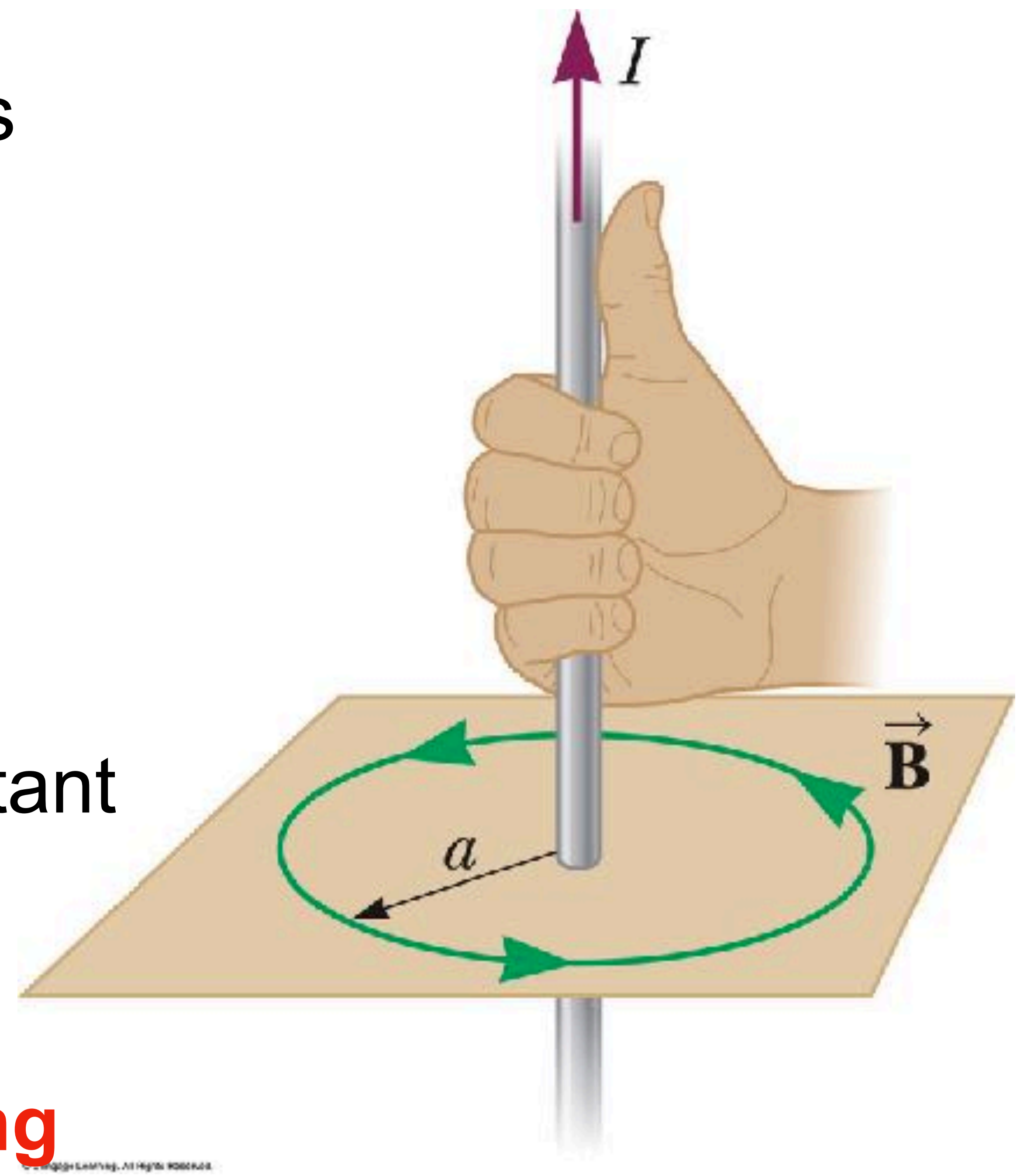
Magnetic Field for a Long, Straight Conductor: Direction

The magnetic field lines are circles concentric with the wire.

The field lines lie in planes perpendicular to the wire.

The magnitude of the field is constant on any circle of radius *a.*

The right-hand rule for **determining the direction of the field** is shown.



Ampere's Law

- Remembering that the magnetic field in a long, straight current carrying conductor is:

$$B = \frac{\mu_0 I}{2\pi r}$$

- This equation is only valid for long straight wires. In general the relationship between current in a wire of any shape, and its magnetic field around it was derived by Andre Marie Ampere.

- For any arbitrary closed path around a current enclosed by the area of the closed path:

$$\boxed{\oint \vec{B} \bullet d\vec{l} = \mu_0 \overrightarrow{I_{encl}}}$$

Ampere's Law

Ampère's law is useful in calculating the magnetic field of a **highly symmetric configuration carrying a steady current.**

magnetic field

small length element along the path

permeability of free space

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \vec{I}_{encl}$$

integral over a closed path

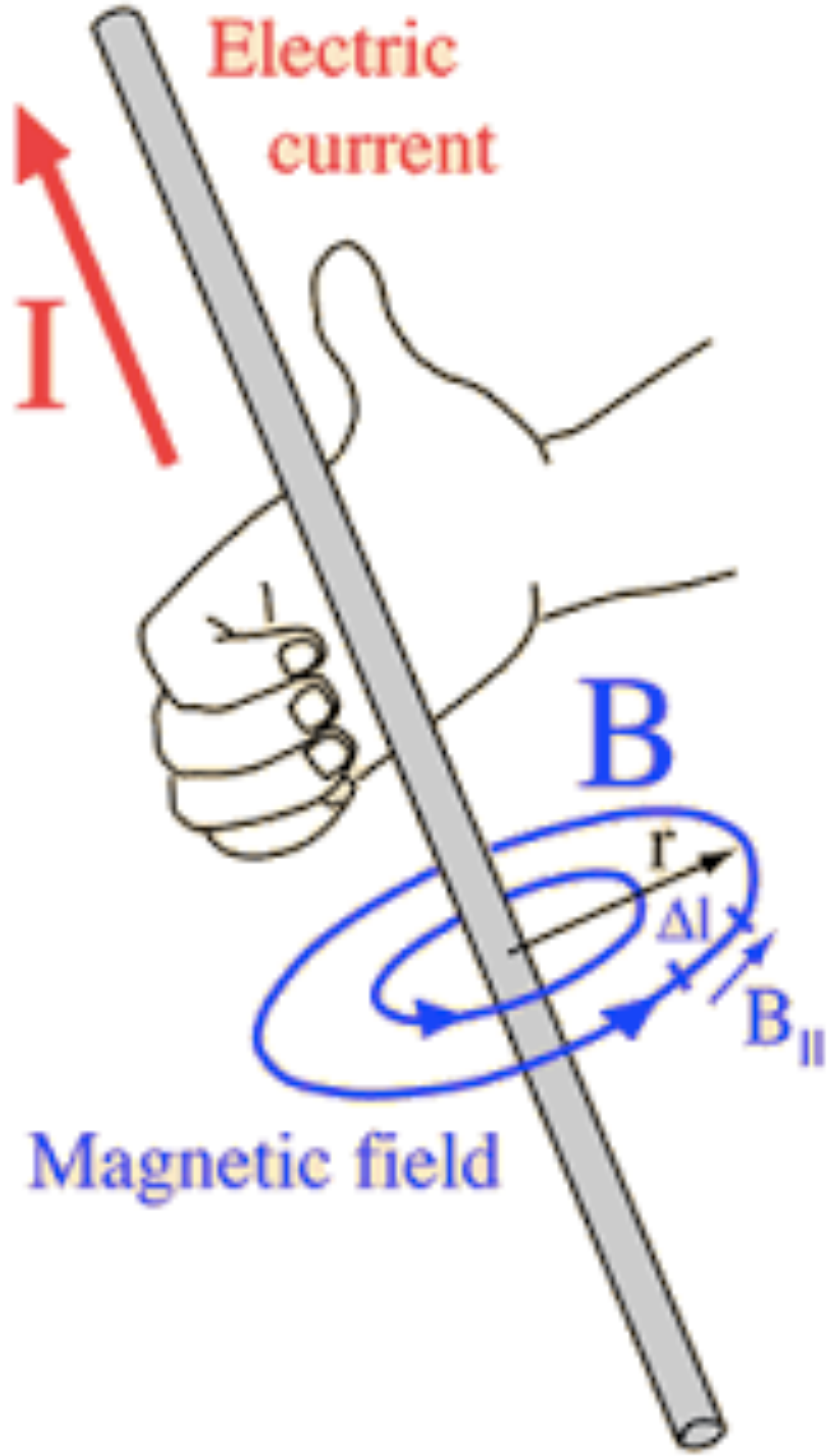
total current through any surface bounded by the closed path

Ampere's law describes the creation of magnetic fields by all **continuous** current configurations.

Put the thumb of your right hand in the direction of the current through the amperian loop and your fingers curl in the **direction you should integrate around the loop.**

Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \vec{I}_{encl}$$

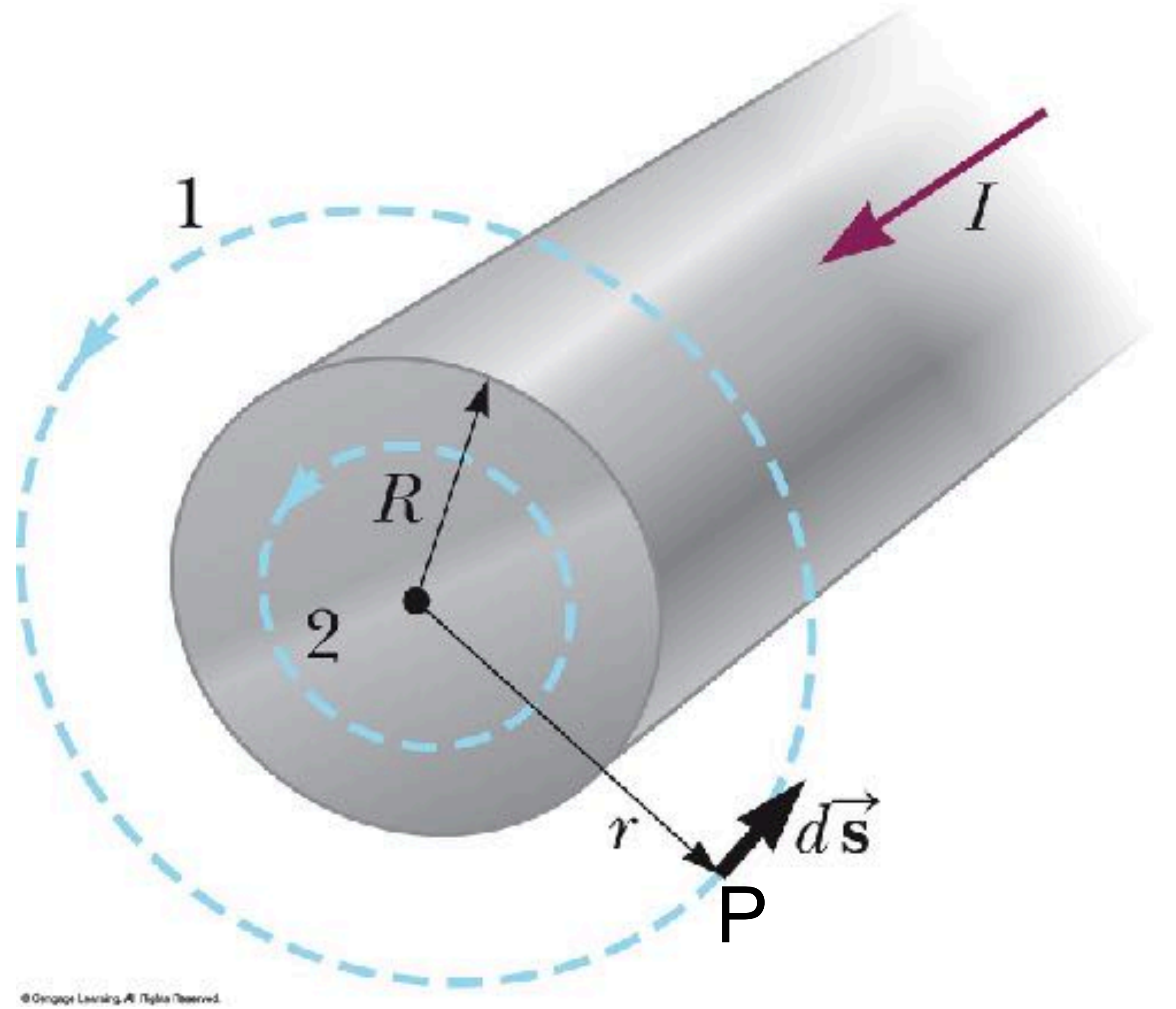


Field Due to a Long Straight Wire – From Ampere's Law

Calculate B at point P.

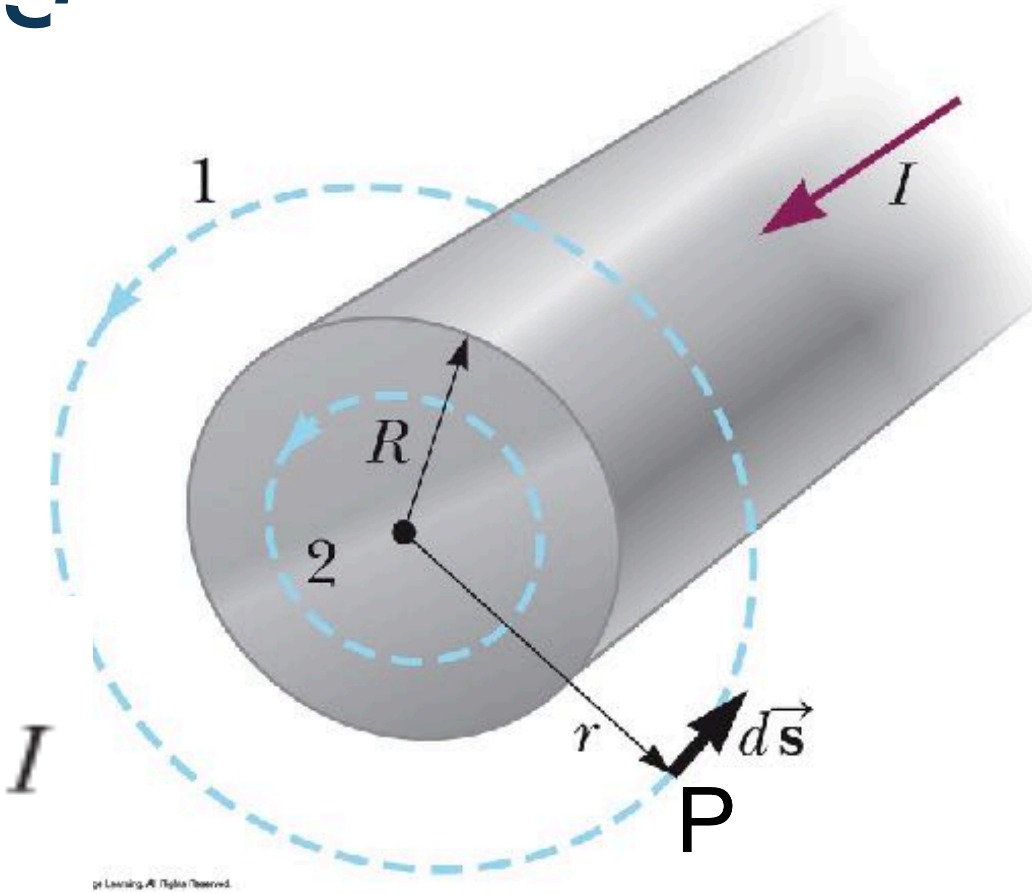
The current is **uniformly distributed through the cross section of the wire.**

Since the wire has a high degree of symmetry, the problem can be categorized as a Ampère's Law problem.



- For $r \geq R$, this should be the same result as obtained from the Biot-Savart Law.

Field Due to a Long Straight Wire – From Ampere's Law



Outside of the wire, r > R:

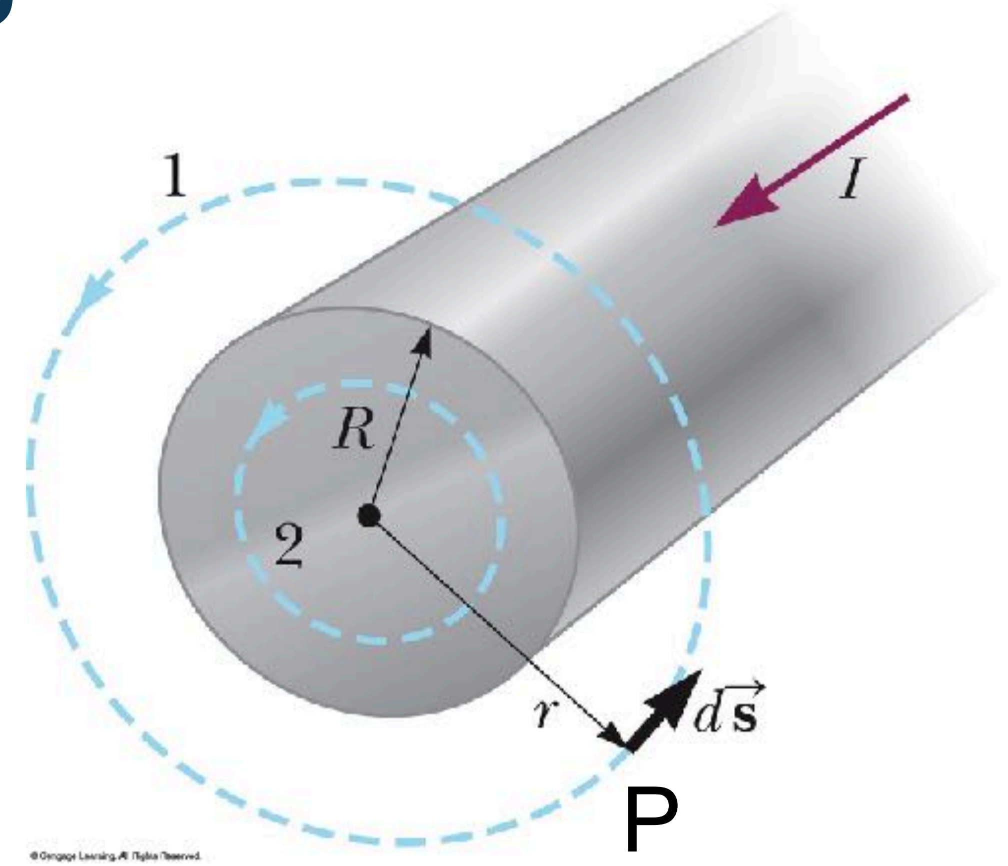
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r) = \mu_0 I$$

$$B = \boxed{\frac{\mu_0 I}{2\pi r}} \quad (\text{for } r \geq R)$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \vec{I}_{encl}$$

This is what we found with Biot Savart!

Field Due to a Long Straight Wire – From Ampere's Law

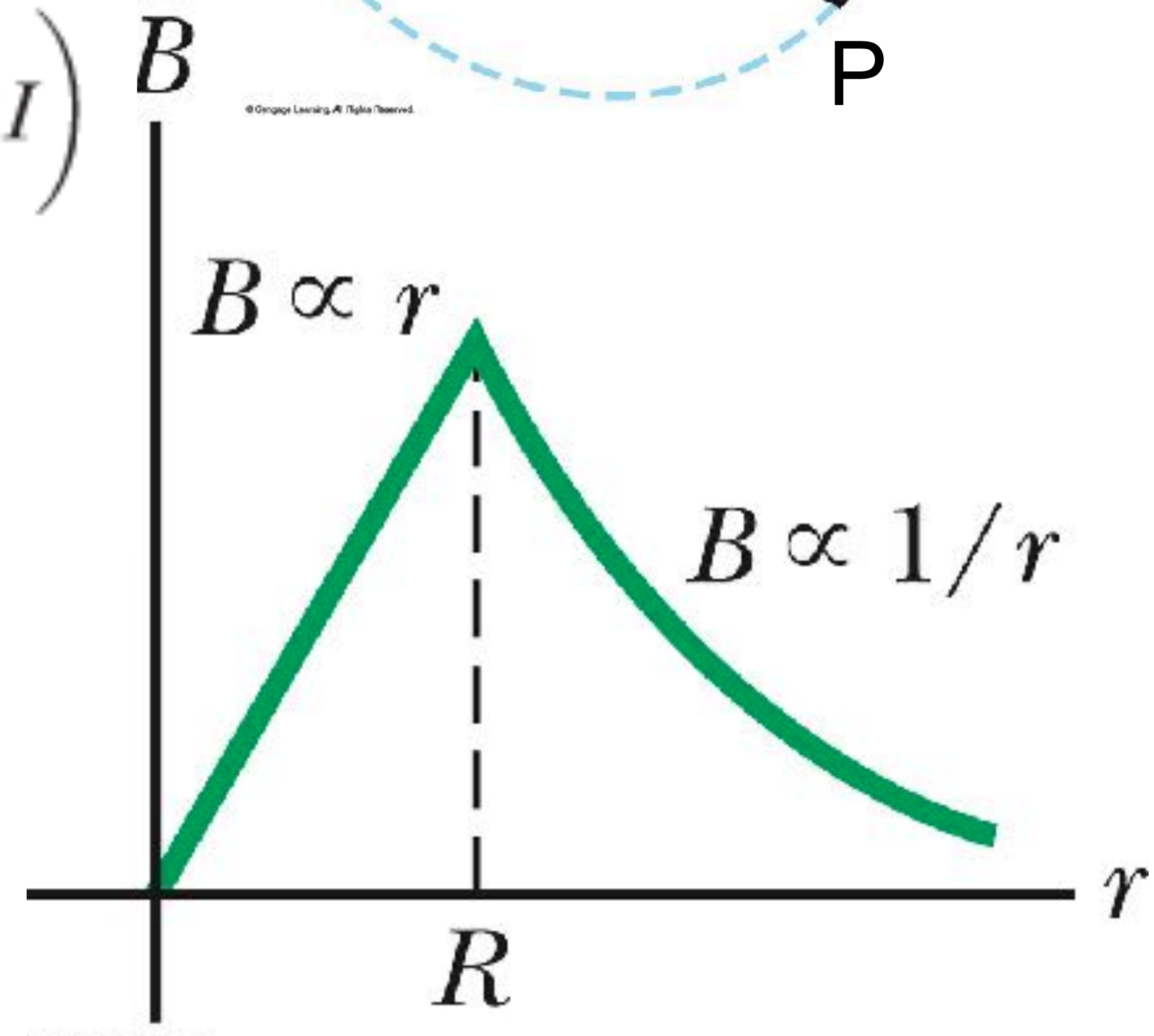


Inside the wire, we need I', the **current inside the amperian circle.**

$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2} \qquad I' = \frac{r^2}{R^2} I$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I\right)$$

$$B = \boxed{\left(\frac{\mu_0 I}{2\pi R^2}\right) r} \qquad (\text{for } r < R)$$



The field is proportional to r inside the wire.

The field varies as 1/r outside the wire.

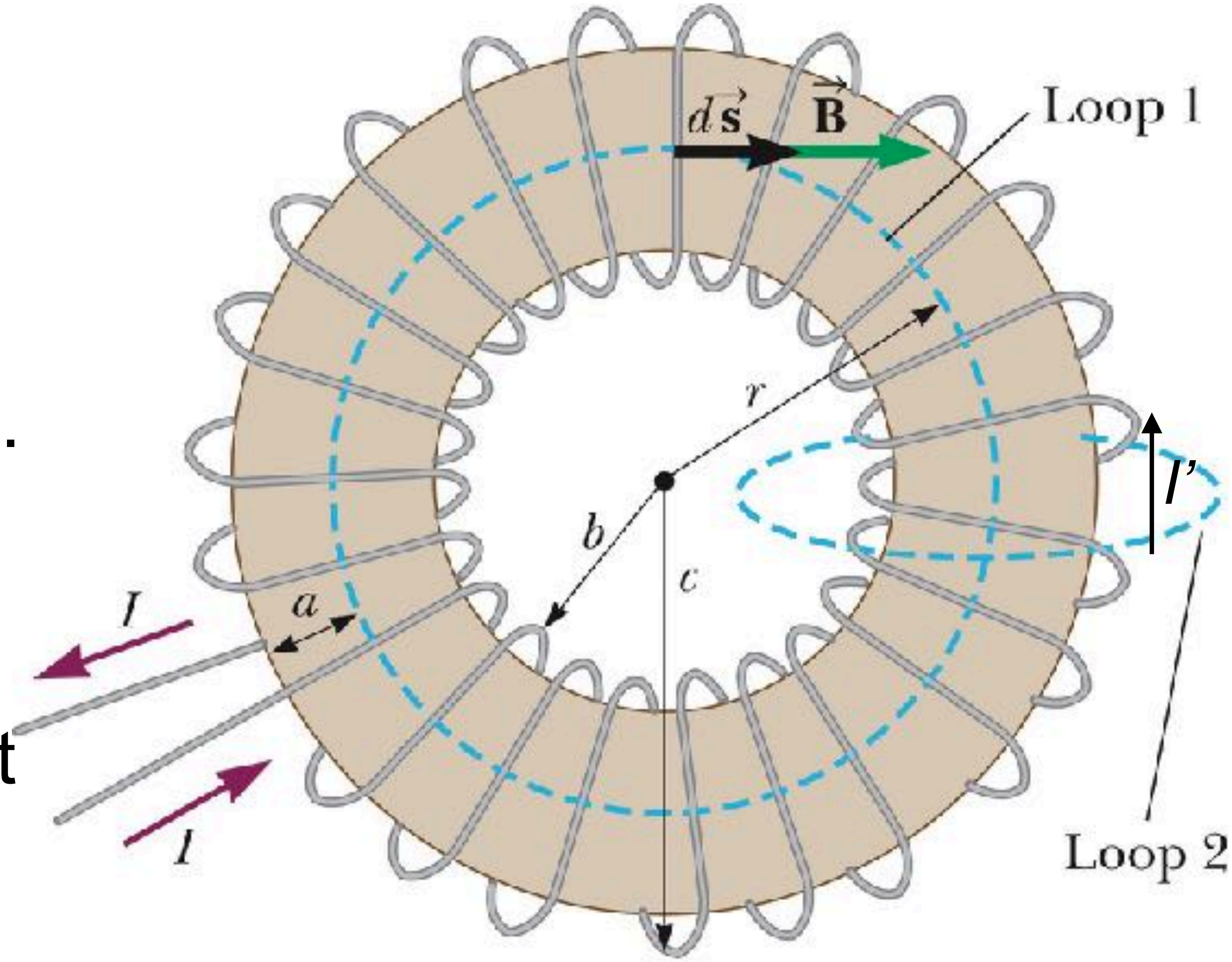
Both equations are equal at r = R.

Magnetic Field of a Toroid

Find the field at a point at distance r from the center of the toroid.

The toroid has N turns of wire.

The magnitude of the field is constant on loop 1 (by symmetry) and tangent to it at all points.



$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r) = \mu_0 N I$$

$$B = \boxed{\frac{\mu_0 N I}{2\pi r}}$$

The wire passes through the loop N times.

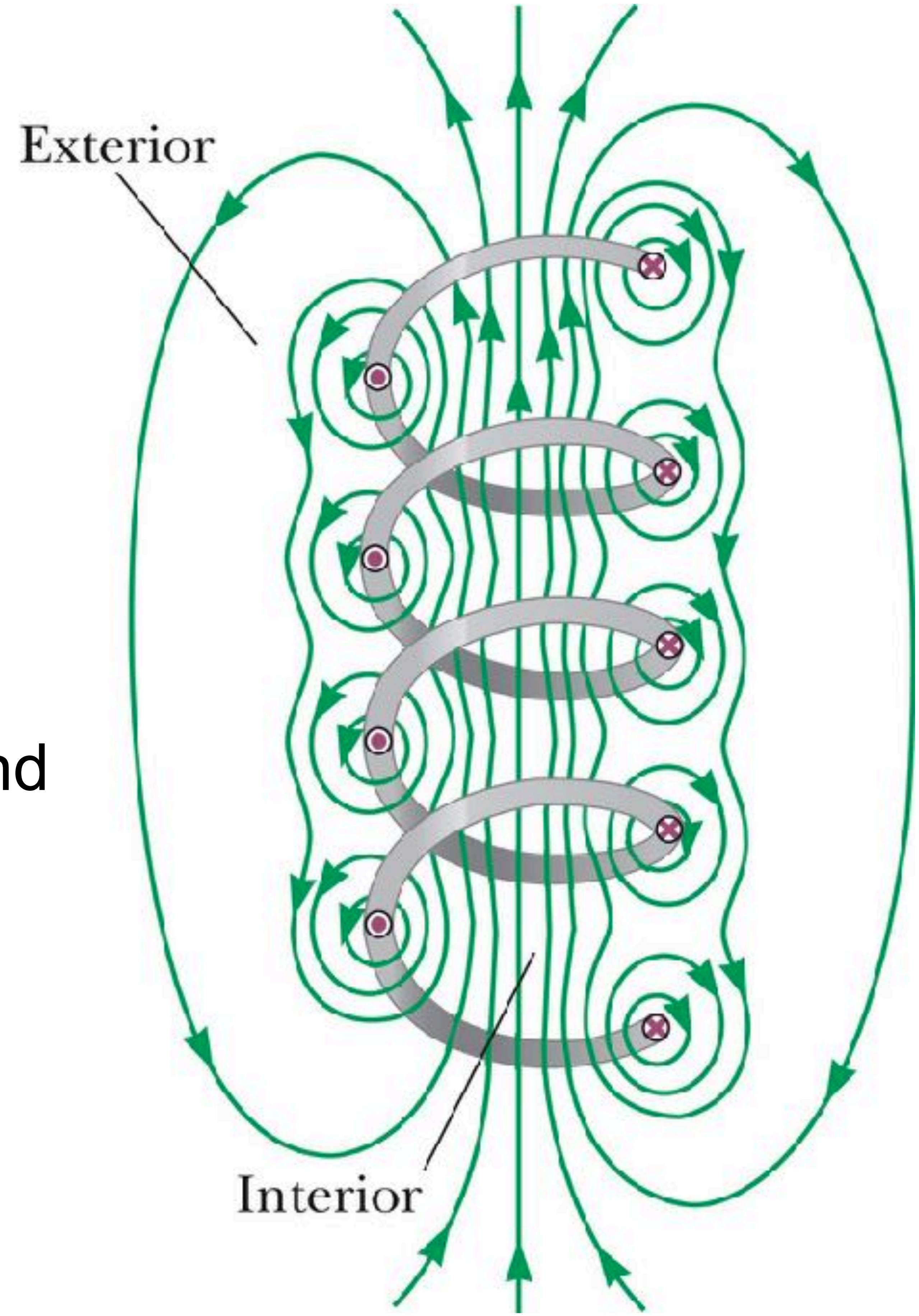
Current forms collection of loops (slide 11)

Magnetic Field of a Solenoid

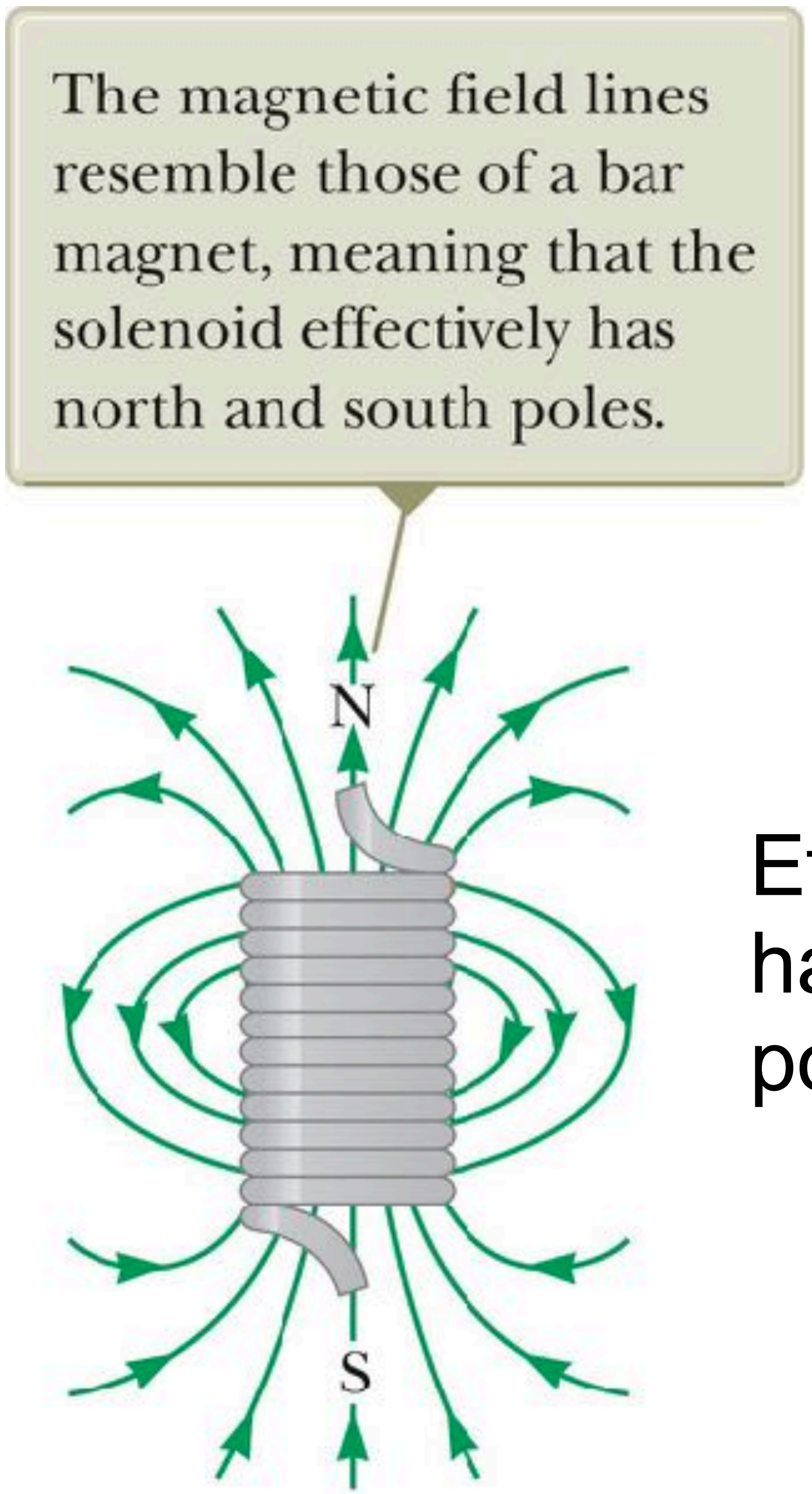
A solenoid is a long wire wound in the form of a helix.

The field lines in the interior are
- Nearly parallel to each other
- Uniformly distributed
- Close together

This indicates the field is strong and almost uniform.



Magnetic Field of a Tightly Wound Solenoid

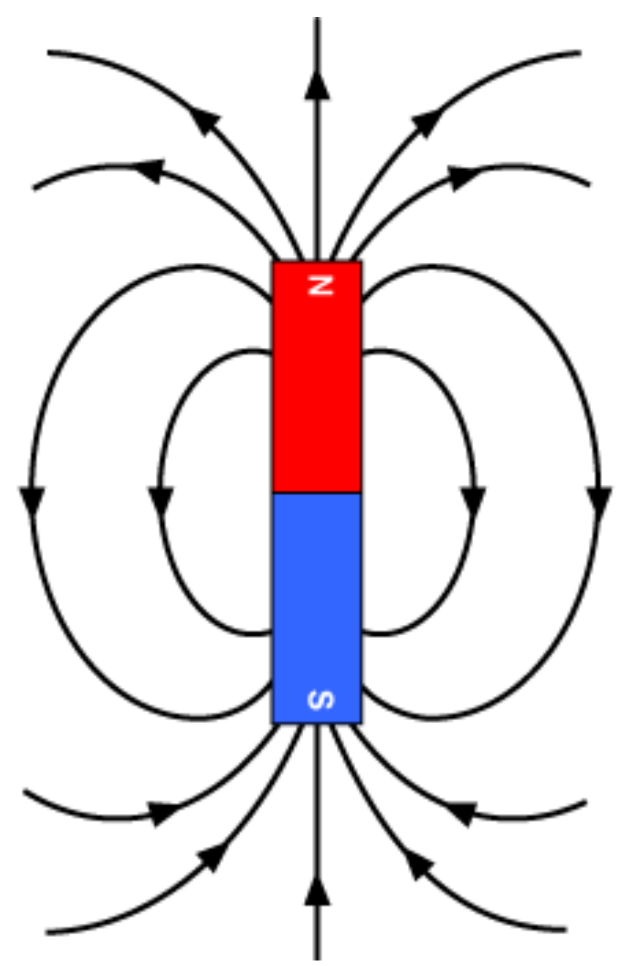


The field distribution is similar to that of a bar magnet.

As the length of the solenoid increases,

- The interior field becomes more uniform.
- The exterior field becomes weaker.

Effectively the solenoid has a north and south pole.



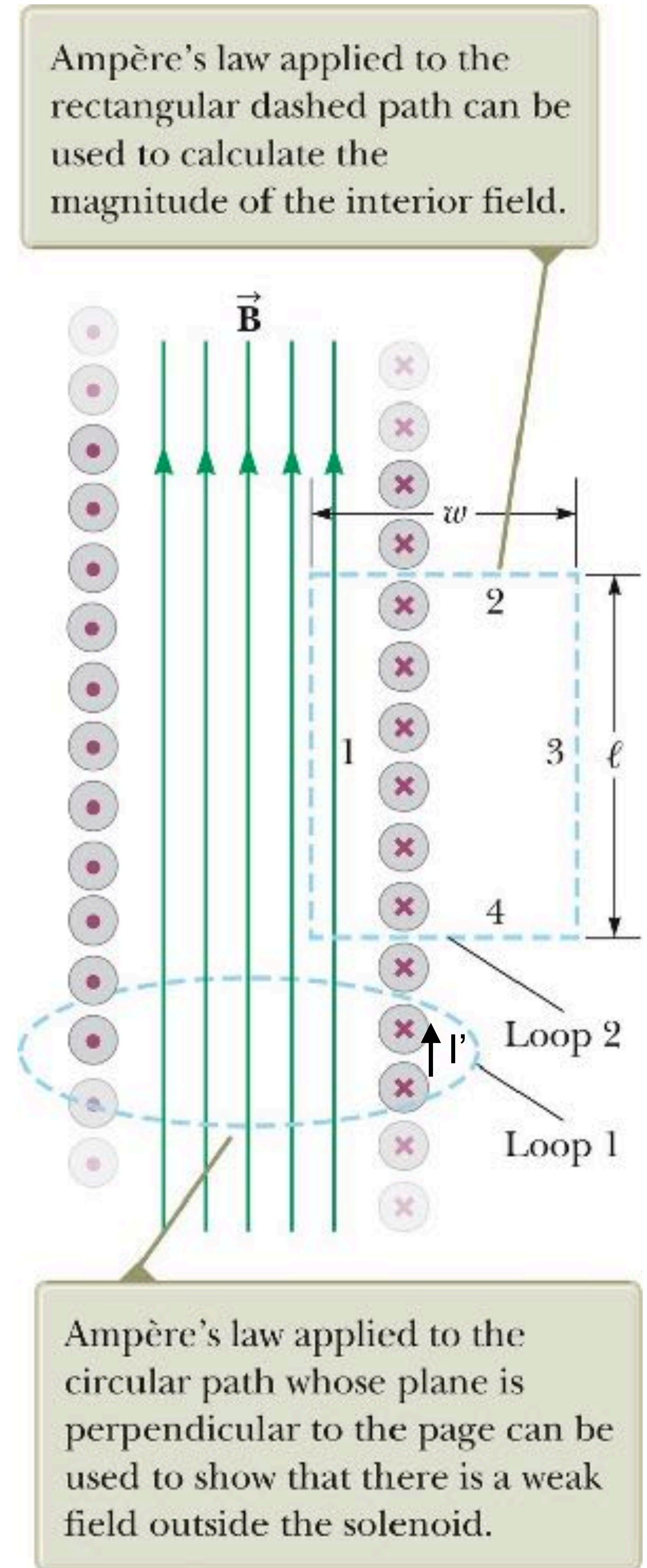
Ideal Solenoid

An ideal solenoid is approached when:

- The turns are closely spaced.
- The length is much greater than the radius of the turns.

2 B fields:

- the external field: it is due to the current moving from coil to coil (loop 1). It is a very weak field with circular field lines, and you can use Ampere's law on that loop to calculate the field
- the field in the interior of the solenoid. You can use the rectangular amperian loop to calculate this field.



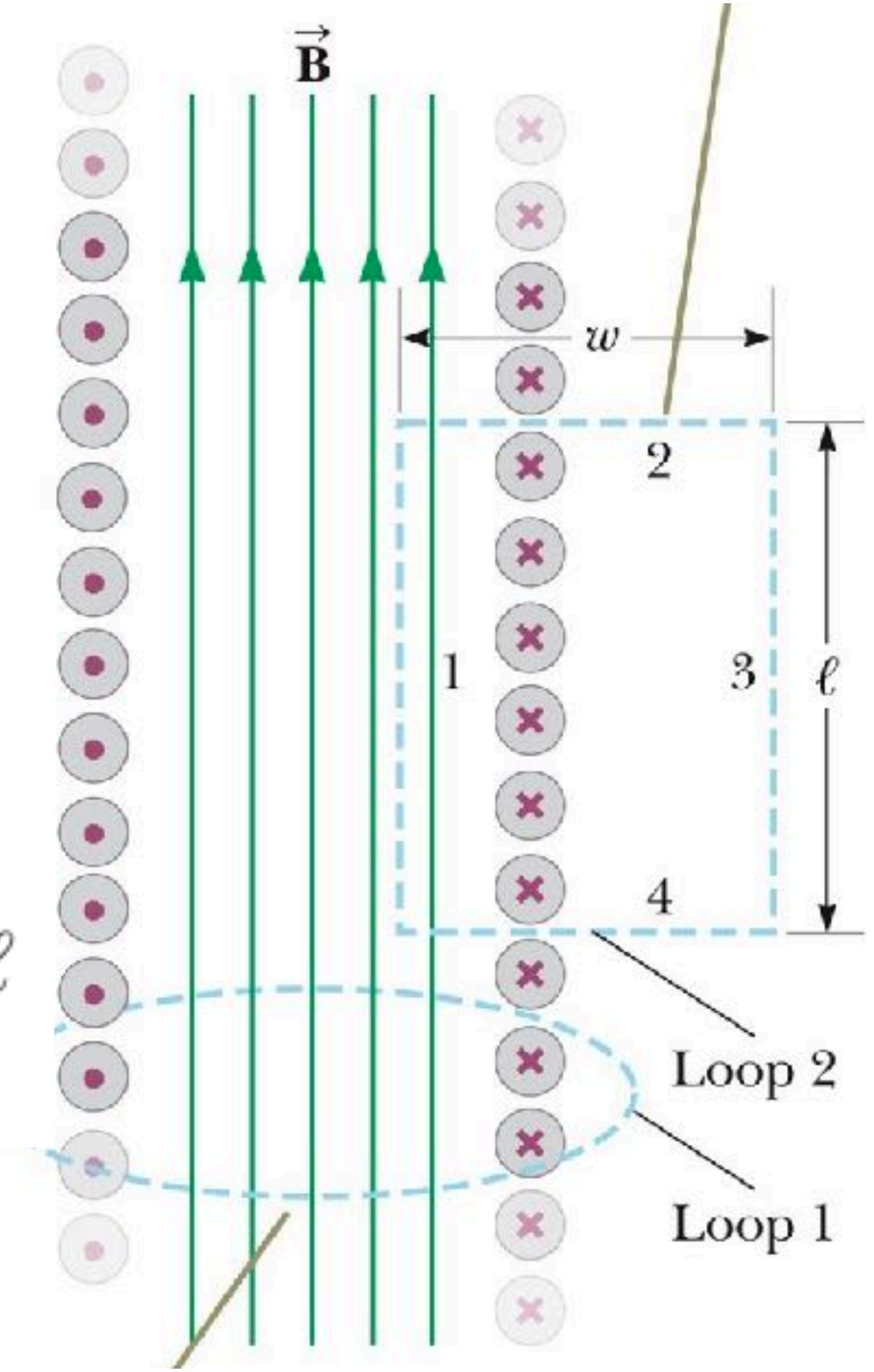
Ampere's Law Applied to a Solenoid

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \vec{I}_{encl}$$

Apply Ampere's law on each side:
Side 2 and 4: B and dl are perpendicular, so the dot product is 0
Side 3: the field here is the external field. B and dl are perpendicular.

Side 1: B and dl are parallel

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{\text{path 1}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \int_{\text{path 1}} ds = B\ell$$



Ampere's Law Applied to a Solenoid

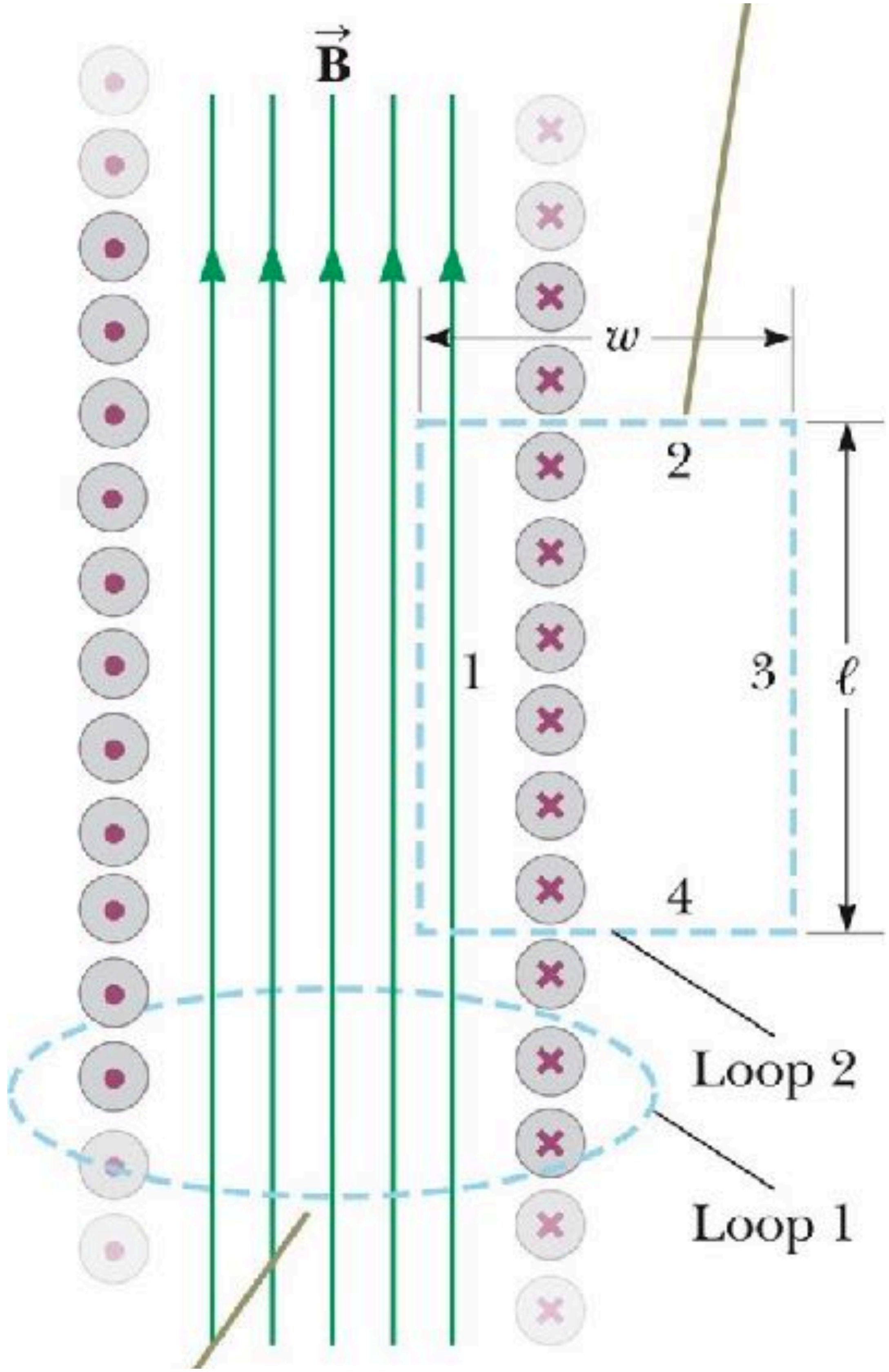
The total current through the rectangular path equals the current through each turn multiplied by the number of turns.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 NI$$

$$\boxed{B = \mu_0 \frac{N}{\ell} I = \mu_0 n I}$$

- $n = N / \ell$ is the number of turns per unit length.

This is valid only at points near the center of a very long solenoid.

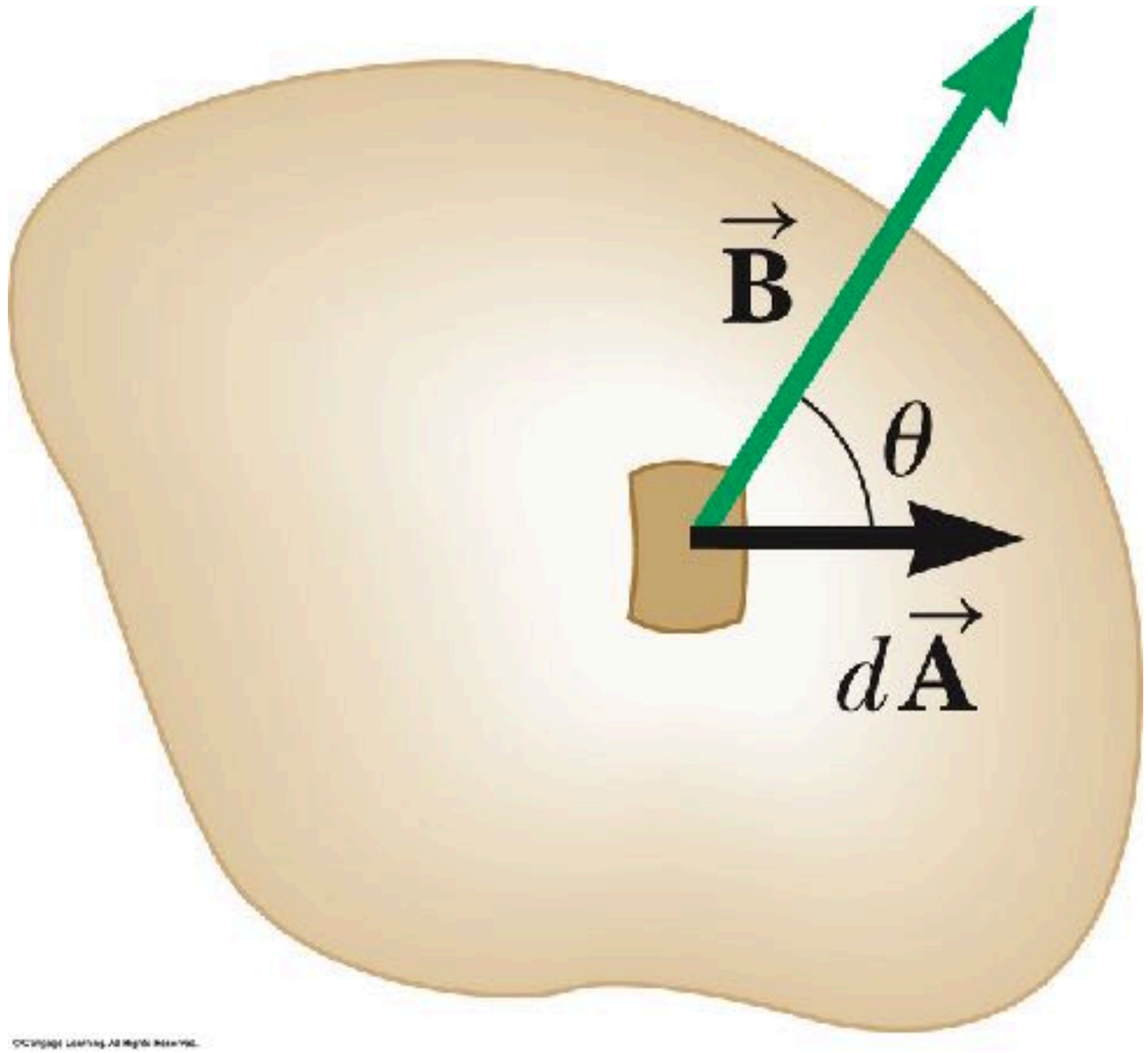


Magnetic Flux

The magnetic flux associated with a magnetic field is defined in a way similar to electric flux.

Consider an area element dA on an arbitrarily shaped surface.

d**A** is a vector that is **perpendicular to the surface** and has a magnitude equal to the area dA = normal to the surface.



The magnetic flux Φ_B is the amount of magnetic field going through the surface:

$$\boxed{\phi_B = \int \vec{B} \cdot d\vec{A}}$$

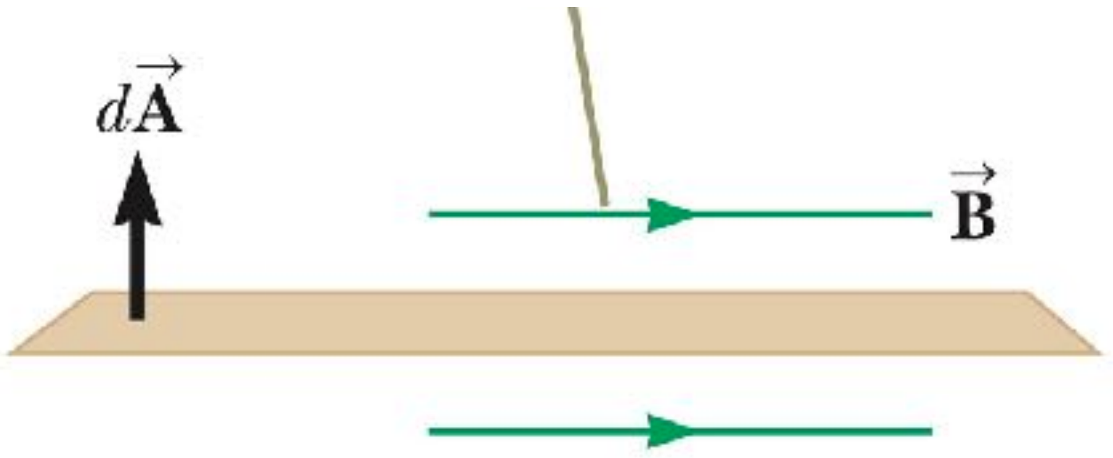
The unit of magnetic flux is $T{\cdot}m^2$ = Wb

- Wb is a weber

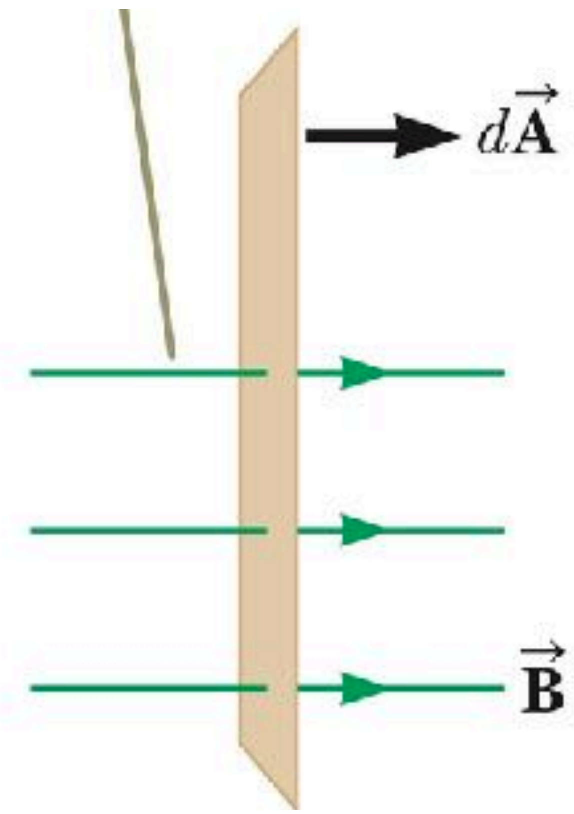
Magnetic Flux Through a Plane

$$\phi_B = \int \vec{B} \cdot d\vec{A}$$

The magnetic flux is $\Phi_B = BA \cos \theta$.



In this case, the field is parallel to the plane and perpendicular to the normal and $\Phi_B = 0$.



In this case, the field is perpendicular to the plane and parallel to the normal and $\Phi = BA$.

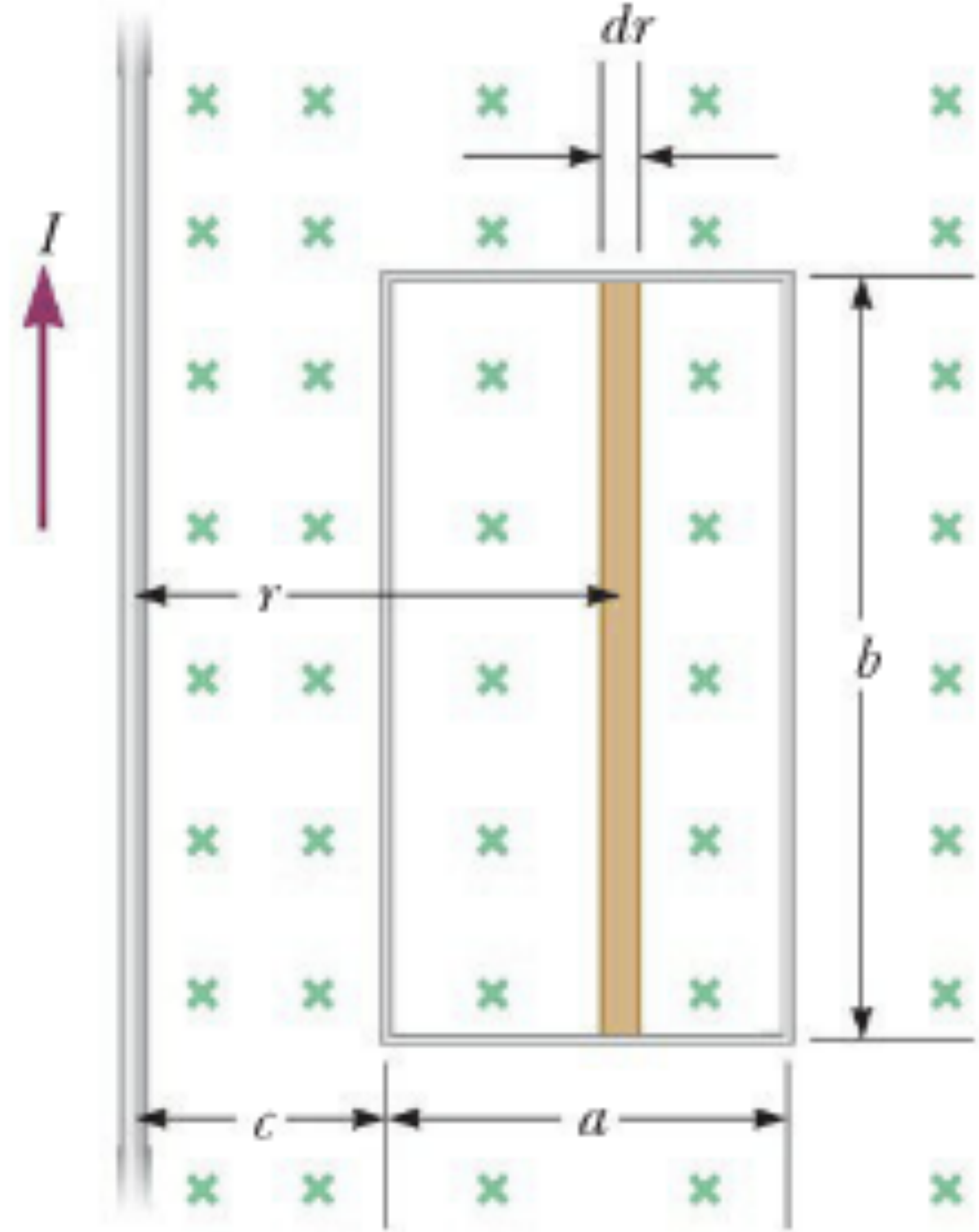
- This is the maximum value of the flux.

Magnetic Flux through a rectangular loop

Find the flux through the loop due to the wire

I is up, use RHR to find field direction
—> circle around the wire
Apply Gauss'law to find the flux through the wire.

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$



Magnetic Flux through a rectangular loop

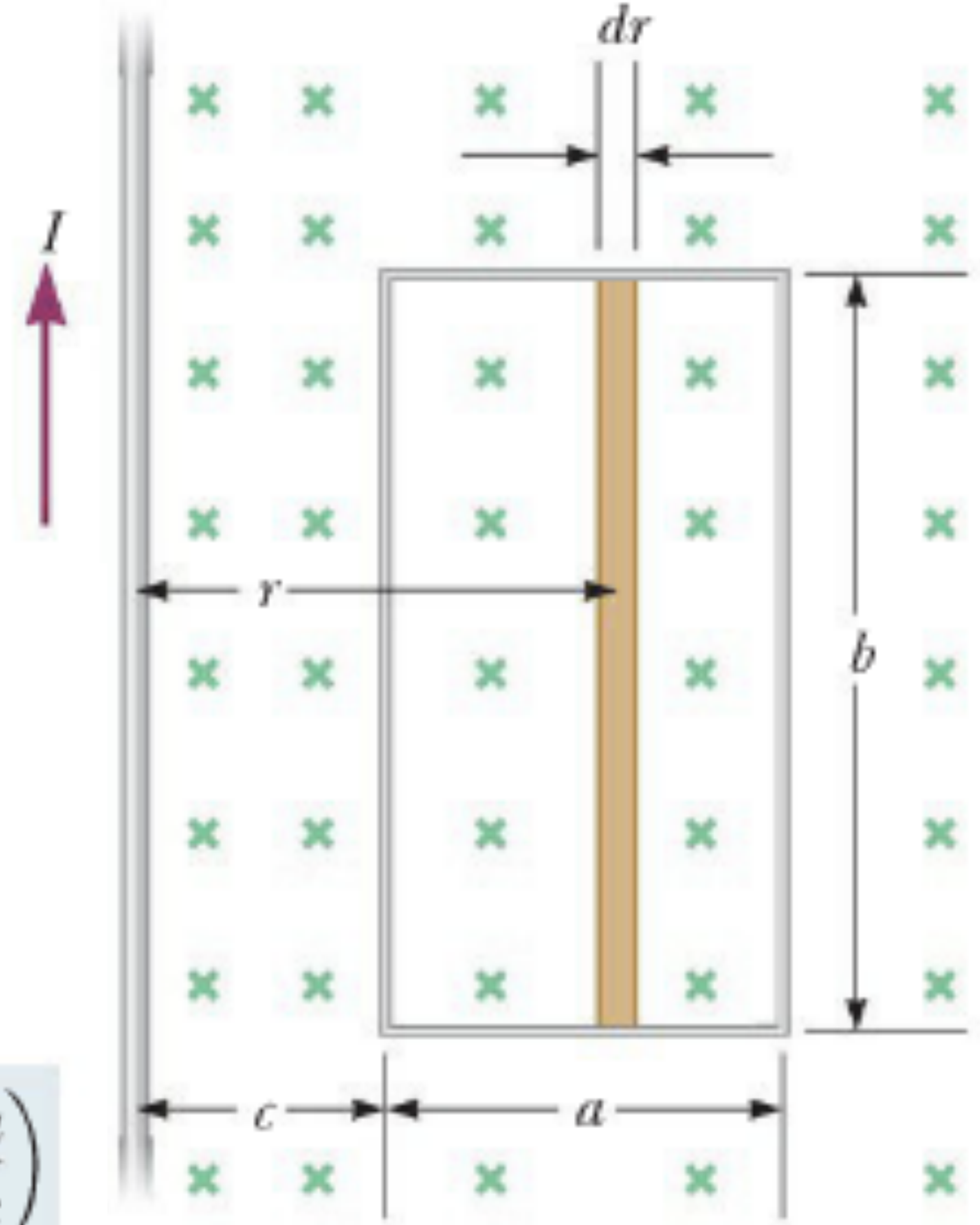
$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$= \int B\, dA = \int \frac{\mu_0 I}{2\pi r}\, dA$$

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r}\, b\, dr = \frac{\mu_0 I b}{2\pi} \int \frac{dr}{r}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi} \ln r \Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c}\right) = \boxed{\frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{c}\right)}$$



Gauss' Law in Magnetism

Magnetic fields do not begin or end at any point.

- Magnetic field lines are continuous and form closed loops.
- **The number of lines entering a surface equals the number of lines leaving the surface.**

Gauss' law in magnetism says the magnetic flux through any closed surface is always zero:

$$\boxed{\oint \overrightarrow{B} \cdot d\overrightarrow{A} = 0}$$

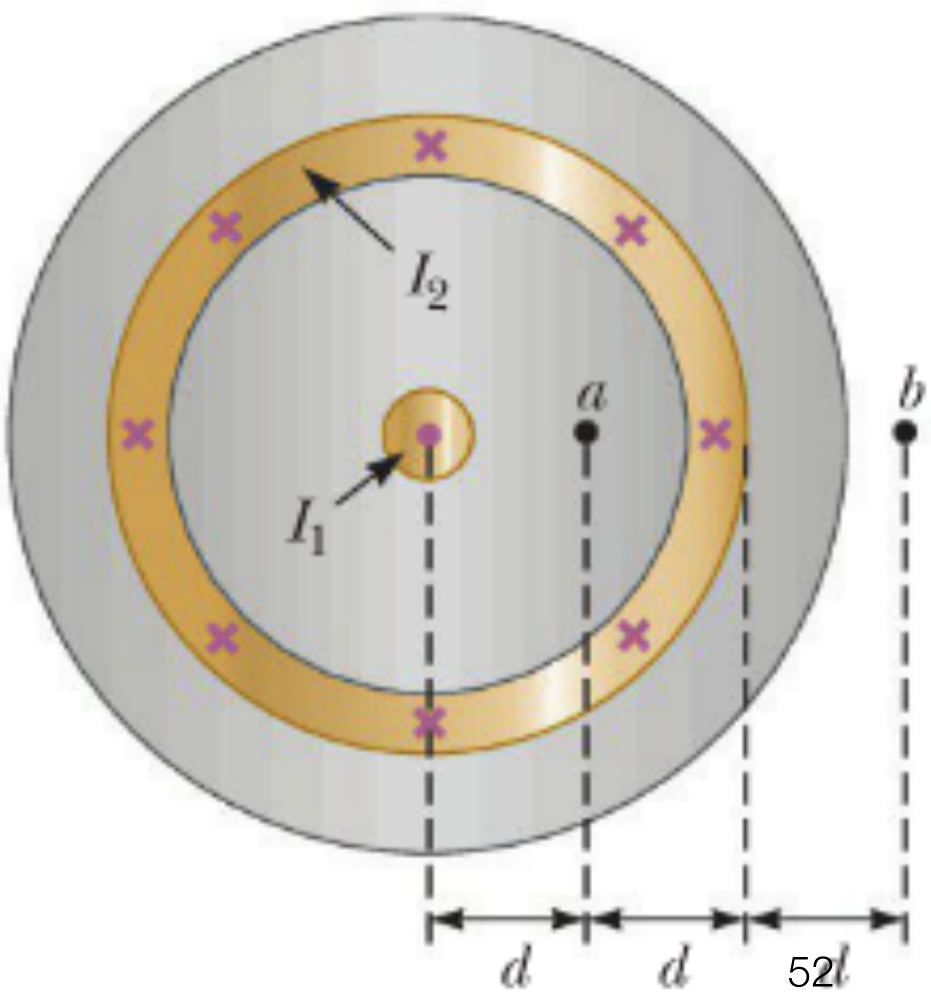
Maxwell's 2nd equation!

THIS DOES NOT MEAN THAT THE FIELD IS 0!!!

This indicates that isolated magnetic poles (monopoles) can't exist.

Example Problem #6

31. W Figure P30.31 is a cross-sectional view of a coaxial cable. The center conductor is surrounded by a rubber layer, an outer conductor, and another rubber layer. In a particular application, the current in the inner conductor is $I_1 = 1.00$ A out of the page and the current in the outer conductor is $I_2 = 3.00$ A into the page. Assuming the distance $d = 1.00$ mm, determine the magnitude and direction of the magnetic field at (a) point a and (b) point b.



Example Problem #6: Solution

(a) From Ampère's law, the magnetic field at point a is given by $B_a = \dfrac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)(1.00\ \mathrm{A})}{2\pi\left(1.00 \times 10^{-3}\ \mathrm{m}\right)}$$

$$= \boxed{200\ \mu\mathrm{T}\ \text{toward top of page}}$$

(b) Similarly at point b: $B_b = \dfrac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b. Taking out of the page as positive, $I_b = 1.00\ \mathrm{A} - 3.00\ \mathrm{A} = -2.00$ A, or $I_b = 2.00$ A into the page. Therefore,

$$B_b = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)(2.00\ \mathrm{A})}{2\pi\left(3.00 \times 10^{-3}\ \mathrm{m}\right)}$$

$$= \boxed{133\ \mu\mathrm{T}\ \text{toward bottom of page}}$$

Example Problem #7

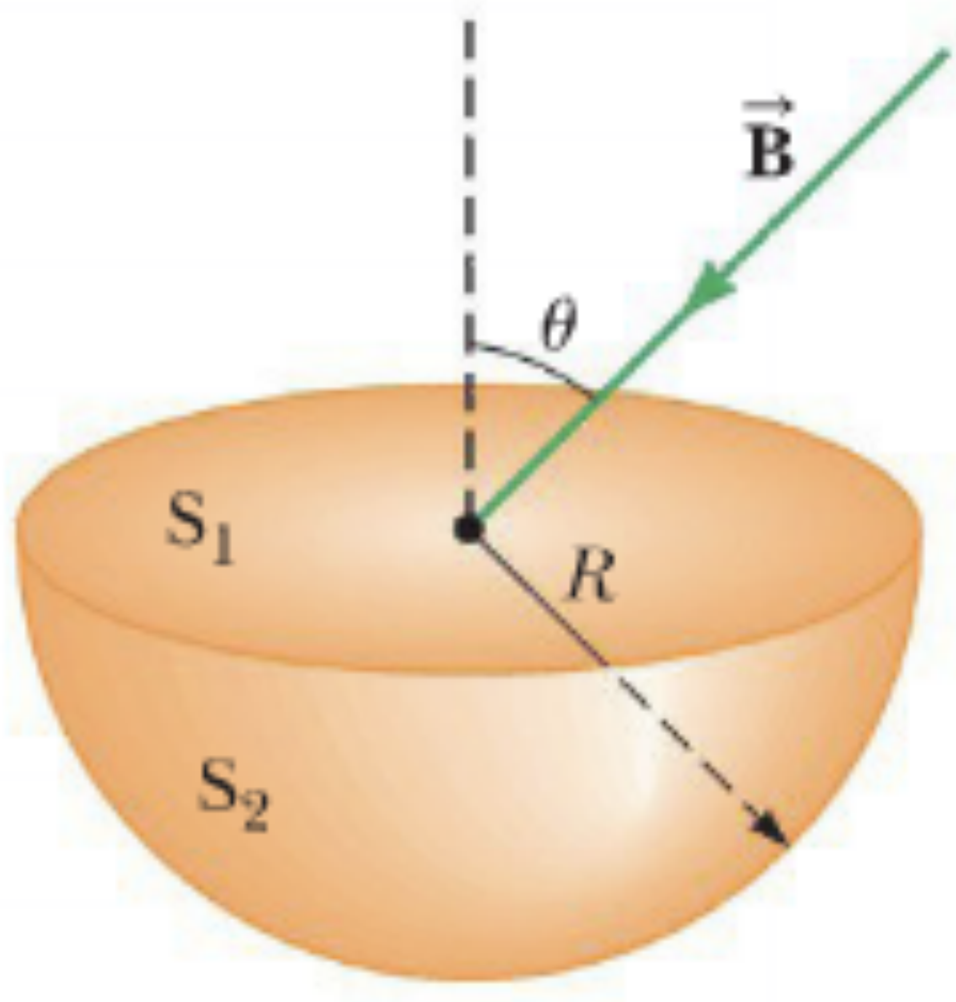
Solenoids allow for non-zero A but zero B to exist (outside of them). Can that change the state of a particle? (push it towards the cylinder or away?)

Example Problem #7: Solution

YES, but not in classical electromagnetism! And neither KE nor p (momentum) can be changed, only the PHASE of the wave(/particle) in QM - This is called the AHARONOV-BOHM EFFECT and it requires a loop

Example Problem #8

46. Consider the hemispherical closed surface in Figure P30.46. The hemisphere is in a uniform magnetic field that makes an angle θ with the vertical. Calculate the magnetic flux through (a) the flat surface S_1 and (b) the hemispherical surface S_2.



Example Problem #8: Solution

(a) The magnetic flux through the flat surface S_1 is

$$(\Phi_B)_{\text{flat}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = B\pi R^2 \cos(180 - \theta) = \boxed{-B\pi R^2 \cos\theta}$$

(b) The net flux out of the closed surface is zero:

$$(\Phi_B)_{\text{flat}} + (\Phi_B)_{\text{curved}} = 0$$

Therefore,

$$(\Phi_B)_{\text{curved}} = \boxed{B\pi R^2 \cos\theta}$$

Example Problem #9

43. W A single-turn square loop of wire, 2.00 cm on each edge, carries a clockwise current of 0.200 A. The loop is inside a solenoid, with the plane of the loop perpendicular to the magnetic field of the solenoid. The solenoid has 30.0 turns/cm and carries a clockwise current of 15.0 A. Find (a) the force on each side of the loop and (b) the torque acting on the loop.

Example Problem #9: Solution

(a) The field produced by the solenoid in its interior is given by

$$\vec{\mathbf{B}} = \mu_0 nI\left(-\hat{\mathbf{i}}\right) = \left(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}\right)\left(\frac{30.0}{10^{-2}\ \mathrm{m}}\right)(15.0\ \mathrm{A})\left(-\hat{\mathbf{i}}\right)$$

$$\vec{\mathbf{B}} = -\left(5.65 \times 10^{-2}\ \mathrm{T}\right)\hat{\mathbf{i}}$$

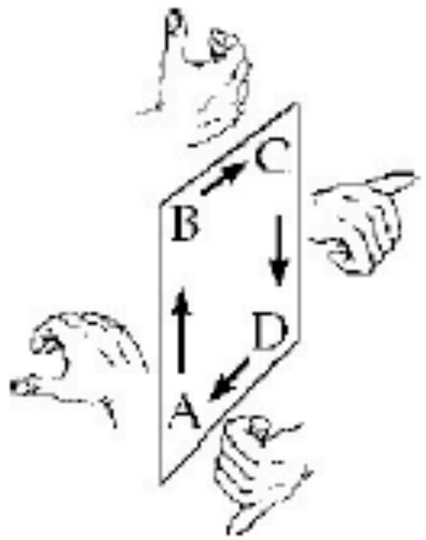
The force exerted on side AB of the square current loop is

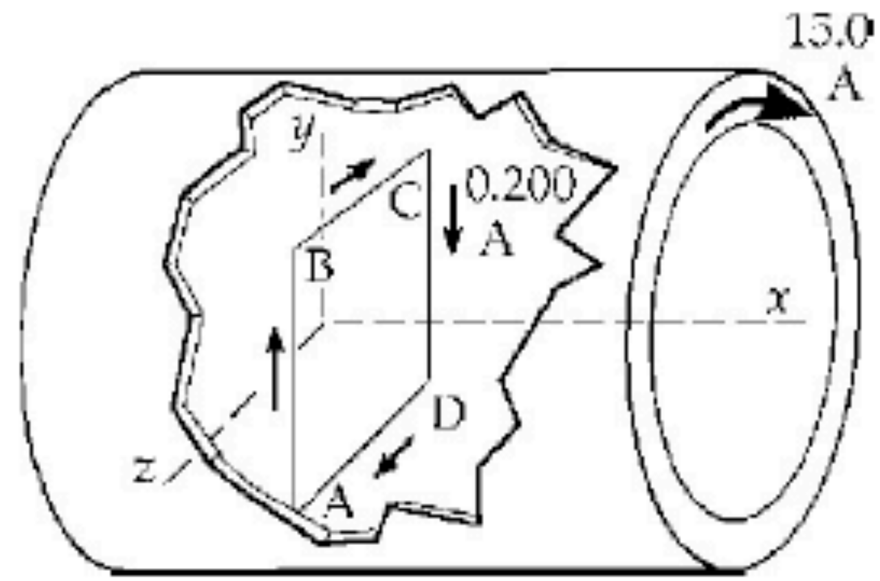
$$\left(\vec{\mathbf{F}}_B\right)_{\mathrm{AB}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}} = (0.200\ \mathrm{A}) \times \left[\left(2.00 \times 10^{-2}\ \mathrm{m}\right)\hat{\mathbf{j}} \times \left(5.65 \times 10^{-2}\ \mathrm{T}\right)\left(-\hat{\mathbf{i}}\right)\right]$$

$$\left(\vec{\mathbf{F}}_B\right)_{\mathrm{AB}} = \left(2.26 \times 10^{-4}\ \mathrm{N}\right)\hat{\mathbf{k}}$$

Example Problem #9: solution

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of $\boxed{226\ \mu\text{N directed away from the center of the loop}}$.





(b) From the above result, it is seen that the net torque exerted on the square loop by the field of the solenoid is $\boxed{\text{zero}}$. More formally, the magnetic dipole moment of the square loop is given by

$$\vec{\mu} = I\vec{\mathbf{A}} = (0.200\ \text{A})\left(2.00\times10^{-2}\ \text{m}\right)^2\left(-\hat{\mathbf{i}}\right) = -80.0\ \mu\text{A}\cdot\text{m}^2\ \hat{\mathbf{i}}$$

The torque exerted on the loop is then

$$\vec{\tau} = \vec{\mu}\times\vec{\mathbf{B}} = \left(-80.0\ \mu\text{A}\cdot\text{m}^2\ \hat{\mathbf{i}}\right)\times\left(-5.65\times10^{-2}\ \text{T}\ \hat{\mathbf{i}}\right) = \boxed{0}$$