Electric potential - Chapter 24

HW03 is up on WebAssign, due Thursday 02/08

The point of this lecture

Electromagnetism has been connected to the study of forces in previous chapters.

In this lecture, electromagnetism will be linked to energy.

By using an energy approach, problems that were insoluble using forces can be solved.

The concept of **potential energy is of great value in the study of electricity.**

Because the electrostatic force is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy.

This will enable the definition of ***electric potential.***

So far we know…

- A charge creates an electric field detectable by dropping a positive test charge in the field
- Any charge dropped in a field will move following electric field lines
- The charge **moves** because it is subject to an electric **force**.

→ In other words, the charge moves because the electric field exerts a **work** on the charge.

Reminder: What is Work ? PHYSICS 1!!

= characterizes the influence of a force on a system
= how much force you must apply on a system to **make it move**

Work ≠ 0 Work ≠ 0 Work = 0

You must take into account the vector nature of the applied force: its magnitude and its direction

Work is done **by** a force **on** an object

$$W = \int \vec{F} \cdot d\vec{s}$$

Work and Potential Energy

Work W is linked to potential energy U.

PHYSICS 1!!

$W = \Delta K$ work - kinetic energy theorem

$\Delta K + \Delta U = 0$ conservation of energy

potential energy = storage energy

→ $W = -\Delta U$

$$\boxed{W = \int \vec{F} \cdot d\vec{s} = -\Delta U}$$

ds is a small displacement tangent at each point to the path of integration

In physics 1, the force is mechanical

In physics 2, it's the electric force F=qE

Work by the electric force on a charge

Work involves motion.

A charge placed in an E field will move, it will follow the electric field lines.

The field exerts a work on the charge.

$$W = \int \vec{F} \cdot d\vec{s} = -\Delta U \qquad \vec{F} = q\vec{E}$$

$$W = -\Delta U = q_0 \int \vec{E} \cdot d\vec{s}$$

$$\boxed{\Delta U = -q_0 \int \vec{E} \cdot d\vec{s}}$$

→ **electric potential energy**

Electric Potential Energy

Remember that really what matters is the difference in potential energy, because potential energy is defined by **the change between two points.**

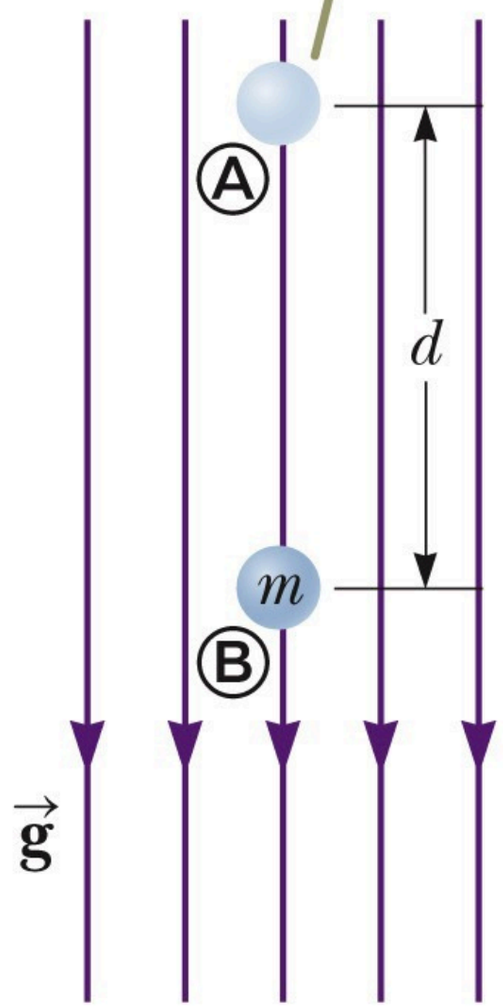
← U=mgh is really ΔU = mgh - 0

← define potential energy here to be 0

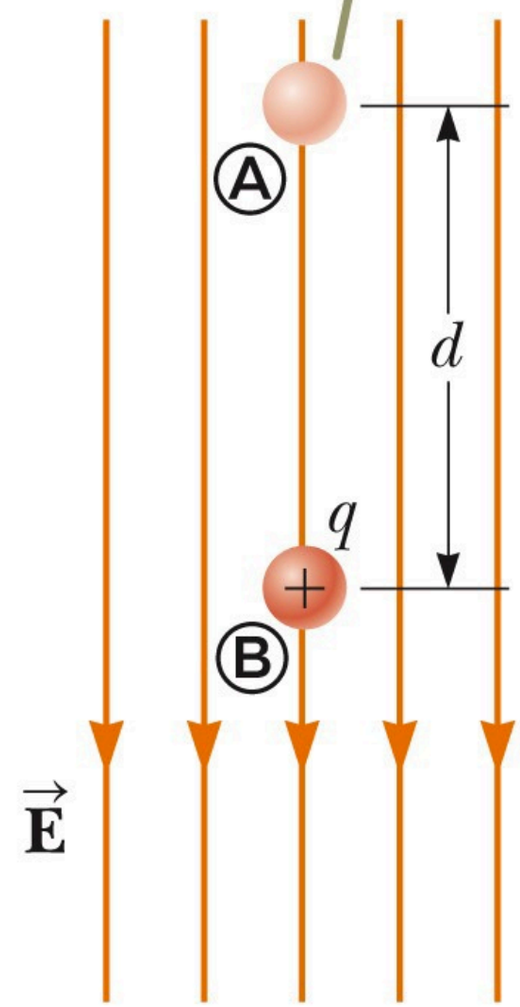
- Therefore the electric potential energy is defined between two charges.
- For a potential energy to exist, there must be a **system of two or more charges.**
- The potential energy belongs to the system and **changes only if a charge is moved relative to the rest of the system**
- The potential energy is **characteristic of the charge-field system.**

Electric potential energy

When an object with mass moves from point Ⓐ to point Ⓑ, the gravitational potential energy of the object–field system decreases.



When a positive charge moves from point Ⓐ to point Ⓑ, the electric potential energy of the charge–field system decreases.



Directions

$$\Delta U = -q_0 \int \vec{E} \cdot d\vec{s}$$

System = positive charge in electric field , *ΔU is negative*

—> charge moves in direction of field

—> system loses electric potential energy

—-> the field does work on the charge

—-> the charged particle gains kinetic energy because of **conservation of energy**

System = negative charge in electric field , *ΔU is negative*

—> charge should move in opposite direction of field, in which case it loses electric potential energy

—> system gains electric potential energy if the charge moves in the direction of the field (Δ*U* positive)

—-> for it to move in the direction of the field, an external agent must do positive work on the charge.

$$\underbrace{\Delta U}_{\text{negative (decreases with increasing displacement ds)}} = -\underbrace{q_0}_{\text{positive since q0 is negative}} \int \underbrace{\vec{E} \cdot d\vec{s}}_{\text{negative since E and ds are in opposite direction}}$$

Potential Energy and Potential

$$\Delta U = -q_0 \int \vec{E} \cdot d\vec{s}$$

We can define a new quantity **the potential V as the potential energy per unit charge,**

$$\boxed{V = \frac{U}{q_o}}$$

V = electric potential
U = potential energy
q_0 = test charge

- The potential is characteristic of the field only.
- The potential is independent of any charges that may be placed in the field.
- The potential has a value at every point in an electric field.

The electric potential is a **SCALAR** quantity (it has no direction)

Change of potential

The difference in potential is the meaningful quantity.

As in physics 1, **we often take the value of the potential to be zero at some convenient point in the field.**

The potential difference between two points exists solely because of a source charge and depends on the source charge distribution.

$$\Delta V = \frac{\Delta U}{q_0} = -\int \vec{E} \cdot d\vec{s}$$

[V] = [J]/[C] = [N/C][m]

unit of energy: joule J
unit of charge: coulomb C
unit of potential: J/C = V
V is a volt

- It takes one joule of work to move a 1 coulomb charge through a potential difference of 1 volt.

In addition, 1 N/C = 1 V/m

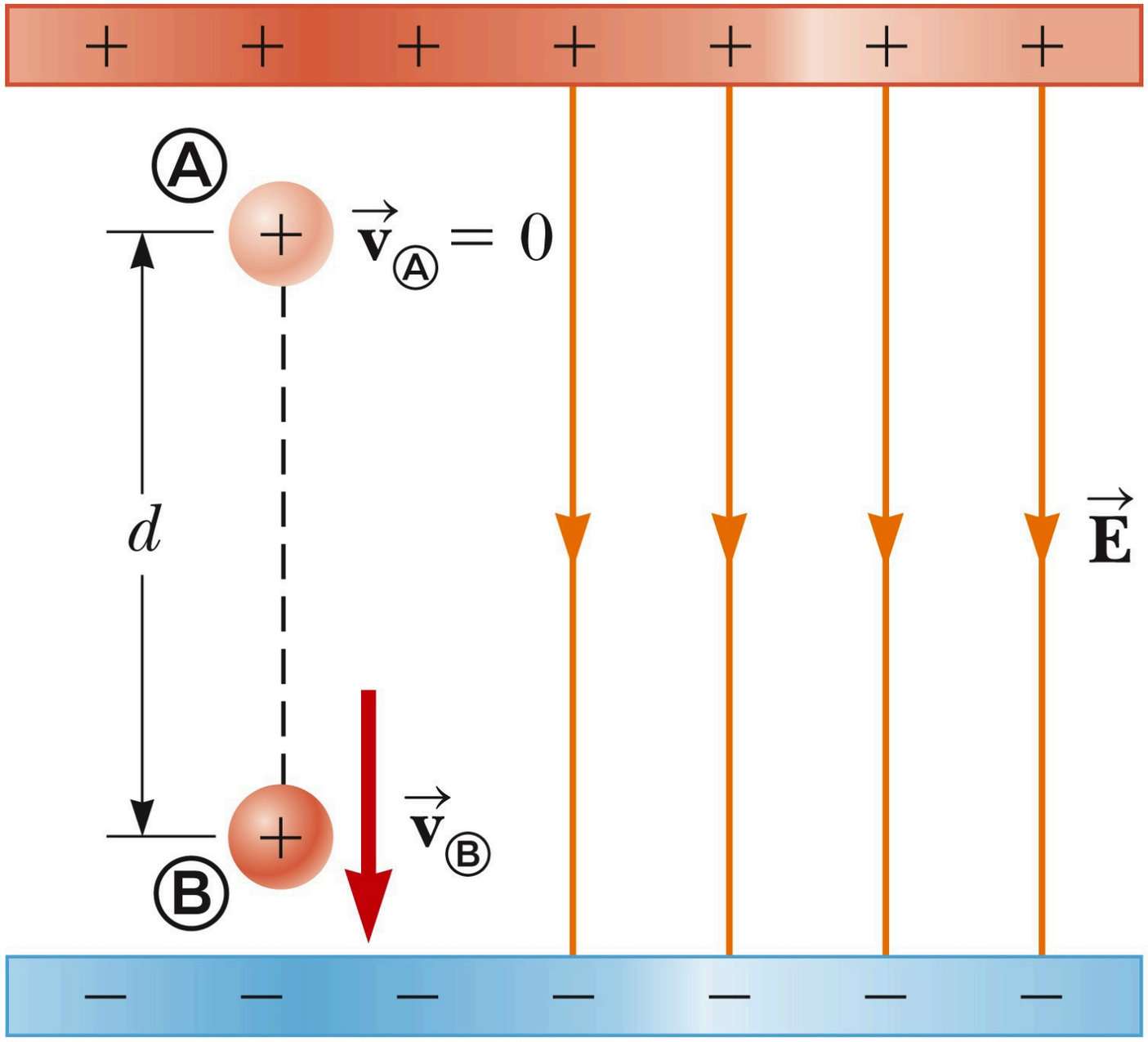
- This indicates we can interpret the **electric field as a measure of the rate of change of the electric potential with respect to position**.

Example: charged particle in uniform field

- A positive charge is released from rest and moves in the direction of the electric field.
- The change in **potential** is negative.

$$\Delta V = \frac{\Delta U}{q_0} = -\int \vec{E} \cdot d\vec{s}$$

- The change in **potential energy** is negative.
- The force and acceleration are in the direction of the field.
- **Conservation of Energy can be used to find its speed.**



Voltage

Electric potential is described by many terms.

The most common term is *voltage.*

A voltage applied to a device or across a device is the same as the potential difference across the device.

- The voltage is **NOT** something that moves through a device.

Electron volts (eV)

Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt.

One ***electron-volt*** is defined as **the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt.**

- $1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$

Potential difference in a uniform field

The equations for electric potential between two points A and B can be simplified if the electric field is uniform:

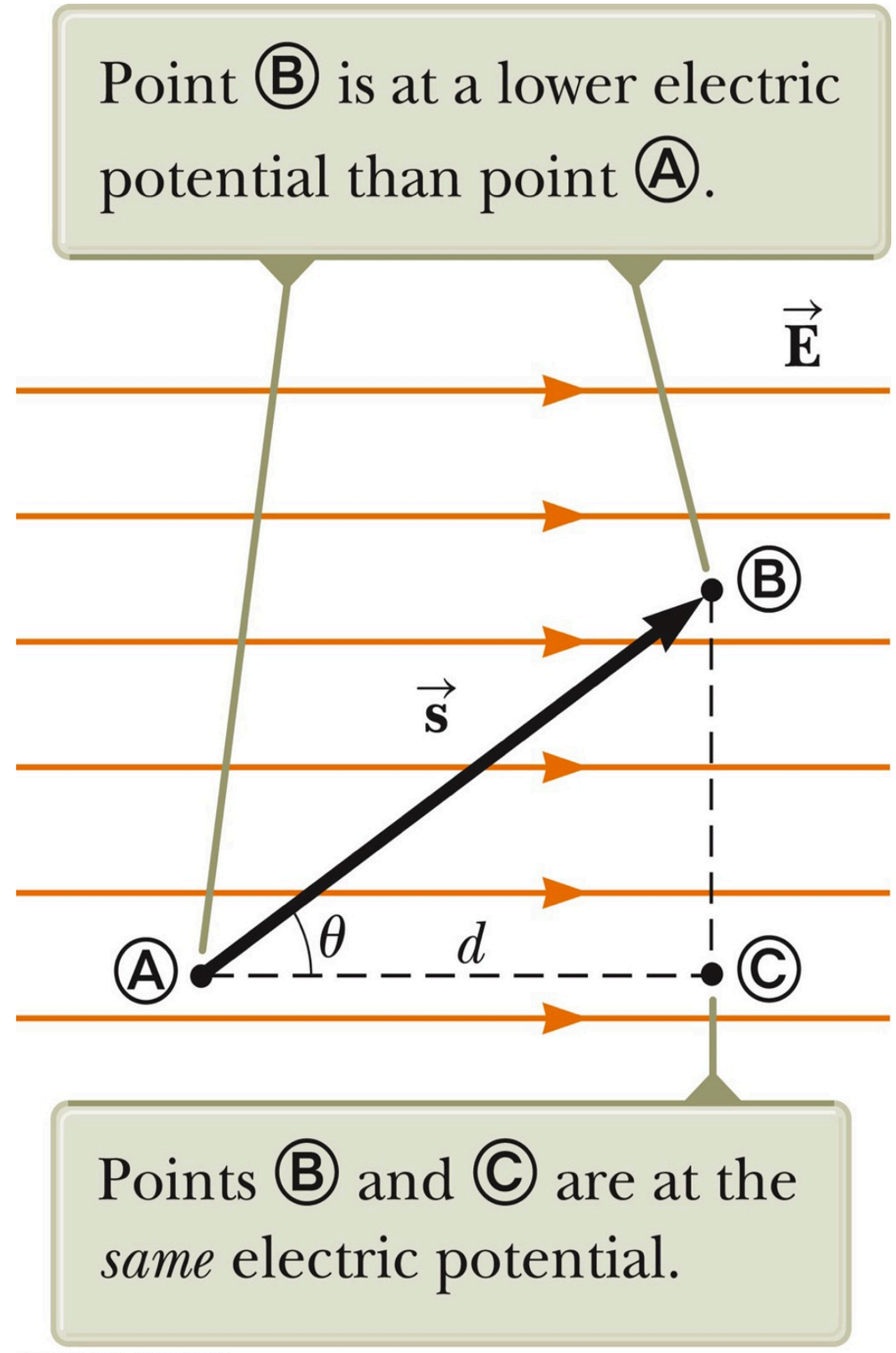
$$V_B - V_A = \Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

$$= \boxed{-Ed}$$

The displacement points from A to B and is **parallel to the field lines.**

The negative sign indicates that the electric potential at point *B* is lower than at point *A.*

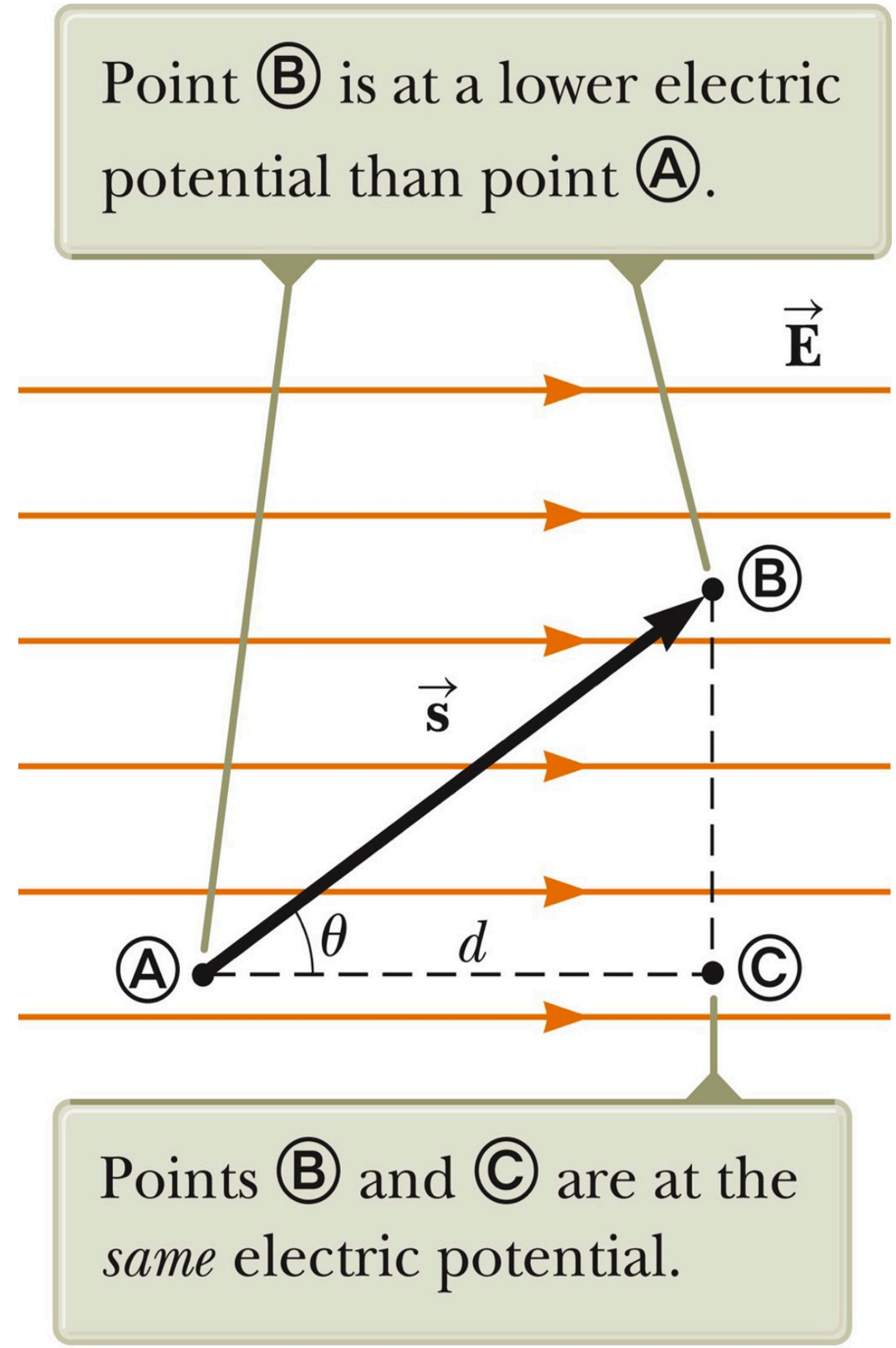
- **Electric field lines always point in the direction of decreasing electric potential.**



Equipotentials

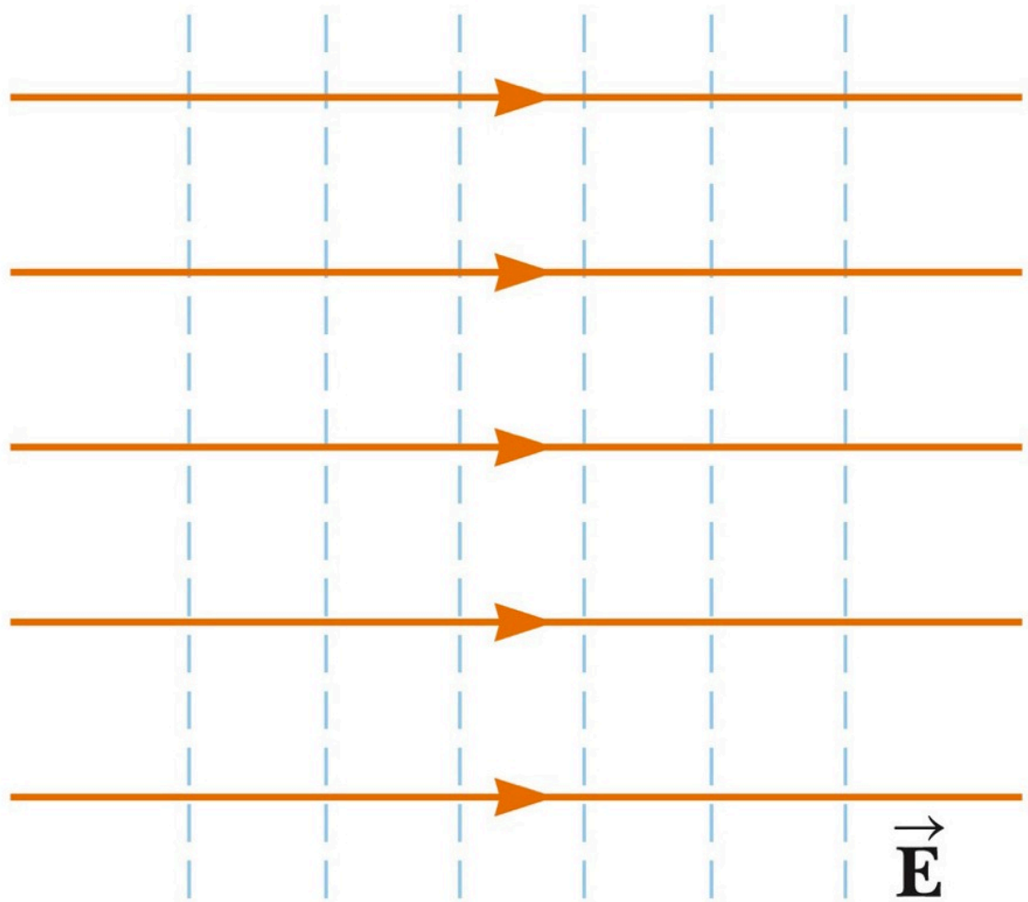
- **All points in a plane perpendicular to a uniform electric field are at the same electric potential.**

The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential.



Equipotentials

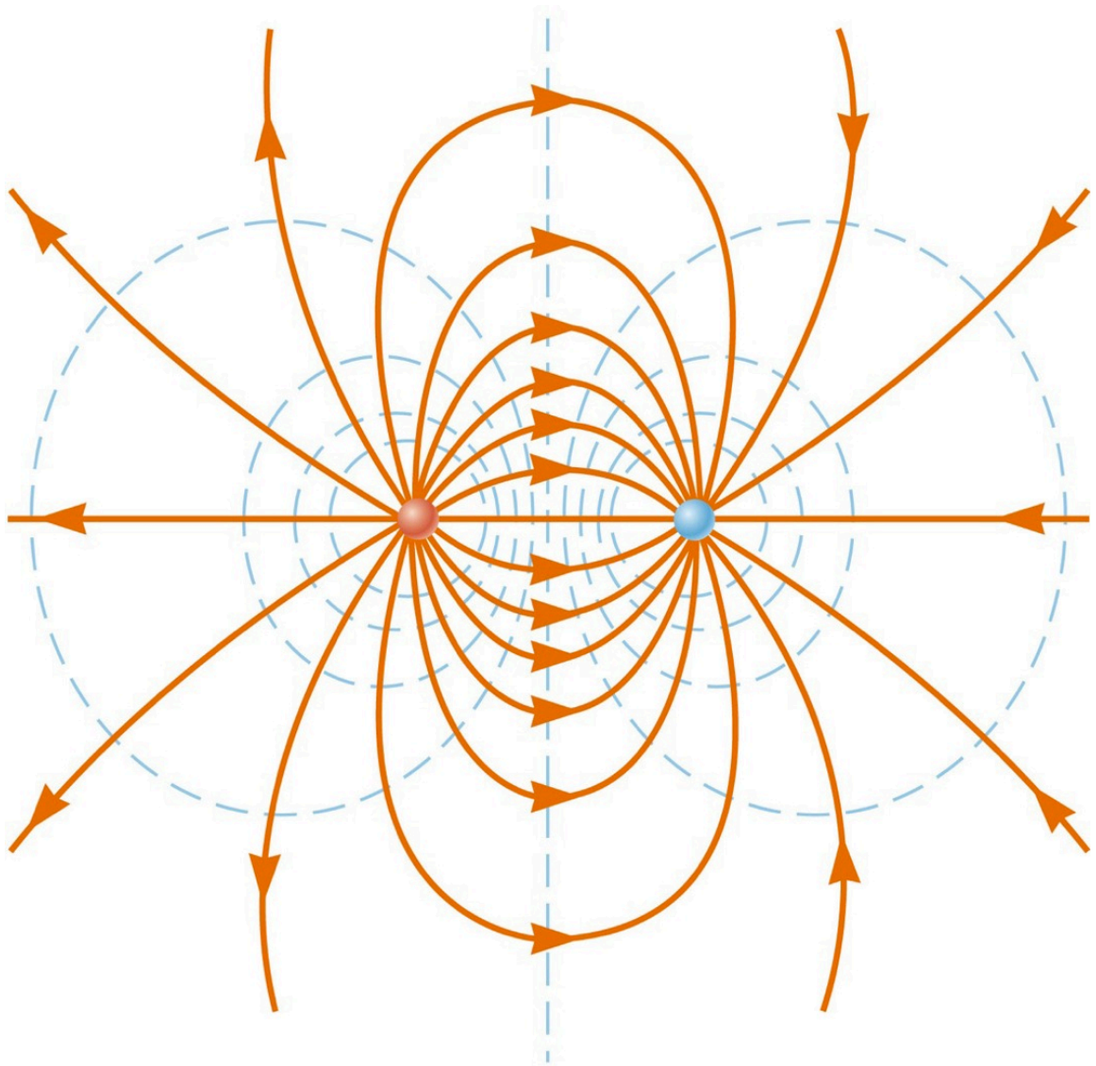
A uniform electric field produced by an infinite sheet of charge



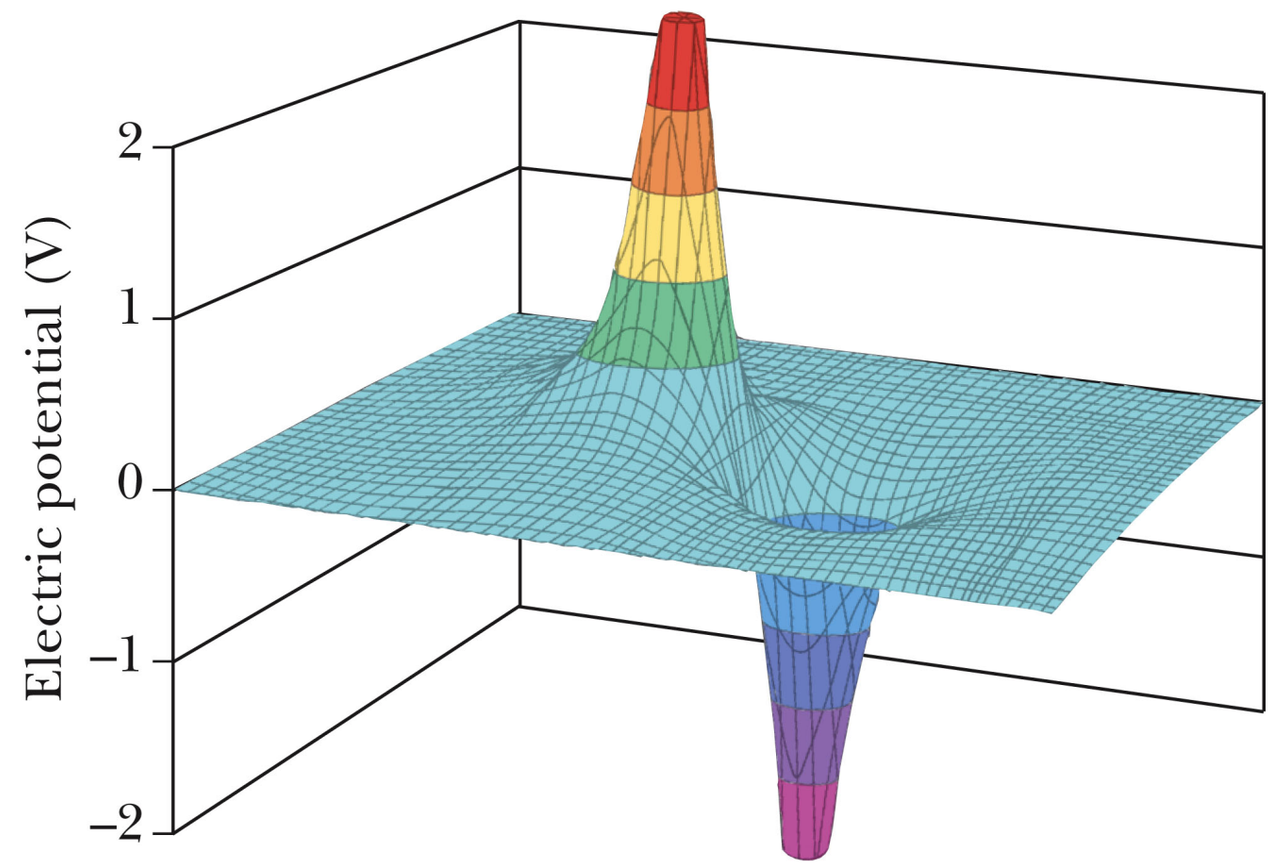
The equipotential lines are everywhere perpendicular to the field lines.

Equipotentials

An electric field produced by an electric dipole



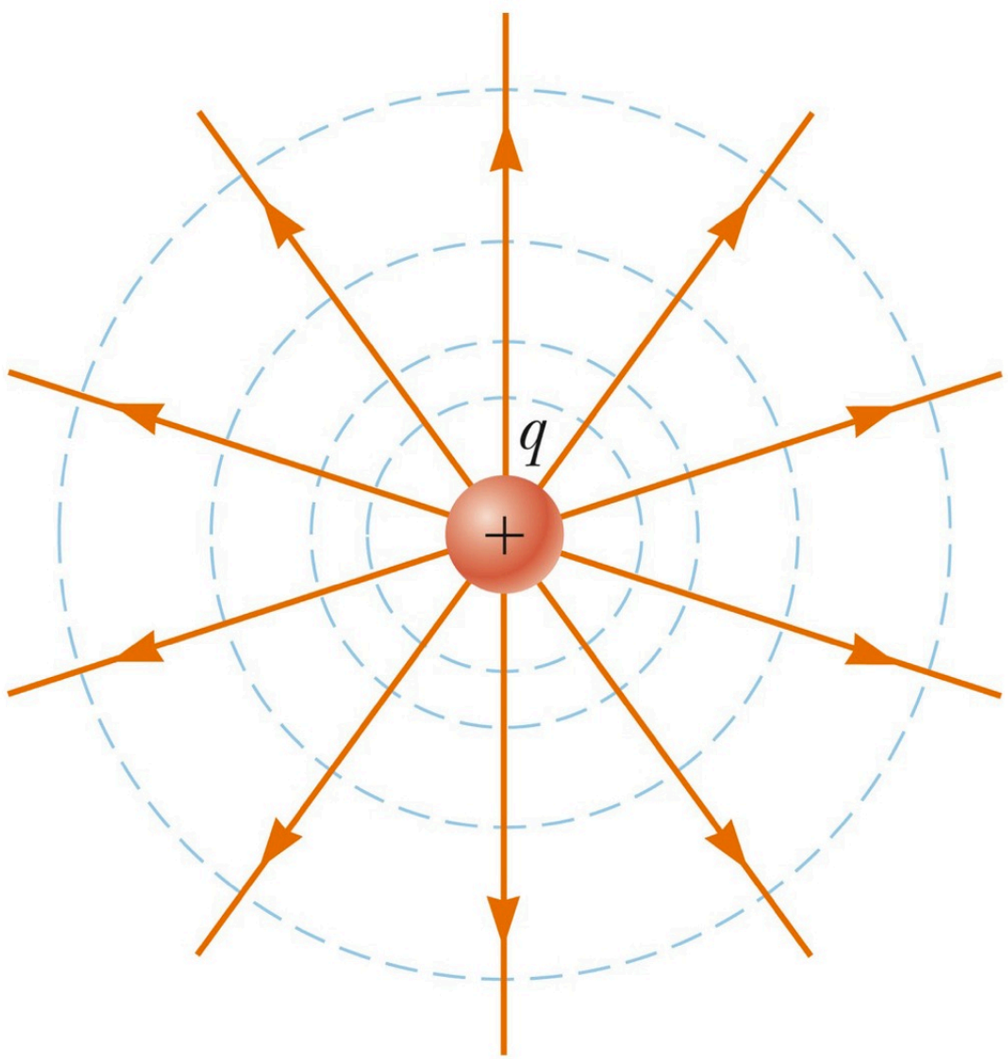
The steep slope between the charges represents the strong electric field in this region.



The equipotential lines are everywhere perpendicular to the field lines.

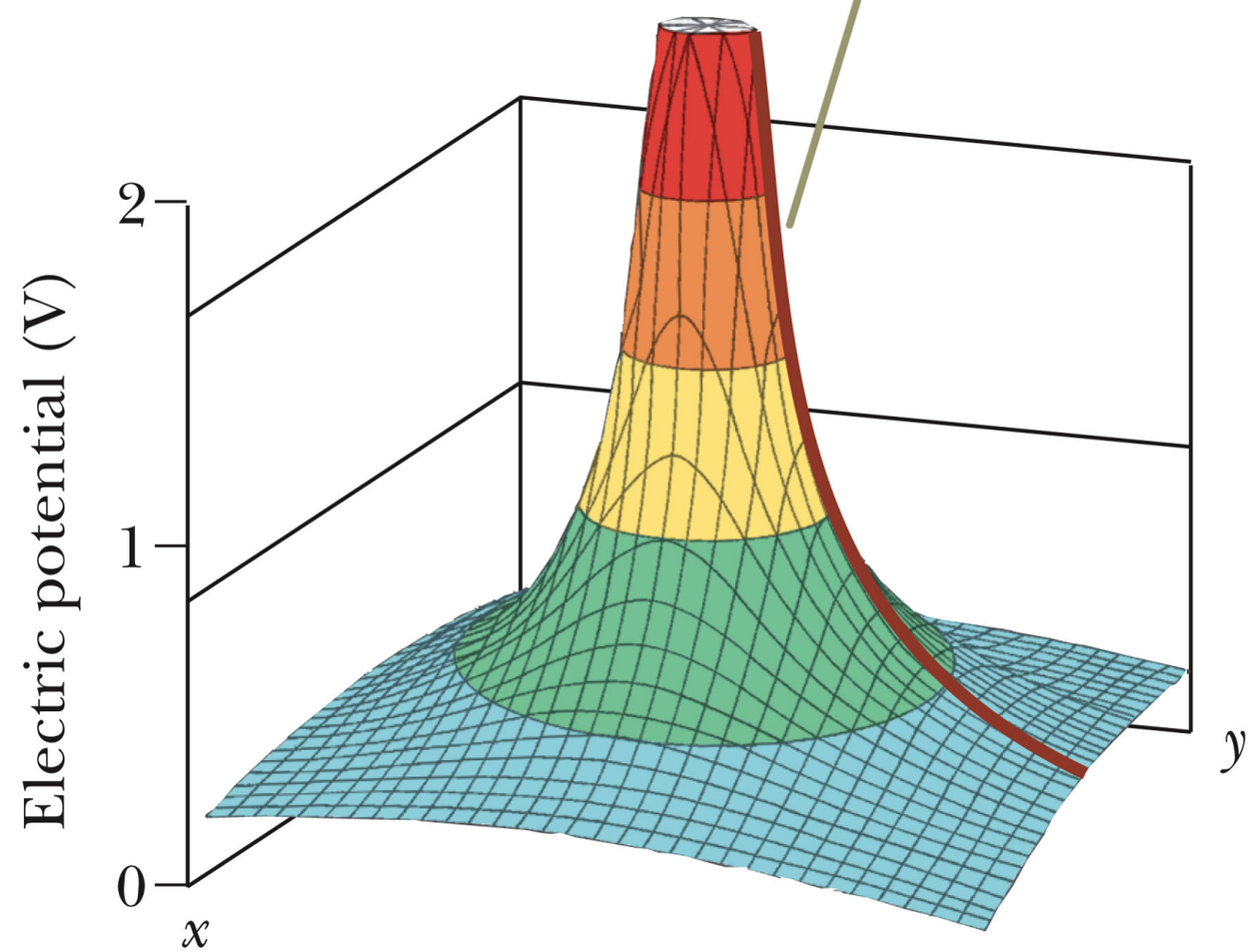
Equipotential for a Point Charge

A spherically symmetric electric field produced by a point charge



b

The red-brown curve shows the $1/r$ nature of the electric potential as given by Equation 25.11.



25s->24s (ed. change)

Example: potential due to point charge at distance r

It is customary to choose a reference potential of $V = 0$ at $r = \infty$.

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

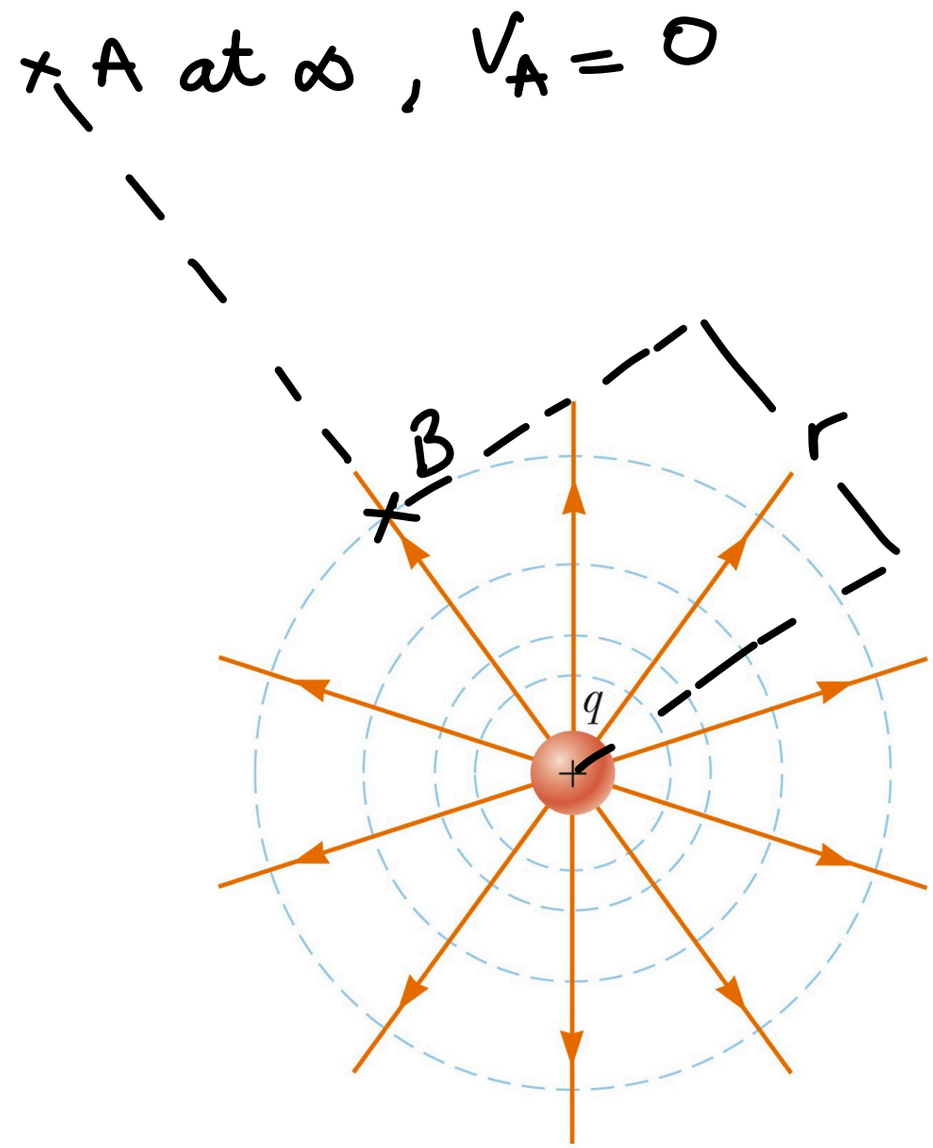
$$V_B - 0 = -\int_A^B \vec{E} \cdot d\vec{s}$$

$$V_B = -\int_\infty^r k_e \frac{q}{r^2} dr$$

$$= -k_e q [\frac{-1}{r}]_\infty^r$$

$$= -k_e q [\frac{-1}{r} - \frac{-1}{\infty}]$$

$$\boxed{= +k_e \frac{q}{r}}$$



Example: point charges

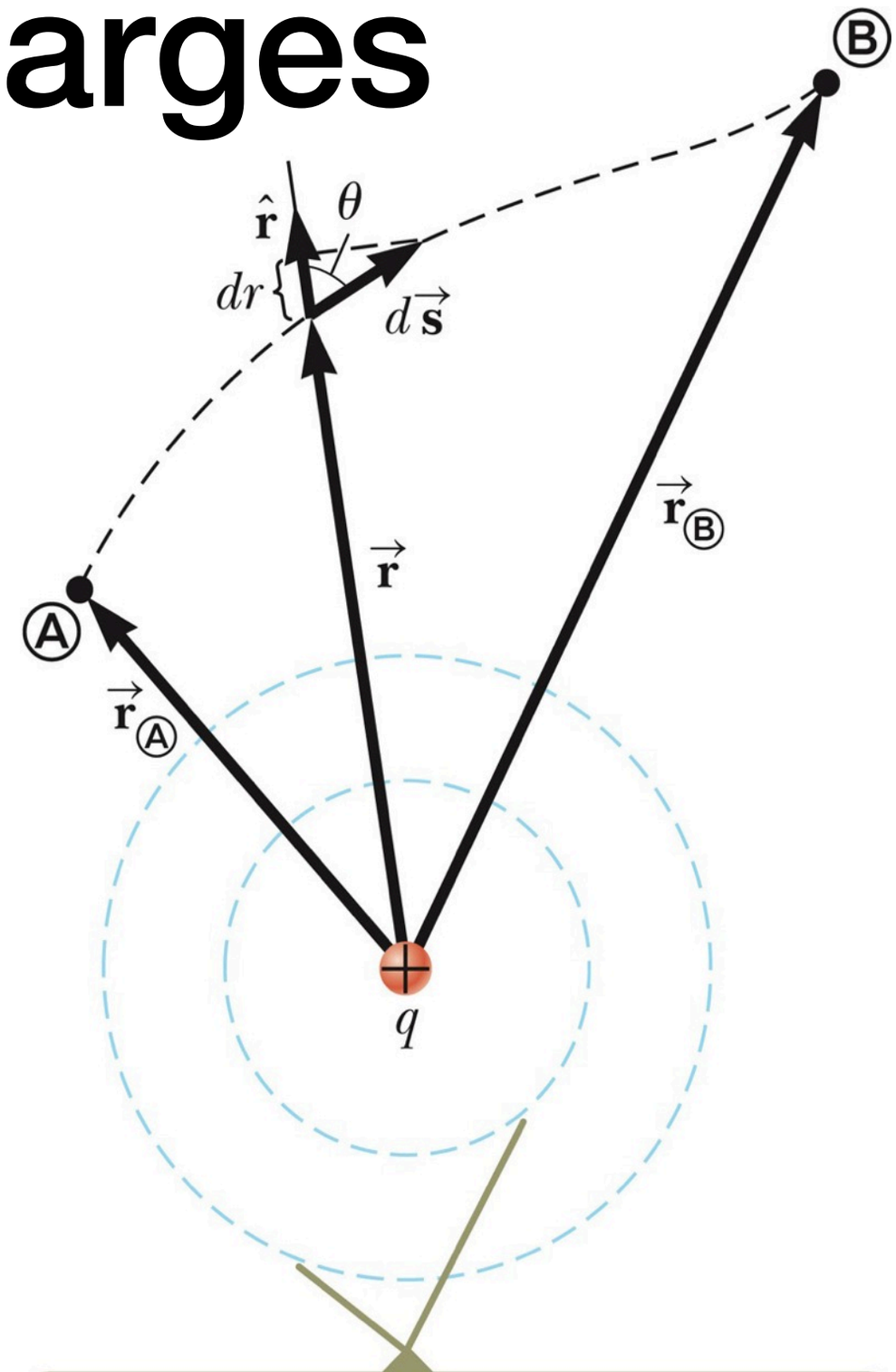
An isolated positive point charge produces a field directed radially outward.

What is the potential difference between points A and B?

The electric potential is independent of the path between points *A* and *B*.

$$\Delta V = \frac{k_e q}{r}$$

$$\boxed{V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]}$$



The two dashed circles represent intersections of spherical equipotential surfaces with the page.

Electric potential of multiple charges

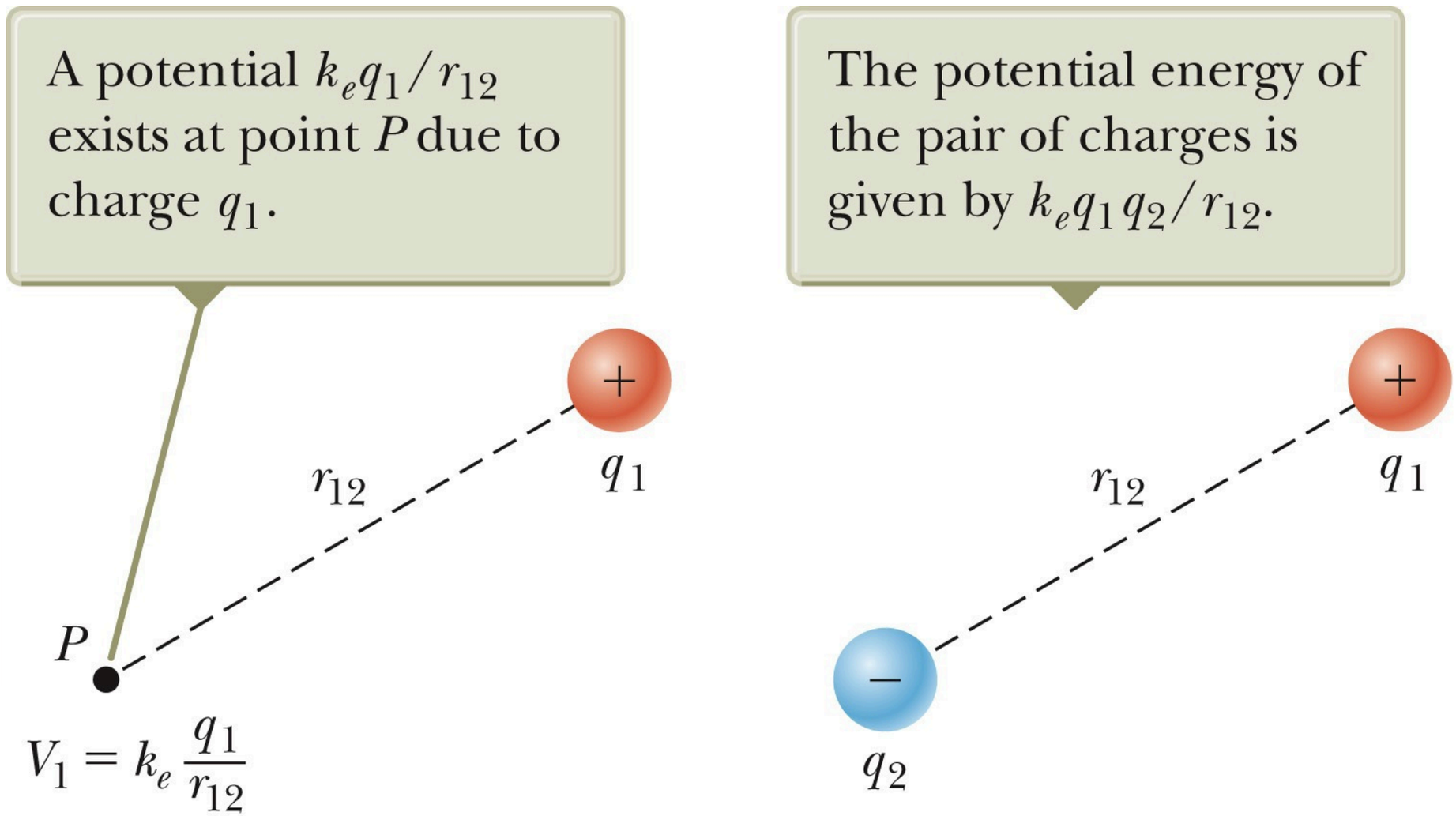
$$\boxed{V = k_e \sum_i \frac{q_i}{r_i}}$$

Potential Energy of Multiple Charges

$$V = \frac{U}{q_o}$$

U=Vq

$$U = k_e \frac{q_1 q_2}{r_{12}}$$



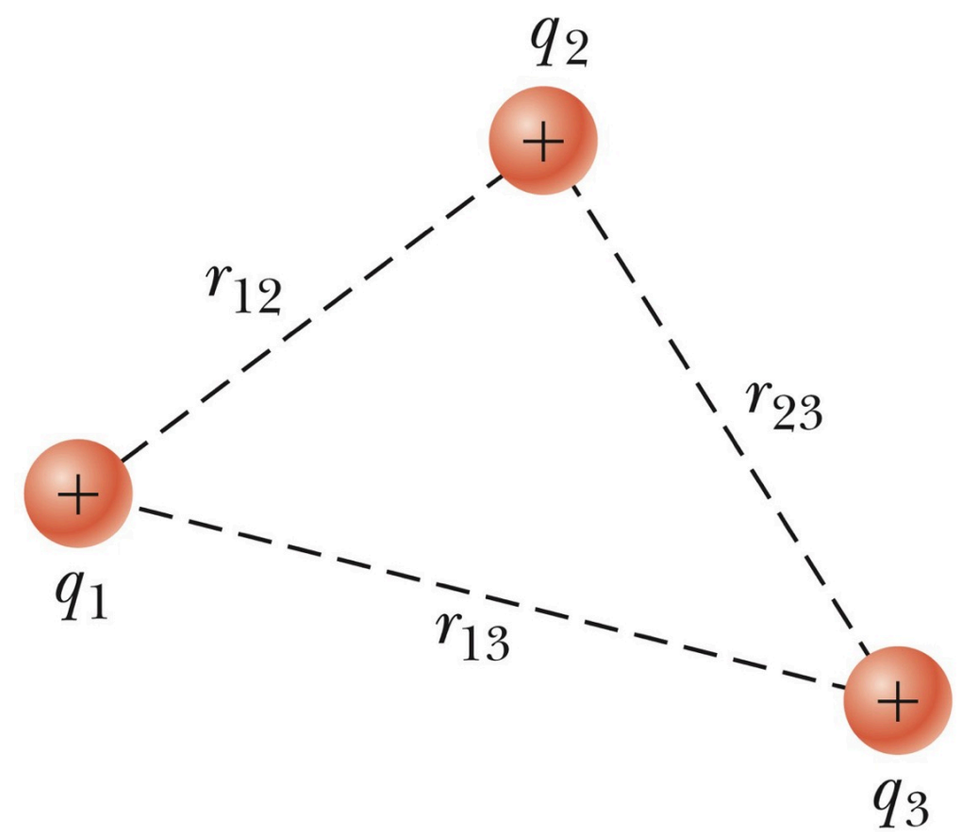
If the two charges are the same sign, *U* is positive and work must be done to bring the charges together.

If the two charges have opposite signs, *U* is negative and work is done to keep the charges apart.

Example: multiple charges

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

The potential energy of this system of charges is given by Equation 25.14.



Example Problem 1

1. Oppositely charged parallel plates are separated
M by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field between the plates? (b) What is the magnitude of the force on an electron between the plates? (c) How much work must be done on the electron to move it to the negative plate if it is initially positioned 2.90 mm from the positive plate?

Example Problem 1: Solution

(a) From Equation 25.6,

$$E = \frac{|\Delta V|}{d} = \frac{600 \text{ J/C}}{5.33 \times 10^{-3} \text{ m}} = \boxed{1.13 \times 10^{5} \text{ N/C}}$$

(b) The force on an electron is given by

$$F = |q| E = \left(1.60 \times 10^{-19} \text{ C}\right)\left(1.13 \times 10^{5} \text{ N/C}\right) = \boxed{1.80 \times 10^{-14} \text{ N}}$$

(c) Because the electron is repelled by the negative plate, the force used to move the electron must be applied in the direction of the electron's displacement. The work done to move the electron is

$$W = F \cdot s \cos\theta = \left(1.80 \times 10^{-14} \text{ N}\right)\left[(5.33 - 2.90) \times 10^{-3} \text{ m}\right]\cos 0°$$

$$= \boxed{4.37 \times 10^{-17} \text{ J}}$$

Example Problem 2

11. An insulating rod having linear charge density $\lambda = 40.0\ \mu\text{C/m}$ and linear mass density $\mu = 0.100$ kg/m is released from rest in a uniform electric field $E = 100$ V/m directed perpendicular to the rod (Fig. P25.11). (a) Determine the speed of the rod after it has traveled 2.00 m. (b) **What If?** How does your answer to part (a) change if the electric field is not perpendicular to the rod? Explain.

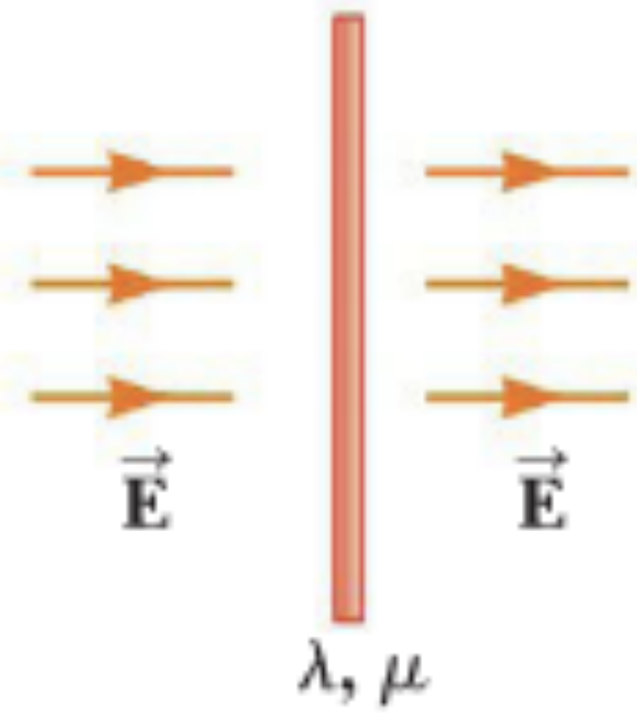


Figure P25.11

Example Problem 2: Solution

Arbitrarily take $V = 0$ at the initial point. Then at distance d downfield, where L is the rod length, $V = -Ed$ and $U_e = -\lambda LEd$.

(a) The rod-field system is isolated:

$$K_i + U_i = K_f + U_f$$

$$0 + 0 = \frac{1}{2} m_{\text{rod}} v^2 - qV$$

$$0 = \frac{1}{2} \mu L v^2 - \lambda LEd$$

$$\frac{1}{2} \mu L v^2 = \lambda LEd$$

Solving for the speed gives

$$v = \sqrt{\frac{2\lambda Ed}{\mu}} = \sqrt{\frac{2\left(40.0 \times 10^{-6}\ \text{C/m}\right)(100\ \text{N/C})(2.00\ \text{m})}{\left(0.100\ \text{kg/m}\right)}}$$

$$= \boxed{0.400\ \text{m/s}}$$

(b) $\boxed{\text{The same.}}$ Each bit of the rod feels a force of the same size as before.

Example Problem 3

25. Two particles each with charge $+2.00\ \mu C$ are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric potential on the y axis at $y = 0.500$ m. (b) Calculate the change in electric potential energy of the system as a third charged particle of $-3.00\ \mu C$ is brought from infinitely far away to a position on the y axis at $y = 0.500$ m.

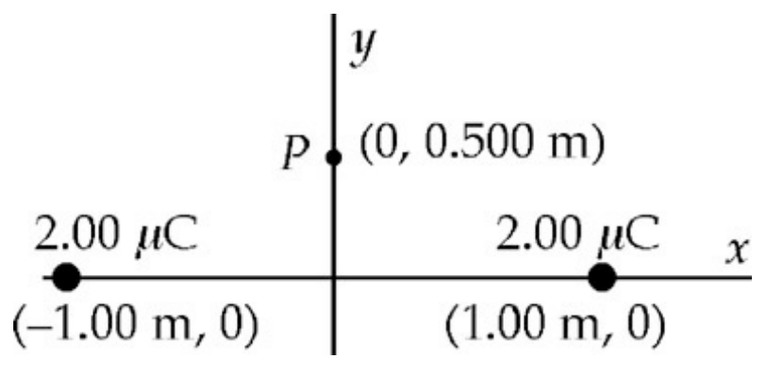
Example Problem 3: Solution

(a) The electric potential at $y = 0.500$ m on the y axis is given by

$$V = \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2} = 2\left(\frac{k_e q}{r}\right)$$

$$V = 2\left(\frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(2.00\times10^{-6}\ \mathrm{C}\right)}{\sqrt{(1.00\ \mathrm{m})^2+(0.500\ \mathrm{m})^2}}\right)$$

$$V = 3.22\times10^{4}\ \mathrm{V} = \boxed{32.2\ \mathrm{kV}}$$



ANS. FIG. P25.25

(b) The change in potential energy of the system when a third charge is brought to this point is

$$U = qV = \left(-3.00\times10^{-6}\ \mathrm{C}\right)\left(3.22\times10^{4}\ \mathrm{J/C}\right) = \boxed{-9.65\times10^{-2}\ \mathrm{J}}$$

Example Problem 4

17. Two particles, with charges of 20.0 nC and −20.0 nC, are placed at the points with coordinates (0, 4.00 cm) and (0, −4.00 cm) as shown in Figure P25.17. A particle with charge 10.0 nC is located at the origin. (a) Find the electric potential energy of the configuration of the three fixed charges. (b) A fourth particle, with a mass of 2.00×10^{-13} kg and a charge of 40.0 nC, is released from rest at the point (3.00 cm, 0). Find its speed after it has moved freely to a very large distance away.

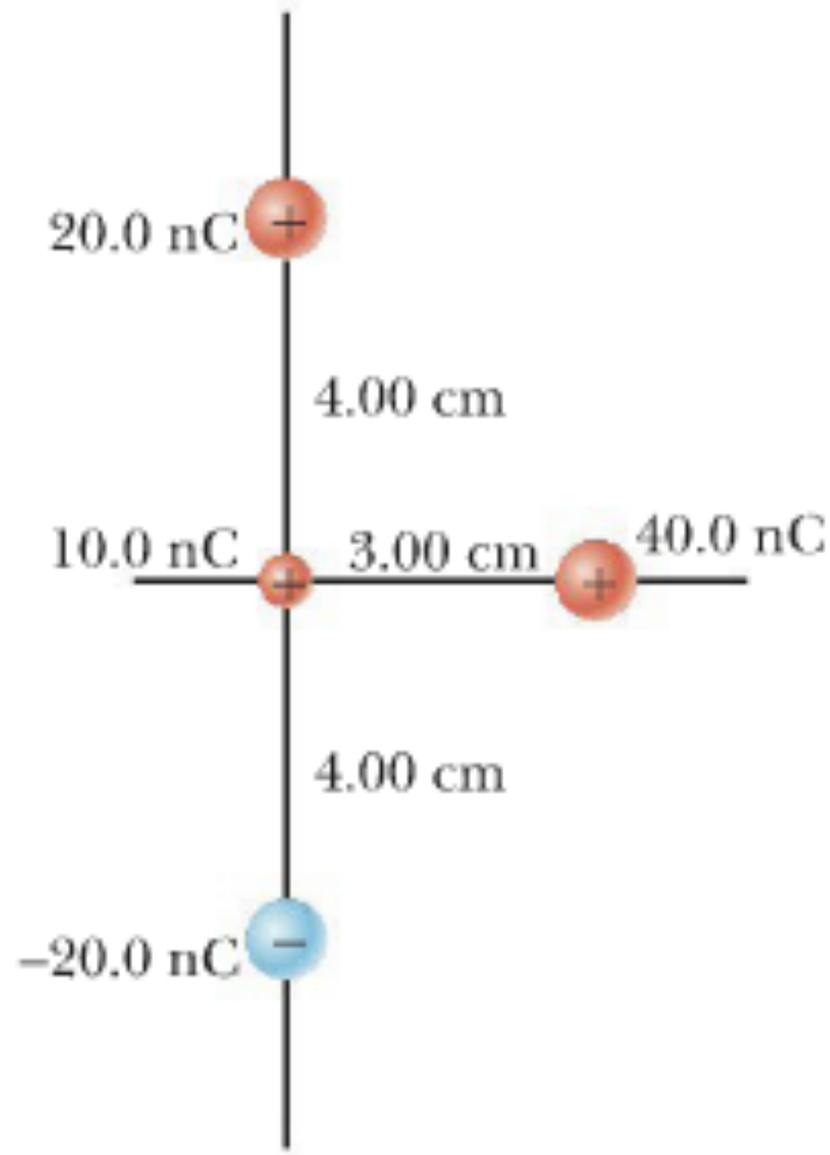


Figure P25.17

Example Problem 4: Solution

(a) In an empty universe, the 20.0-nC charge can be placed at its location with no energy investment. At a distance of 4.00 cm, it creates a potential

$$V_1 = \frac{k_e q_1}{r} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(20.0\times10^{-9}\ \mathrm{C}\right)}{0.040\ 0\ \mathrm{m}} = 4.50\ \mathrm{kV}$$

To place the 10.0-nC charge there we must put in energy

$$U_{12} = q_2 V_1 = \left(10.0\times10^{-9}\ \mathrm{C}\right)\left(4.50\times10^{3}\ \mathrm{V}\right) = 4.50\times10^{-5}\ \mathrm{J}$$

Next, to bring up the –20.0-nC charge requires energy

$$\begin{aligned} U_{23} + U_{13} &= q_3 V_2 + q_3 V_1 = q_3\left(V_2 + V_1\right) \\ &= \left(-20.0\times10^{-9}\ \mathrm{C}\right)\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right) \\ &\qquad \times\left(\frac{10.0\times10^{-9}\ \mathrm{C}}{0.040\ 0\ \mathrm{m}} + \frac{20.0\times10^{-9}\ \mathrm{C}}{0.080\ 0\ \mathrm{m}}\right) \\ &= -4.50\times10^{-5}\ \mathrm{J} - 4.50\times10^{-5}\ \mathrm{J} \end{aligned}$$

At this point, V1 is calculated at the location of charge q3

The total energy of the three charges is

$$U_{12} + U_{23} + U_{13} = \boxed{-4.50\times10^{-5}\ \mathrm{J}}$$

same as formula on slide 24

Example Problem 4: Solution

(b) The three fixed charges create this potential at the location where the fourth is released:

$$V = V_1 + V_2 + V_3$$
$$= \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)$$
$$\times\left(\frac{20.0\times10^{-9}\ \mathrm{C}}{\sqrt{(0.040\ 0\ \mathrm{m})^2+(0.030\ 0\ \mathrm{m})^2}}\right.$$
$$\left.+\frac{10.0\times10^{-9}\ \mathrm{C}}{0.030\ 0\ \mathrm{m}}-\frac{20.0\times10^{-9}\ \mathrm{C}}{\sqrt{(0.040\ 0\ \mathrm{m})^2+(0.030\ 0\ \mathrm{m})^2}}\right)$$
$$V = 3.00\times10^{3}\ \mathrm{V}$$

Energy of the system of four charged objects is conserved as the fourth charge flies away:

$$\left(\frac{1}{2}mv^2+qV\right)_i=\left(\frac{1}{2}mv^2+qV\right)_f$$
$$0+\left(40.0\times10^{-9}\ \mathrm{C}\right)\left(3.00\times10^{3}\ \mathrm{V}\right)=\frac{1}{2}\left(2.00\times10^{-13}\ \mathrm{kg}\right)v^2+0$$
$$v=\sqrt{\frac{2\left(1.20\times10^{-4}\ \mathrm{J}\right)}{2\times10^{-13}\ \mathrm{kg}}}=\boxed{3.46\times10^{4}\ \mathrm{m/s}}$$

(stop here?)

Summary

$$W = \int \vec{F} \cdot d\vec{s} = -\Delta U$$

Electric force does work to move a charge

$$V = \frac{U}{q_o}$$

V, the potential, is the potential energy per unit charge

$$\Delta V = \frac{\Delta U}{q_0} = -\int \vec{E} \cdot d\vec{s}$$

Potential is linked to potential energy and is linked to electric field, through work.
The - sign means that electric field lines always point in the direction of decreasing potential.

$$V = +k_e \frac{q}{r}$$

Potential due to a **point charge**, at a distance r

Finding the electric field using the electric potential

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

so:

$$\boxed{E_x = -\frac{dV}{dx}}$$

Similar statements would apply to the *y* and *z* components.

Equipotential surfaces must always be perpendicular to the electric field lines passing through them.

Finding the electric field using the electric potential

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

Continuous charge distribution

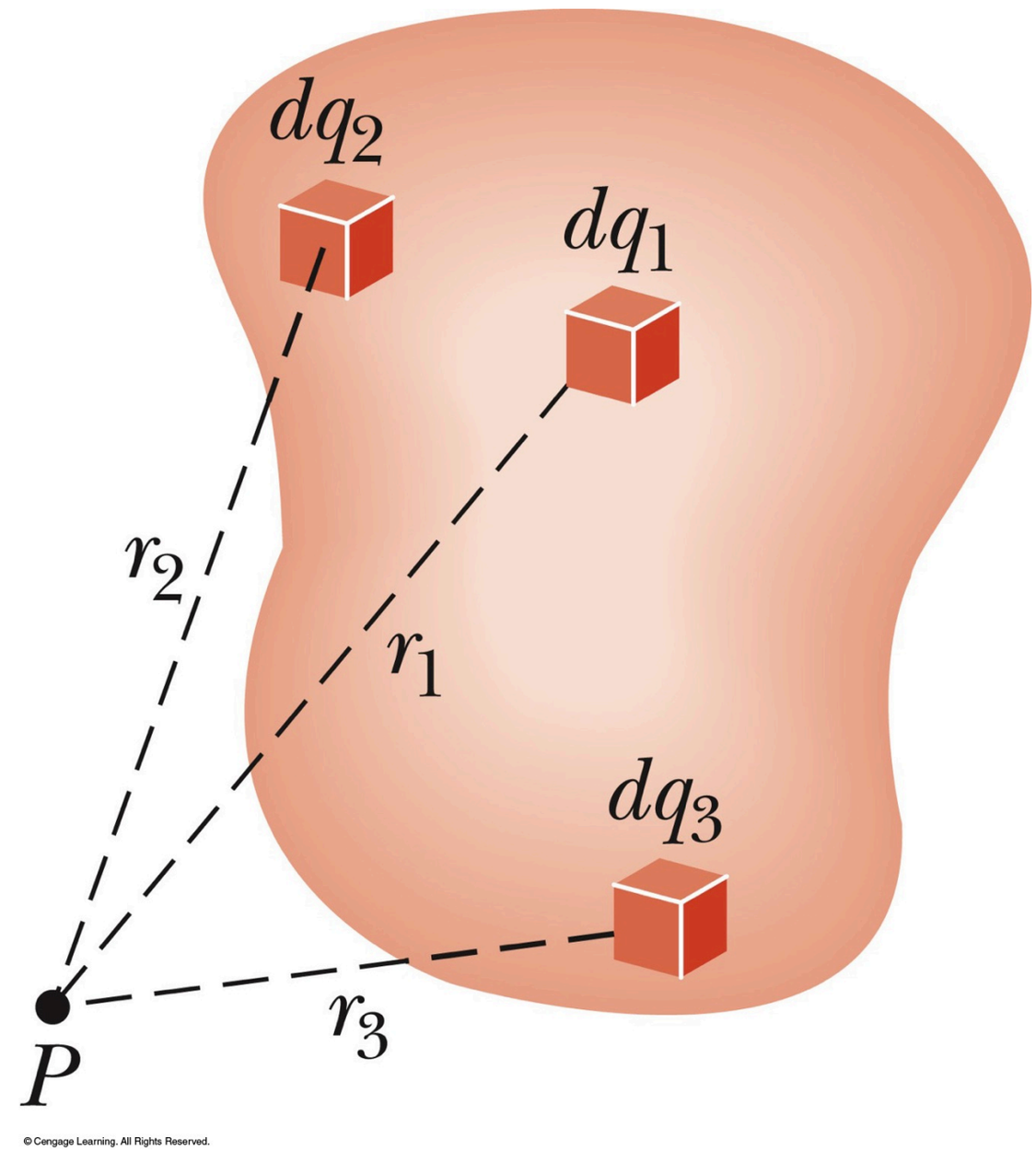
Consider a small charge element *dq*

- Treat it as a point charge.

The potential at some point due to this charge element is

$$dV = k_e \frac{dq}{r}$$

$$V = k_e \int \frac{dq}{r}$$



- This value for *V* uses the reference of *V* = 0 when *P* is infinitely far away from the charge distributions.

E field of a uniform ring of charge

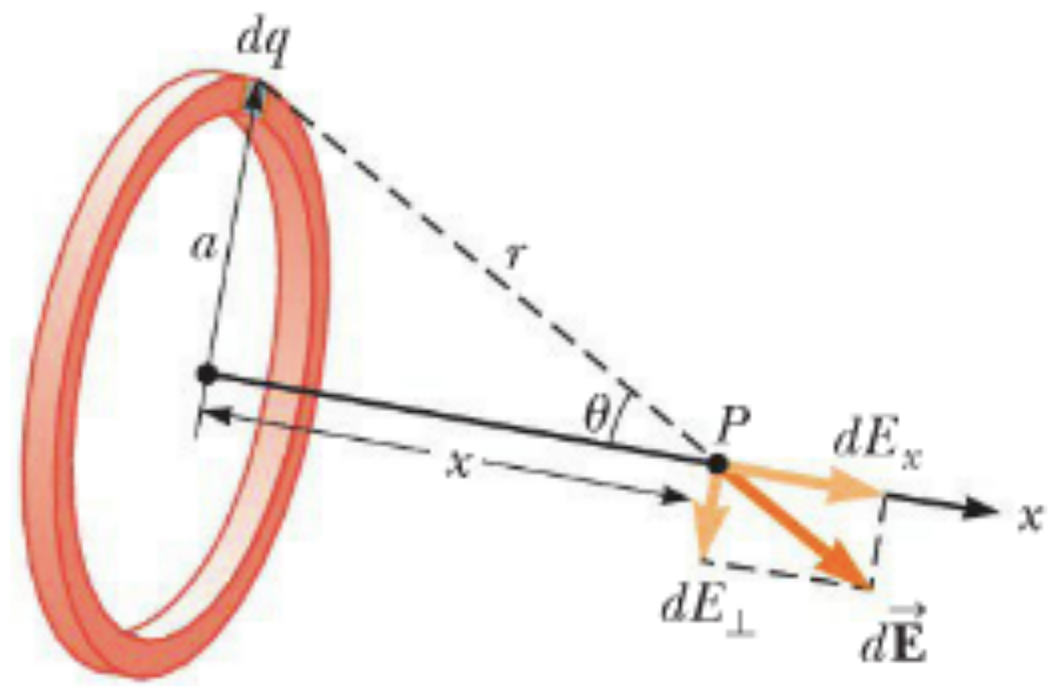
$$(1) \quad dE_x = k_e \frac{dq}{r^2} \cos\theta = k_e \frac{dq}{a^2 + x^2} \cos\theta$$

$$(2) \quad \cos\theta = \frac{x}{r} = \frac{x}{(a^2 + x^2)^{1/2}}$$

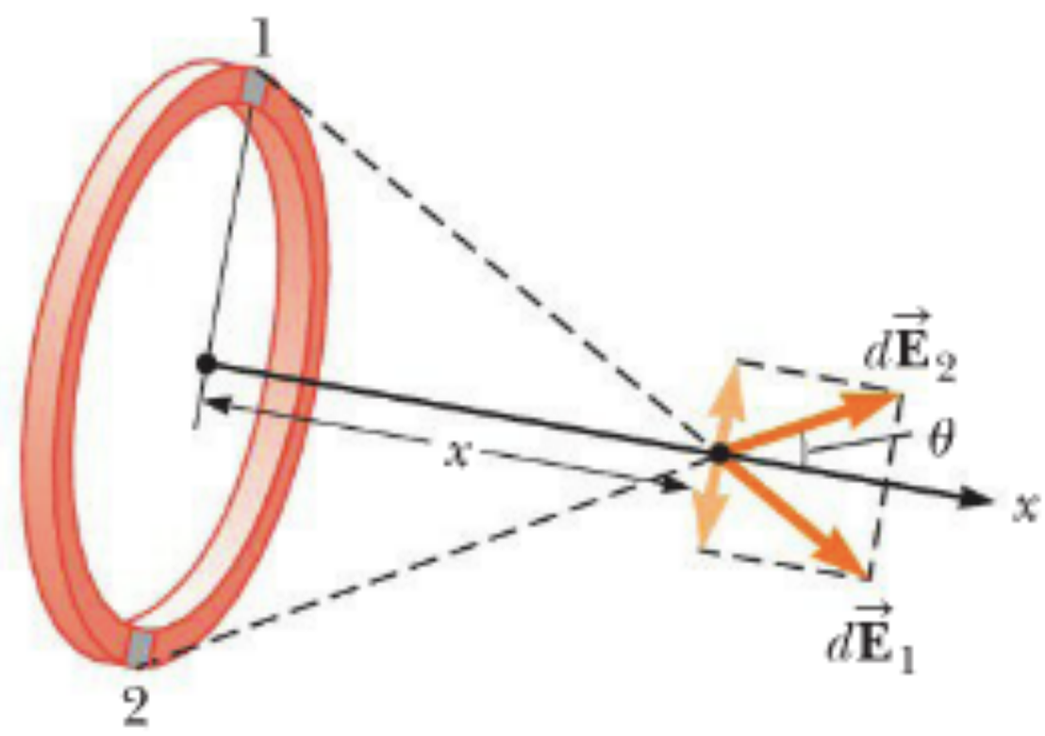
$$dE_x = k_e \frac{dq}{a^2 + x^2} \left[\frac{x}{(a^2 + x^2)^{1/2}} \right] = \frac{k_e x}{(a^2 + x^2)^{3/2}} dq$$

$$E_x = \int \frac{k_e x}{(a^2 + x^2)^{3/2}} dq = \frac{k_e x}{(a^2 + x^2)^{3/2}} \int dq$$

$$(3) \quad E = \frac{k_e x}{(a^2 + x^2)^{3/2}} Q$$



from earlier lectures



Uniformly Shaped Ring

P is located on the perpendicular central axis of the uniformly charged ring .

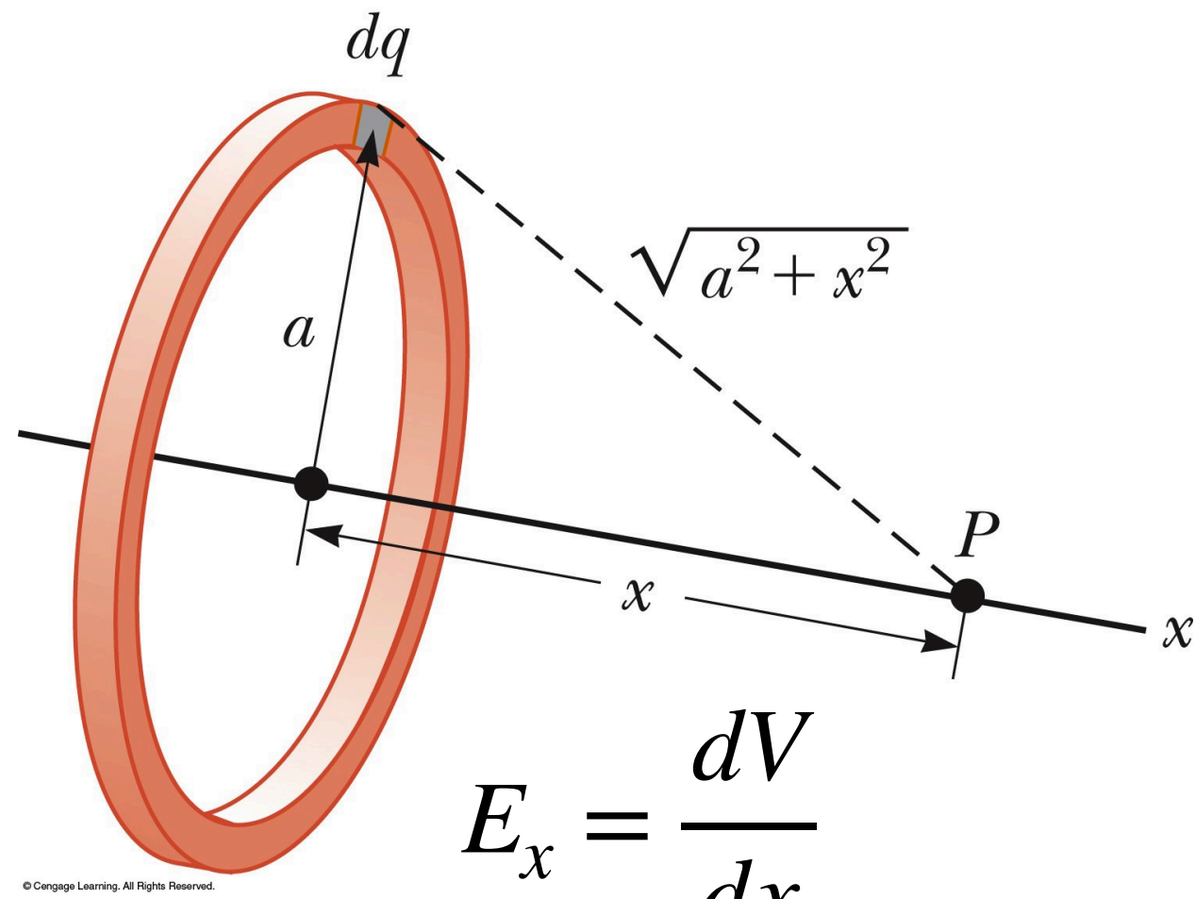
The symmetry of the situation means that all the charges on the ring are the same distance from point P.

- The ring has a radius *a* and a total charge *Q*.

The potential and the field are given by

$$V = k_e \int \frac{dq}{r} = \frac{k_e Q}{\sqrt{a^2 + x^2}}$$

$$E_x = \frac{k_e x}{\left(a^2 + x^2\right)^{3/2}} Q$$



$$E_x = \frac{dV}{dx}$$

Unlike the electric field, the potential due to every single small charge distribution dq is the same.

This is why you do not need to express dq in terms of r and θ.

E field of uniformly charged disk

$$dE = k_e \frac{dq}{d^2}$$

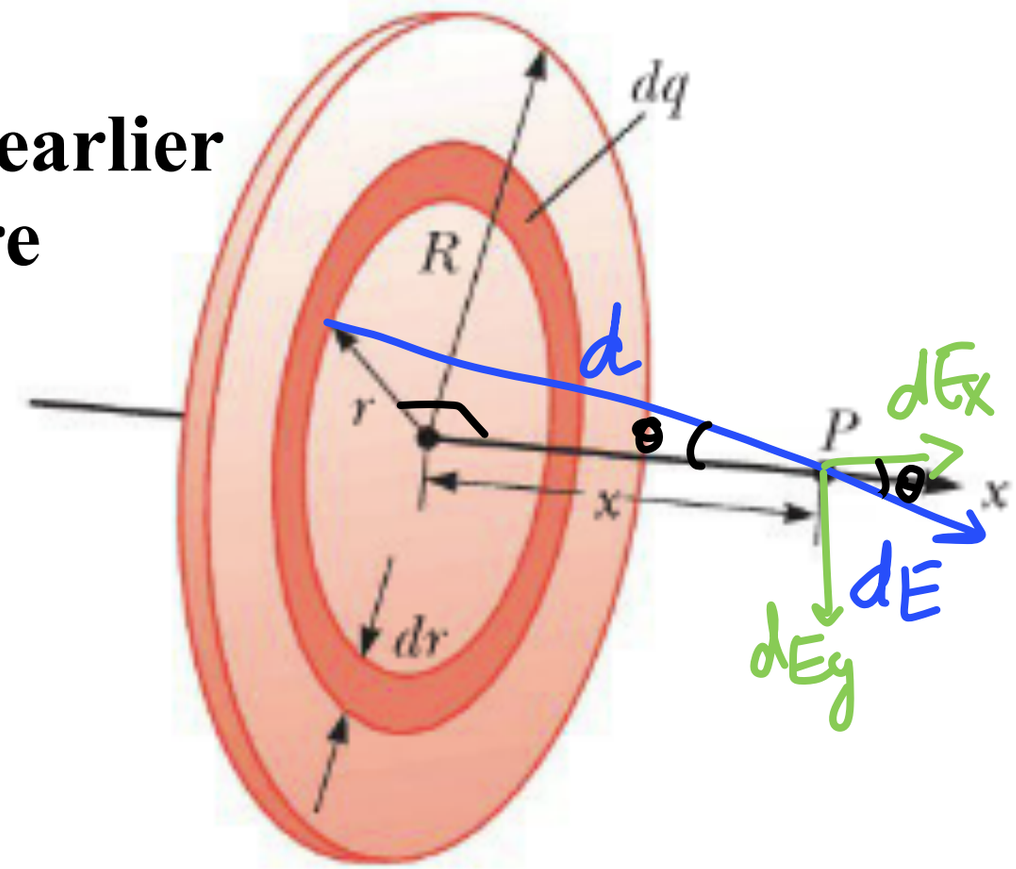
$$dE_x = dE \cos\theta$$

$$dE_x = k_e \frac{dq}{d^2} \cos\theta = k_e \frac{dq}{x^2 + r^2} \cos\theta$$

$$dq = \sigma\, dA = \sigma(2\pi r\, dr) = 2\pi\sigma r\, dr$$

$$dE_x = \frac{k_e x}{(r^2 + x^2)^{3/2}} (2\pi\sigma r\, dr)$$

from earlier lecture



$$\cos\theta = \frac{x}{d} = \frac{x}{\sqrt{r^2 + x^2}}$$

$$\cos\theta = \frac{dE_x}{dE}$$

$$E_x = k_e x \pi\sigma \int_0^R \frac{2r\, dr}{(r^2 + x^2)^{3/2}}$$

$$= k_e x \pi\sigma \int_0^R (r^2 + x^2)^{-3/2} d(r^2)$$

*Note the change in variable

$$= k_e x \pi\sigma \left[\frac{-2}{(r^2 + x^2)^{1/2}}\right]_0^R = k_e x \pi\sigma(-\frac{2}{(r^2 + x^2)^{1/2}} = 2\pi k_e \sigma \left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]$$

Uniformly Shaped Disk

$$dq = \sigma\, dA = \sigma(2\pi r\, dr) = 2\pi\sigma r\, dr$$

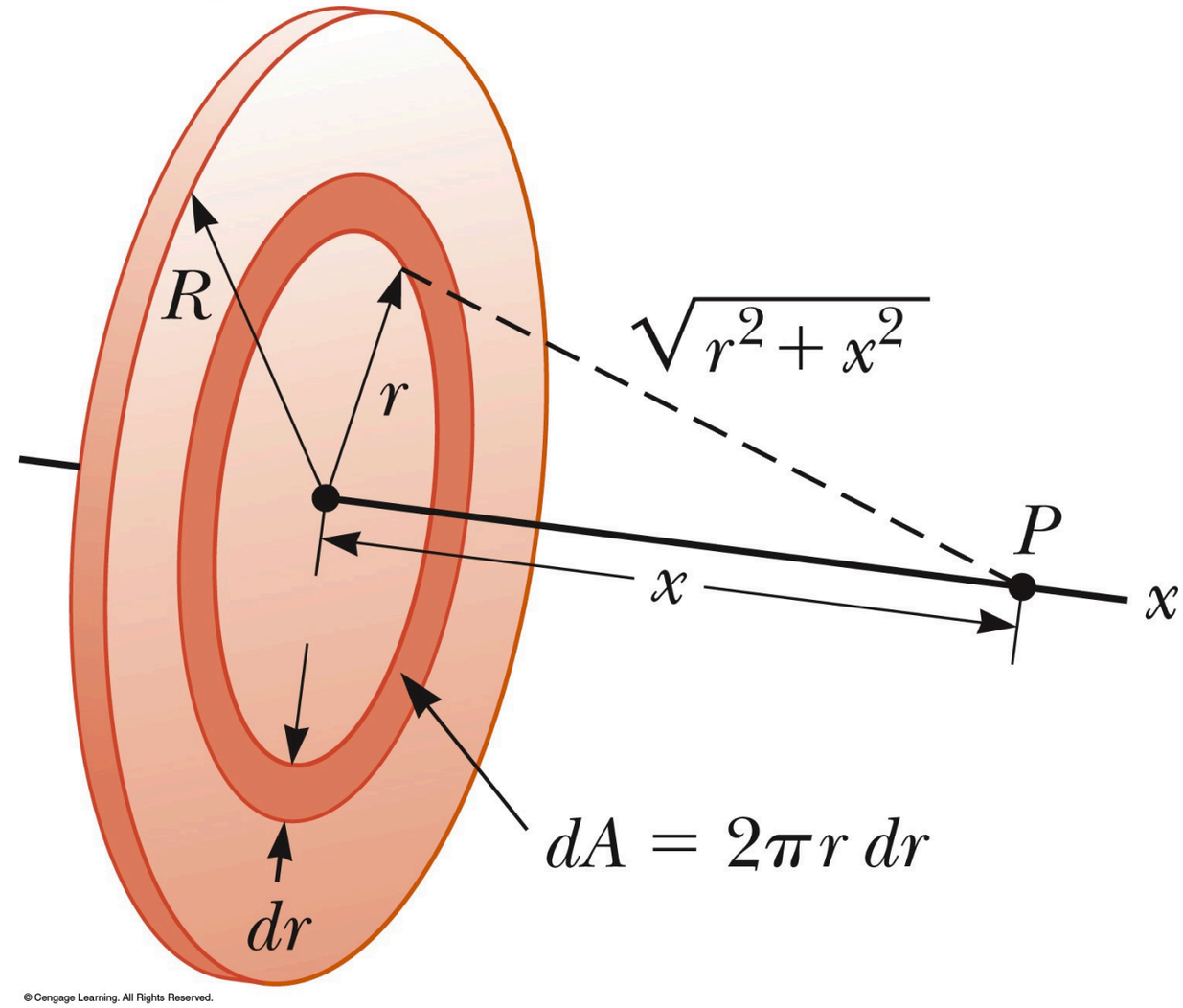
$$dV = \frac{k_e\, dq}{\sqrt{r^2 + x^2}} = \frac{k_e 2\pi\sigma r\, dr}{\sqrt{r^2 + x^2}}$$

$$V = \pi k_e \sigma \int_0^R \frac{2r\, dr}{\sqrt{r^2 + x^2}}$$

$$= \pi k_e \sigma \int_0^R (r^2 + x^2)^{-1/2}\, 2r\, dr$$

$$V = 2\pi k_e \sigma[(R^2 + x^2)^{1/2} - x]$$

$$E_x = -\frac{dV}{dx} = 2\pi k_e \sigma\left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]$$



$$V = k_e \int \frac{dq}{r}$$

$$dV = k_e \frac{dq}{r}$$

Finite Line of Charge

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}}$$

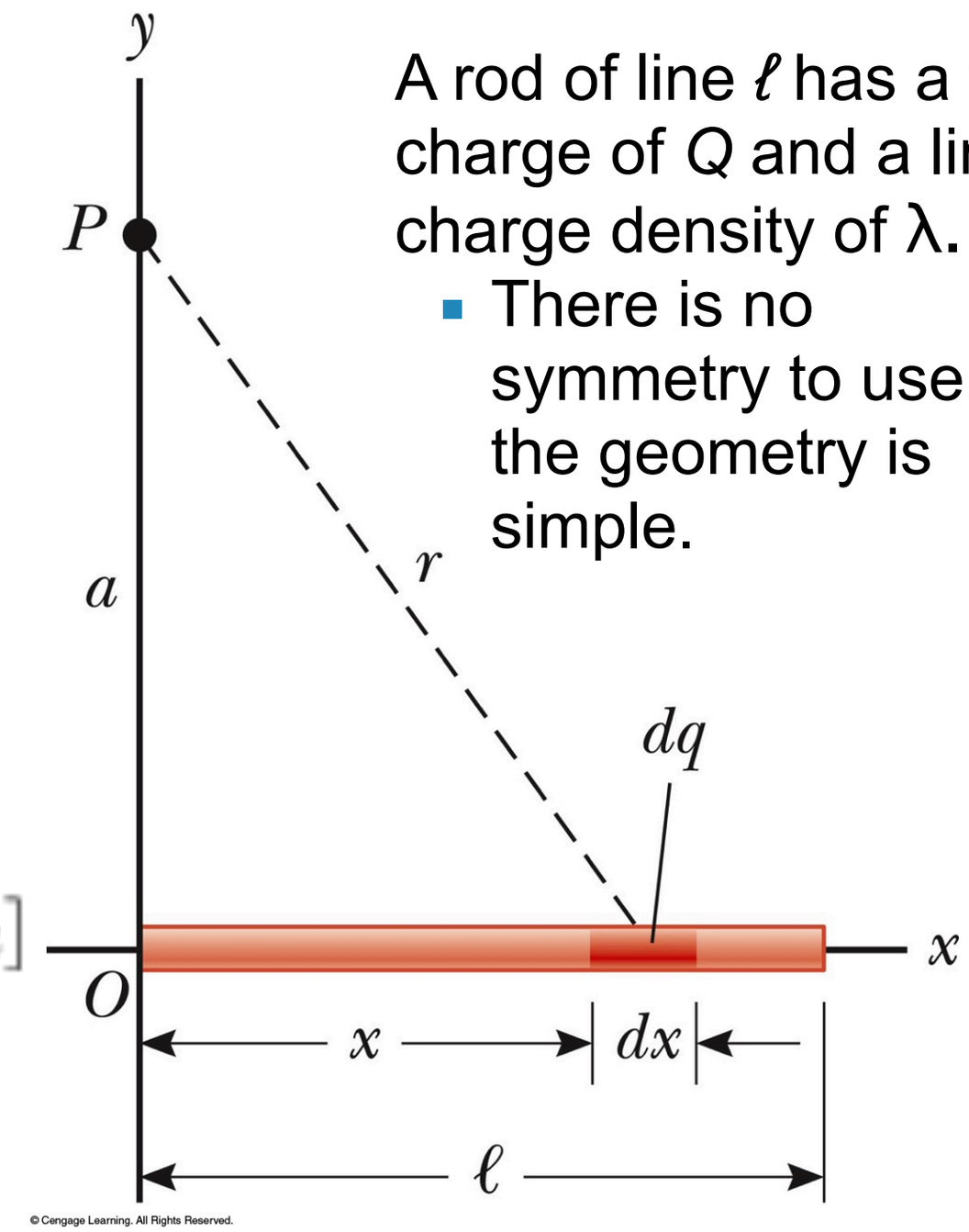
$$V = \int_0^{\ell} k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}}$$

$$V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2 + x^2}}$$

$$= k_e \frac{Q}{\ell} \ln\left(x + \sqrt{a^2 + x^2}\right)\Big|_0^{\ell}$$

$$V = k_e \frac{Q}{\ell}\left[\ln\left(\ell + \sqrt{a^2 + \ell^2}\right) - \ln a\right]$$

$$= k_e \frac{Q}{\ell} \ln\left(\frac{\ell + \sqrt{a^2 + \ell^2}}{a}\right)$$



A rod of line ℓ has a total charge of Q and a linear charge density of λ.

- There is no symmetry to use, but the geometry is simple.

Can you find the E field from this?

Charged Conductor

<u>At the surface:</u> E is always perpendicular to the displacement so **E**.d**s**=0

***V* is constant everywhere on the surface of a charged conductor in equilibrium.**

- $\Delta V = 0$ between any two points on the surface

The surface of any charged conductor in electrostatic equilibrium is an equipotential surface.

<u>Inside the conductor:</u> E=0, so V=constant

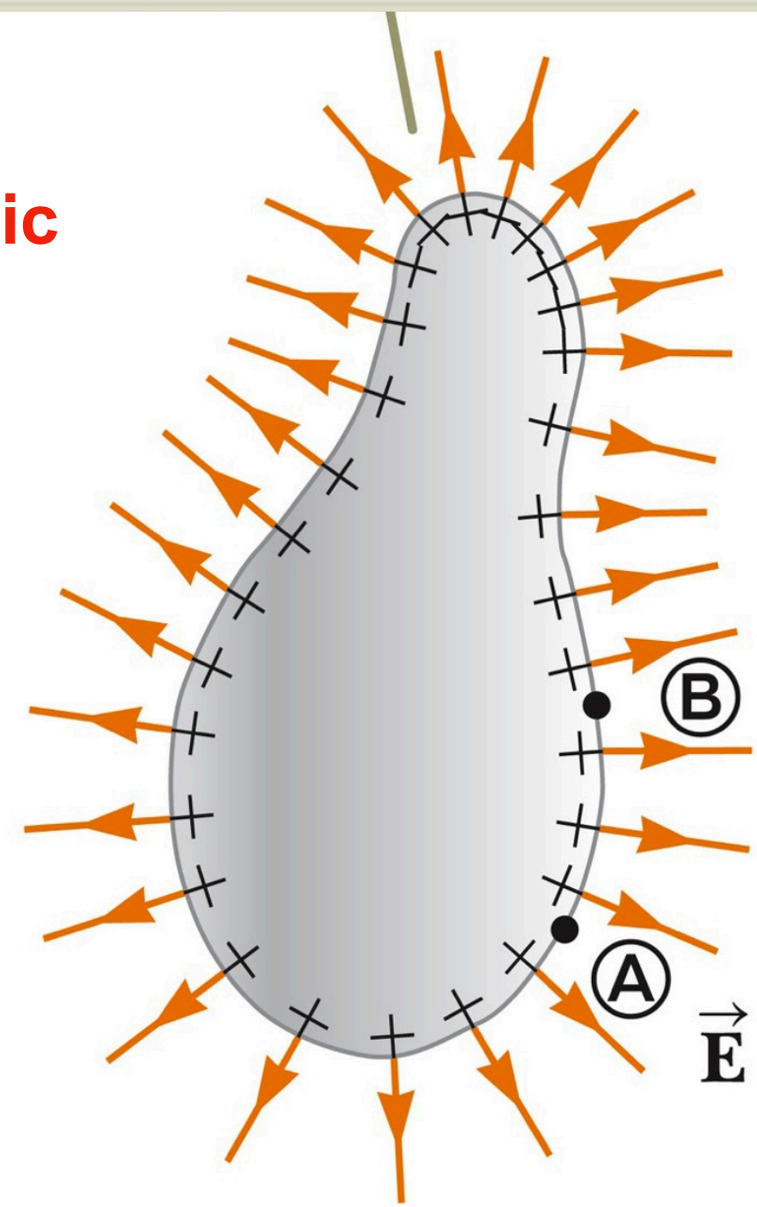
The electric potential is constant everywhere inside the conductor and **equal to the value at the surface.**

The charge density is high where the radius of curvature is small

- And low where the radius of curvature is large

The electric field is large near the convex points having small radii of curvature and reaches very high values at sharp points.

Notice from the spacing of the positive signs that the surface charge density is nonuniform.



Cavity in a conductor

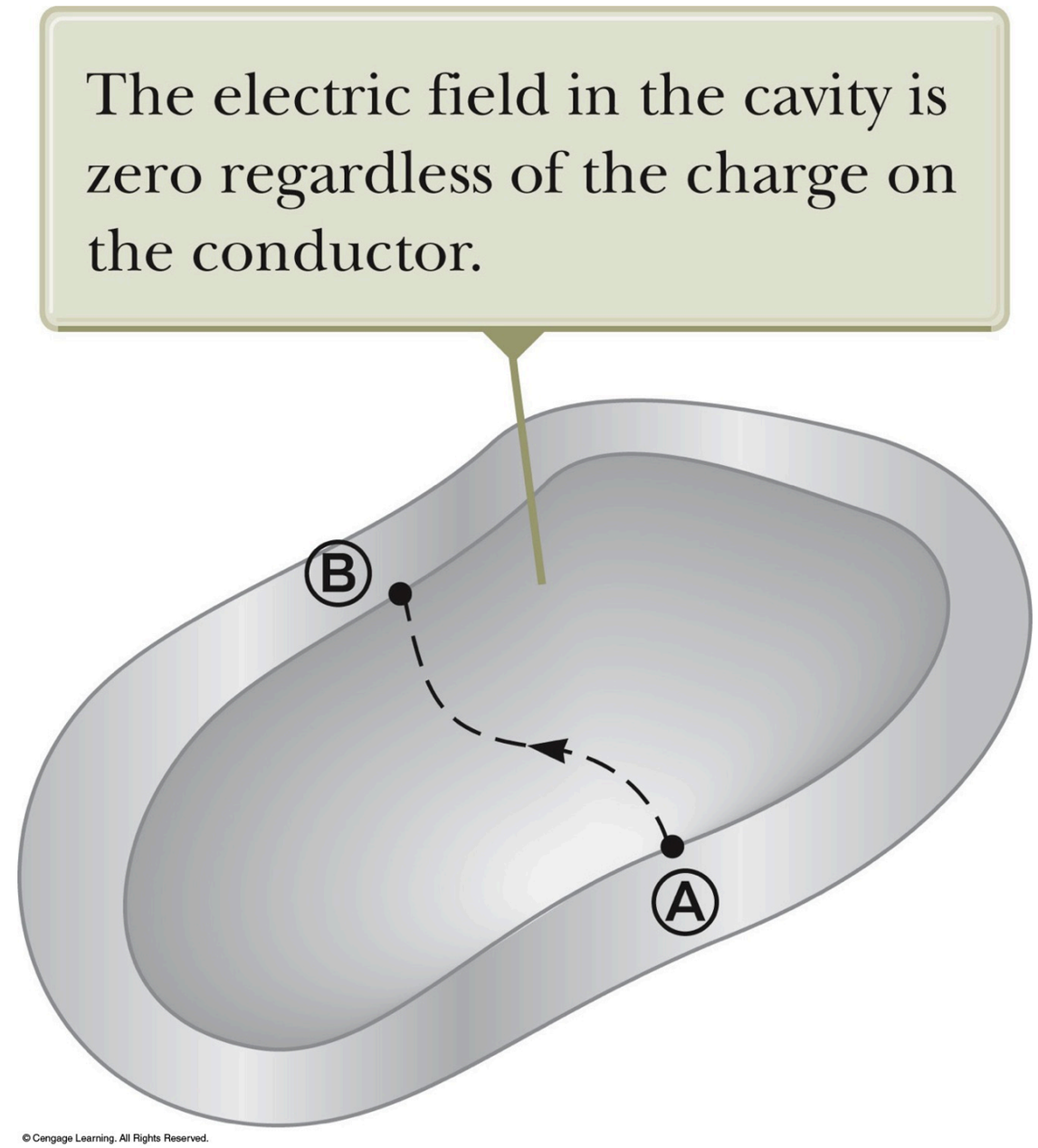
Assume an irregularly shaped cavity is inside a conductor.

Assume no charges are inside the cavity.

The electric field inside does not depend on the charge distribution on the outside surface of the conductor.

For all paths between *A* and *B*,

$$\Delta V = 0$$



A cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity.

Example Problem 5

47. A wire having a uniform linear charge density λ is bent **W** into the shape shown in Figure P25.47. Find the electric potential at point O.

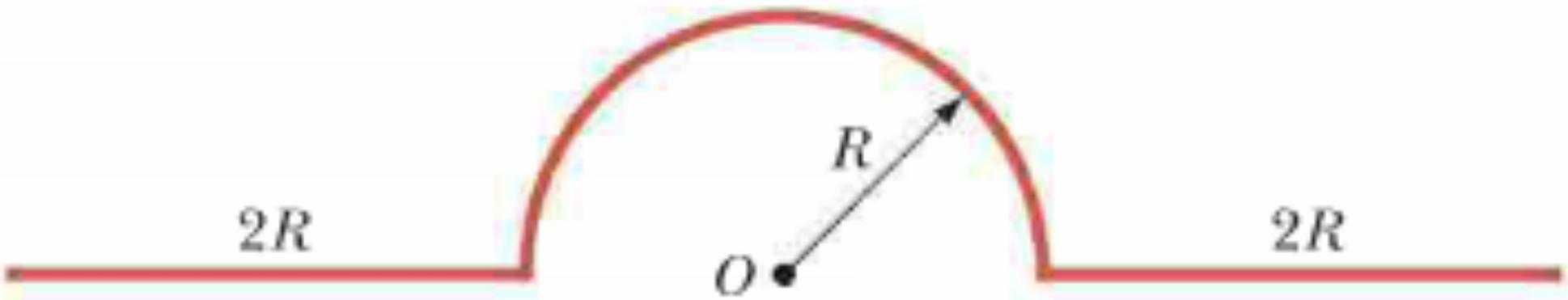


Figure P25.47

Example Problem 5 - Solution

$$V = k_e \int_{\text{all charge}} \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda dx}{-x} + k_e \int_{\text{semicircle}} \frac{\lambda ds}{R} + k_e \int_{R}^{3R} \frac{\lambda dx}{x}$$

$$V = -k_e \lambda \ln(-x)\Big|_{-3R}^{-R} + \frac{k_e \lambda}{R} \pi R + k_e \lambda \ln x \Big|_{R}^{3R}$$

$$V = k_e \lambda \ln \frac{3R}{R} + k_e \lambda \pi + k_e \lambda \ln 3 = \boxed{k_e \lambda (\pi + 2 \ln 3)}$$