Capacitance & Dielectrics Chapter 25

HW03 is up on WebAssign, due Thursday 02/08

What is a battery?

Device that provides charges, more precisely, electrons.

2 terminals : cathode (-) and anode (+)

the cathode is a source of electrons that when connected to an external circuit will flow and give out energy.

A battery contains charges and therefore creates an electric field.

Both terminals have different charges, therefore there is a potential difference between them.

A battery is a source of potential difference.

It is because there is a potential difference that current can flow.



Circuits and Circuit Elements

Electric circuits are the basis for the vast majority of the devices used in society.

Circuit elements must be connected with wires to form electric circuits.

Circuits describe the flow of electricity through different circuit elements.

The circuit elements are represented by **symbols**.

The circuits themselves are represented by **diagrams**.



Basic circuit

A flashlight = a light bulb
 powered by a battery



To draw the circuit, identify the circuit elements:

- the light bulb
- the battery
- the wires

symbol for battery:

symbol for light bulb:







In an electric circuit, **it's the electrons that move and create a current.** They move from the - to the + end of the battery.

Is this a correct circuit diagram?



NO! You must respect conventions and symbols.

What this is a nice schematical visualization of a circuit, but it is not a diagram. 5

Capacitors

Capacitors are devices that store electric charge.

1 of the most basic circuit element

In general, capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse.

Defibrillators

- When cardiac fibrillation occurs, the heart produces a rapid, irregular pattern of beats
- A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern.



Capacitors

A capacitor consists of **two conductors**.

- These conductors are called plates.
- A capacitor can be charged or not
- When the capacitor is charged, the plates carry charges of equal magnitude and opposite directions.
- A potential difference exists between the plates due to the charge.



When the capacitor is charged, the conductors carry charges of equal magnitude and opposite sign.



Parallel plate capacitor

You have 2 plates that are parallel, each of surface area A and separated by a distance d

Initially, conductors are in electrostatic equilibrium. So the plates have no charges.

If you want to charge (or discharge) a capacitor, you need to move electrons.

To move electrons, you need to create a field.

To create a field, you need charges —>you need a battery.

Remember field between plates:

from Gauss's law



8



How to charge a capacitor

The battery establishes an electric field in the connecting wires

--> electrons in the wires start moving.

On the wire connected to the negative terminal of the battery:

electrons will move from the wire to the plate (away from the battery)

—> after a while the conducting wire and plate are in equilibrium, and the plate is negatively charged

On the wire connected to the positive terminal of the battery:

electrons will move from the plate to the battery terminal —> after a while the plate is positively charged (the electrons have moved away, leaving a net positive charge).

In its final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery. The amount of charge is the same on both plates.

When the capacitor is connected to the terminals of a battery, electrons transfer between the plates and the wires so that the plates become charged.



Charge and Voltage Considerations

If the capacitor remains connected to a battery, the voltage across the capacitor necessarily remains the same.

If the capacitor is disconnected from the battery, the capacitor is an isolated system and the charge remains the same.



Capacitance

The **capacitance**, *C*, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors.

$$C \equiv \frac{Q}{\Delta V}$$

The SI unit of capacitance is the farad (F).

The farad is a large unit, typically you will see microfarads (μ F) and picofarads (pF).

Capacitance will always be a positive quantity

The capacitance of a given capacitor is constant.

The capacitance is a measure of the capacitor's ability to store charge .

 The capacitance of a capacitor is the amount of charge the capacitor can store per unit of potential difference.

Capacitance for an isolated sphere

Assume a spherical charged conductor with radius *r*.

The sphere will have the same capacitance as it would if there were a conducting sphere of infinite radius, concentric with the original sphere.

Assume V = 0 for the infinitely large shell

$$C = \frac{Q}{\Delta V} \qquad \Delta V = k_e \frac{q}{r}$$
$$C = \frac{Q}{k_e Q/r}$$
$$C = \frac{r}{k_e} = 4\pi\epsilon_0 r$$

Note, this is independent of the charge on the sphere and its potential.

Capacitance for parallel plates

The charge density on the plates is $\sigma = Q/A$.

- A is the area of each plate, the area of each plate is equal
- Q is the charge on each plate, equal with opposite signs

The electric field is uniform between the plates and zero elsewhere.



The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates.

Capacitance of a cylindrical Capacitor Remember Gauss's law lectu

Remember Gauss's law lecture slide 33 (when taking a cylindrical gaussian surface)

-Q

l

$$E = 2k_e \frac{\lambda}{r}$$

$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$V_b - V_a = -\int_a^b E_r dr = -2k_e \lambda \int_a^b \frac{dr}{r} = -2k_e \lambda \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{(2k_e Q/\ell) \ln(b/a)} = \frac{\ell}{2k_e \ln(b/a)}$$
(26.4)

Capacitance of a spherical capacitor

See Gaus' law lecture slide 29

$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$V_{b} - V_{a} = -\int_{a}^{b} E_{r} dr = -k_{e} Q \int_{a}^{b} \frac{dr}{r^{2}} = k_{e} Q \left[\frac{1}{r}\right]_{a}^{b}$$

(1)
$$V_b - V_a = k_e Q \left(\frac{1}{b} - \frac{1}{a}\right) = k_e Q \frac{a-b}{ab}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{|V_b - V_a|} = \frac{ab}{k_e(b - a)}$$
 (26.6)



Parallel Circuits and Circuits in series

3 ways of building a circuit:

- with the elements in parallel
- with the elements in series
- with some elements in parallel and some in series.

elements in series means they are on the same wire, one after the other.



3 capacitors in series

elements in parallel means they are parallel to each other.

same potential difference across both capacitors!



2 capacitors in parallel

Capacitors in parallel

When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged.

The flow of charges ceases when the voltage across the capacitors equals that of the battery.

The potential difference across the capacitors is the same.

- And each is equal to the voltage of the battery
- $\Delta V_1 = \Delta V_2 = \Delta V$

The capacitors reach their maximum charge when the flow of charge ceases. The total charge is equal to the sum of the charges on the capacitors.

•
$$Q_{tot} = Q_1 + Q_2$$

A pictorial representation of two capacitors connected in parallel to a battery



Equivalent capacitance (C in ||)

The capacitors can be replaced with one capacitor with a capacitance of $C_{eq.}$

 The equivalent capacitor must have exactly the same external effect on the circuit as the original capacitors.

$$C_{eq} = C_1 + C_2 + C_3 + \dots$$

The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors.

A circuit diagram showing the two capacitors connected in parallel to a battery

A circuit diagram showing the equivalent capacitance of the capacitors in parallel





Capacitors in series

When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery.

As this negative charge accumulates on the right plate of C_2 , an equivalent amount of negative charge is removed from the left plate of C_2 , leaving it with an excess positive charge.

All of the right plates gain charges of – Q and all the left plates have charges of +Q.

A pictorial representation of two capacitors connected in series to a battery C_1 C_9 ΔV_1 ΔV_{2} -0-Q ΔV

Equivalent capacitance (C in series)

An equivalent capacitor can be found that performs the same function as the series combination. The charges are all the same.

 $Q_1 = Q_2 = Q$

The potential differences add up to the battery voltage.

 $\Delta V_{tot} = \Delta V_1 + \Delta V_2 + \dots$

The equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

The equivalent capacitance of a series combination is **always less** than any individual capacitor in the combination.

A circuit diagram showing the two capacitors connected in series to a battery

A circuit diagram showing the equivalent capacitance of the capacitors in series





Equivalent capacitance example



The 1.0- μ F and 3.0- μ F capacitors are in parallel as are the 6.0- μ F and 2.0- μ F capacitors.

These parallel combinations are in series with the capacitors next to them.

The series combinations are in parallel and the final equivalent capacitance can be found.

stop here?

Example Problem 1

noisy capacitor discharge (a YouTube demo)

Example Problem 1: Solution

https://www.youtube.com/shorts/MQ&0.74mTabs $C = \frac{15.6 \text{ pF}}{k_e (b-a)} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.140 \text{ m} - 0.0700 \text{ m})}{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.140 \text{ m} - 0.0700 \text{ m})}$

$$\Delta V = \frac{Q}{C} = \frac{4.00 \times 10^{-6} \text{ C}}{1.56 \times 10^{-11} \text{ F}} = 2.57 \times 10^{5} \text{ V} = 257 \text{ kV}$$

Example Problem 2

MINDBLOWER: Casimir plates in a vacuum

Example Problem 2: Solution

https://physicsworld.com/a/the-casimir-effect-a-force-from-nothing/

$$= 1.36 \times 10^{-12} \text{ F} = \boxed{1.36 \text{ pF}}_{25}$$

Example Problem 3

9. An air-filled capacitor consists of two parallel plates,
M each with an area of 7.60 cm², separated by a distance of 1.80 mm. A 20.0-V potential difference is applied to these plates. Calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.

Example 1 Problem 3: Solution

(a) The potential difference between two points in a uniform electric field is $\Delta V = Ed$, so

$$E = \frac{\Delta V}{d} = \frac{20.0 \text{ V}}{1.80 \times 10^{-3} \text{ m}} = \boxed{1.11 \times 10^4 \text{ V/m}}$$

(b) The electric field between capacitor plates is $E = \frac{\sigma}{\epsilon_0}$, so $\sigma = \epsilon_0 E$:

$$\sigma = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.11 \times 10^4 \text{ V/m}) = 9.83 \times 10^{-8} \text{ C/m}^2$$
$$= 98.3 \text{ nC/m}^2$$

(c) For a parallel-plate capacitor,
$$C = \frac{\epsilon_0 A}{d}$$
:

$$C = \frac{\left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)\left(7.60 \times 10^{-4} \text{ m}^2\right)}{1.80 \times 10^{-3} \text{ m}}$$
$$= 3.74 \times 10^{-12} \text{ F} = \boxed{3.74 \text{ pF}}$$

(d) The charge on each plate is $Q = C\Delta V$:

$$Q = (3.74 \times 10^{-12} \text{ F})(20.0 \text{ V}) = \boxed{74.7 \text{ pC}}_{27}$$

Example Problem 4

13. Two capacitors, $C_1 = 5.00 \ \mu\text{F}$ and $C_2 = 12.0 \ \mu\text{F}$, are connected in parallel, and the resulting combination is connected to a 9.00-V battery. Find (a) the equivalent capacitance of the combination, (b) the potential difference across each capacitor, and (c) the charge stored on each capacitor.

Example Problem 4: Solution

(a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$$C_{\rm eq} = C_1 + C_2 = 5.00 \ \mu F + 12.0 \ \mu F = 17.0 \ \mu F$$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$$\Delta V = \boxed{9.00 \text{ V}}$$

(c)
$$Q_5 = C\Delta V = (5.00 \ \mu F)(9.00 \ V) = 45.0 \ \mu C$$

$$Q_{12} = C\Delta V = (12.0 \ \mu F)(9.00 \ V) = 108 \ \mu C$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00 \ \mu F} + \frac{1}{12.0 \ \mu F}$$

Example Problem 5

19. For the system of four capacitors shown in Figure P26.19, find (a) the equivalent capacitance of the system, (b) the charge on each capacitor, and (c) the potential difference across each capacitor.



Example Problem 5. Solution $\frac{4}{3}C$

(a) The equivalent capacitance of the series combination in the upper branch is

$$\frac{1}{C_{\text{upper}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3.00 \ \mu\text{F}} + \frac{1}{6.00 \ \mu\text{F}} \rightarrow C_{\text{upper}} = 2.00 \ \mu\text{F}$$

Likewise, the equivalent capacitance of the series combination in the lower branch is

$$\frac{1}{C_{\text{lower}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.00 \ \mu\text{F}} + \frac{1}{4.00 \ \mu\text{F}} \rightarrow C_{\text{lower}} = 1.33 \ \mu\text{F}$$

These two equivalent capacitances are connected in parallel with each other, so the equivalent capacitance for the entire circuit is

$$C_{eq} = C_{upper} + C_{lower} = 2.00 \ \mu F + 1.33 \ \mu F = 3.33 \ \mu F$$

$$Q_3 = Q_6 = Q_{upper} = C_{upper} (\Delta V) = (2.00 \ \mu\text{F})(90.0 \ \text{V}) = 180 \ \mu\text{C}$$

Example Problem 5: Solution

(b) Note that the same potential difference, equal to the potential difference of the battery, exists across both the upper and lower branches. Each of the capacitors in series combination holds the same charge as that on the equivalent capacitor. For the upper branch:

$$Q_3 = Q_6 = Q_{upper} = C_{upper} (\Delta V) = (2.00 \ \mu F)(90.0 \ V) = 180 \ \mu C \ s$$

so, $|180 \ \mu\text{C}$ on the 3.00- μF and the 6.00- μF capacitors

For the lower branch:

$$Q_2 = Q_4 = Q_{\text{lower}} = C_{\text{lower}} (\Delta V) = (1.33 \ \mu\text{F})(90.0 \ \text{V}) = 120 \ \mu\text{C}$$

so, $|120 \ \mu\text{C}$ on the 2.00- μF and 4.00- μF capacitors

Example Problem 5: Solution

(c) The potential difference across each of the capacitors in the circuit is:

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120 \ \mu C}{2.00 \ \mu F} = \boxed{60.0 \ V}$$
$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180 \ \mu C}{3.00 \ \mu F} = \boxed{60.0 \ V}$$

60.0 V across the 3.00- μ F and the 2.00- μ F capacitors

$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{120 \ \mu C}{4.00 \ \mu F} = \boxed{30.0 \ V}$$
$$\Delta V_6 = \frac{Q_6}{C_6} = \frac{180 \ \mu C}{6.00 \ \mu F} = \boxed{30.0 \ V}$$

30.0 V across the 6.00- μ F and the 4.00- μ F capacitors

$$\left[\frac{1}{2C} + \frac{1}{C}\right]^{-1} = \left[\frac{2C}{2C}\right]^{3\beta}$$

Example Problem 6

26. Find (a) the equivalent capacitance of the capacitors in Figure P26.26, (b) the charge on each capacitor, and (c) the potential difference across each capacitor.







(a) First, we replace the parallel combination between points b and c by its equivalent capacitance,

 $C_{\rm bc} = 2.00 \ \mu \text{F} + 6.00 \ \mu \text{F} = 8.00 \ \mu \text{F}$.

Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_{\rm ab}} + \frac{1}{C_{\rm bc}} + \frac{1}{C_{\rm cd}} = \frac{3}{8.00 \ \mu \rm F}$$



ANS. FIG. P26.26

giving

$$C_{\rm eq} = \frac{8.00 \ \mu \rm F}{3} = 2.67 \ \mu \rm F$$

$$Q_{ab} = Q_{bc} = Q_{cd} = C_{eq} (\Delta V_{ad}) = (2.67 \ \mu\text{F})(9.00 \ \text{V}) = 24.0 \ \mu\text{C}$$
$$\Delta V_{bc} = \frac{Q_{bc}}{Q_{bc}} = \frac{24.0 \ \mu\text{C}}{Q_{bc}} = 3.00 \ \text{V}$$

Example 3 Problem 6: Solution

(b) The charge on each capacitor in the series is the same as the charge on the equivalent capacitor:

$$Q_{\rm ab} = Q_{\rm bc} = Q_{\rm cd} = C_{\rm eq} (\Delta V_{\rm ad}) = (2.67 \ \mu \text{F})(9.00 \text{ V}) = 24.0 \ \mu \text{C}$$

Then, note that $\Delta V_{bc} = \frac{Q_{bc}}{C_{bc}} = \frac{24.0 \ \mu C}{8.00 \ \mu F} = 3.00 \text{ V}$. The charge on each

capacitor in the original circuit is:

On the 8.00 μ F between a and b:

 $Q_8 = Q_{ab} = \boxed{24.0 \ \mu C}$

On the 8.00 μ F between c and d:

$$Q_8 = Q_{cd} = 24.0 \ \mu C$$

On the 2.00 μ F between b and c:

$$Q_2 = C_2 (\Delta V_{\rm bc}) = (2.00 \ \mu \text{F})(3.00 \text{ V}) = 6.00 \ \mu \text{C}$$

On the 6.00 μ F between b and c:

$$Q_6 = C_6 (\Delta V_{bc}) = (6.00 \ \mu F)(3.00 \ V) = \boxed{18.0 \ \mu C}$$

$Q_{2} = C_{2} (\Delta V_{bc}) = (2.00 \ \mu F)(3.00 \ V) = [0.00 \ \mu C]$ Example Problem 6: Solution $Q_{6} = C_{6} (\Delta V_{bc}) = (6.00 \ \mu F)(3.00 \ V) = [18.0 \ \mu C]$

(c) We earlier found that $\Delta V_{bc} = 3.00 \text{ V}$. The two $8.00 \,\mu\text{F}$ capacitors have the same voltage: $\Delta V_8 = \Delta V_8 = \frac{Q}{C} = \frac{24.0 \,\mu\text{C}}{8.00 \,\mu\text{F}} = 3.00 \text{ V}$, so we

conclude that the potential difference across each capacitor is the same: $\Delta V_8 = \Delta V_2 = \Delta V_6 = \Delta V_8 = \boxed{3.00 \text{ V}}$.

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}.$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1 (C_p - C_1)}$$

$$C_{1}^{2} - C_{1}C_{p} + C_{p}C_{s} = 0$$

$$C_{1} = \frac{C_{p} \pm \sqrt{C_{p}^{2} - 4C_{p}C_{s}}}{2} = \frac{1}{2}C_{p} \pm \sqrt{\frac{1}{4}C_{p}^{2} - C_{p}C_{s}}$$

HW04 is up on WebAssign

It is due on Thursday 02/15

Energy in Capacitor

Before the switch is closed, the energy is stored in the battery.

When the switch is closed, the energy becomes electric potential energy, which is related to the separation of the positive and negative charges on the plates.

$$\Delta U = -q_0 \int \vec{E} \cdot d\vec{s}$$

A capacitor can be described as a device that stores energy as well as charge.



Energy stored in Capacitor

Assume the capacitor is being charged and, at some point, has a charge q on it. The work needed to transfer a charge from one plate to the other is

$$dW = \Delta V dq = \frac{q}{C} dq$$
 $W = \Delta U = q \Delta V$

The work required is the area of the tan rectangle (remember an integral is the area under the curve).

The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

The work done in charging the capacitor appears as electric potential energy U:

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

The work required to move charge dq through the potential difference ΔV across the capacitor plates is given approximately by the area of the shaded rectangle.



Energy in parallel plate capacitor

The energy can be considered to be stored in the electric field .

For a parallel-plate capacitor, the energy can be expressed in terms of the field as

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2 \qquad \qquad C = \frac{\epsilon_0 A}{d}$$
$$\Delta V = Ed$$
$$U = 0.5 (\epsilon_0 A d)E^2.$$

It can also be expressed in terms of the energy density (energy per unit volume)

$$u_E = \frac{1}{2} \varepsilon_0 E^2$$
.

Capacitors with dielectrics

A dielectric is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance.

Dielectrics include rubber, glass, and waxed paper

With a dielectric, the capacitance becomes $C = \kappa C_{o_1}$

- The capacitance increases by the factor κ when the dielectric completely fills the region between the plates.
- к is the dielectric constant of the material.
- C₀ is the capacitance in vacuum

Dielectrics: how they work

When an external electric field is applied, the molecules partially align with the field.

The charged edges of the dielectric can be modeled as an additional pair of parallel plates establishing an electric field \vec{E}_{ind} in the direction opposite that of \mathbf{E}_0 .

A smaller field, means a smaller potential, and a greater capacitance.

Dielectrics

For a parallel-plate capacitor, $C = \kappa (\epsilon_0 A) / d$

In theory, *d* could be made very small to create a very large capacitance.

In practice, there is a limit to *d*.

 d is limited by the electric discharge that could occur through the dielectric medium separating the plates.

For a given *d*, the **maximum voltage** that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material (how much E field or voltage a dielectric can take before breaking down, i.e before becoming a conductor).

Dielectrics provide the following advantages:

- Increase in capacitance
- Increase the maximum operating voltage
- Possible mechanical support between the plates
 - This allows the plates to be close together without touching.
 - This decreases *d* and increases *C*.

Some Dielectric Constants and Dielectric Strengths

Table 26.1 Approximate Dielectric Constants and Dielectric Strengths

of Various Materials at Room Temperature

Material	Dielectric Constant ĸ	Dielectric Strength ^a (10 ⁶ V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	
Water	80	

"The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

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Example Problem 7

How much chemical (electrical) energy is stored in Lithium (batteries)?

Example Problem 7: Solution $\Delta V = \sqrt{\frac{2U_E}{C}} = \sqrt{\frac{2(300 \text{ J})}{20.0 \times 10^{-6} \text{ F}}} = 4.47 \times 10^3 \text{ V}$

43 MJ per kg (cf. Gasoline, at 46 MJ / kg approximately)

$$U_{E} = \frac{1}{2}C(\Delta V)^{2} = \frac{1}{2}(3.00 \ \mu\text{F})(6.00 \ \text{V})^{2} = \boxed{54.0 \ \mu\text{J}}$$
$$Q = C\Delta V = (150 \times 10^{-12} \ \text{F})(10 \times 10^{3} \ \text{V}) = 1.50 \times 10^{-6} \ \text{C} = \boxed{1.50 \ \mu\text{C}}$$
$$U_{E} = \frac{1}{2}C(\Delta V)^{2},$$
$$\Delta V = \sqrt{\frac{2U_{E}}{C}} = \sqrt{\frac{2(250 \times 10^{-6} \ \text{J})}{150 \times 10^{-12} \ \text{F}}} = 1.83 \times 10^{3} \ \text{V} = \boxed{1.83 \ \text{kV}}$$

Example Problem 8

34. Two capacitors, $C_1 = 18.0 \ \mu\text{F}$ and $C_9 = 36.0 \ \mu\text{F}$, are connected in series, and a 12.0-V battery is connected across the two capacitors. Find (a) the equivalent capacitance and (b) the energy stored in this equivalent capacitance. (c) Find the energy stored in each individual capacitor. (d) Show that the sum of these two energies is the same as the energy found in part (b). (e) Will this equality always be true, or does it depend on the number of capacitors and their capacitances? (f) If the same capacitors were connected in parallel, what potential difference would be required across them so that the combination stores the same energy as in part (a)? (g) Which capacitor stores more energy in this situation, C_1 or C_9 ?

Example Problem 8: Solution

(a) The equivalent capacitance of a series combination of C_1 and C_2 is

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{18.0 \ \mu \rm{F}} + \frac{1}{36.0 \ \mu \rm{F}} \rightarrow C_{\rm eq} = \boxed{12.0 \ \mu \rm{F}}$$

(b) This series combination is connected to a 12.0-V battery, the total stored energy is

$$U_{E, eq} = \frac{1}{2} C_{eq} (\Delta V)^2 = \frac{1}{2} (12.0 \times 10^{-6} \text{ F}) (12.0 \text{ V})^2 = \boxed{8.64 \times 10^{-4} \text{ J}}$$

$$Q_1 = Q_2 = Q_{\text{total}} = C_{\text{eq}} (\Delta V) = (12.0 \ \mu\text{F})(12.0 \ \text{V})$$

= 144 μ C = 1.44 × 10⁻⁴ C

$$-2$$
 $(1 44 + 10^{-4} C)^2$

(c) Capacitors in series carry the same charge as their equivalent capacitor. The charge stored on each of the two capacitors in the series combination is

$$Q_1 = Q_2 = Q_{\text{total}} = \mathcal{E}_{\text{eq}}(AV) = (12.0 \mu F)(12.0 V)$$
$$= 144 \mu C = 1.44 \times 10^{-4} C$$

and the energy stored in each of the individual capacitors is:

18.0 μ F capacitor:

$$U_{EE1} = \frac{Q_{11}^{22}}{2C_{11}} = \frac{\left((1.44 \times 10^{-4} \text{ C}) \right)^2}{2\left((18.0 \times 10^{-6} \text{ F}) \right)} = 57765 \times 100^{+4} \text{ J}$$

36.0 μ F capacitor:

$$\mathcal{U}_{E^{2}}^{I} = \frac{Q_{2}^{2}}{2C_{2}} = \frac{\left(1.44 \times 10^{-4} \text{ C}\right)^{2}}{2\left(36.9 \times 10^{-6} \text{ F}\right)} = 2.88 \times 10^{-4} \text{ J}$$

- (d) $\begin{array}{c} U_{E1}^{F} + U_{E2}^{F} = 5.76 \times 10^{-4}] \pm 2.88 \times 10^{-4} J = 8.64 \times 10^{-4} \overline{J} = U_{E_{i}eq'} \\ \text{which is one reason why the 12.0 μ capacitor is considered which is one reason why the 12.0 μ capacitor is considered to be equivalent to the two capacitors. \\ \text{to be equivalent to the two capacitors.} \end{array}$
- (e) The total energy of the equivalent capacitance will always equal The total energy of the equivalent capacitance will always equal the sum of the energies stored in the individual capacitors. the sum of the energies stored in the individual capacitors.

 $U_{E1} + U_{E2} = 5.76 \times 10$ J + 2.88 × 10 J = 8.64 × 10 J = $U_{E, eq}$

which is one reason why the 12.0 μ F capacitor is considered

to be equivalent to the two capacitors.

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the sum of the energies stored in the individual capacitors.

(f) If C_1 and C_2 were connected in parallel rather than in series, the equivalent capacitance would be $C_{eq} = C_1 + C_2 = 18.0 \,\mu\text{F} + 36.0 \,\mu\text{F}$ = 54.0 μ F. If the total energy stored in this parallel combination is to be the same as stored in the original series combination, it is necessary that

$$\frac{1}{2}C_{\rm eq}(\Delta V)^2 = U_{E,\rm eq}$$

From which we obtain

$$\Delta V = \sqrt{\frac{2U_{E, eq}}{C_{eq}}} = \sqrt{\frac{2(8.64 \times 10^{-4} \text{ J})}{54.0 \times 10^{-6} \text{ F}}} = 5.66 \text{ V}$$

(g) Because the potential difference is the same across the two capacitors when connected in parallel, and $U_E = \frac{1}{2}C(\Delta V)^2$,

the larger capacitor C_2 stores more energy.

$$U_{E} = \frac{1}{2}C(\Delta V)^{2} + \frac{1}{2}C(\Delta V)^{2} = C(\Delta V)^{2}_{52}$$

Example Problem 9

44. The voltage across an air-filled parallel-plate capacitor is measured to be 85.0 V. When a dielectric is inserted and completely fills the space between the plates as in Figure P26.44, the voltage drops to 25.0 V. (a) What is the dielectric constant of the inserted material? (b) Can you identify the dielectric? If so, what is it? (c) If the dielectric does not completely fill the space between the plates, what could you conclude about the voltage across the plates?

Example Problem 9: Solution

Note that the charge on the plates remains constant at the original (a) value, Q_0 , as the dielectric is inserted. Thus, the change in the potential difference, $\Delta V = Q^2 / \overline{C}$, is due to a change in capacitance alone. The ratio of the final and initial capacitances is

$$\frac{C_f}{C_i} \frac{C_f \kappa \in \mathcal{K} \neq \mathcal{A} \neq \mathcal{A}}{\overline{C_i} \in \mathcal{A} \neq \mathcal{A} \neq \mathcal{A} \neq \mathcal{A}} = \kappa$$

and
$$\frac{C_f}{C_i} \frac{\underline{C}_f Q_0 / (\Delta V)}{C_i Q_0} \Delta V (\Delta V)_i \Delta V (\Delta V)_i (\Delta V)_i (\Delta V)_i (\Delta V)_i \Delta V (\Delta V)_i \Delta V)_i (\Delta V)_i \Delta V (\Delta V) (\Delta V)_i \Delta V (\Delta V) (\Delta V$$

Thus, the dielectric constant of the inserted material is $|\kappa = 3.40|$.

- The material is probably |nylon| (see Table 26.1). (b)
- (C)The presence of a dielectric weakens the field between plates, and the weaker field, for the same charge on the plates, results in a smaller potential difference. If the dielectric only partially filled the space between the plates, the field is weakened only within the dielectric and not in the remaining air-filled space, so the potential difference would not be as small. The voltage would lie $C = \frac{1}{\frac{1}{d}} = \frac{2.10(8.85 \times 10^{-5} \text{ m})(1.75 \times 10^{-4} \text{ m}^2)}{4.00 \times 10^{-5} \text{ m}} = 8.13 \times 10^{-11} \text{ F}$

$$\Lambda V = + E_1 q_1 - \pi 60.0 \times 10^6 V / m) (1.00 \times 10^{-5} m) - 2.40 kV$$