Current and Resistance

Chapter 26-27

HW04 is up on WebAssign, due Thursday 02/15

Electric Current

Most practical applications of electricity deal with **electric currents.**

- The electric charges move through some region of space.

Electric current is the **rate of flow of charge through some region of space.**

The SI unit of current is the **ampere** (A).

- 1 A = 1 C / s

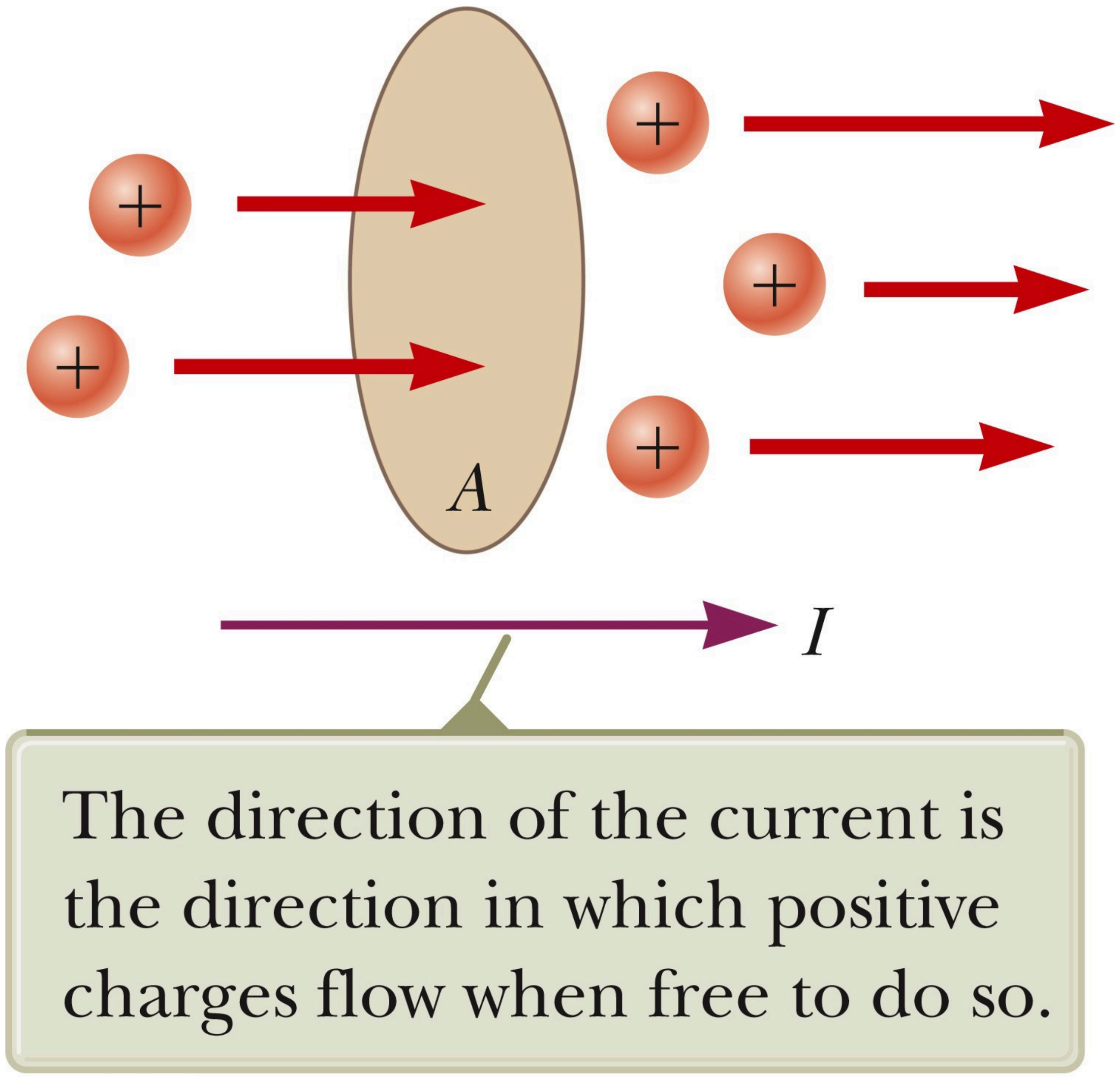
The symbol for electric current is ***I***.

Average Electric Current

Assume charges are moving perpendicular to a surface of area *A*.

If ΔQ is the amount of charge that passes through *A* in time Δt, then the average current is

$$\boxed{I_{avg} = \frac{\Delta Q}{\Delta t}}$$



The charged particles passing through the surface could be positive, negative or both.
It is conventional to **assign to the current the same direction as the flow of positive charges.**

It is common to refer to any moving charge as a *charge carrier.*

Instantaneous Electric Current

If the rate at which the charge flows varies with time, the instantaneous current, I, is defined as the differential limit of average current as $\Delta t \to 0$.

$$\boxed{I \equiv \frac{dQ}{dt}}$$

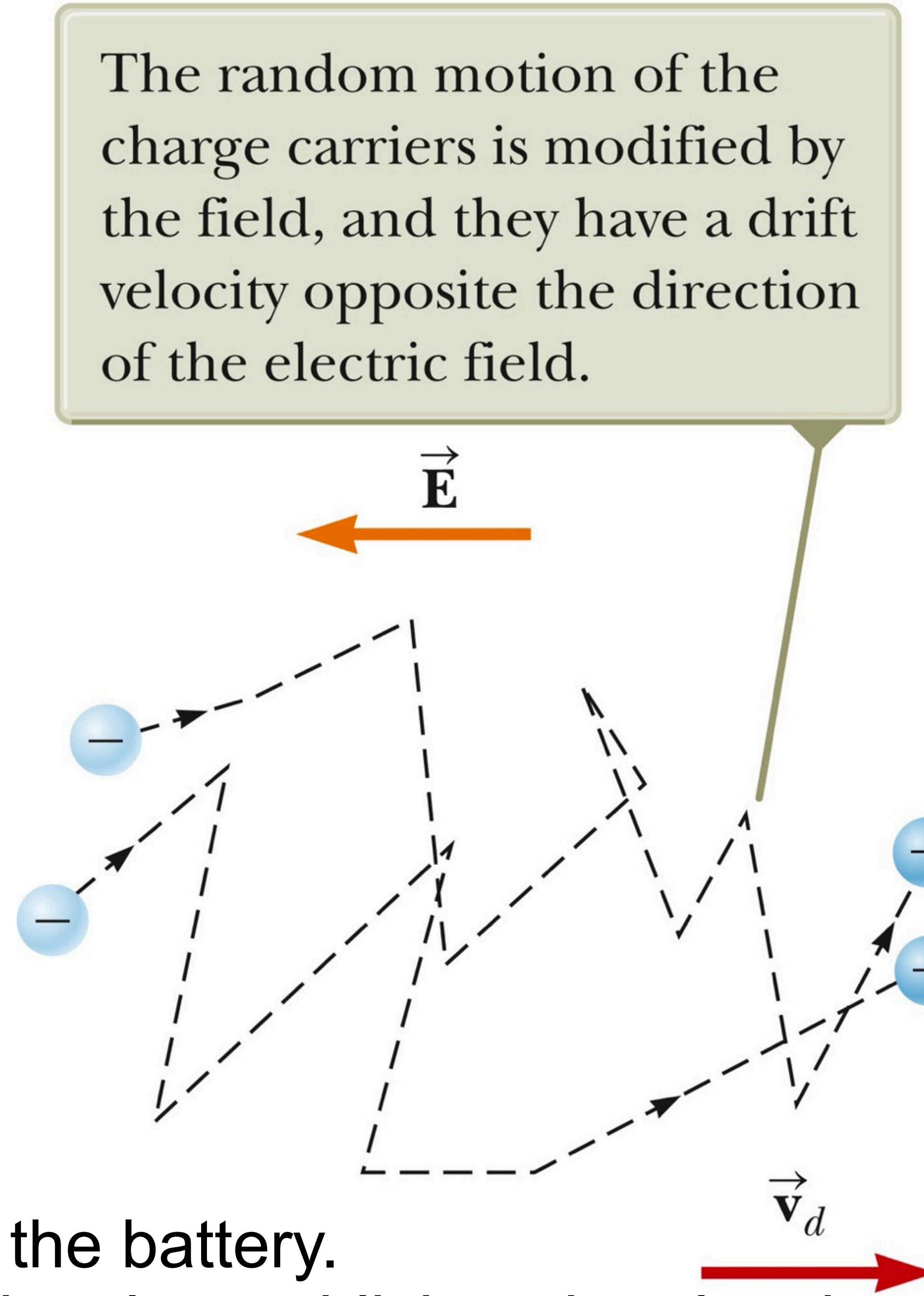
What creates a current?

Take a conductor, such as a wire.

Apply a potential difference.

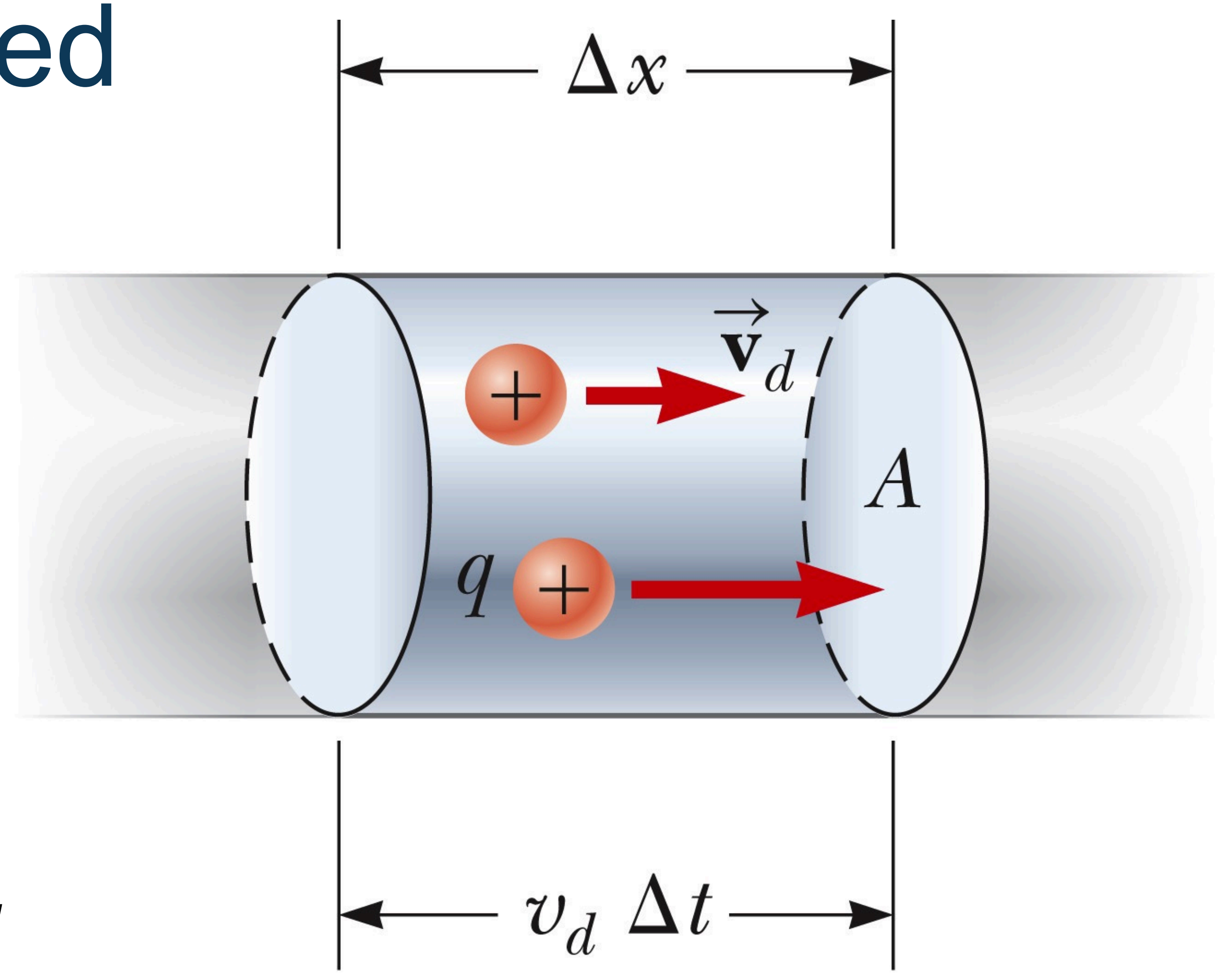
This creates an electric field in the conductor which exerts an electric force on the electrons.

The electric force causes the electrons to move in the wire in the opposite direction to a field and creates a current.



<u>Note:</u>
The electrons are already in the wire.
They respond to the electric field set up by the battery.
The battery does not supply the electrons, it only establishes the electric field.

Current and Drift Speed



Cylindrical conductor of cross-sectional area *A*.
q is the individual charge
n is the charge volume density = number of mobile charge carriers per unit volume.

The total charge is $\Delta Q = (nA\Delta x)q$

If **v_d is the speed at which the carriers move**, then $v_d = \Delta x / \Delta t$ and $\Delta x = v_d \, \Delta t$

Rewrite: $\Delta Q = (nAv_d \, \Delta t)q$

$$I_{\text{ave}} = \Delta Q/\Delta t = nqv_dA$$

$$\boxed{v_d = \frac{I_{avg}}{nqA}}$$

v_d is an average speed called the **drift speed.**

Current Density

J is the **current density** of a conductor.

It is defined as the **current per unit area.**

$$\boxed{J = \frac{I}{A} = nqv_d}$$

- This expression is **valid only if** the current density is uniform and *A* is perpendicular to the direction of the current.

J has SI units of A/m^2

The current density is in the direction of the positive charge carriers.

Conductivity

A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor.

For some materials, **the current density is directly proportional to the field.**

The constant of proportionality, σ, is called the **conductivity** of the conductor.

$$\boxed{J = \sigma E}$$

This tells you how well a material conducts electricity.

This is the mathematical form of **Ohm's law**.

Ohm's Law

Ohm's law states that for many materials, **the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.**

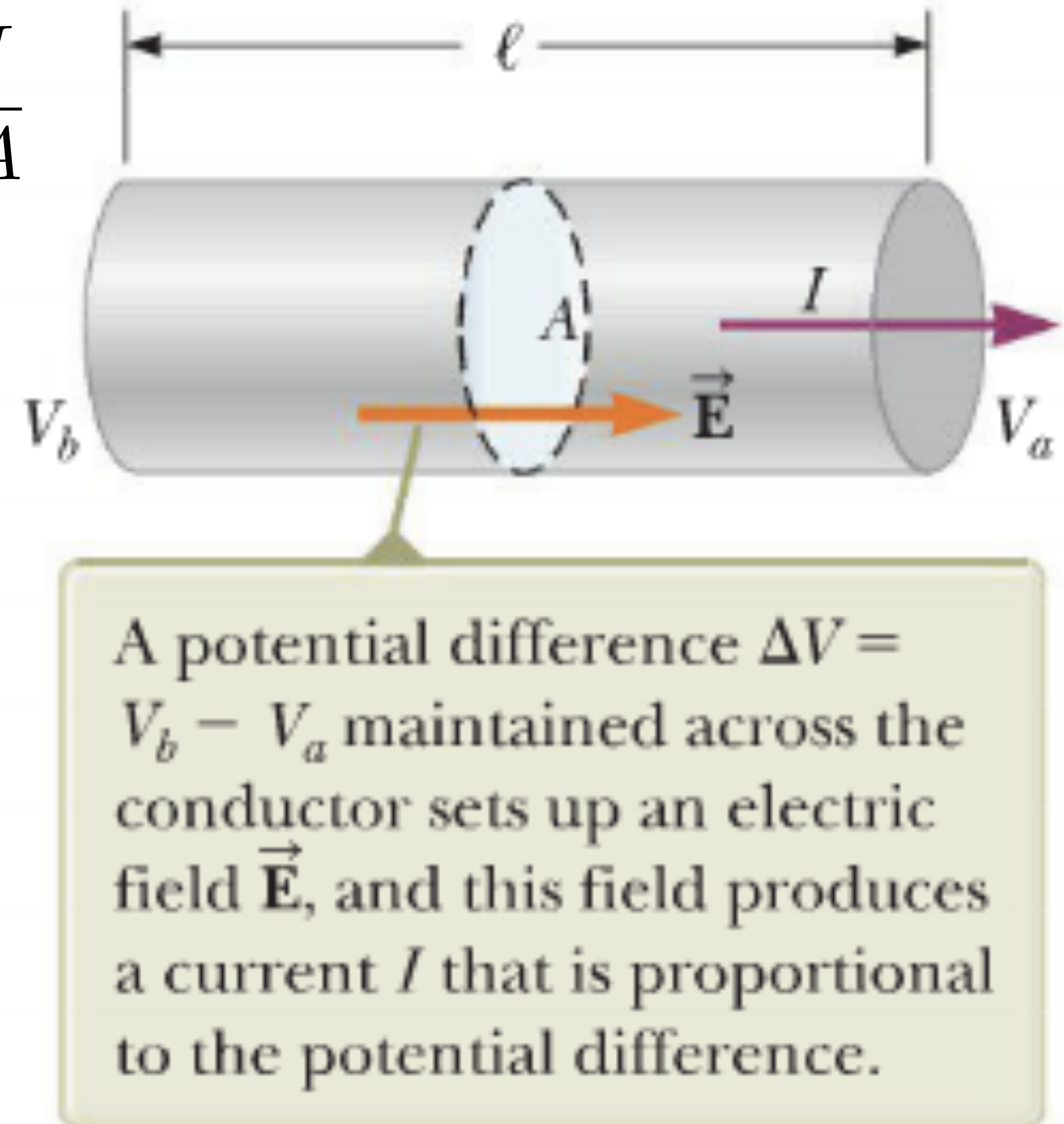
$$\frac{J}{E} = \sigma$$

- Most metals obey Ohm's law
- Materials that obey Ohm's law are said to be ***ohmic***
- Not all materials follow Ohm's law
  - Materials that do not obey Ohm's law are said to be ***nonohmic***.

Ohm's law is **not** a fundamental law of nature.
Ohm's law is an empirical relationship **valid only for certain materials.**

Ohm's Law rewritten

$$\frac{J}{E} = \sigma \qquad \Delta V = E\ell \qquad J = \frac{I}{A}$$



$$J = \sigma E$$

$$J = \sigma \frac{\Delta V}{\ell}$$

$$\Delta V = \frac{\ell J}{\sigma}$$

$$\boxed{\Delta V = \frac{\ell I}{\sigma A}}$$

The potential difference is proportional to the current!

Resistance

In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor. The constant of proportionality is called the **resistance** of the conductor.

$$\Delta V = \frac{\ell I}{\sigma A}$$

$$\boxed{R \equiv \frac{\Delta V}{I}}$$

Define: $R = \dfrac{\ell}{\sigma A}$

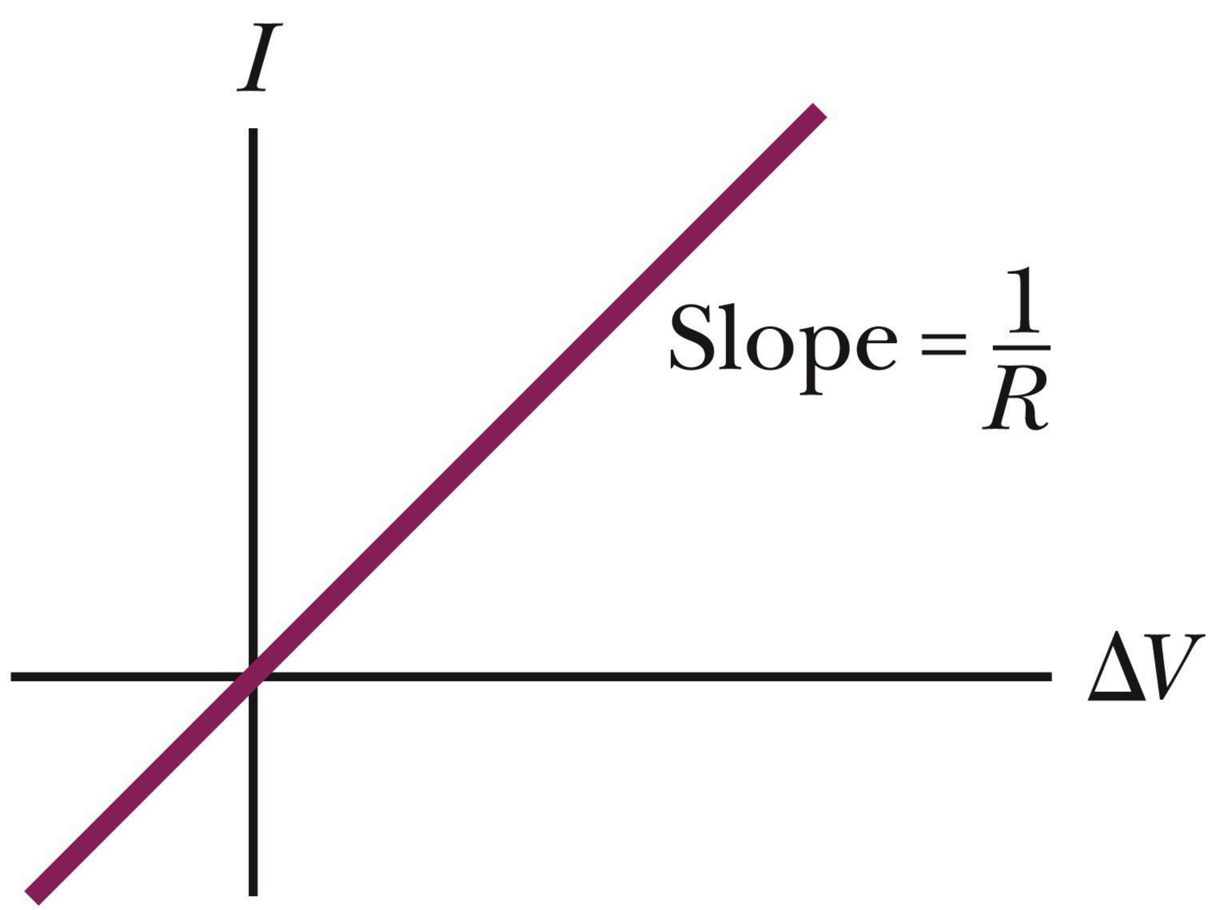
Ohm's law rewritten

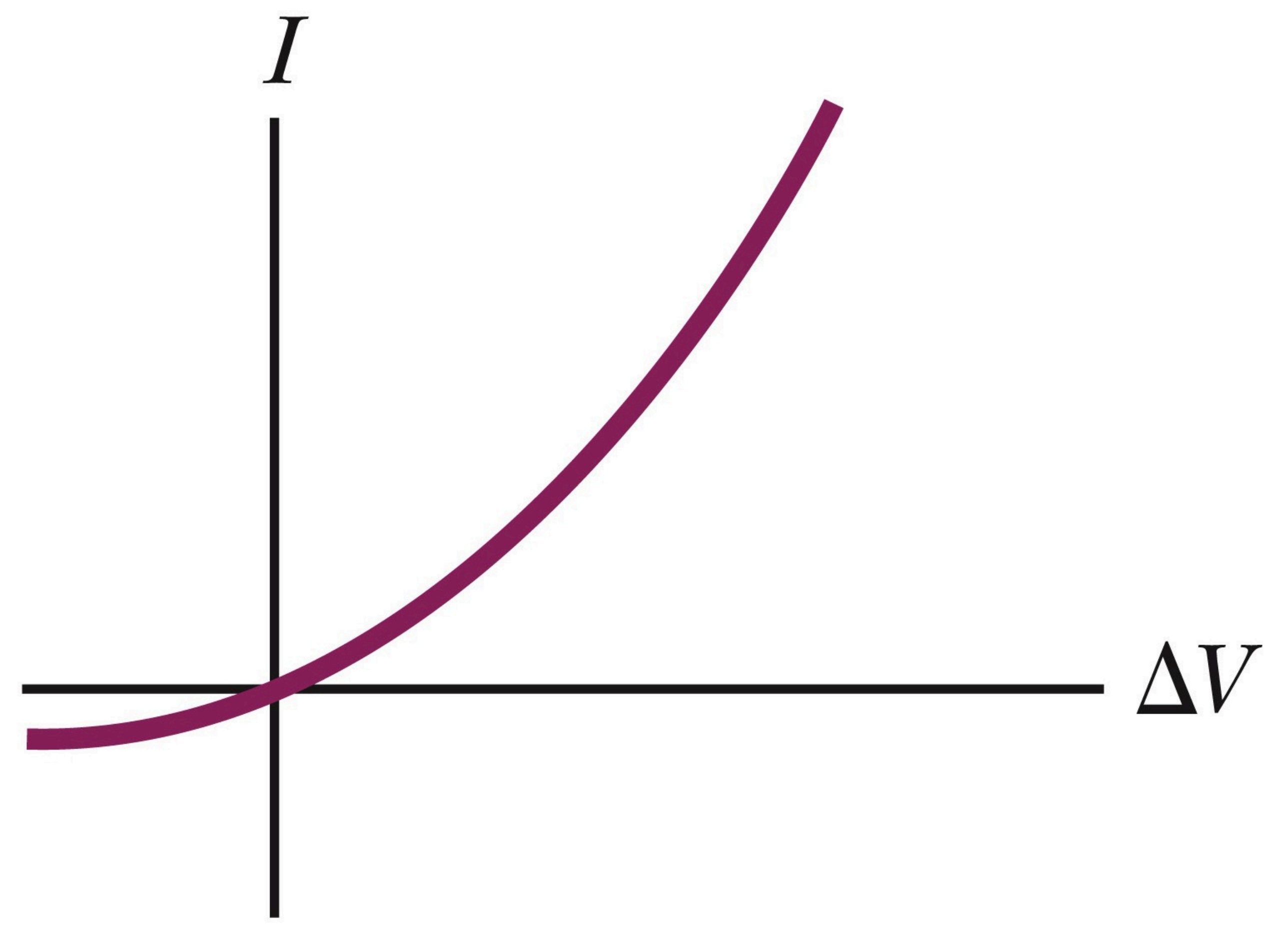
SI units of resistance are ***ohms*** (Ω).

- 1 Ω = 1 V / A

Resistance in a circuit arises due to collisions between the electrons carrying the current with the fixed atoms inside the conductor.

Ohmic vs Nonohmic material





Ohmic: resistance is constant over a wide range of voltages.

The relationship between current and voltage is linear.

The slope is related to the resistance.

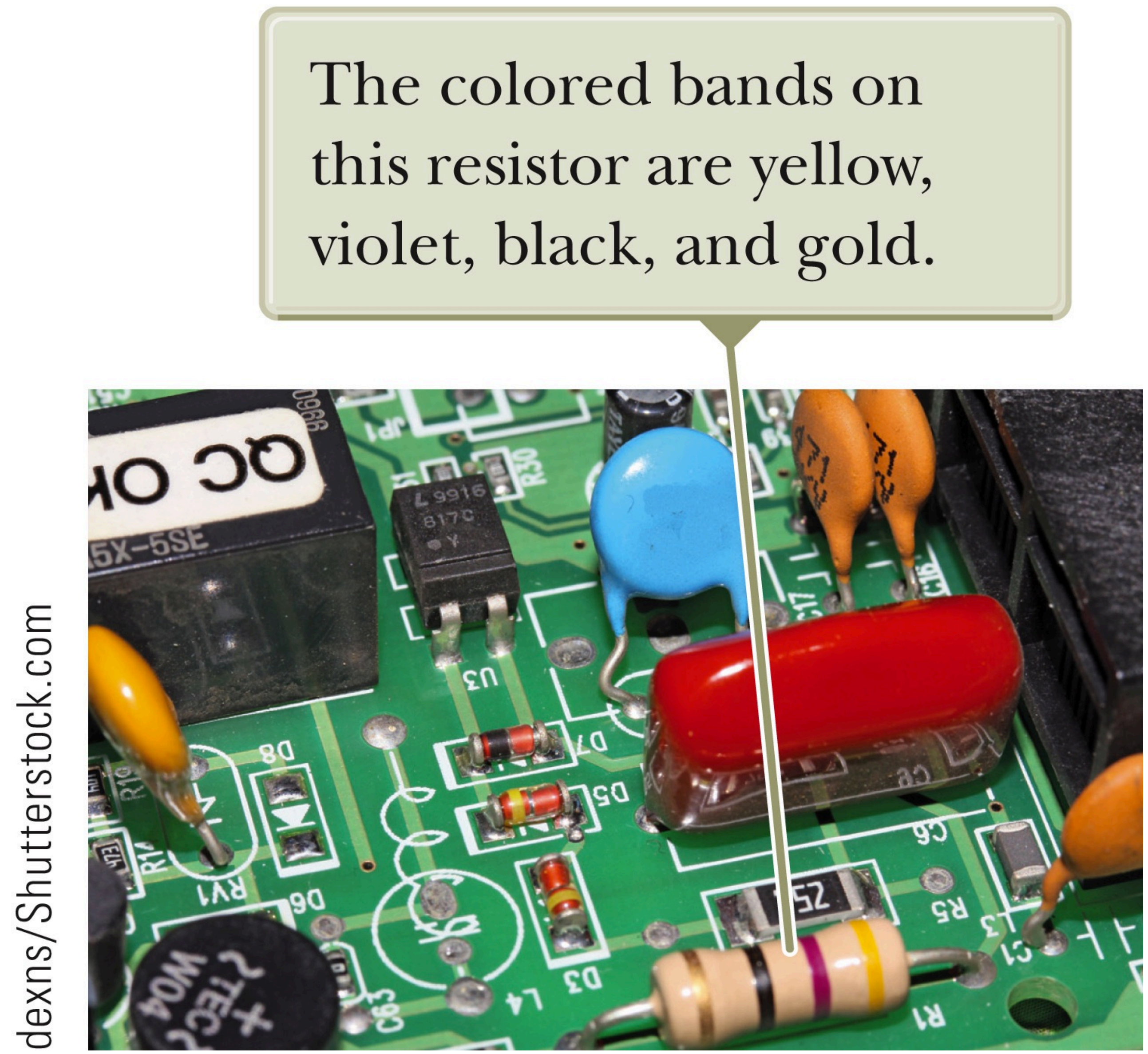
Nonohmic materials are those whose resistance changes with voltage or current.

The current-voltage relationship is nonlinear.

Resistors

Most electric circuits use **circuit elements called resistors** **to control the current in the various parts of the circuit.**

symbol:



Values of resistors are normally indicated by colored bands.

- The first two bands give the first two digits in the resistance value.
- The third band represents the power of ten for the multiplier band.
- The last band is the tolerance.

Resistor color code

Table 27.1 Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

Resistor color code example

Red (=2) and blue (=6) give the first two digits: 26
Green (=5) gives the power of ten in the multiplier: 10^5
The value of the resistor then is $26 \times 10^5\ \Omega$ (or 2.6 MΩ)
The tolerance is 10% (silver = 10%) or $2.6 \times 10^5\ \Omega$

Resistivity

The inverse of the conductivity is the **resistivity**:

$$\boxed{\rho = \frac{1}{\sigma}}$$

Resistivity has SI units of ohm-meters ($\Omega \cdot \text{m}$)
Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature.

Resistance is also related to resistivity:

$$\boxed{R = \rho \frac{\ell}{A}}$$

The resistance of a material depends on its **geometry** and its **resistivity**.

An **ideal conductor** would have **zero** resistivity.
An **ideal insulator** would have **infinite** resistivity.

Resistivity Values

Table 27.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] α [(°C)$^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Resistance and Temperature

Over a limited temperature range, **the resistivity of a conductor varies approximately linearly with the temperature.**

$$\boxed{\rho = \rho_o[1+\alpha(T - T_o)]}$$

- ρ_o is the resistivity at some reference temperature T_o (usually 20° C)
- α is the **temperature coefficient of resistivity**
  - SI units of α are $^oC^{-1}$

The temperature coefficient of resistivity can be expressed as

$$\boxed{\alpha = \frac{1}{\rho_o}\frac{\Delta\rho}{\Delta T}}$$

Since the resistance is proportional to the resistivity, you can find the effect of temperature on resistance.

$$\boxed{R = R_o[1 + \alpha(T - T_o)]}$$

Use of this property enables precise temperature measurements through careful monitoring of the resistance of a probe made from a particular material: **thermometer**!

Conductors and Semi-conductors

Conductors are materials that exhibit an increase in resistivity with an increase in temperature.

α is positive

Semiconductors are materials that exhibit a decrease in resistivity with an increase in temperature.

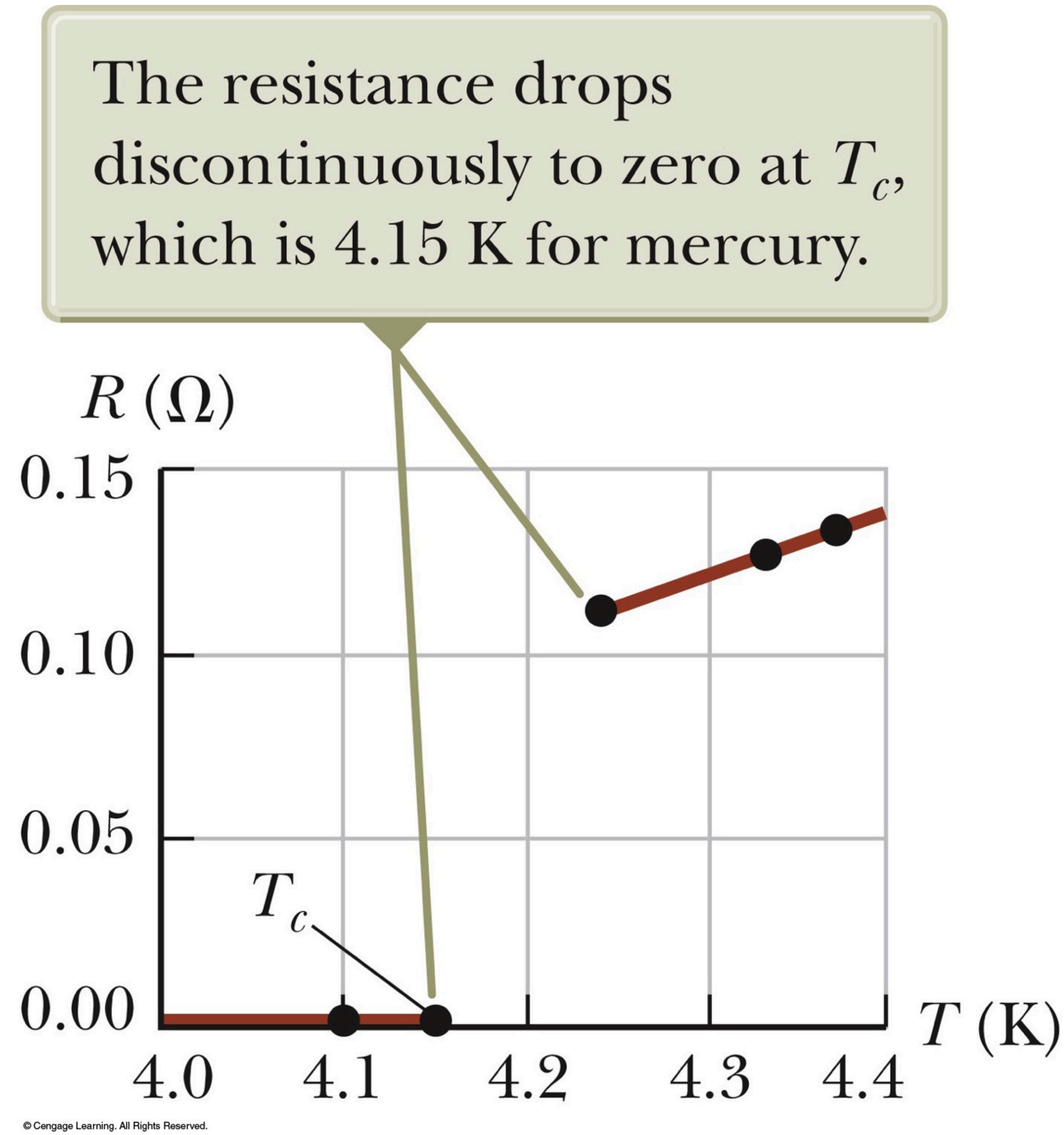
α is negative

There is an increase in the density of charge carriers at higher temperatures.

Superconductors

A class of materials and compounds whose **resistances fall to virtually zero below a certain temperature,** T_C.

- T_C is called the **critical temperature.**



The value of T_C is sensitive to:

- chemical composition
- pressure
- molecular structure

Once a current is set up in a superconductor, it persists without any applied voltage.

- Since $R = 0$

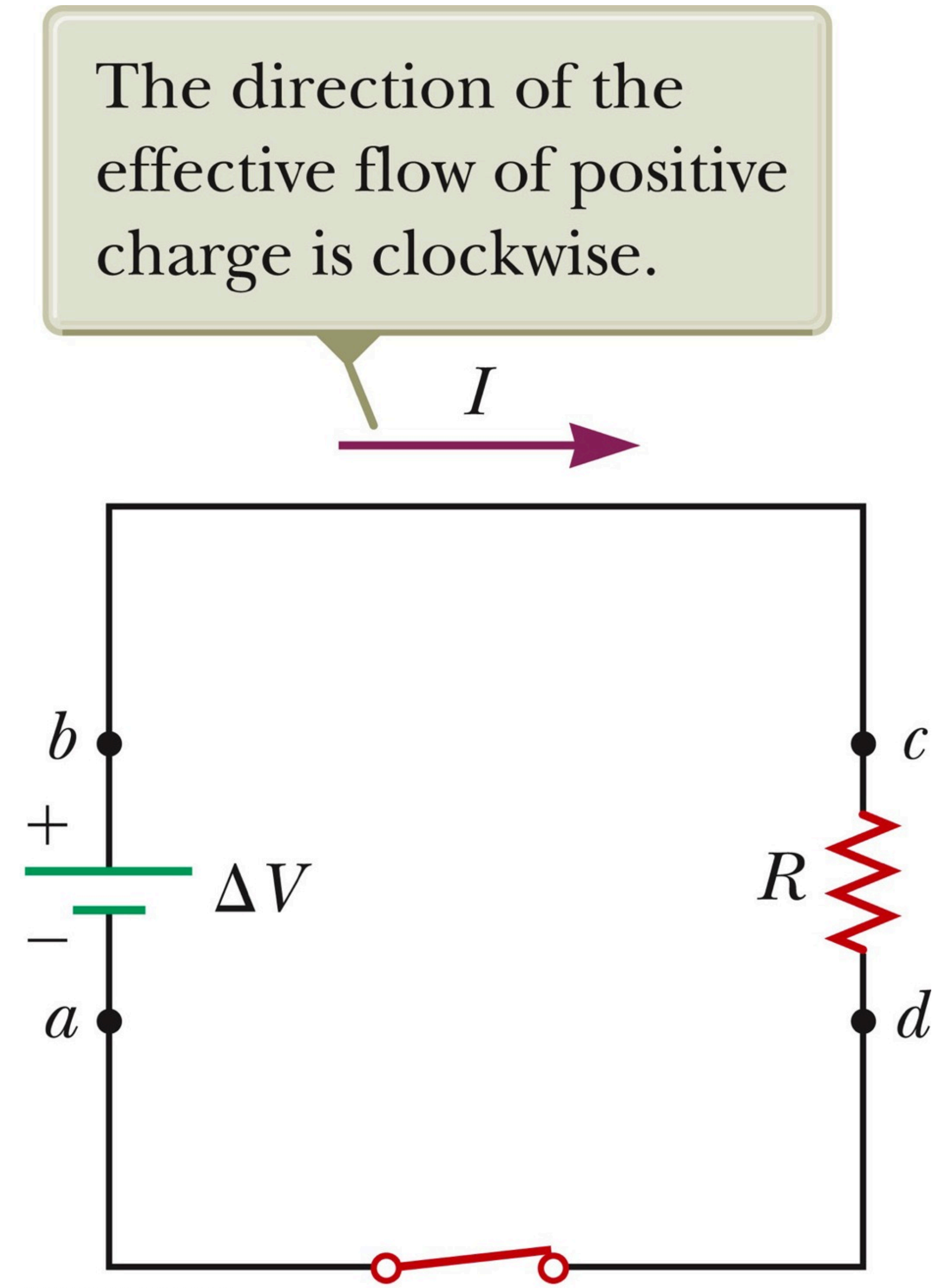
Energy considerations

As a charge moves from *a* to *b*, the electric potential energy of the system increases.

- The chemical energy in the battery must decrease by this same amount.

This electric potential energy is transformed into internal energy in the resistor.

- Corresponds to increased vibrational motion of the atoms in the resistor



The rate at which the system's potential energy decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor.

The resistor is normally in contact with the air, so its increased temperature will result in a transfer of energy by heat into the air.

After some time interval, the resistor reaches a constant temperature.

- The input of energy from the battery is balanced by the output of energy by heat and radiation.

Electrical power

The **power** is the **rate at which the energy is delivered to the resistor.**

$$\boxed{P = I\,\Delta V}$$

Applying Ohm's Law ($\Delta V=RI$), alternative expressions can be found:

$$\boxed{P = I\,\Delta V = I^2 R = \frac{(\Delta V)^2}{R}}$$

Units: I is in A (ampere), R is in Ω (ohms), ΔV is in V (volts), and P is in W (watts)

A few important remarks…

A single electron is moving at the drift velocity in the circuit.

- It may take hours for an electron to move completely around a circuit.

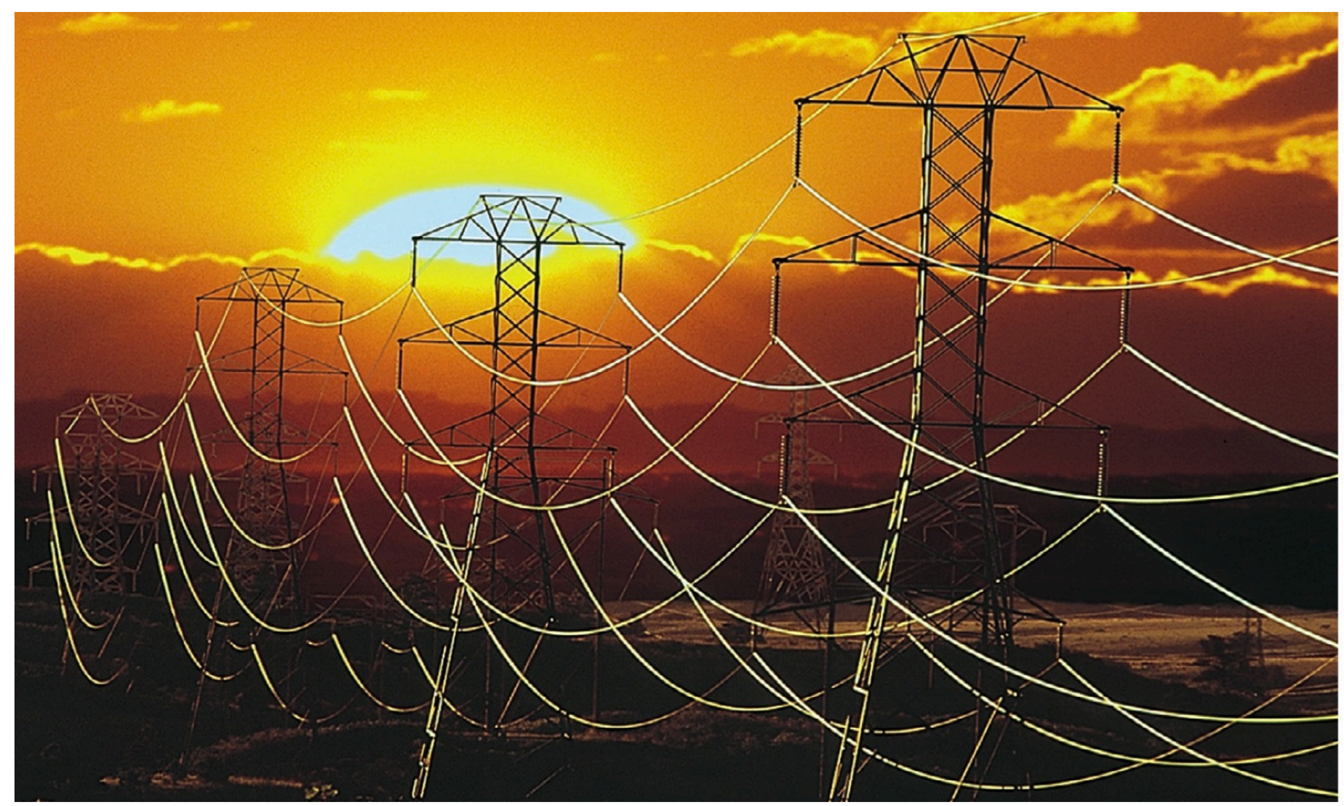
The current is the same everywhere in the circuit.

- Current is not “used up” anywhere in the circuit

The charges flow in the same rotational sense at all points in the circuit.

Real power lines have resistance.

Power companies transmit electricity at high voltages and low currents to minimize power losses.



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Example Problems #1 & #2 (a and b)

9. The quantity of charge q (in coulombs) that has passed
W through a surface of area $2.00\ \text{cm}^2$ varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds. (a) What is the instantaneous current through the surface at $t = 1.00$ s? (b) What is the value of the current density?

Example Problems #1,2 Solution

We are given $q = 4t^3 + 5t + 6$. The area is

$$A = \left(2.00\ \text{cm}^2\right)\left(\frac{1.00\ \text{m}}{100\ \text{cm}}\right)^2 = 2.00\times 10^{-4}\ \text{m}^2$$

(a) $$I(1.00\ \text{s}) = \left.\frac{dq}{dt}\right|_{t=1.00\ \text{s}} = \left.\left(12t^2 + 5\right)\right|_{t=1.00\ \text{s}} = \boxed{17.0\ \text{A}}$$

(b) $$J = \frac{I}{A} = \frac{17.0\ \text{A}}{2.00\times 10^{-4}\ \text{m}^2} = \boxed{85.0\ \text{kA/m}^2}$$

Example Problem #4 (Example 3: room temp superC)

31. M (a) A 34.5-m length of copper wire at 20.0°C has a radius of 0.25 mm. If a potential difference of 9.00 V is applied across the length of the wire, determine the current in the wire. (b) If the wire is heated to 30.0°C while the 9.00-V potential difference is maintained, what is the resulting current in the wire?

Example Problem #4: Solution

(a) The resistance at 20.0°C is

$$R_0 = \frac{\rho \ell}{A} = \frac{\left(1.7 \times 10^{-8}\ \Omega \cdot \text{m}\right)(34.5\ \text{m})}{\pi\left(0.25 \times 10^{-3}\ \text{m}\right)^2} = 2.99\ \Omega$$

and the current is

$$I = \frac{\Delta V}{R_0} = \frac{9.00\ \text{V}}{3.00\ \Omega} = \boxed{3.01\ \text{A}}$$

(b) At 30.0°C, from Equation 27.20,

$$R = R_0\left[1 + \alpha(\Delta T)\right]$$

$$= (2.99\ \Omega)\left[1 + \left(3.9 \times 10^{-3}\ (°\text{C})^{-1}\right)(30.0°\text{C} - 20.0°\text{C})\right] = 3.10\ \Omega$$

The current is then

$$I = \frac{\Delta V}{R_0} = \frac{9.00\ \text{V}}{3.10\ \Omega} = \boxed{2.90\ \text{A}}$$

Example Problem #5 & 6

39. A certain waffle iron is rated at 1.00 kW when connected to a 120-V source. (a) What current does the waffle iron carry? (b) What is its resistance?

53. M A certain toaster has a heating element made of Nichrome wire. When the toaster is first connected to a 120-V source (and the wire is at a temperature of 20.0°C), the initial current is 1.80 A. The current decreases as the heating element warms up. When the toaster reaches its final operating temperature, the current is 1.53 A. (a) Find the power delivered to the toaster when it is at its operating temperature. (b) What is the final temperature of the heating element?

Example Problem #5 : Solution

(a) From Equation 27.21,

$$P = I\Delta V \rightarrow I = P/\Delta V = \left(1.00\times10^{3}\ \text{W}\right)/(120\ \text{V}) = \boxed{8.33\ \text{A}}$$

(b) From Equation 27.23,

$$P = \Delta V^{2}/R \rightarrow R = \Delta V^{2}/P = (120\ \text{V})^{2}/\left(1.00\times10^{3}\ \text{W}\right) = \boxed{14.4\ \Omega}$$

Example Problem #6: Solution

At operating temperature,

(a) $P = I\Delta V = (1.53\text{ A})(120\text{ V}) = \boxed{184\text{ W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$$R = R_0(1 + \alpha\Delta T)$$

$$\frac{120\text{ V}}{1.53\text{ A}} = \left(\frac{120\text{ V}}{1.80\text{ A}}\right)\left[1 + \left(0.400 \times 10^{-3}\ (^\circ\text{C})^{-1}\right)\Delta T\right]$$

which gives

$$\Delta T = 441^\circ\text{C}$$

and $T = 20.0^\circ\text{C} + 441^\circ\text{C} = \boxed{461^\circ\text{C}}$

stop here?

Resistance (chapter 28)

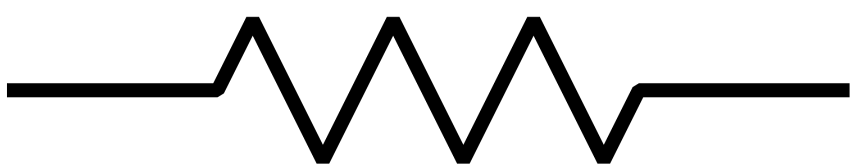
In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor. The constant of proportionality is called the **resistance** of the conductor.

$$\boxed{R \equiv \frac{\Delta V}{I}}$$

$$R = \frac{\ell}{\sigma A}$$

SI units = *ohms* (Ω).

- 1 Ω = 1 V / A

Symbol: 

Remember a resistance "resists" the current, it impedes the flow of charges.

Resistors in Series

= connected end-to-end

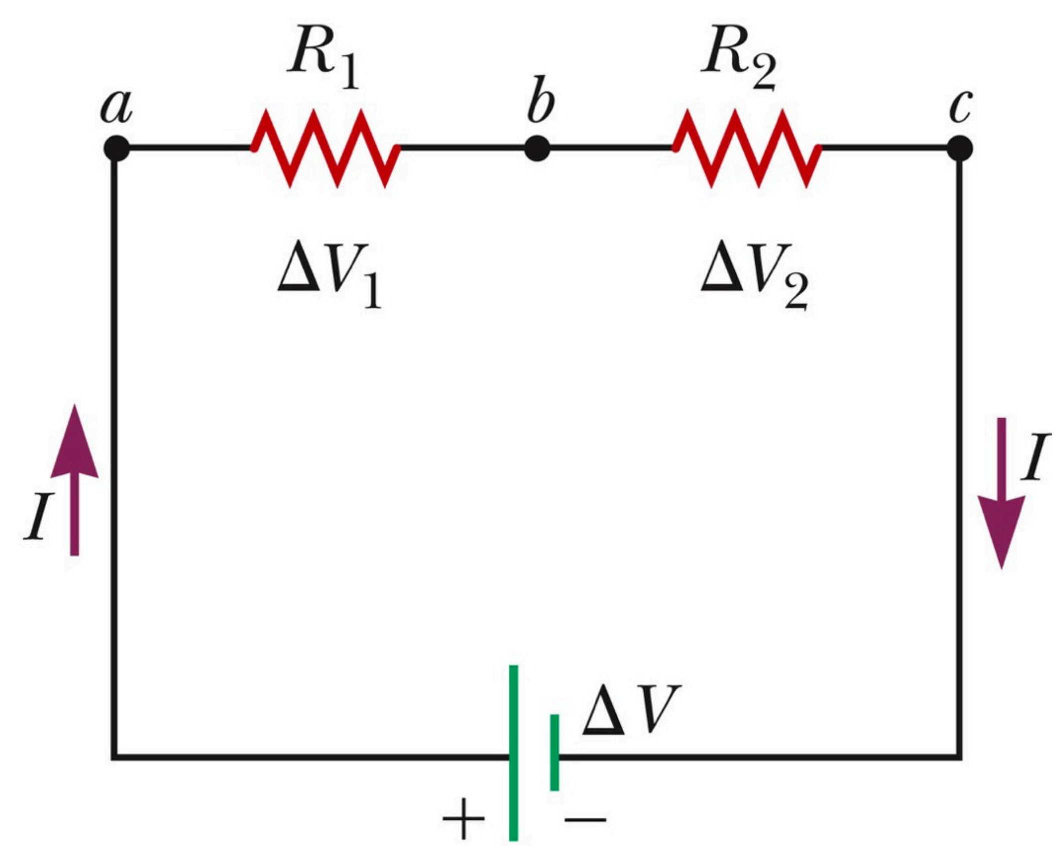
Currents are the same in all the resistors because the amount of charge that passes through one resistor must also pass through the other resistors in the same time interval.

- $I = I_1 = I_2$

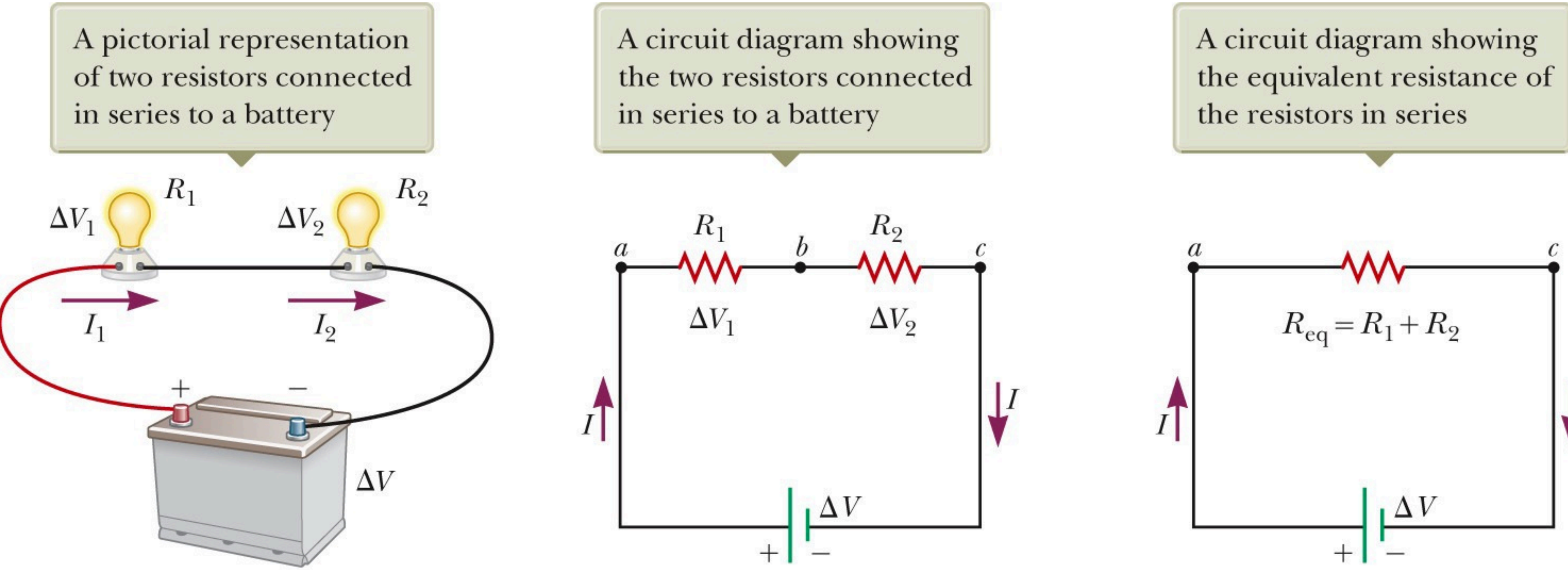
Potential difference will divide among the resistors such that the sum of the potential differences across the resistors is equal to the total potential difference across the combination. This is a consequence of conservation of energy.

- $\Delta V = \Delta V_1 + \Delta V_2 = IR_1 + IR_2$
  $= I\,(R_1 + R_2)$

A circuit diagram showing the two resistors connected in series to a battery



Equivalent Resistance – Series



All three representations are equivalent: two resistors are replaced with their equivalent resistance.

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \ldots}$$

The equivalent resistance of a series combination of resistors is always greater than any individual resistance.

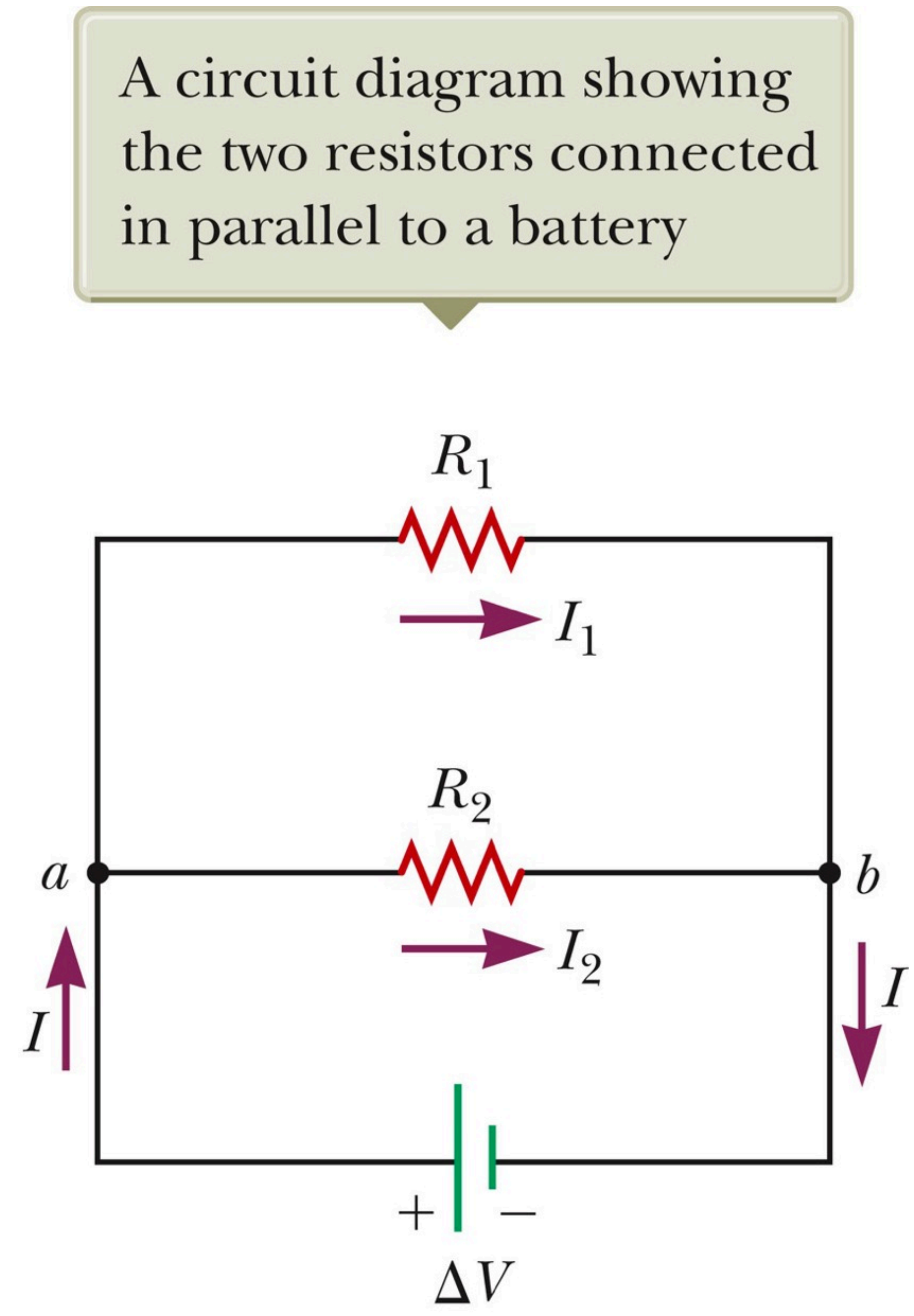
Resistors in Parallel

The potential difference across each resistor is the same because each is connected directly across the battery terminals.

$$\Delta V = \Delta V_1 = \Delta V_2$$

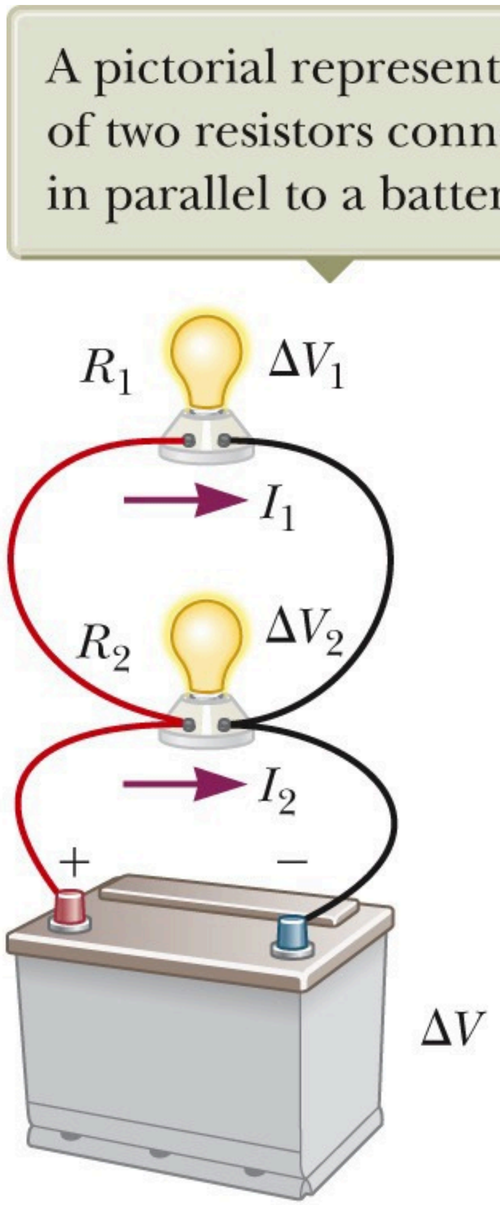
The current, I, that enters a junction must be equal to the total current leaving that junction.

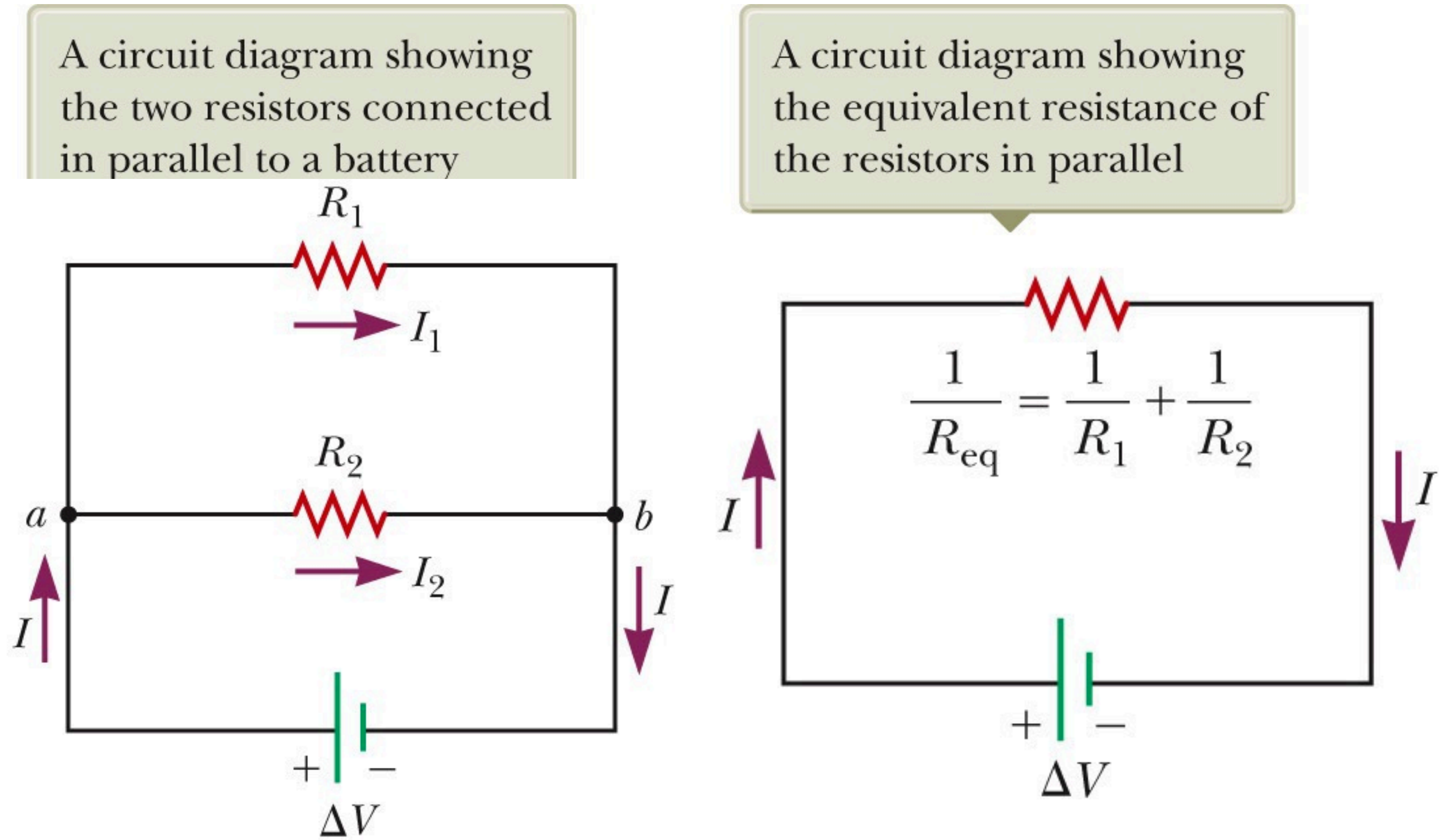
- $I = I_1 + I_2 = (\Delta V_1 / R_1) + (\Delta V_2 / R_2)$
- The currents are generally not the same.
- Consequence of conservation of electric charge



A **junction** is a point where the current can split.

Equivalent Resistance – Parallel





All three diagrams are equivalent: the equivalent resistance replaces the two original resistances.

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistance.

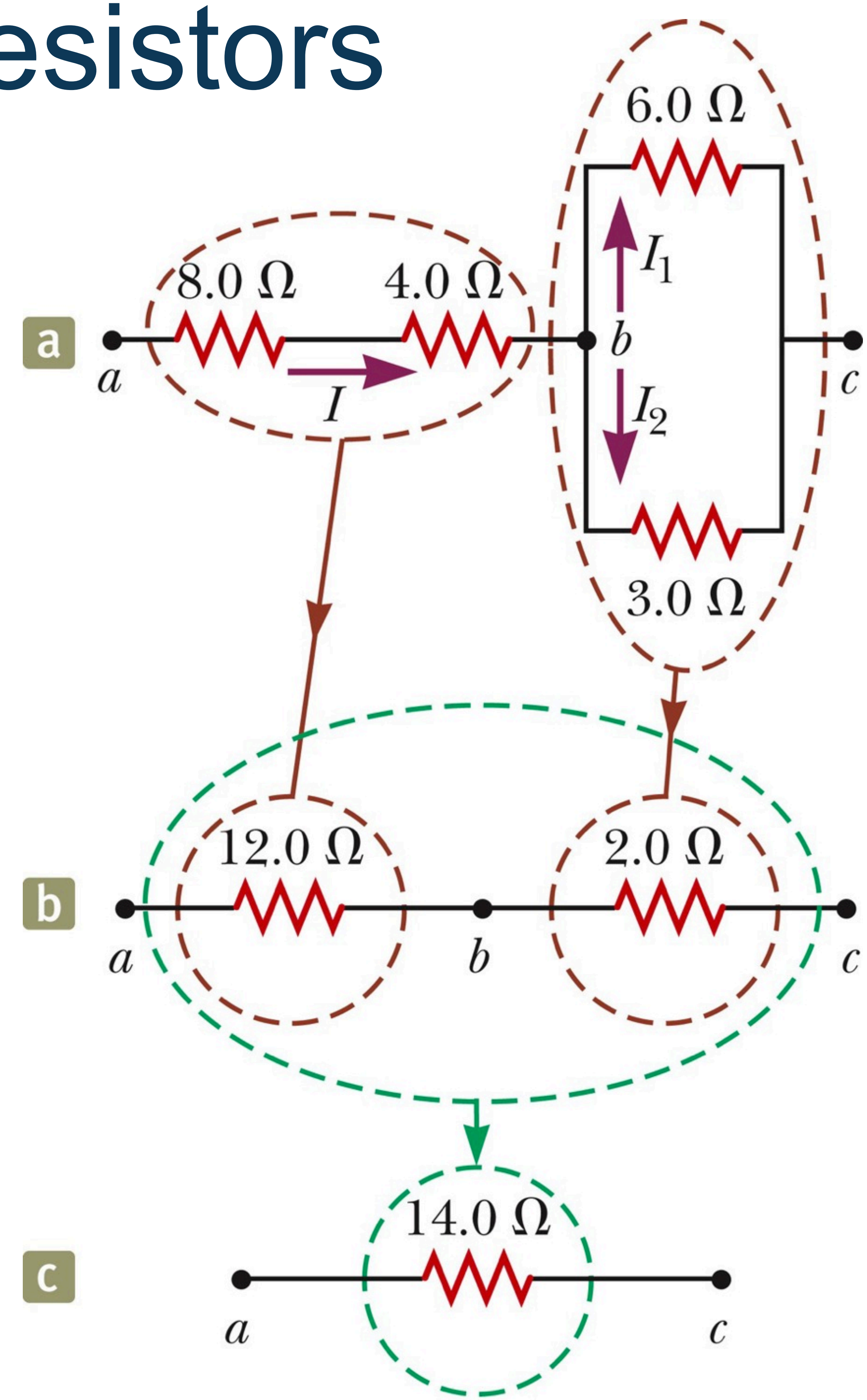
- The equivalent is always less than the smallest resistor in the group.

Combinations of Resistors

The 8.0-Ω and 4.0-Ω resistors are in series and can be replaced with their equivalent, 12.0 Ω.

The 6.0-Ω and 3.0-Ω resistors are in parallel and can be replaced with their equivalent, 2.0 Ω.

These equivalent resistances are in series and can be replaced with their equivalent resistance, 14.0 Ω.



Some Notes on Circuit

A local change in one part of a circuit may result in a global change throughout the circuit.

- For example, changing one resistor will affect the currents and voltages in all the other resistors and the terminal voltage of the battery.

In a series circuit:

- There is one path for the current to take.
- If one device in the circuit creates an open circuit, all devices are inoperative.

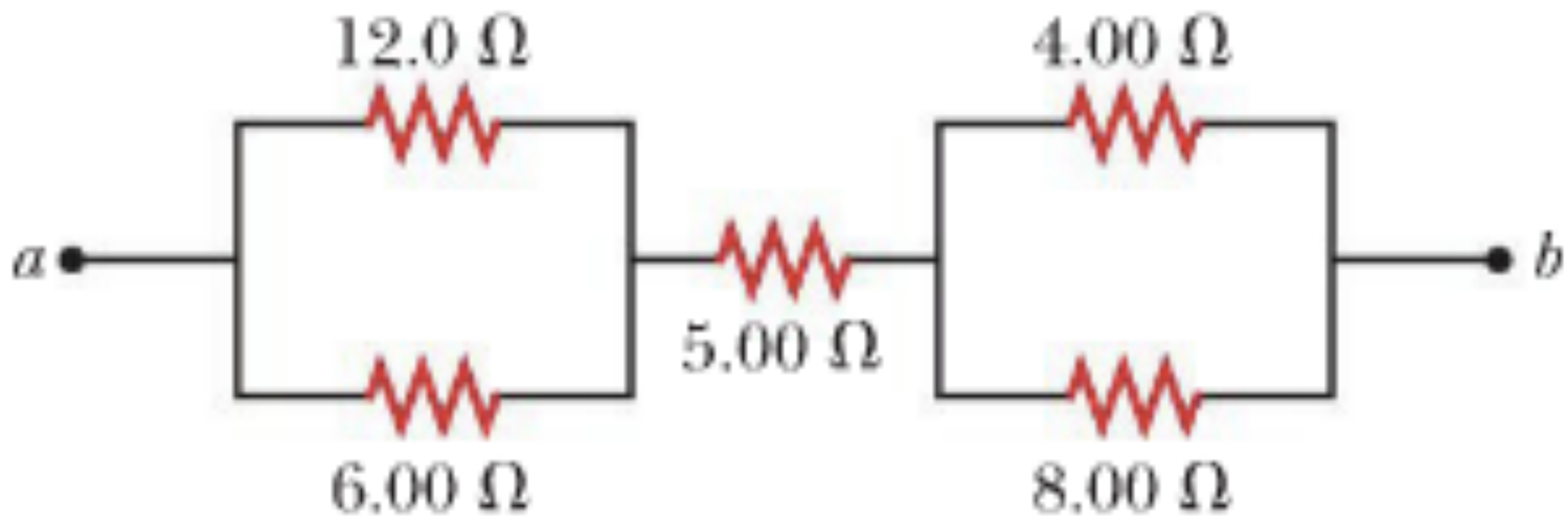
In a parallel circuit:

- There are multiple paths for the current to take.
- The current takes all the paths.
- The lower resistance will have higher currents.
- Even very high resistances will have some currents.
- Each device operates independently of the others so that if one is switched off, the others remain on.
- All of the devices operate on the same voltage.

Household circuits are wired so that electrical devices are connected in parallel.

Example Problem #1

17. Consider the combination of resistors shown in Figure P28.17. (a) Find the equivalent resistance between points *a* and *b*. (b) If a voltage of 35.0 V is applied between points *a* and *b*, find the current in each resistor.



Example Problem #1: Solution

(a) The parallel combination of the 6.0 Ω and 12 Ω resistors has an equivalent resistance of

$$\frac{1}{R_{p1}} = \frac{1}{6.0\ \Omega} + \frac{1}{12\ \Omega} = \frac{2+1}{12\ \Omega}$$

or $$R_{p1} = \frac{12\ \Omega}{3} = 4.0\ \Omega$$

Similarly, the equivalent resistance of the 4.0 Ω and 8.0 Ω parallel combination is

$$\frac{1}{R_{p2}} = \frac{1}{4.0\ \Omega} + \frac{1}{8.0\ \Omega} = \frac{2+1}{8.0\ \Omega}$$

or $$R_{p2} = \frac{8\ \Omega}{3}$$

The total resistance of the series combination between points *a* and *b* is then

$$R_{ab} = R_{p1} + 5.0\ \Omega + R_{p2} = 4.0\ \Omega + 5.0\ \Omega + \frac{8.0}{3}\ \Omega$$

$$= \frac{35}{3}\ \Omega = \boxed{11.7\ \Omega}$$

Example Problem #1: Solution

(b) If $\Delta V_{ab} = 35\text{ V}$, the total current from a to b is $I_{ab} = \Delta V_{ab}/R_{ab} =$ $35\text{ V}/(35\ \Omega/3) = 3.0\text{ A}$ and the potential differences across the two parallel combinations are

$$\Delta V_{p1} = I_{ab}R_{p1} = (3.0\text{ A})(4.0\ \Omega) = 12\text{ V}$$

and $$\Delta V_{p2} = I_{ab}R_{p2} = (3.0\text{ A})\left(\frac{8.0}{3}\ \Omega\right) = 8.0\text{ V}$$

so the individual currents through the various resistors are:

$$I_{12} = \Delta V_{p1}/12\ \Omega = \boxed{1.0\text{ A}}$$

$$I_{6} = \Delta V_{p1}/6.0\ \Omega = \boxed{2.0\text{ A}}$$

$$I_{5} = I_{ab} = \boxed{3.0\text{ A}}$$

$$I_{8} = \Delta V_{p2}/8.0\ \Omega = \boxed{1.0\text{ A}}$$

and $$I_{4} = \Delta V_{p2}/4.0\ \Omega = \boxed{2.0\text{ A}}$$

Example Problem #2

How many Ohm’s Laws are there?

Example Problem #2: Solution

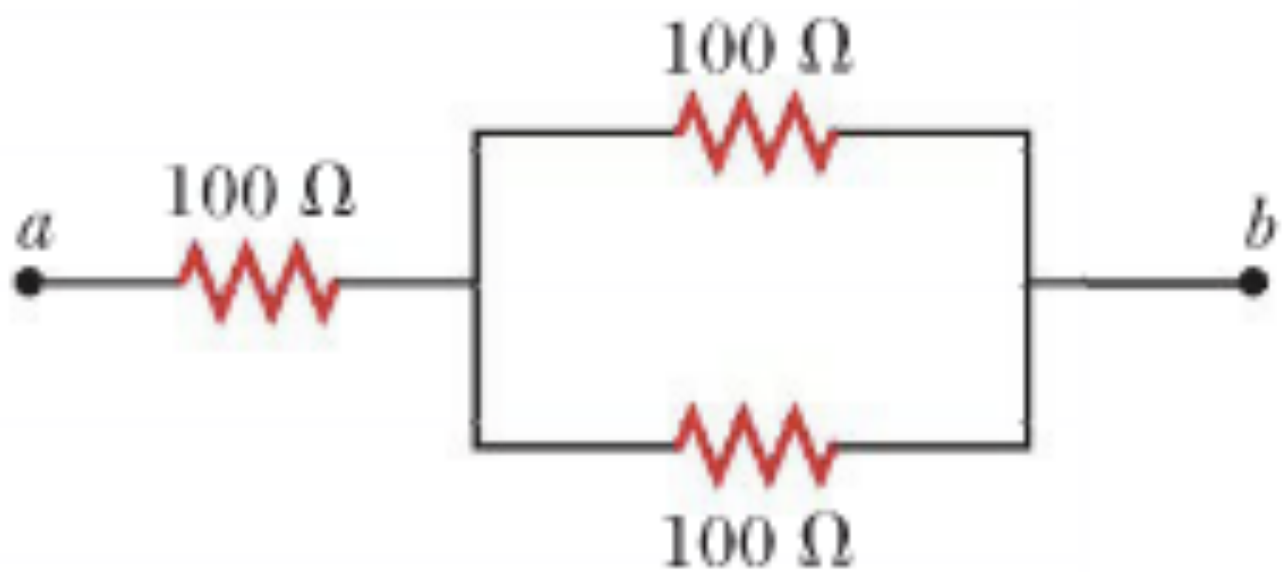
Only ONE! (algebraic re-arrangement possible)

Announcement

HW05 is up on WebAssign, due Thursday 02/22

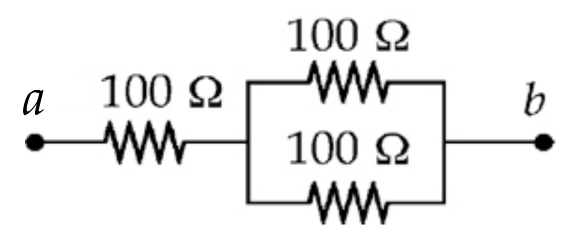
Example Problem #3

5. Three 100-Ω resistors are connected as shown in Figure P28.5. The maximum power that can safely be delivered to any one resistor is 25.0 W. (a) What is the maximum potential difference that can be applied to the terminals *a* and *b*? (b) For the voltage determined in part (a), what is the power delivered to each resistor? (c) What is the total power delivered to the combination of resistors?



Example Problem #3: Solution

(a) Since all the current in the circuit must pass through the series 100-Ω resistor,



ANS. FIG. P28.5

$$P_{\max} = I_{\max}^2 R$$

so $$I_{\max} = \sqrt{\frac{P}{R}} = \sqrt{\frac{25.0\ \mathrm{W}}{100\ \Omega}} = 0.500\ \mathrm{A}.$$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1} \Omega = 150\ \Omega$$

$$\Delta V_{\max} = R_{eq} I_{\max} = \boxed{75.0\ \mathrm{V}}$$

(b) From a to b in the circuit, the power delivered is

$P_{\text{series}} = \boxed{25.0\ \mathrm{W}}$ for the first resistor, and

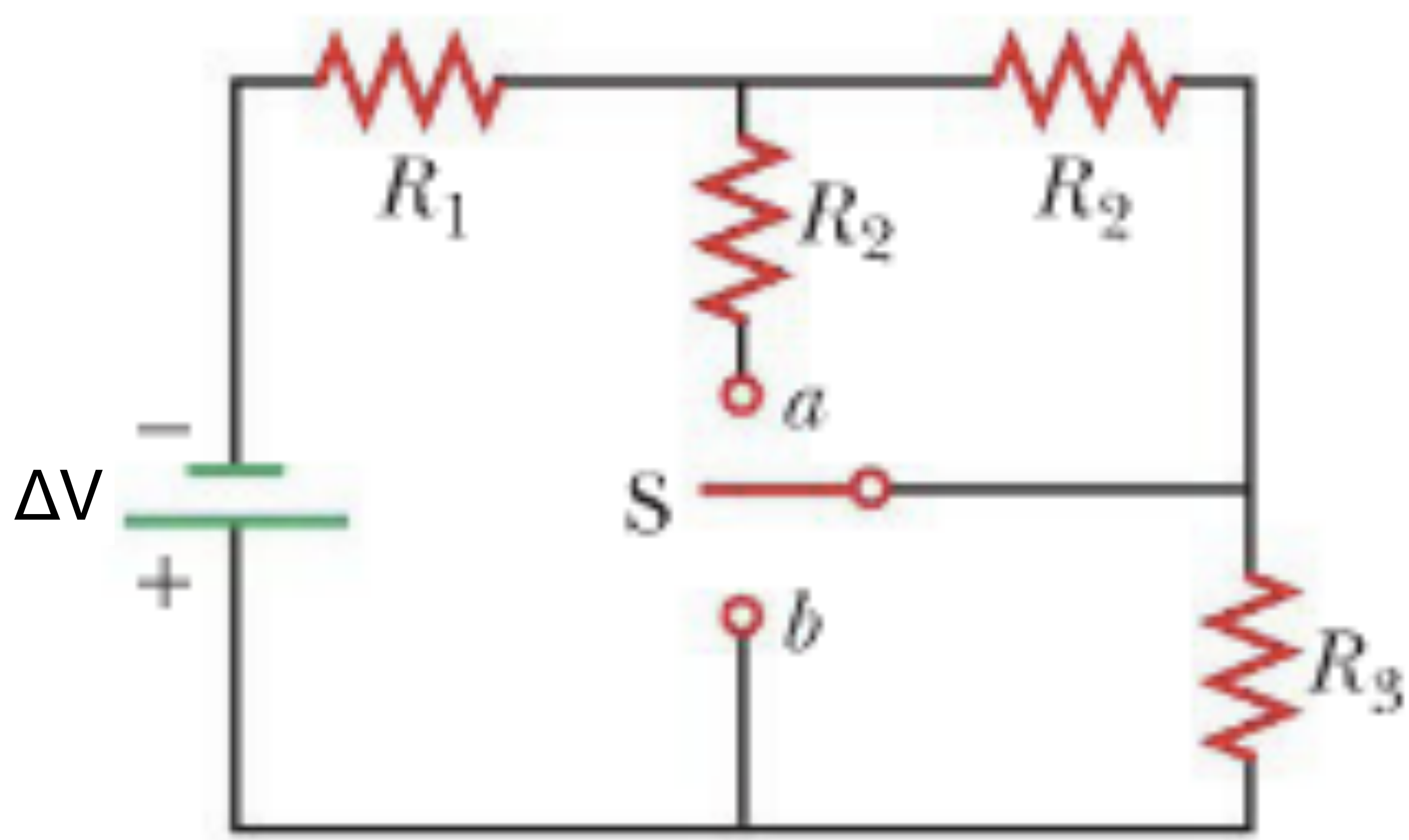
$$P_{\text{parallel}} = I^2 R = (0.250\ \mathrm{A})^2 (100\ \Omega) = \boxed{6.25\ \mathrm{W}}$$

for each of the two parallel resistors.

(c) $P = I\Delta V = (0.500\ \mathrm{A})(75.0\ \mathrm{V}) = \boxed{37.5\ \mathrm{W}}$

Example Problem #4

11. A battery with **a potential difference of 6.00 V** supplies current to the circuit shown in Figure P28.11. When the double-throw switch S is open as shown in the figure, the current in the battery is 1.00 mA. When the switch is closed in position *a*, the current in the battery is 1.20 mA. When the switch is closed in position *b*, the current in the battery is 2.00 mA. Find the resistances (a) R_1, (b) R_2, and (c) R_3.



Example Problem #4: Solution

When S is open, R_1, R_2, and R_3 are in series with the battery. Thus,

$$R_1 + R_2 + R_3 = \frac{6\ \text{V}}{10^{-3}\ \text{A}} = 6\ \text{k}\Omega \qquad \textbf{[1]}$$

When S is closed in position a, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. Thus,

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{6\ \text{V}}{1.2\times 10^{-3}\ \text{A}} = 5\ \text{k}\Omega \qquad \textbf{[2]}$$

When S is closed in position b, R_1 and R_2 are in series with the battery and R_3 is shorted. Thus,

$$R_1 + R_2 = \frac{6\ \text{V}}{2\times 10^{-3}\ \text{A}} = 3\ \text{k}\Omega \qquad \textbf{[3]}$$

Subtracting [3] from [1] gives $R_3 = 3\ \text{k}\Omega$.

Subtracting [2] from [1] gives $R_2 = 2\ \text{k}\Omega$.

Then, from [3], $R_1 = 1\ \text{k}\Omega$.

Answers: (a) $\boxed{R_1 = 1.00\ \text{k}\Omega}$ (b) $\boxed{R_2 = 2.00\ \text{k}\Omega}$ (c) $\boxed{R_3 = 3.00\ \text{k}\Omega}$