Electric Fields

Serway Chapter 22-23(a)

HW01 is already up on WebAssign, due Th 01/25

Maxwell's Equations

1873

IceBreakers: What is Electricity? What does it mean in your life?

- James Clerk Maxwell used observations and other experimental facts as a basis for formulating the laws of electromagnetism.
  - Unified electricity and magnetism

The goal of this class is to understand Maxwell's equations.

The concept of force links the study of electromagnetism to previous study.

The electromagnetic force between charged particles is one of the fundamental forces of nature.

$$\oint \mathbf{E} \bullet d\mathbf{A} = \frac{Q}{\varepsilon_0}$$

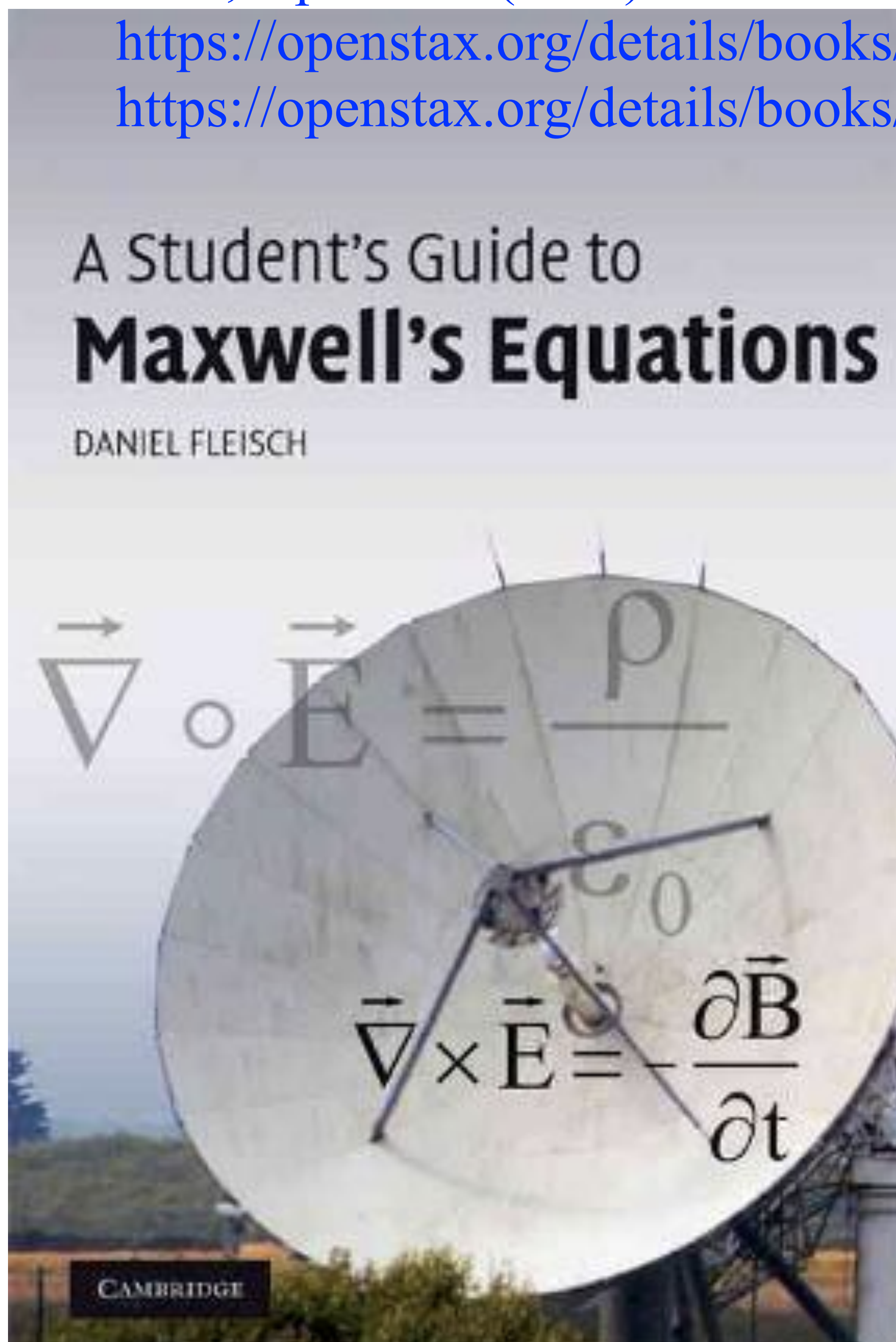
$$\oint \mathbf{B} \bullet d\mathbf{A} = 0$$

$$\oint \mathbf{E} \bullet d\mathbf{s} = -\frac{d\Phi_m}{dt}$$

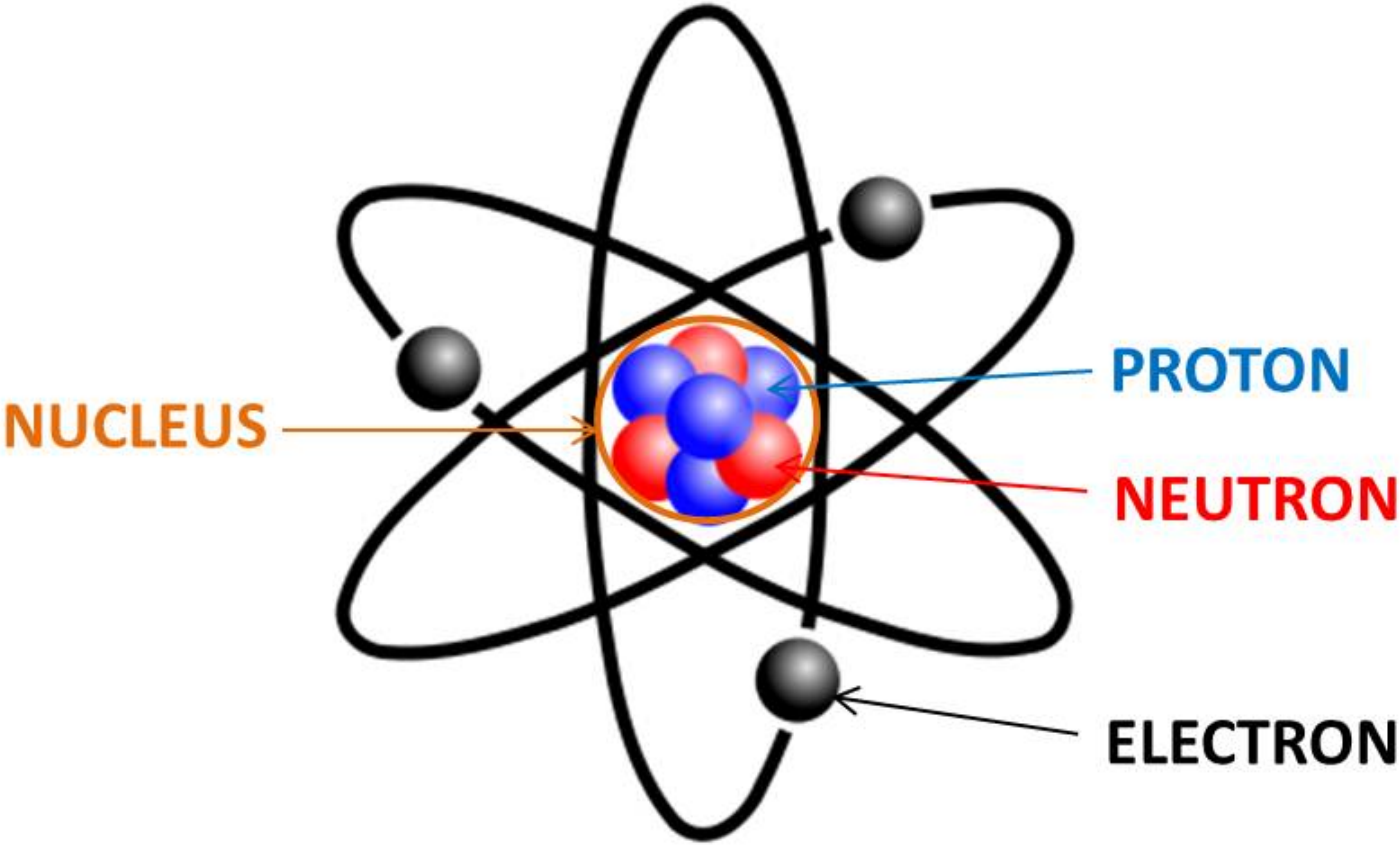
$$\oint \mathbf{B} \bullet d\mathbf{s} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_e}{dt}$$

Buy or borrow this book if you can (not mandatory but strongly suggested)

Also, OpenStax (free!)
https://openstax.org/details/books/university-physics-volume-1
https://openstax.org/details/books/university-physics-volume-2



An atom



Particles Summary

Two kinds of electric charges: **positive and negative**

- Negative charges are the type possessed by electrons.
- Positive charges are the type possessed by protons.

Table 23.1 Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,176\,5 \times 10^{-19}$	$9.109\,4 \times 10^{-31}$
Proton (p)	$+1.602\,176\,5 \times 10^{-19}$	$1.672\,62 \times 10^{-27}$
Neutron (n)	0	$1.674\,93 \times 10^{-27}$

What is the charge of a natural atom?

Electric charges

Charge is **quantized** = it exists as discreet packets

symbol for charge = q

$$\boxed{\mathbf{q = \pm Ne}}$$

- *N* is an integer = the number of charges
- **e is the fundamental unit of charge**
- **$|e| = 1.6 \times 10^{-19}$ C**
- unit of charge is the Coulomb C

- Electron: $q = -e$
- Proton: $q = +e$

Example

(for you)

What is the total charge of a material containing 20 electrons and 10 protons?

$$Q = 20 \times -e + 10 \times e$$

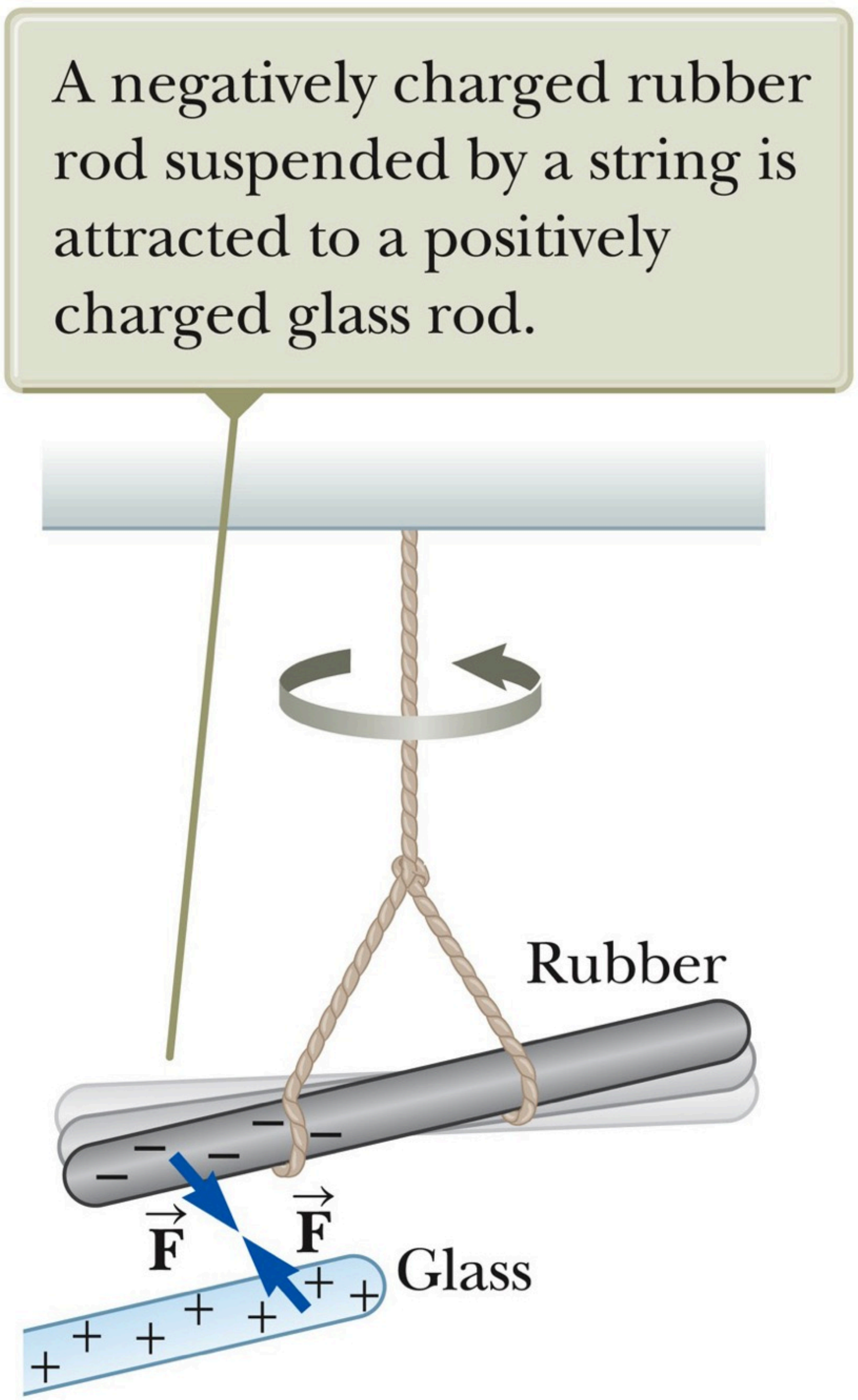
$$= 10 \times -e$$

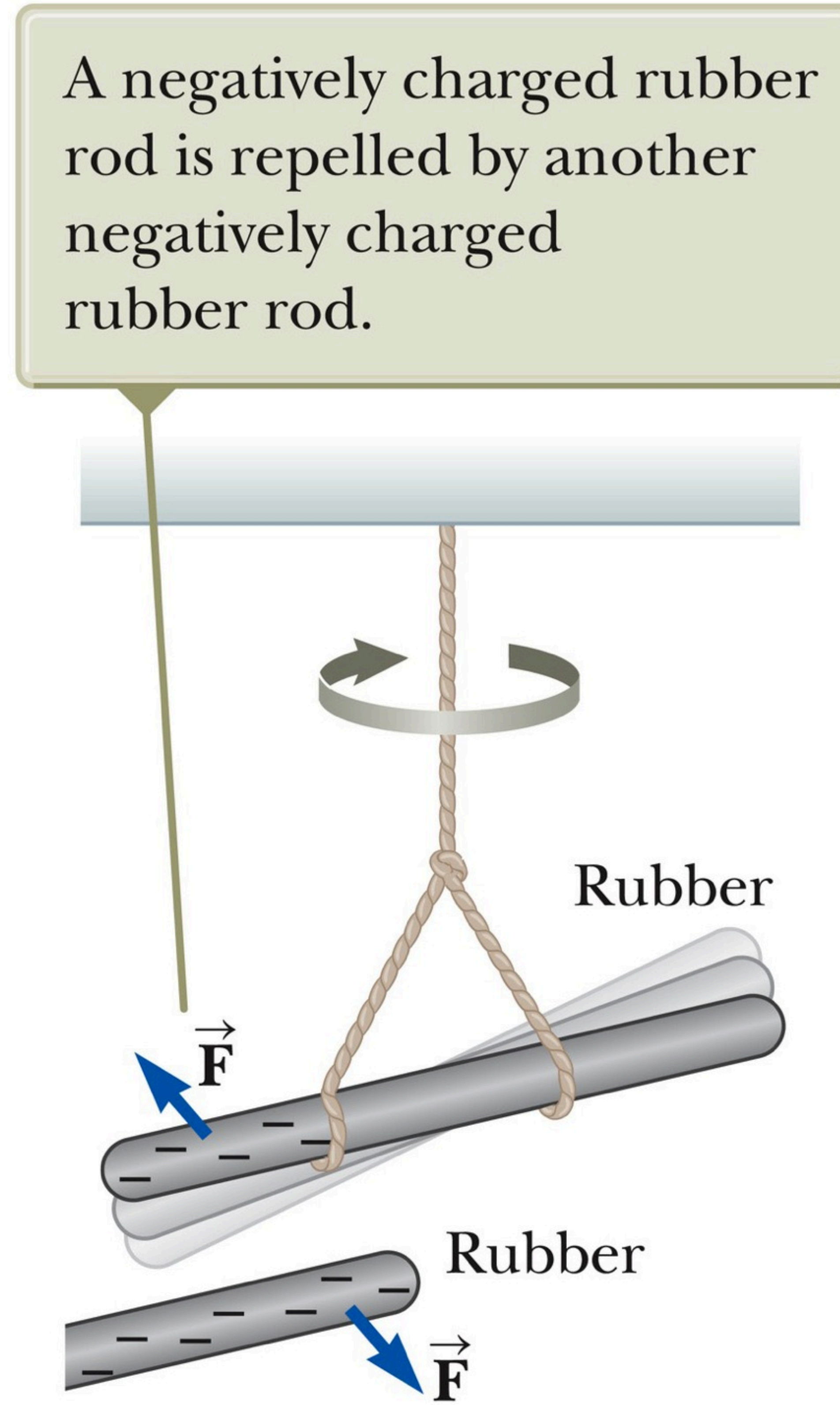
$$= 10 \times -1.6 \times 10^{-19}$$

$$= -1.6 \times 10^{-18}\ C$$

Electric charges

Charges of the same sign repel one another and charges with opposite signs attract one another.





Conservation of charge q

A glass rod is rubbed with silk.

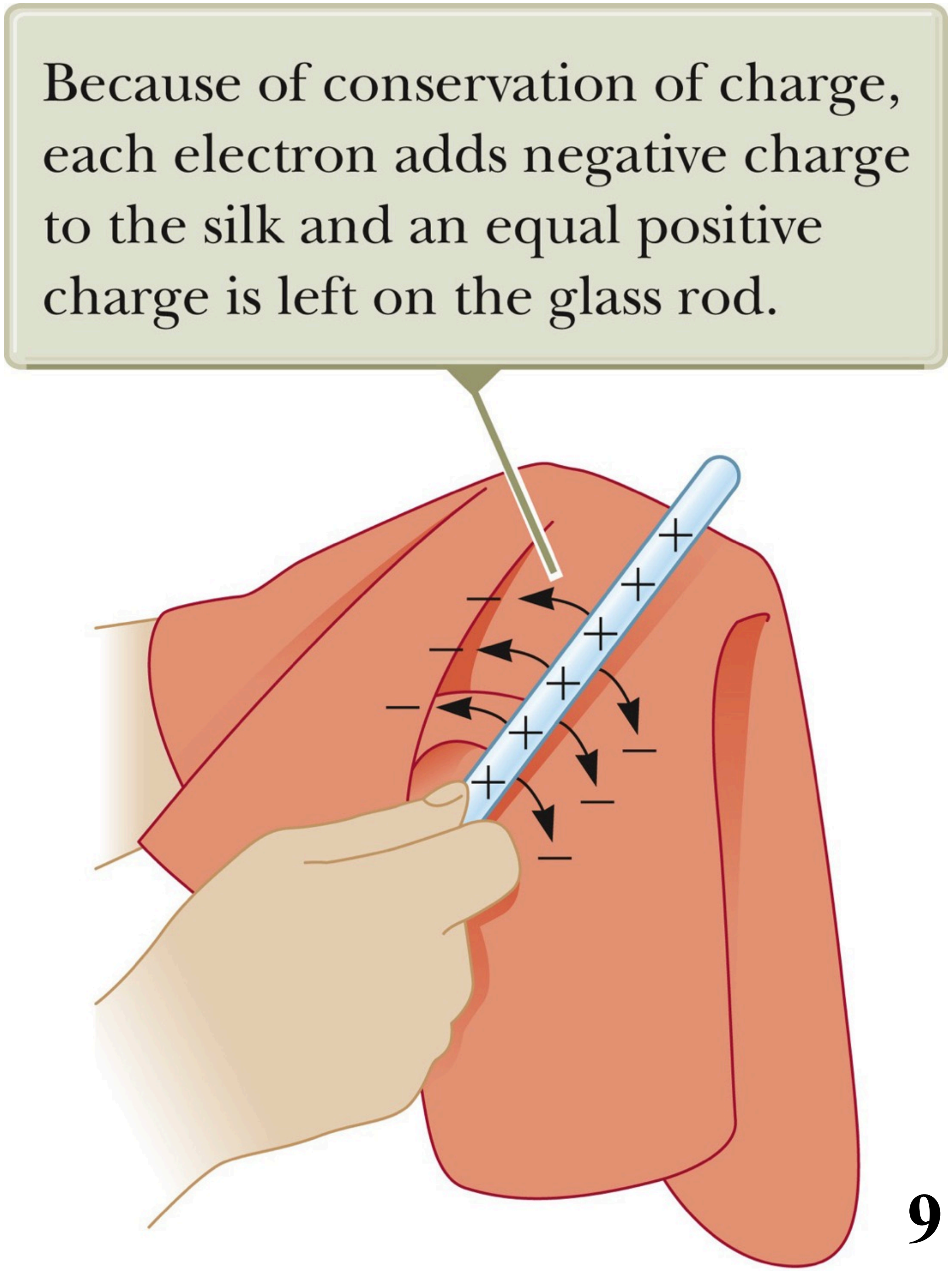
Electrons are transferred from the glass to the silk.

Each electron adds a negative charge to the silk.

An equal positive charge is left on the rod.

Electric charge is always conserved in an isolated system.

- For example, charge is not created in the process of rubbing two objects together.
- The electrification is due to a **transfer of charge** from one object to another.

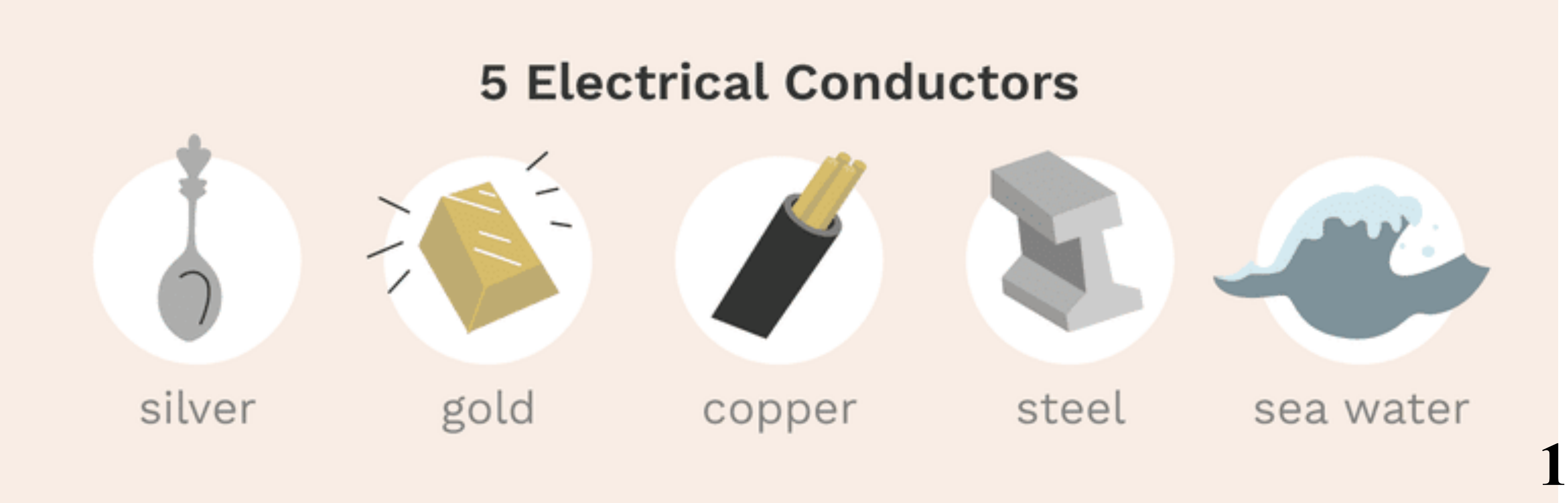


Conductors

Electrical conductors are materials in which **some of the electrons are free electrons.**

- Free electrons are not bound to the atoms.
- These electrons can move relatively freely through the material.
- When a good conductor is charged in a small region, **the charge readily distributes itself over the entire surface of the material.**

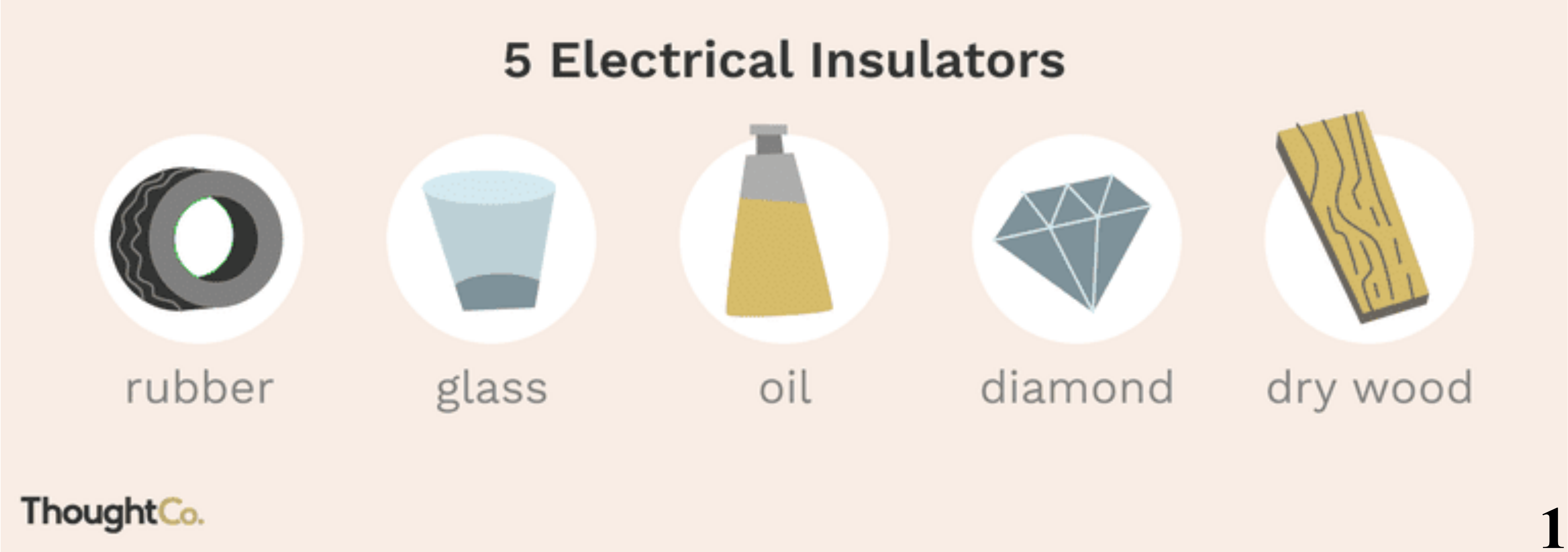
- **metals are conductors**



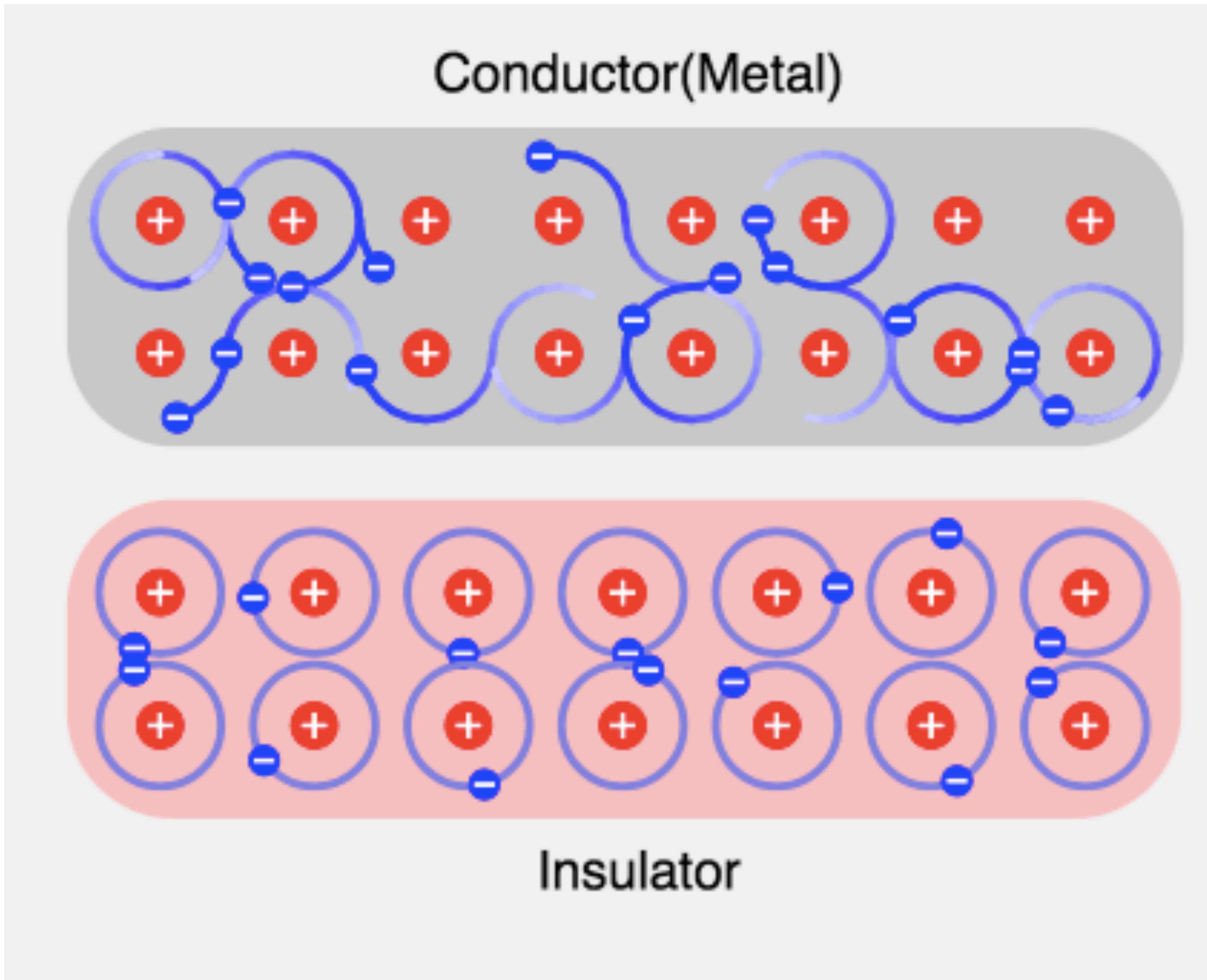
Insulators

Electrical insulators are materials in which **all of the electrons are bound to atoms.**

- These electrons can not move relatively freely through the material.
- When a good insulator is charged in a small region, **the charge is unable to move to other regions of the material.**



Conductor and Insulator



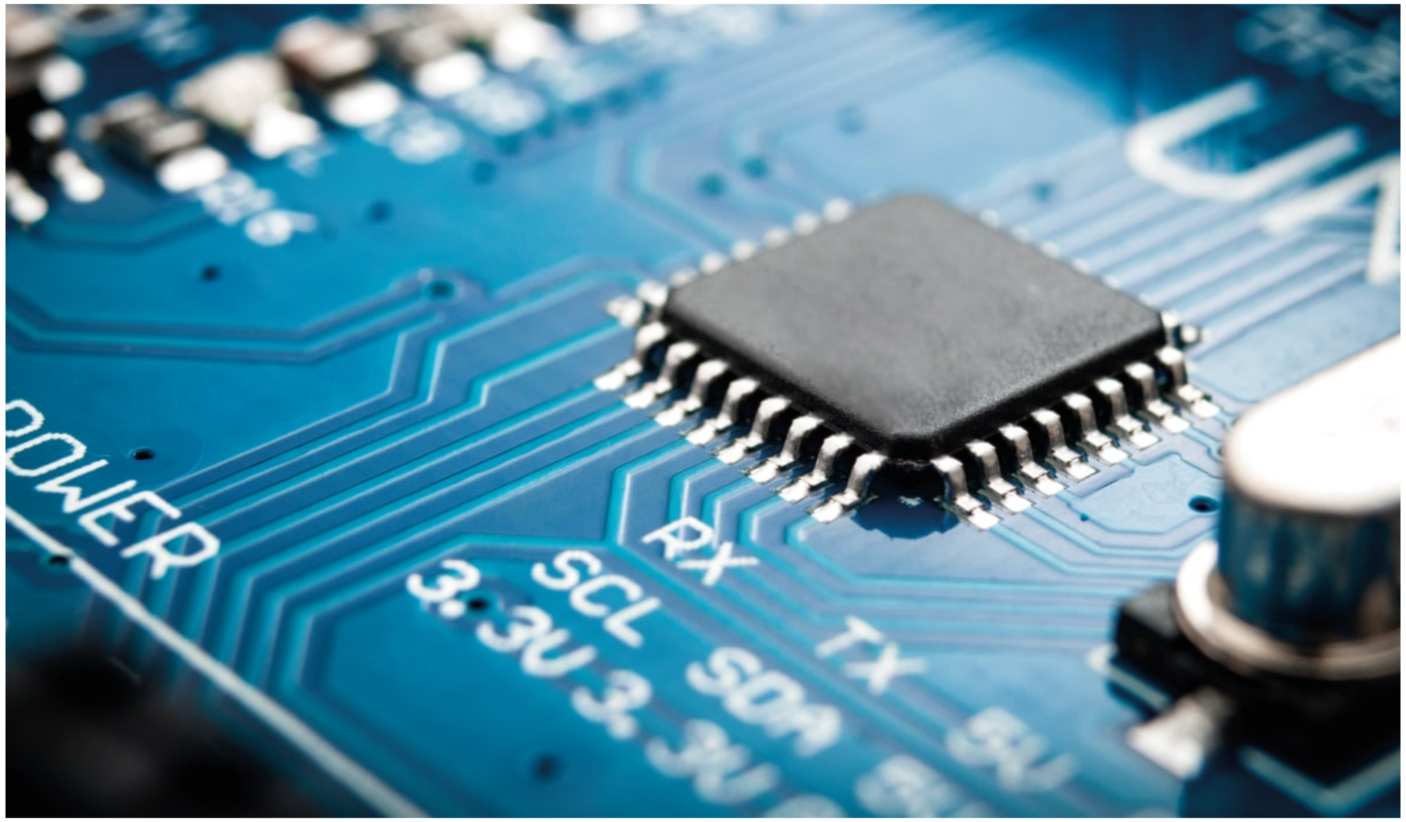
Semi-Conductors

The electrical properties of semiconductors are **somewhere between those of insulators and conductors.**

The electrical properties of semiconductors **can be changed by the addition of controlled amounts of certain atoms to the material: dopants**

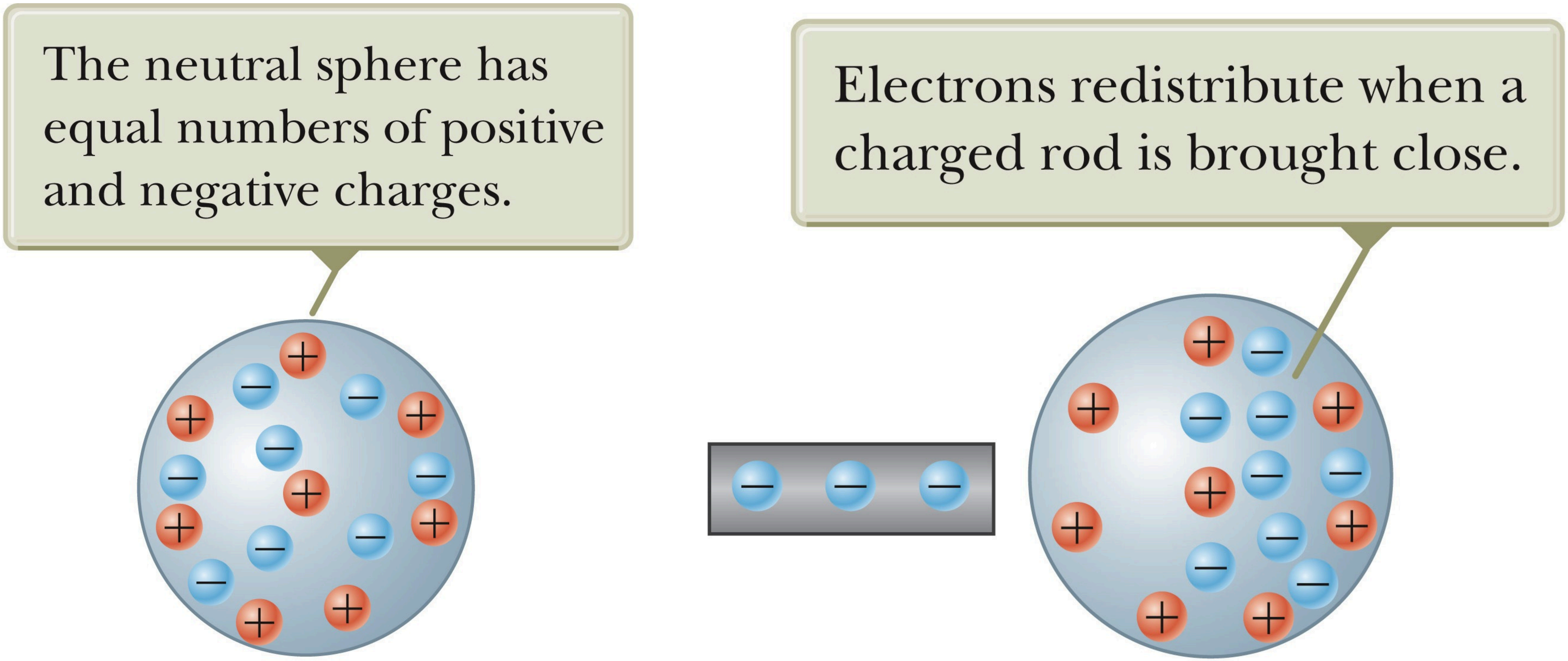
Examples of semiconductor materials include silicon and germanium.

- Semiconductors made from these materials are commonly used in making electronic chips.



Induction (in conductors)

Charging by induction requires **no contact** with the object inducing the charge.



- the bar does **not** touch the sphere.

Induction (in conductors)

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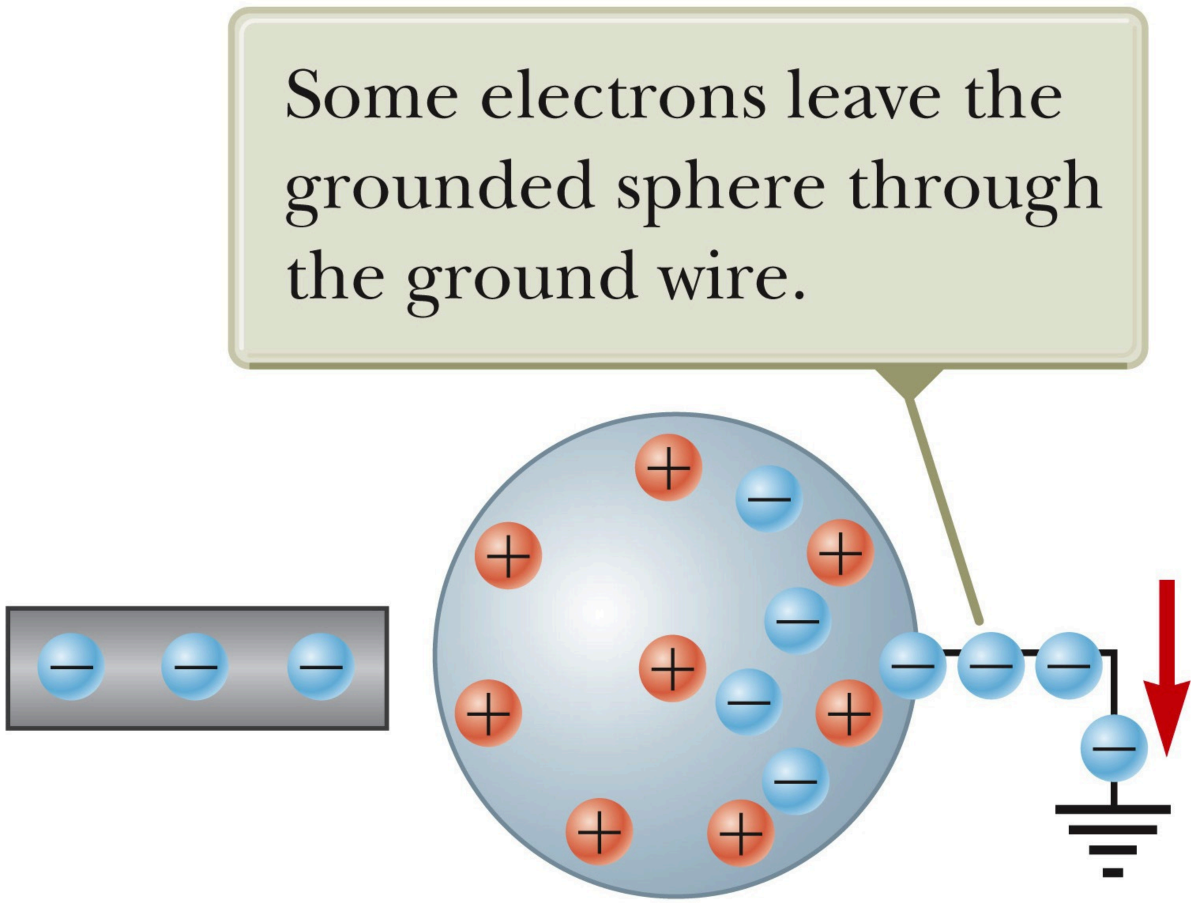
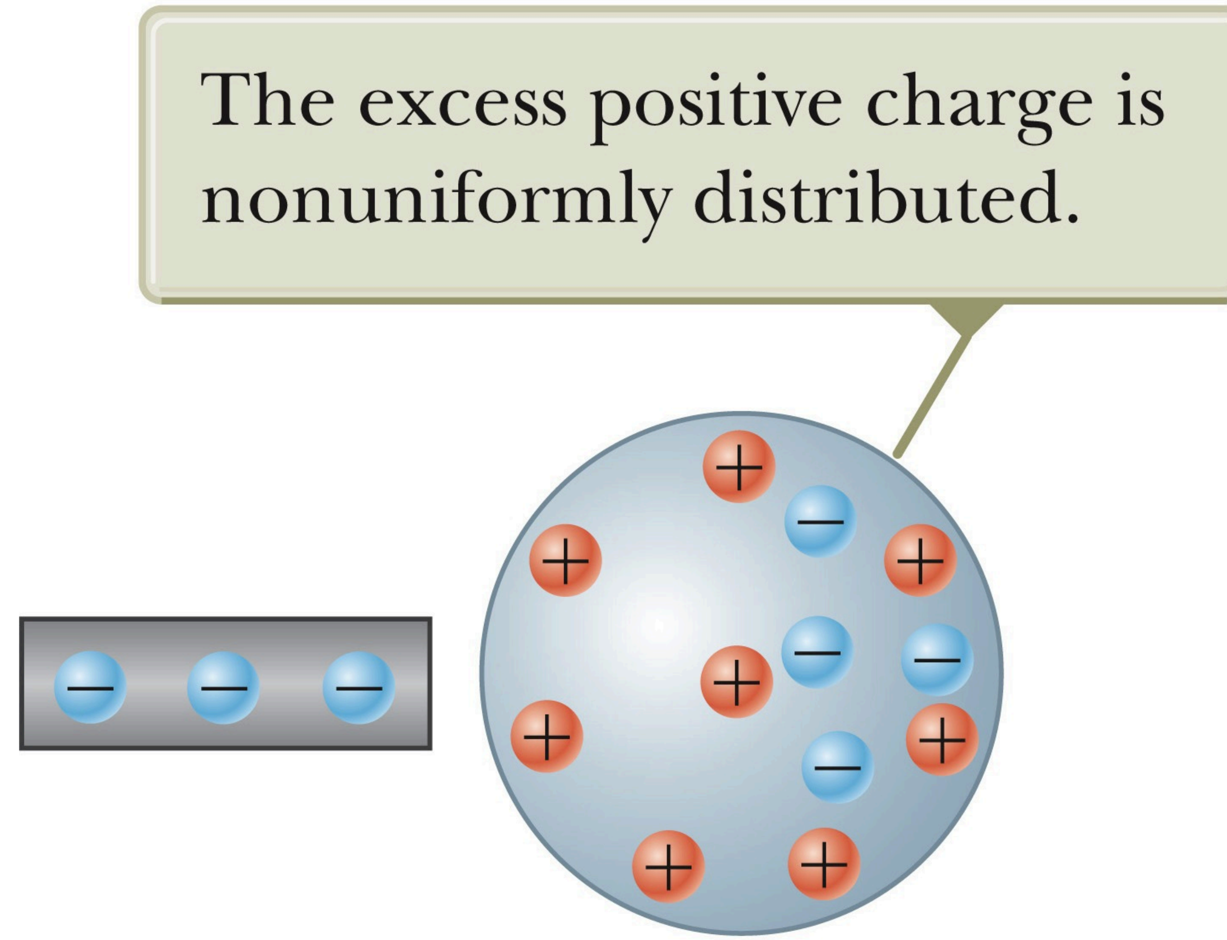


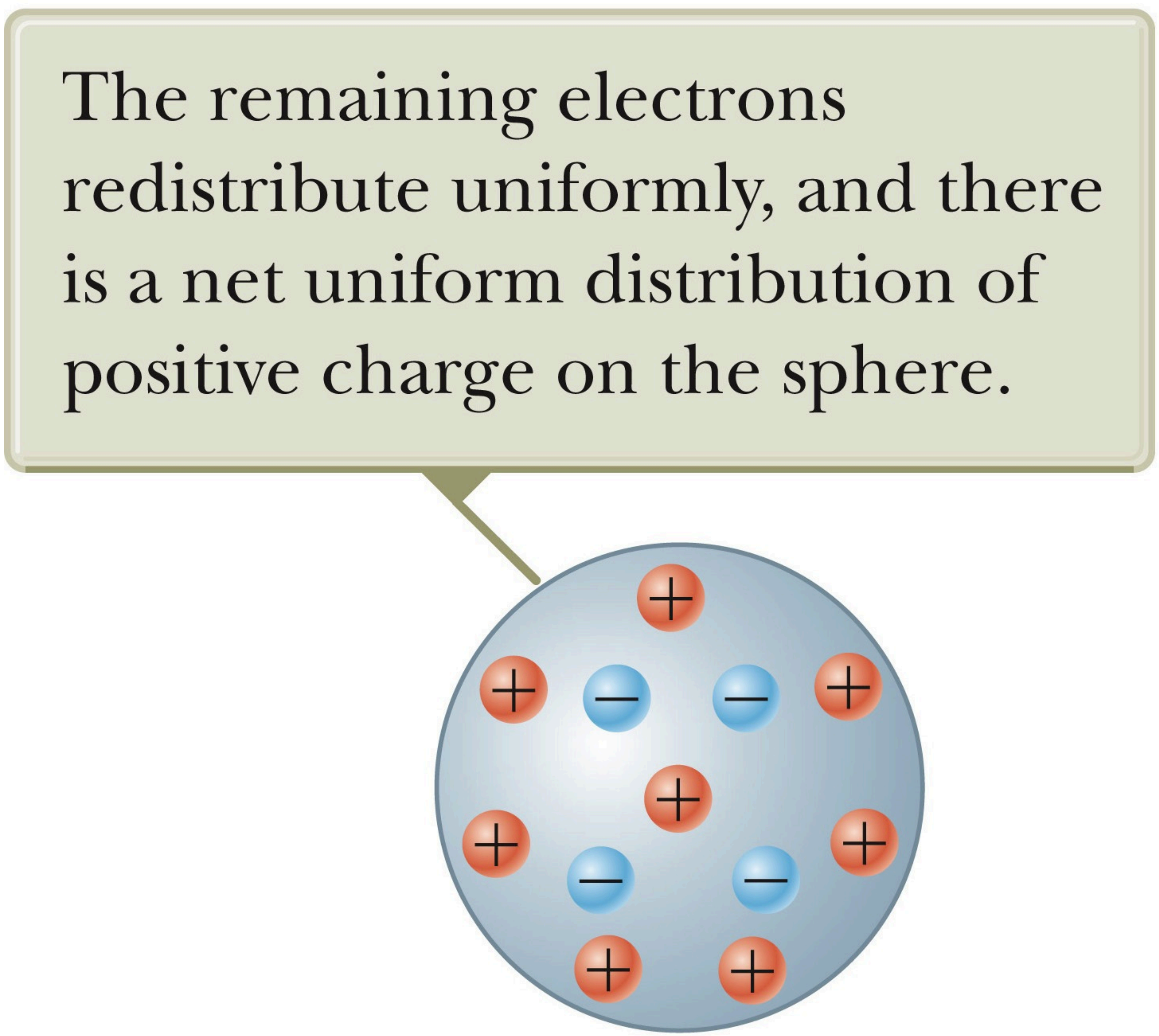
The sphere is grounded.



The positive charge has been ***induced*** in the sphere.

Induction (in conductors)

Charging by induction requires no contact with the object inducing the charge.

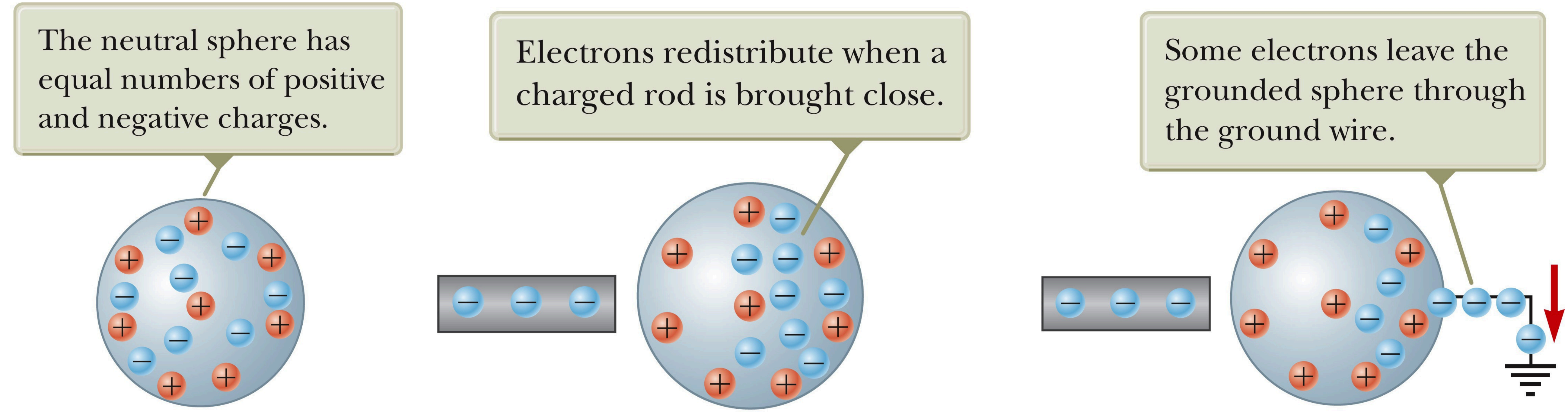


There is still a **net positive charge** on the sphere.

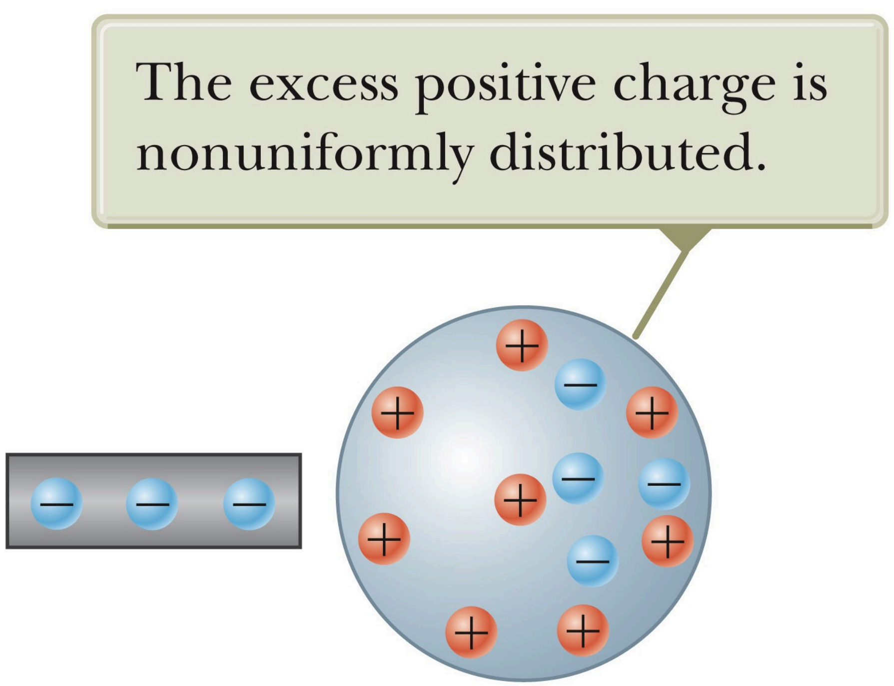
The charge is now **uniformly distributed.**

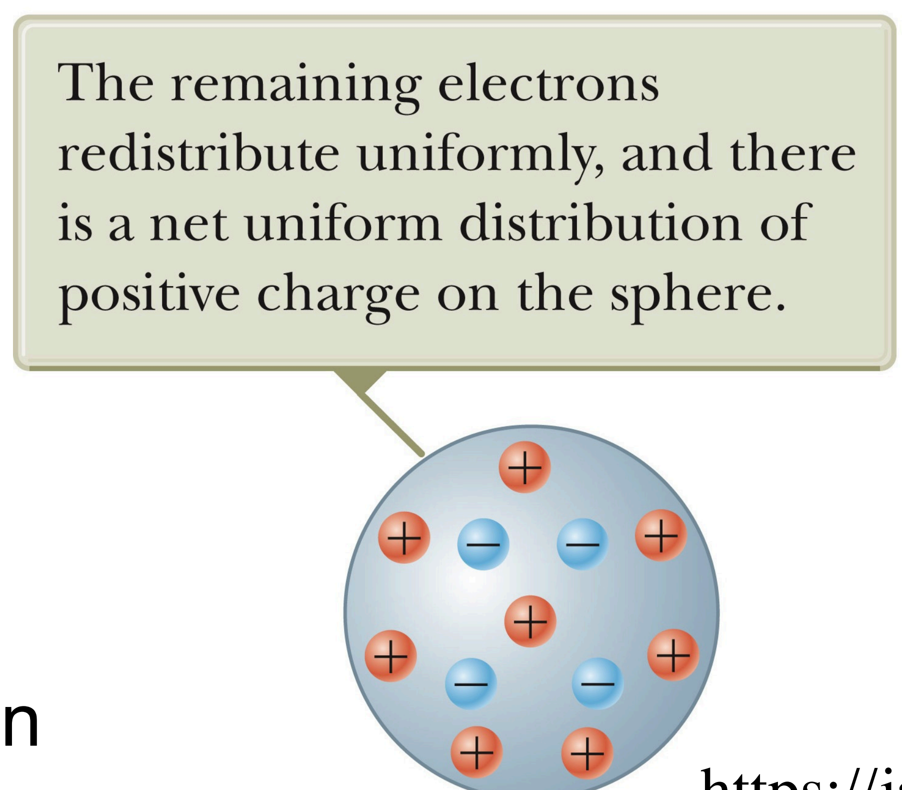
Induction (in conductors)

Charging by induction requires no contact with the object inducing the charge.



- the bar does **not** touch the sphere. The sphere is grounded.





There is still a **net positive charge** on the sphere.

The charge is now **uniformly distributed.**

The positive charge has been ***induced*** in the sphere.

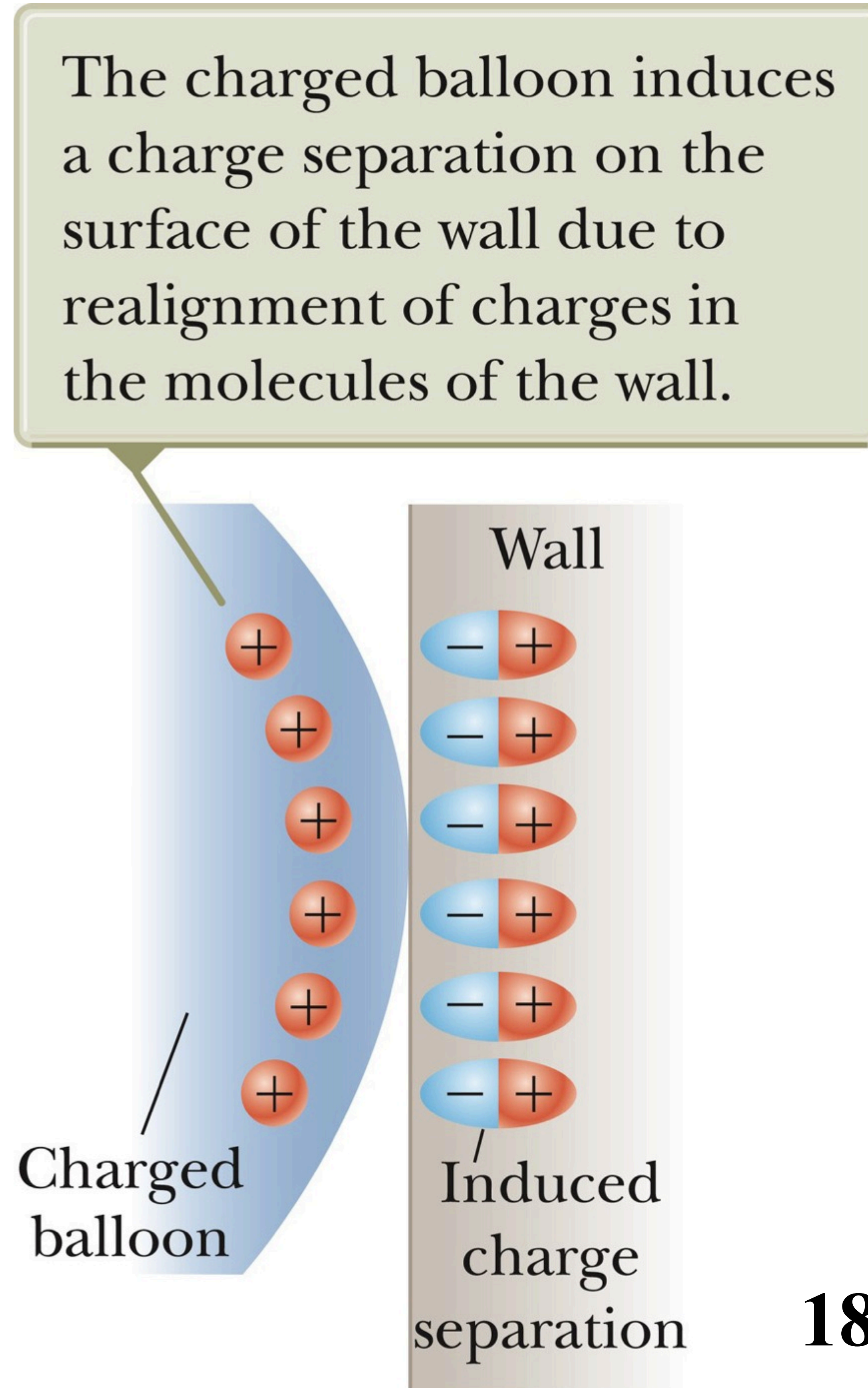
https://javalab.org/en/electroscope_en/

Charge rearrangements in insulators

A process similar to induction can take place in insulators.

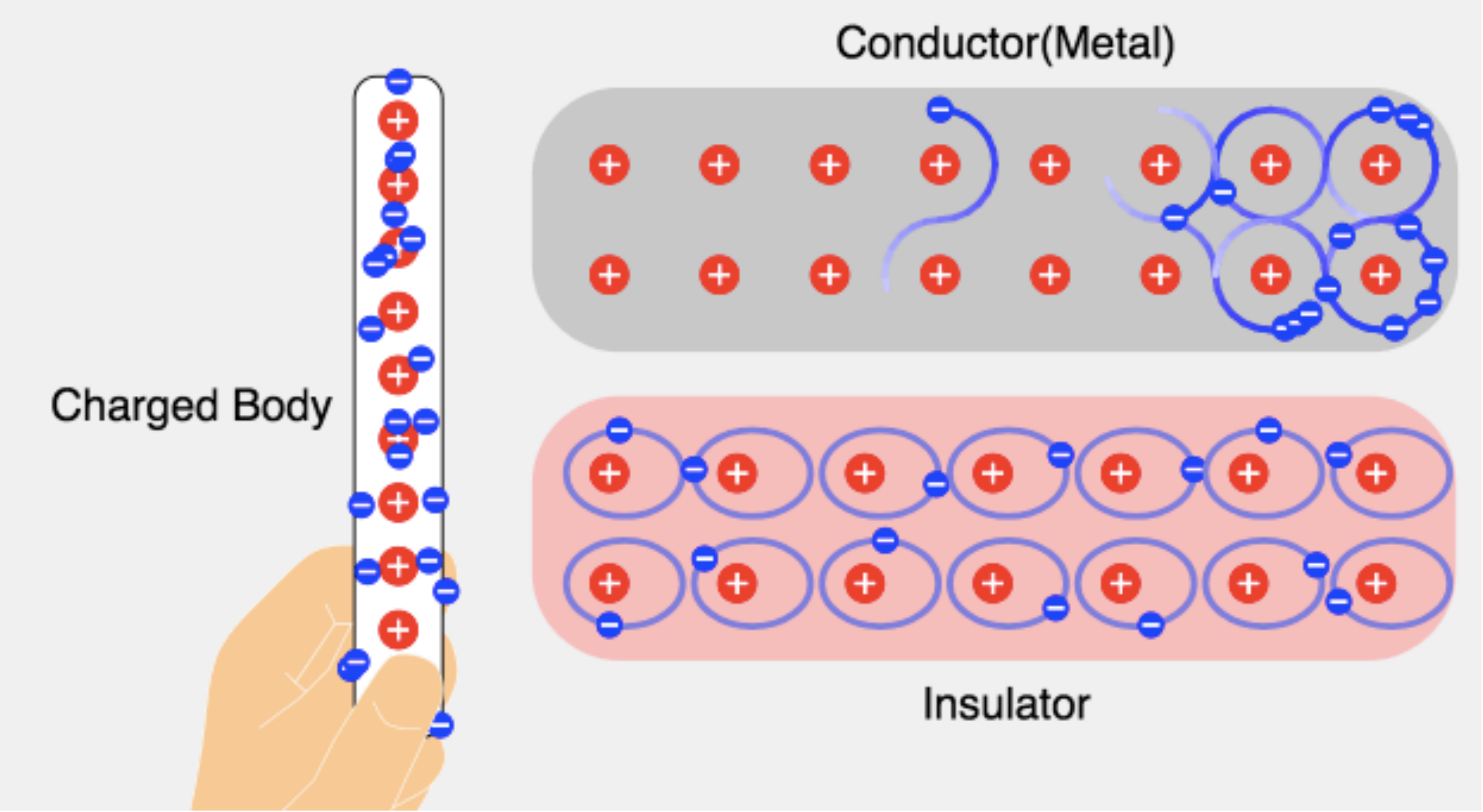
The charges within the molecules of the material are rearranged.

The proximity of the positive charges on the surface of the object and the negative charges on the surface of the insulator results in an attractive force between the object and the insulator.



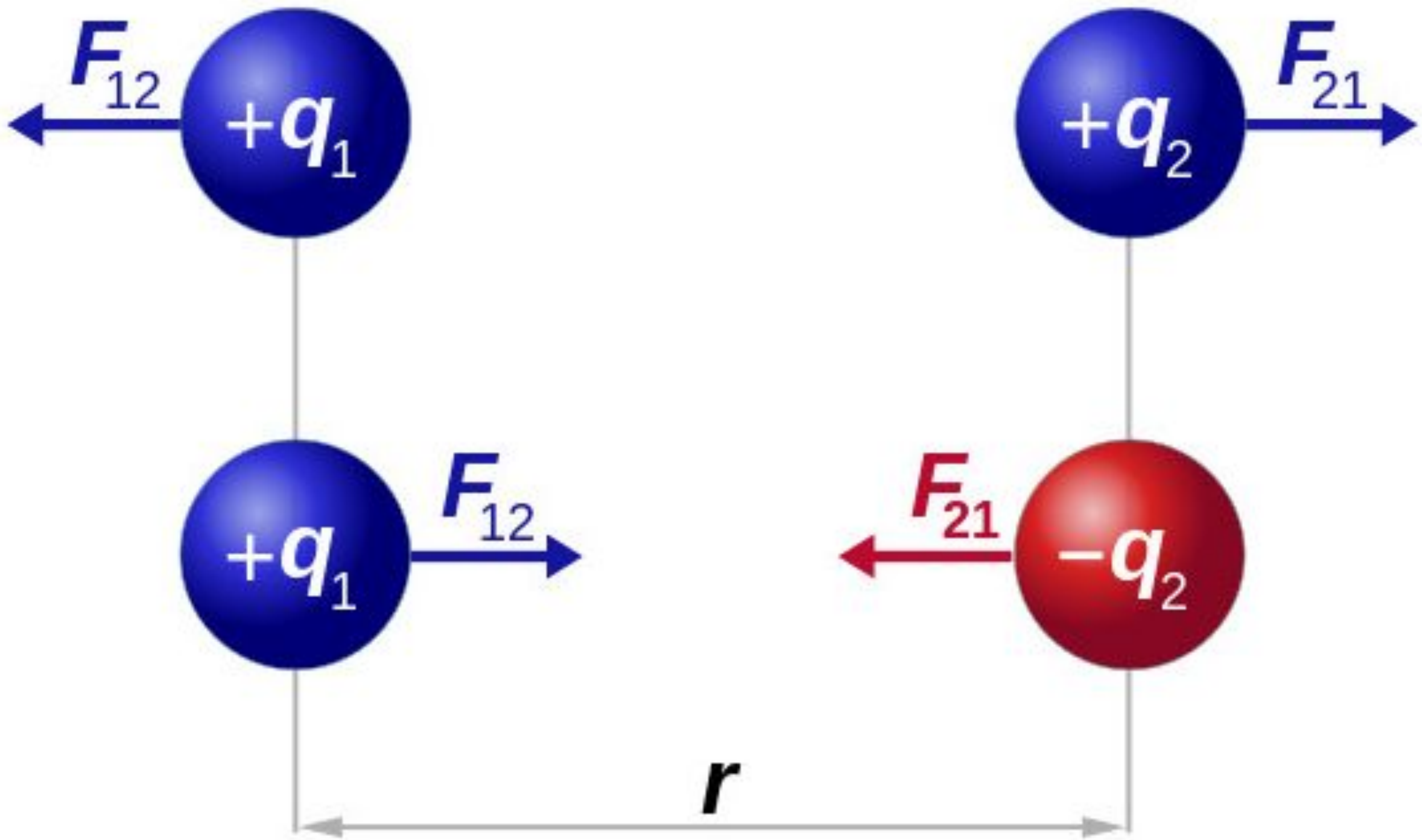
Conductor and Insulator

https://javalab.org/en/conductor_and_insulator_en/



Coulomb's law

= how to define electrical force

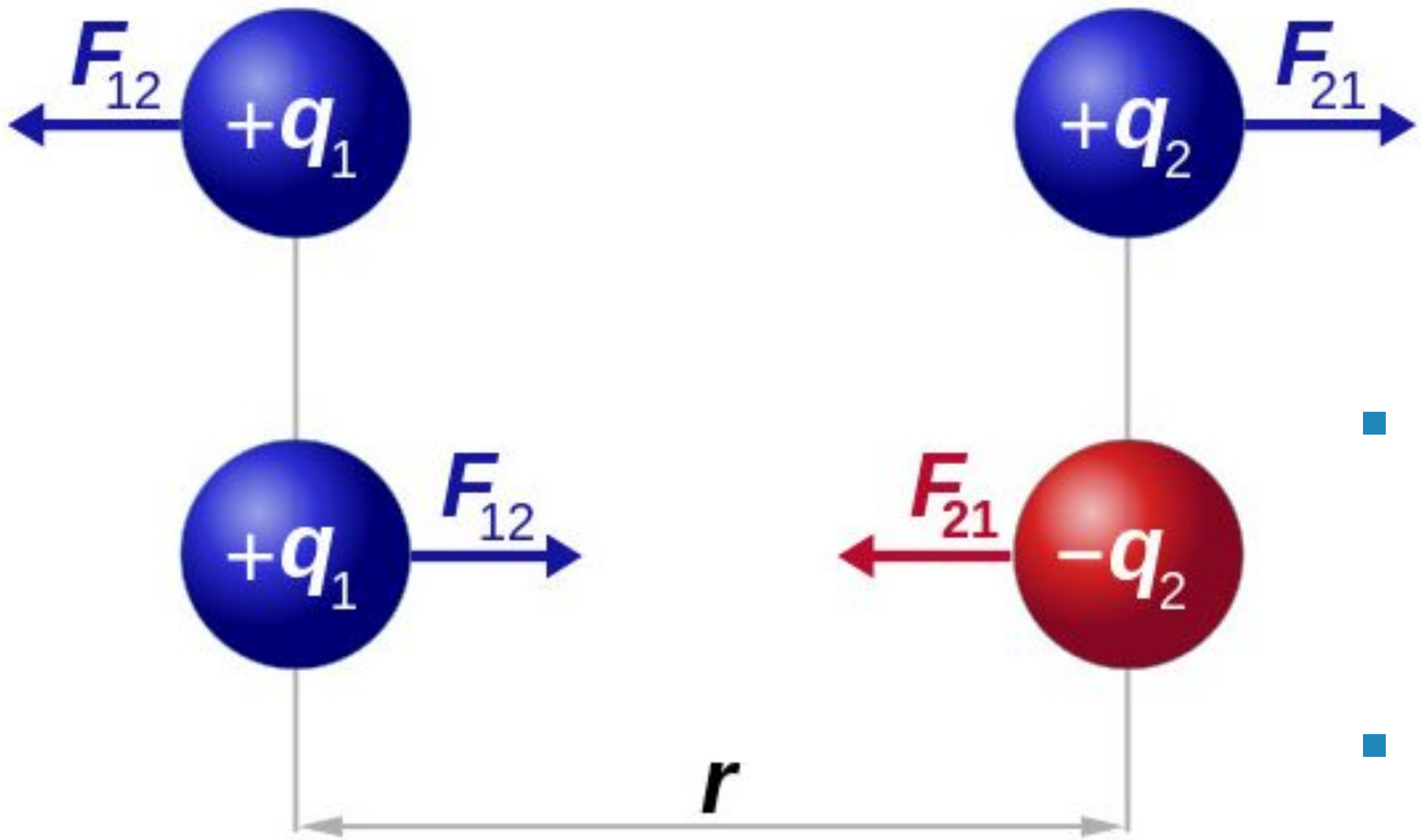


q = ± Ne

- $e = 1.6 \times 10^{-19}$ C

The force is attractive if the charges are of opposite sign.
The force is repulsive if the charges are of like sign.
The force is a conservative force.

Coulomb's law



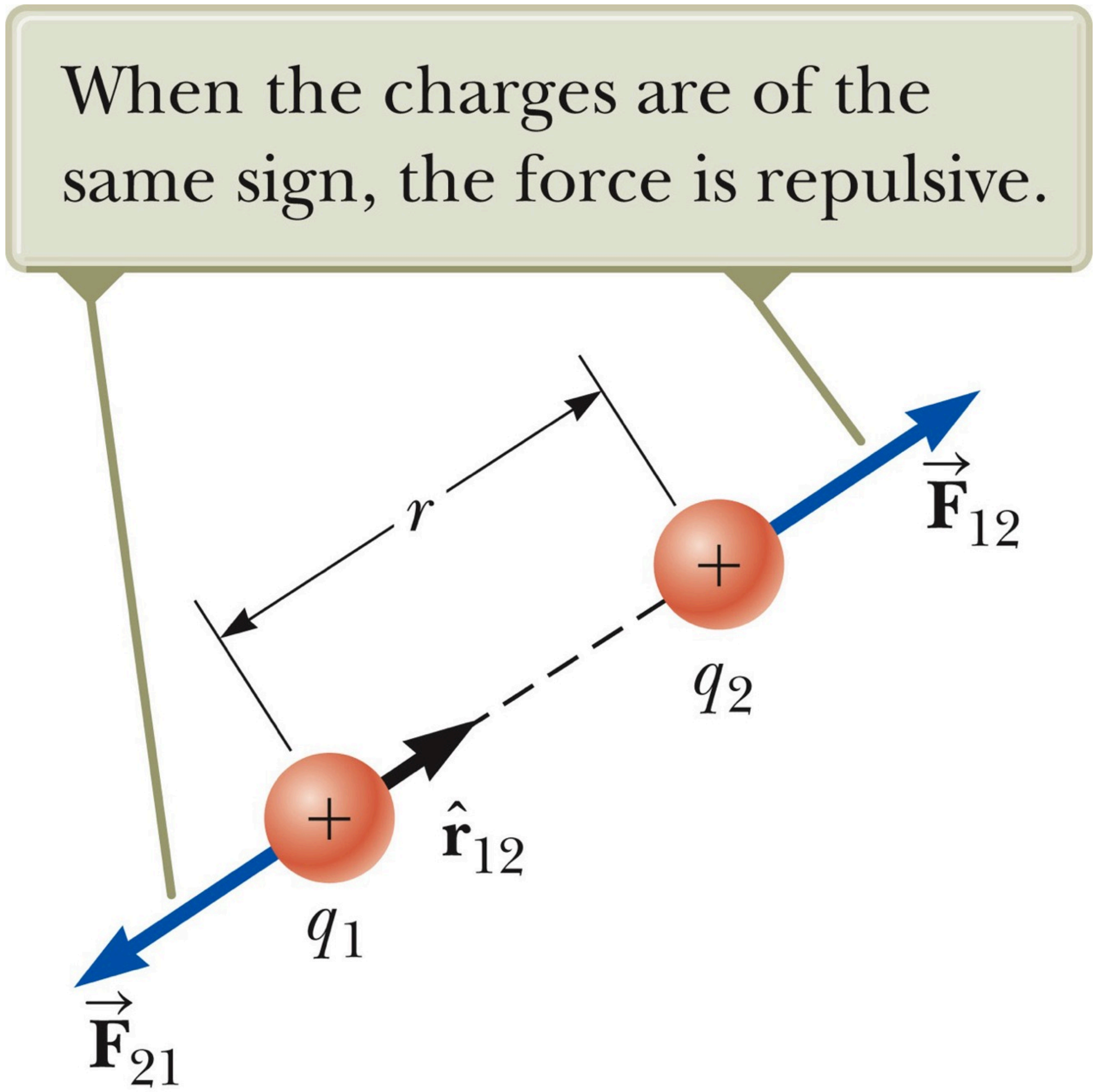
$$\boxed{F_{12} = F_{21} = k_e \frac{|q_1|\,|q_2|}{r^2}}$$

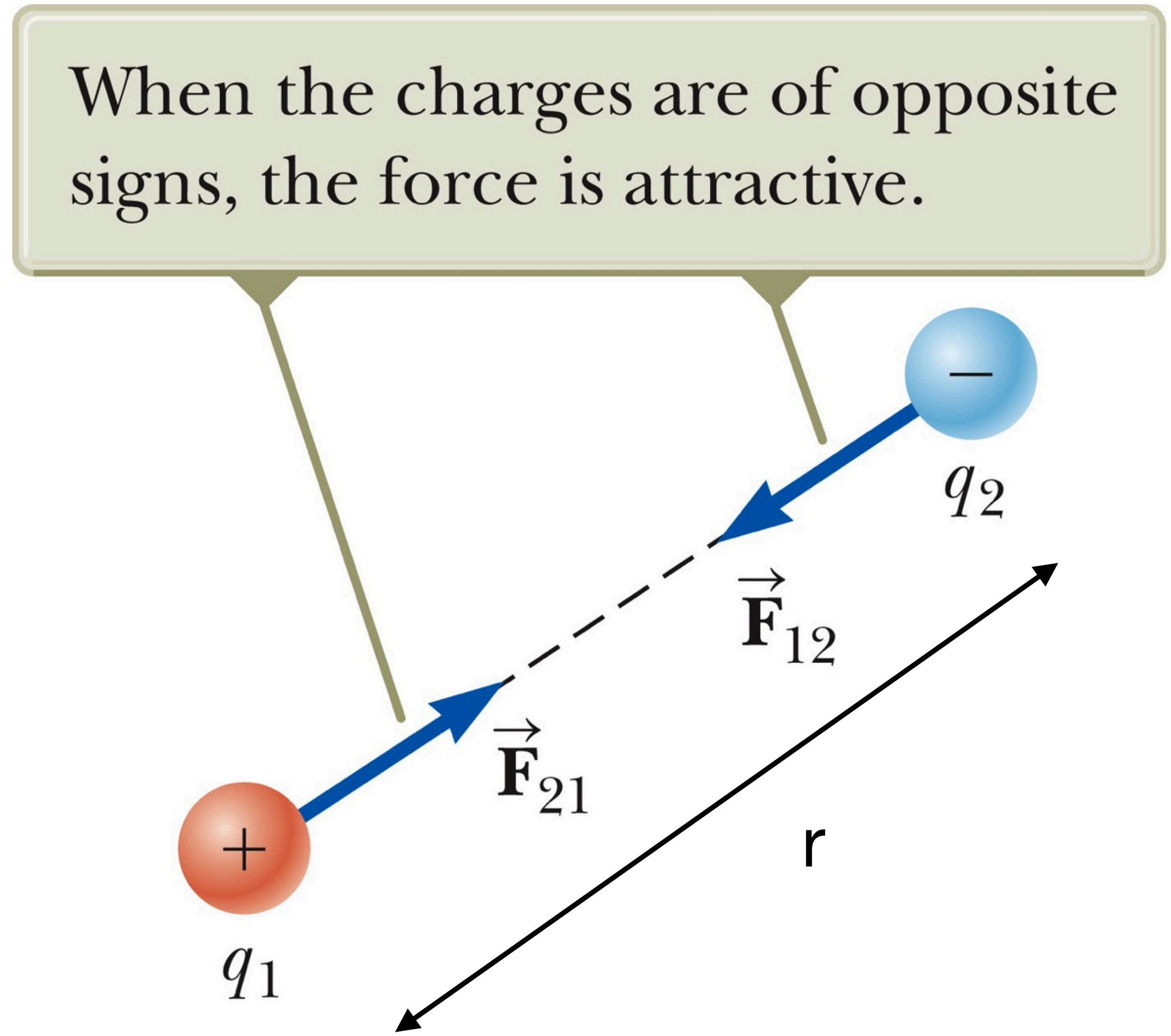
- ε_o is the **permittivity of free space** = describes how well vacuum conducts electricity
- $\varepsilon_o = 8.8542 \times 10^{-12}\ C^2 / N{\cdot}m^2$

k_e is called the **Coulomb constant.**

- $k_e = 8.9876 \times 10^9\ N{\cdot}m^2/C^2 = 1/(4\pi\varepsilon_o)$

Vector nature of the EM force

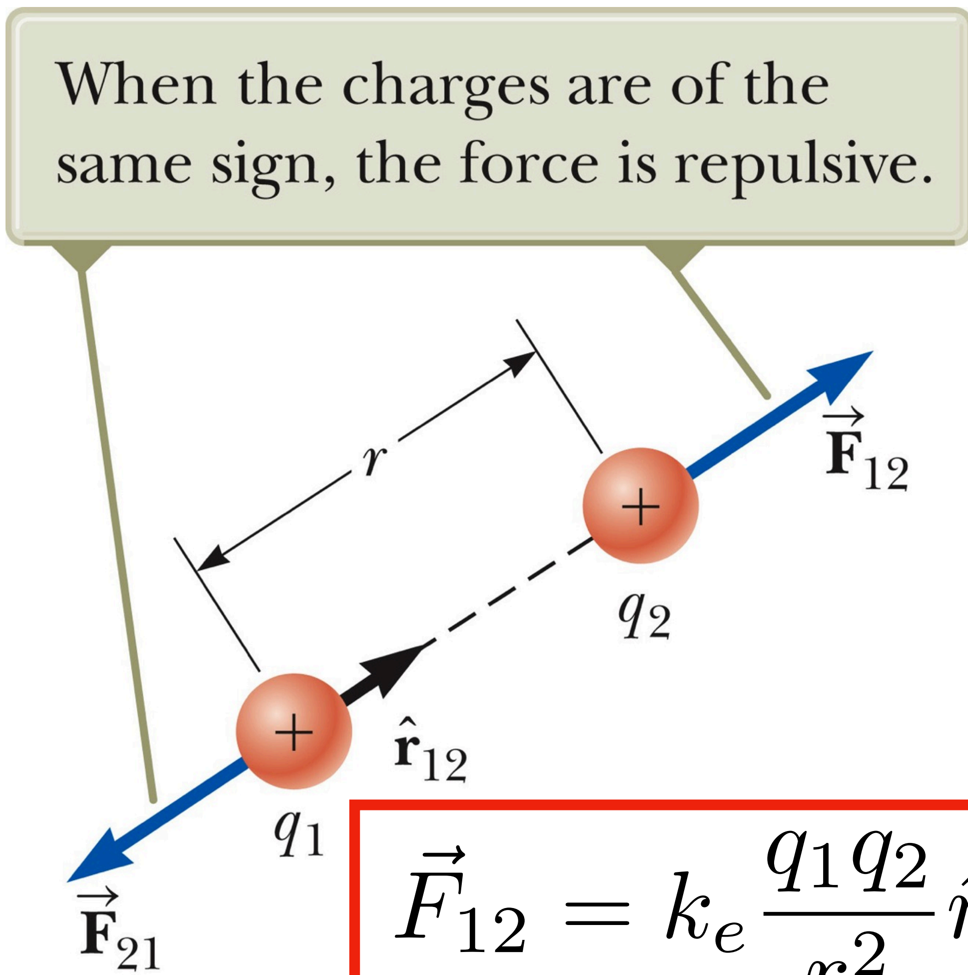




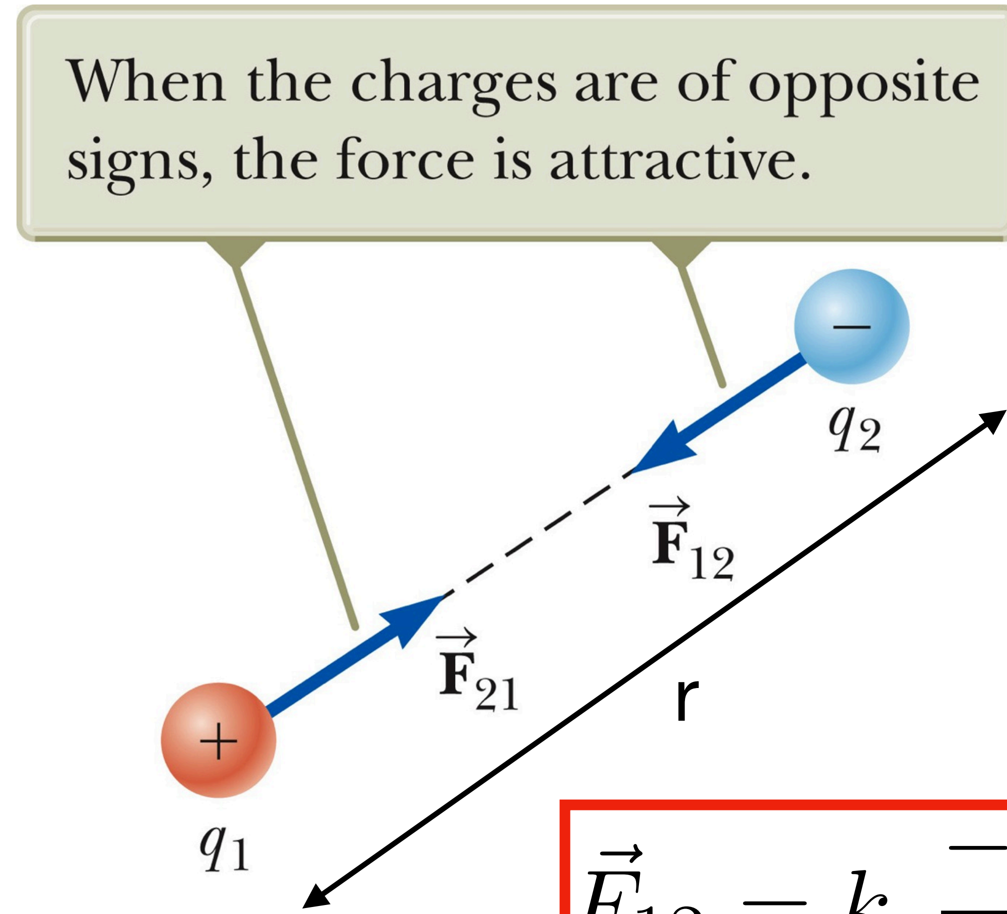
$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$\vec{F}_{12} = k_e \frac{-q_1 q_2}{r^2} \hat{r}_{12}$$

Vector nature of the EM force



$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$



$$\vec{F}_{12} = k_e \frac{-q_1 q_2}{r^2} \hat{r}_{12}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

Newton's 3rd law!

The sign of the product of $q_1 q_2$ gives the ***relative*** direction of the force between q_1 and q_2.

The ***absolute*** direction is determined by the actual location of the charges.

More than 2 charges

The resultant force on any one charge equals the vector sum of the forces exerted by the other individual charges that are present.

- Remember to add the forces *as vectors.*

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$F_3 = F_{13} - F_{23}$$

$$F_3 = k_e \frac{|q_1||q_3|}{r^2} - ke \frac{|q_2||q_3|}{r^2}$$

$$F_3 = k_e\left(\frac{1 \times 1}{0.5^2} - \frac{1 \times 1}{1.5^2}\right)$$

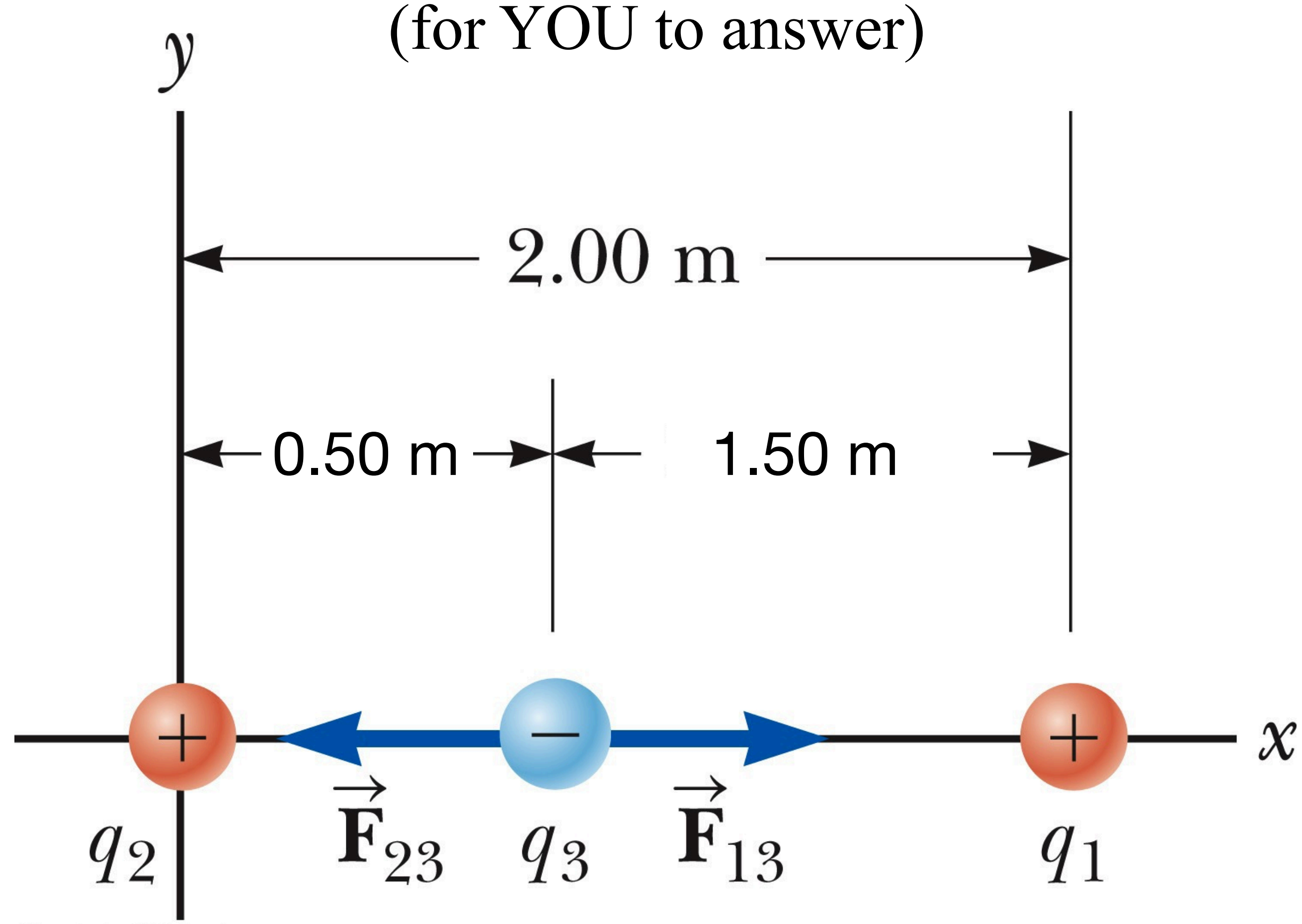
typo: swap 1.5 and 0.5

$$F_3 = -8.9876 \times 10^9 \times 3.56$$

$$F_3 = -3.2 \times 10^{10} N$$

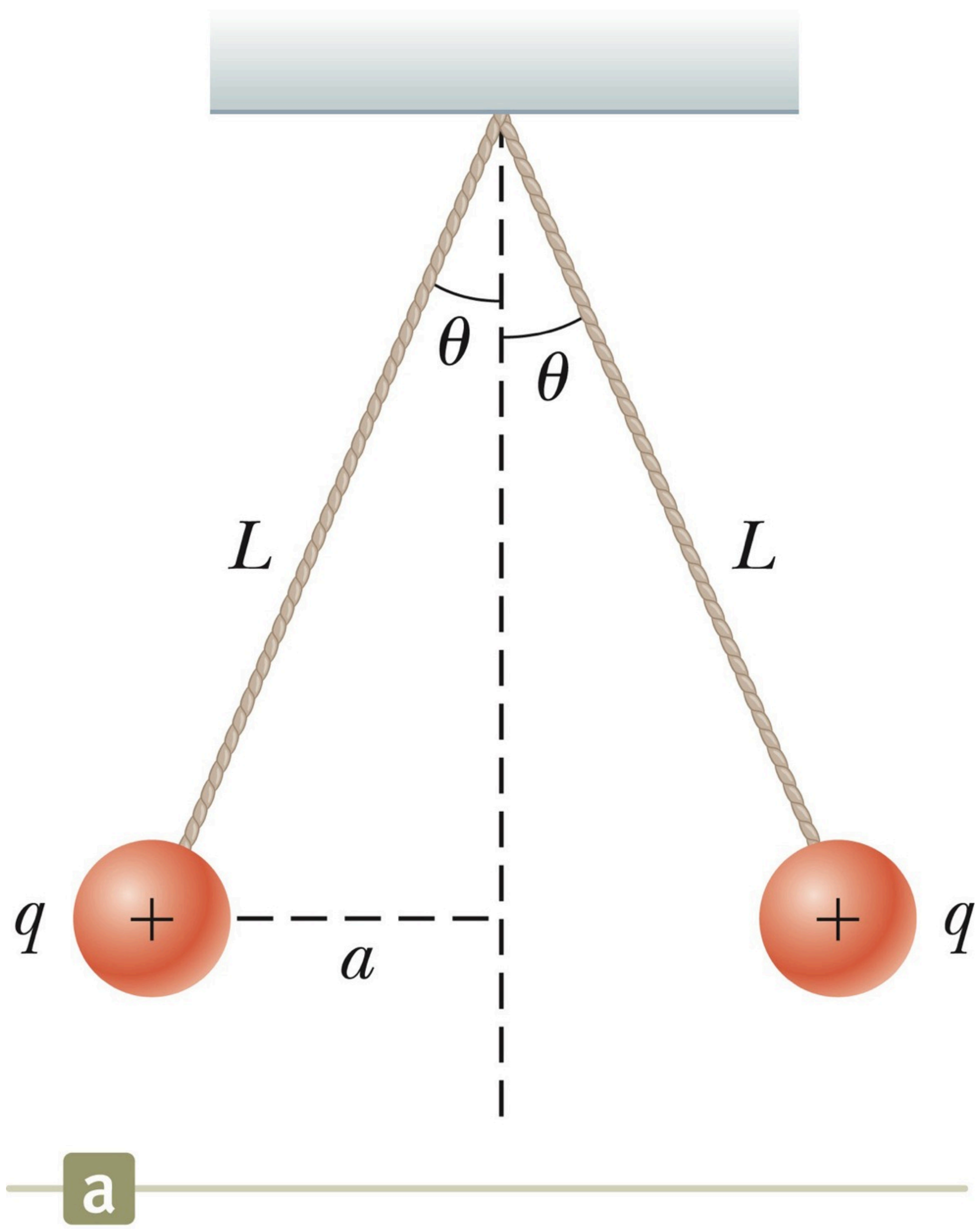
What is the resultant force on q3?

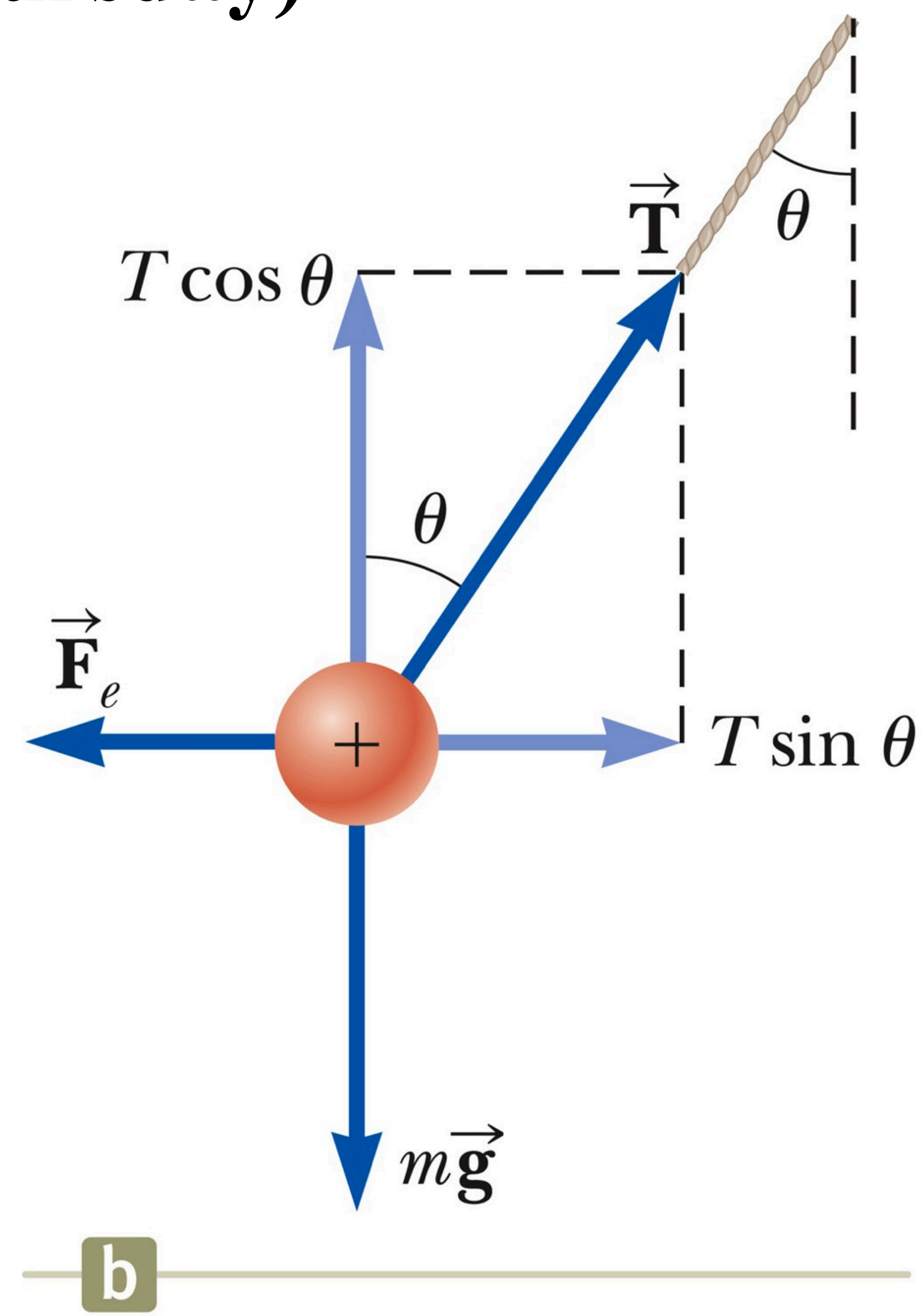
(for YOU to answer)



Electrical force with other forces

(stop here on Thursday)





Example Problem 1

How much power is there in a single lightning bolt?

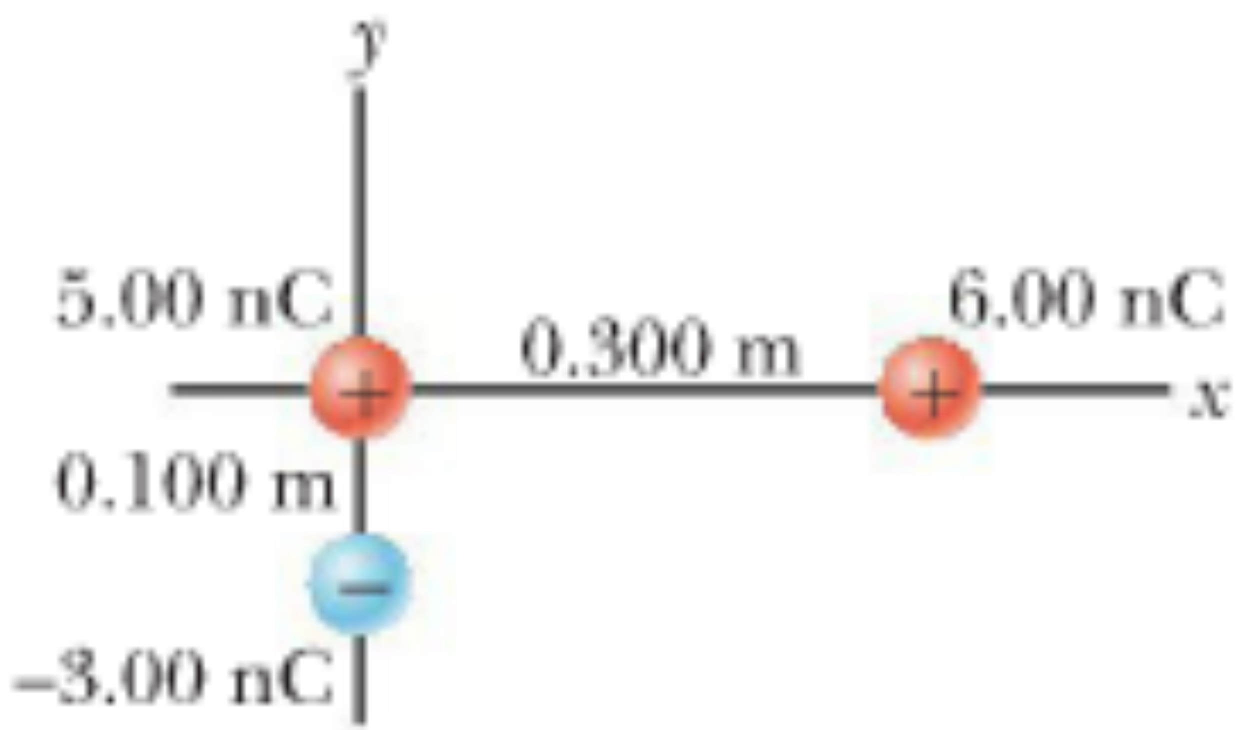
Example Problem 1 - Solution

some "problems" will be factoids

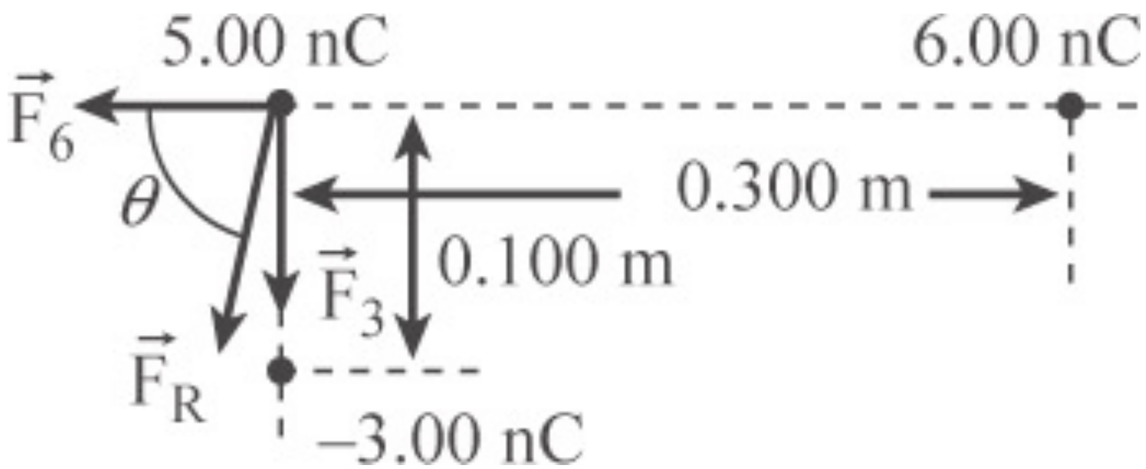
Up to 10 GW

Example Problem 2

11. Three point charges are arranged as shown in Figure M P23.11. Find (a) the magnitude and (b) the direction of the electric force on the particle at the origin.



Example Problem 2 - Solution (1/2)



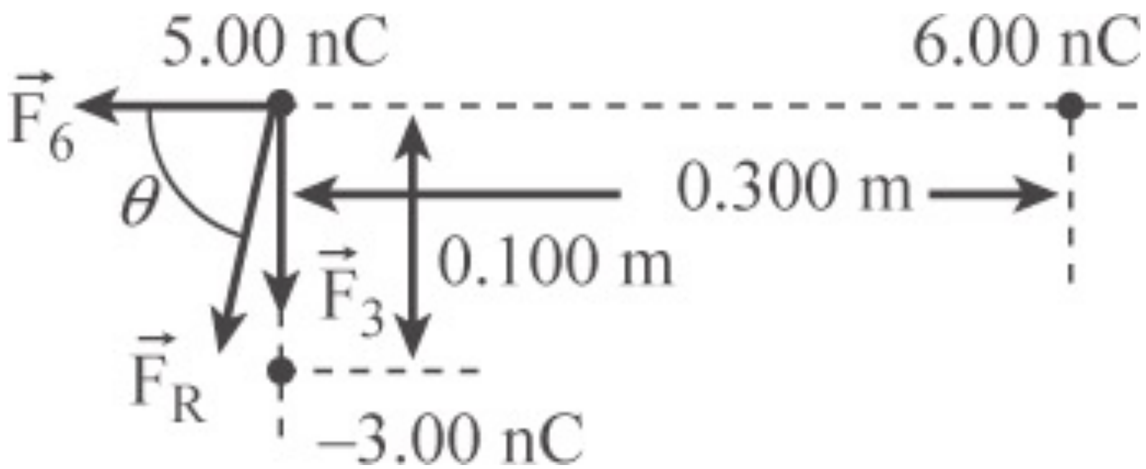
$$F_x = -F_6 = -\left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(6.00\times10^{-9}\ \mathrm{C}\right)\left(5.00\times10^{-9}\ \mathrm{C}\right)}{(0.300\ \mathrm{m})^2}$$

$$= -3.00\times10^{-6}\ \mathrm{N}\quad(\text{to the left})$$

$$F_y = -F_3 = -\left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(3.00\times10^{-9}\ \mathrm{C}\right)\left(5.00\times10^{-9}\ \mathrm{C}\right)}{(0.100\ \mathrm{m})^2}$$

$$= -1.35\times10^{-5}\ \mathrm{N}\quad(\text{downward})$$

Example Problem 2 - Solution (2/2)



(a) The forces are perpendicular, so the magnitude of the resultant is

$$F_R = \sqrt{(F_6)^2 + (F_3)^2} = \boxed{1.38 \times 10^{-5}\ \text{N}}$$

(b) The magnitude of the angle of the resultant is

$$\theta = \tan^{-1}\left(\frac{F_3}{F_6}\right) = 77.5°$$

The resultant force is in the third quadrant, so the direction is

$$\boxed{77.5° \text{ below } -x \text{ axis}}$$

Example Problem 3

16. Two small metallic spheres, each of mass $m = 0.200$ g, are suspended as pendulums by light strings of length L as shown in Figure P23.16. The spheres are given the same electric charge of 7.2 nC, and they come to equilibrium when each string is at an angle of $\theta = 5.00°$ with the vertical. How long are the strings?

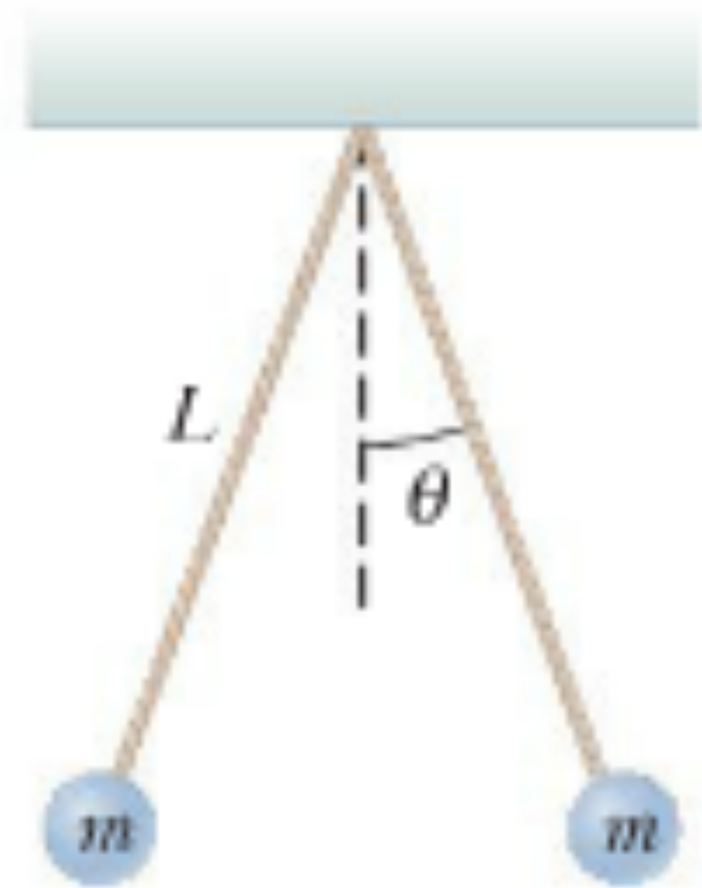
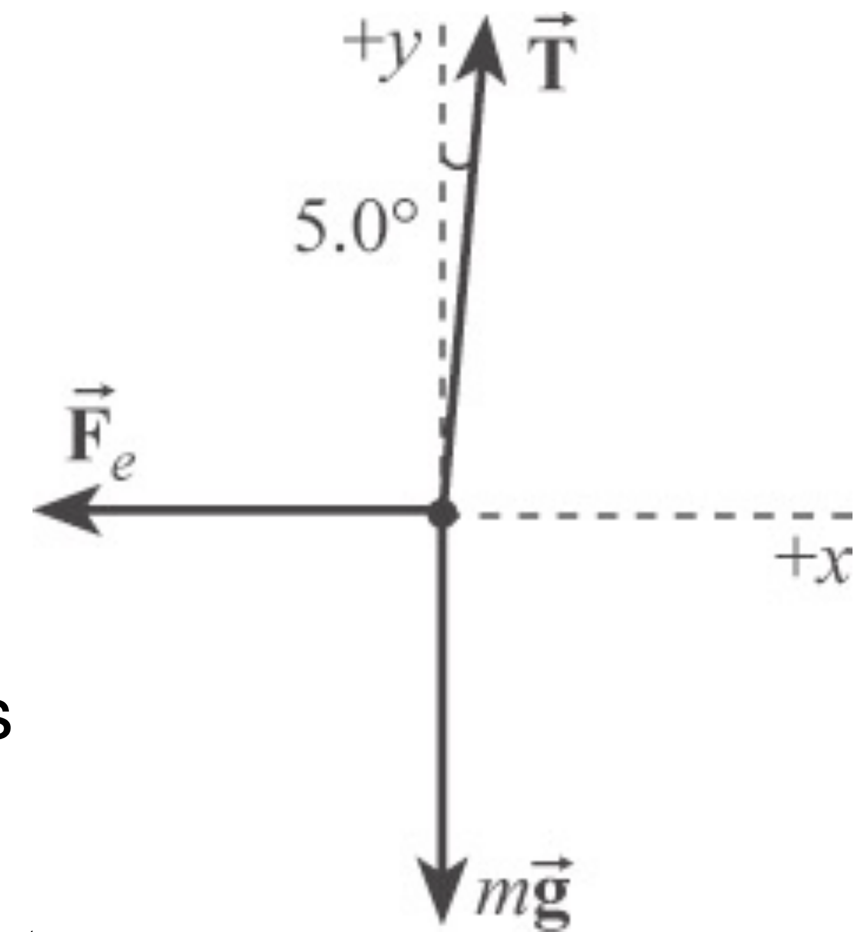


Figure P23.16

Example Problem 3 - Solution

Consider the free-body diagram of one of the spheres shown in ANS. FIG. P23.16. Here, T is the tension in the string and F_e is the repulsive electrical force exerted by the other sphere.



$$\sum F_y = 0 \;\Rightarrow\; T\cos 5.0° = mg$$

or $$T = \frac{mg}{\cos 5.0°}$$

$$\sum F_x = 0 \;\Rightarrow\; F_e = T\sin 5.0° = mg\tan 5.0°$$

At equilibrium, the distance separating the two spheres is $r = 2L\sin 5.0°$.

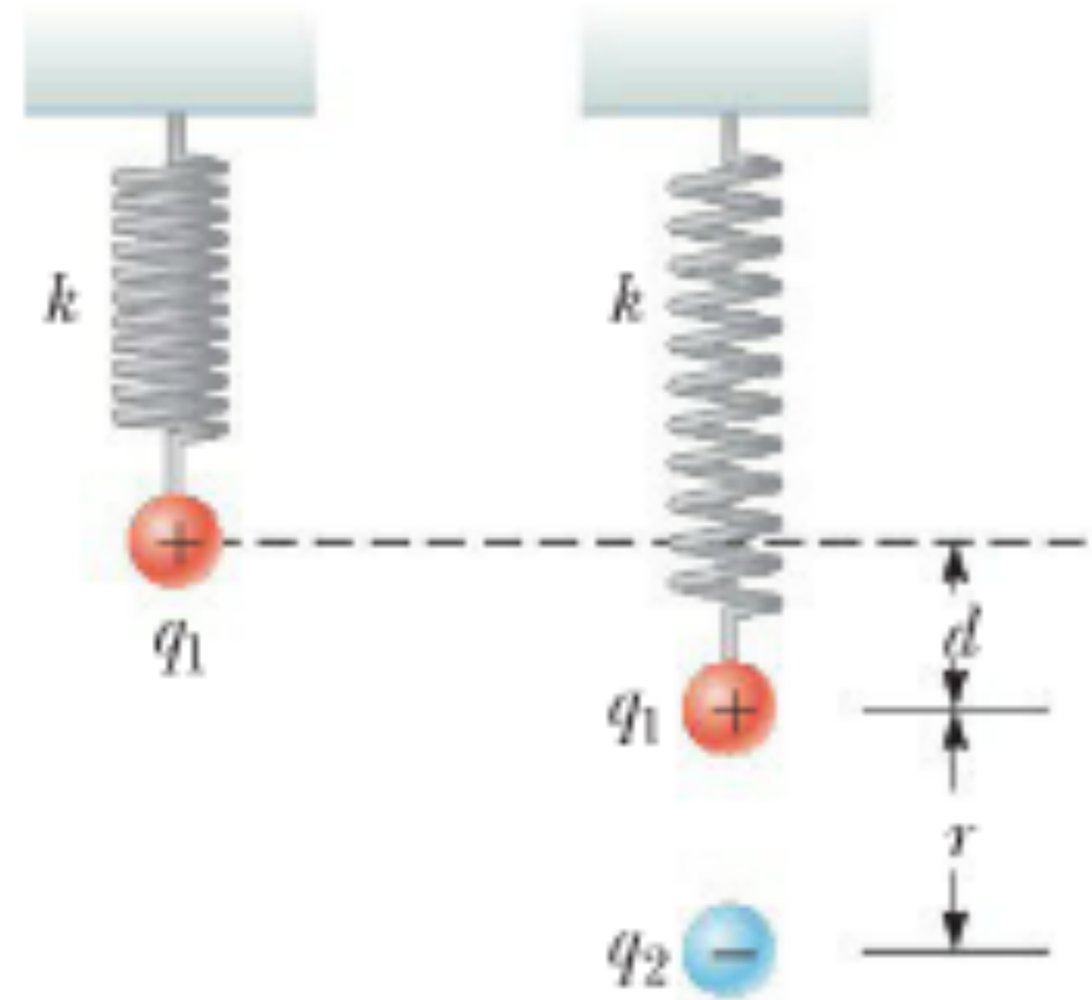
Thus, $F_e = mg\tan 5.0°$ becomes $\dfrac{k_e q^2}{(2L\sin 5.0°)^2} = mg\tan 5.0°$, which yields

$$L = \sqrt{\frac{k_e q^2}{mg\tan 5.0°(2\sin 5.0°)^2}}$$

$$= \sqrt{\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.20\times10^{-9}\ \mathrm{C})^2}{(0.200\times10^{-3}\ \mathrm{kg})(9.80\ \mathrm{m/s^2})\tan 5.0°(2\sin 5.0°)^2}} = \boxed{0.299\ \mathrm{m}}$$

Example Problem 4

62. A small sphere of charge $q_1 = 0.800\ \mu\text{C}$ hangs from the end of a spring as in Figure P23.62a. When another small sphere of charge $q_2 = -0.600\ \mu\text{C}$ is held beneath the first sphere as in Figure P23.62b, the spring stretches by $d = 3.50$ cm from its original length and reaches a new equilibrium position with a separation between the charges of $r = 5.00$ cm. What is the force constant of the spring?



Example Problem 4 - Solution

The downward electric force on the 0.800 μC charge is balanced by the upward spring force:

$$\frac{k_e q_1 q_2}{r^2} = kx$$

solving for the spring constant gives

$$k = \frac{k_e q_1 q_2}{x r^2}$$

$$= \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(0.800 \times 10^{-6} \text{ C}\right)\left(0.600 \times 10^{-6} \text{ C}\right)}{(0.0350 \text{ m})(0.0500 \text{ m})^2}$$

$$= \boxed{49.3 \text{ N/m}}$$

Electric field

Definition: The space around an electrified object - a space in which electric forces act

The electric force is a **field force**.

Field forces can act through space.

- **The effect is produced even with no physical contact between objects.**

Faraday developed the concept of a field in terms of electric fields.

Electric field (cont.)

An **electric field** is the field produced by some charge or charge distribution.

- This charged object is the **source charge.**
- ***If you have a charge you have a field***
- **The presence of the test charge is not necessary for the field to exist.**

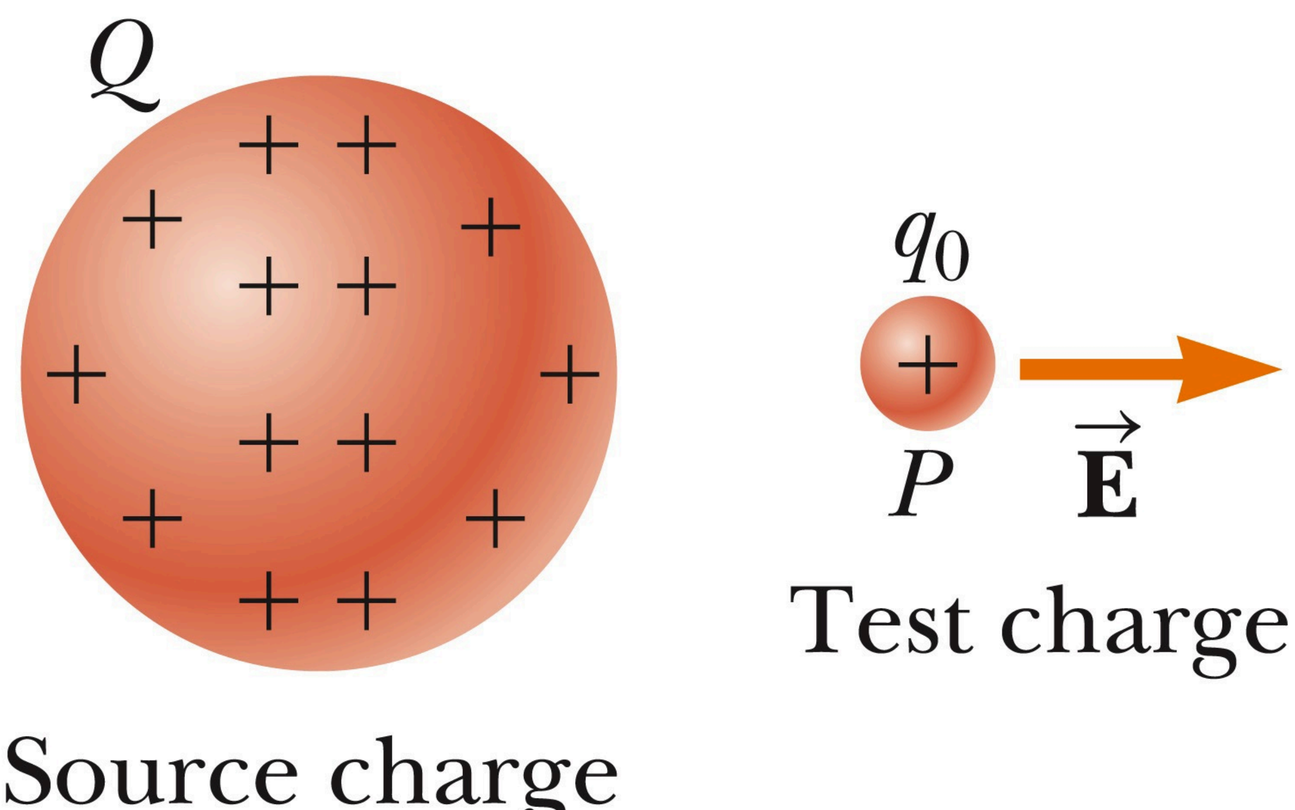
When another charged object, the **test charge**, enters this electric field, an electric force acts on it. The test charge serves as a detector of the field.

While the field can exist anywhere, if you don't have a test charge you can't detect it.

Force and Electric Field

- A source charge creates a field
- A test charge detects the field : the test charge FEELS the effect of the field.
- How do particles feel anything? Through force.
- **The test charge FEELS the effect of the field through the electric force.**
- There is therefore a clear connection between electric force and electric field.

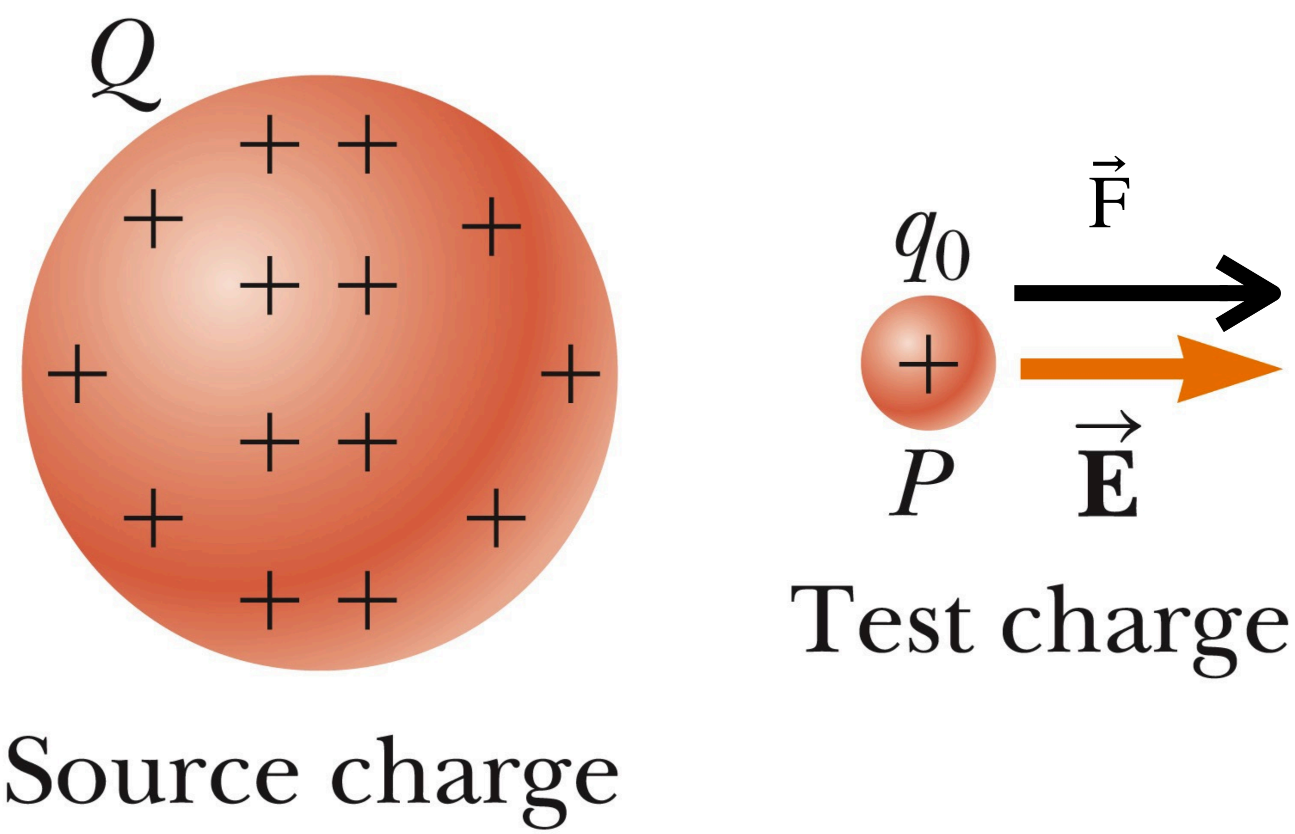
$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$



We can also say that an electric field exists at a point if a test charge at that point experiences an electric force.

Force and Electric Field

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$



$\vec{\mathbf{E}}$ is the electric **field** vector

$\vec{\mathrm{F}}$ is the electric **force**

q_0 is a positive test charge

SI units of $\vec{\mathbf{E}}$: N/C (identical to V/m — Volts come later)

The electric field E is defined as the electric force on a positive test charge per unit charge.

Force and Electric Field Vector

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

$\vec{\mathbf{E}}$ is the electric **field** vector

$\vec{\mathrm{F}}$ is the electric **force**

q_0 is a positive test charge

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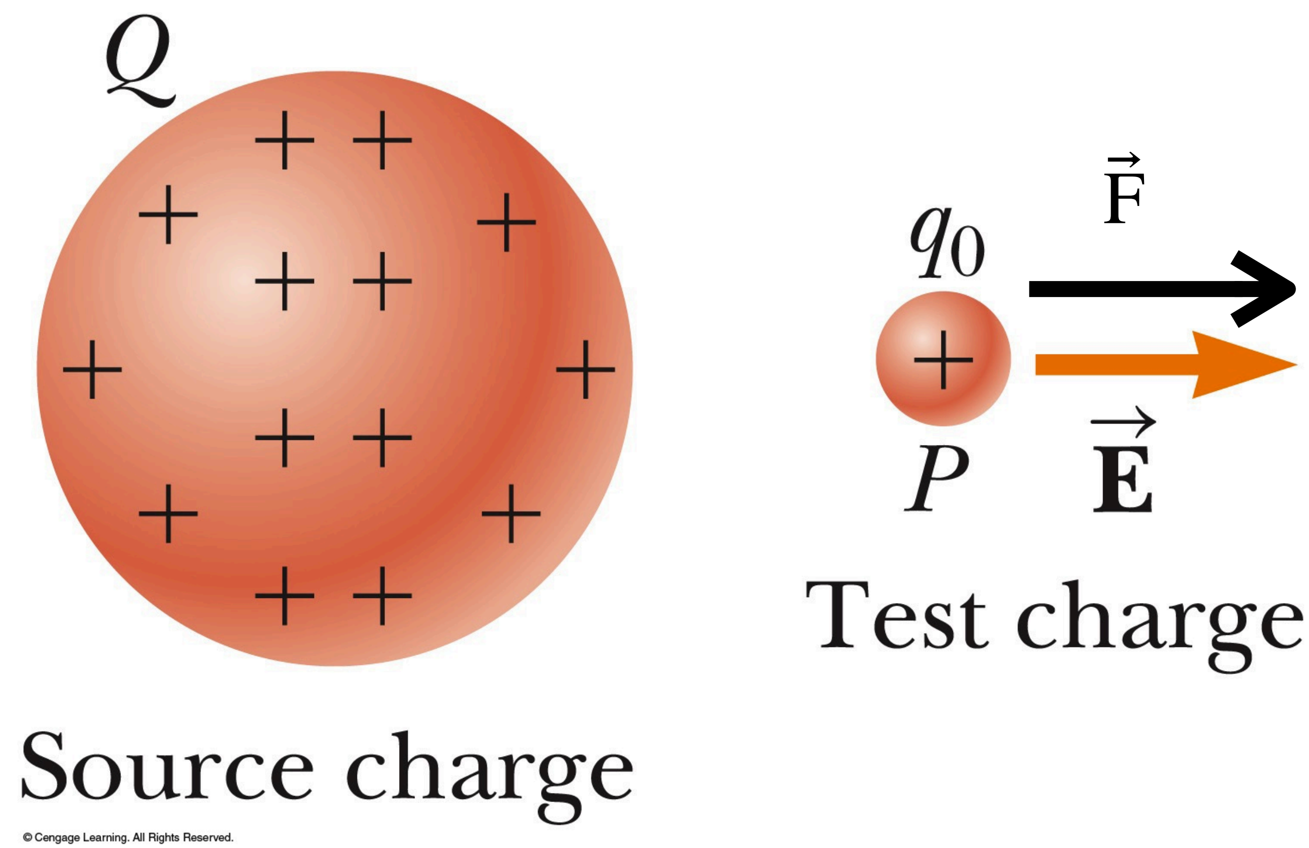


The electric field vector, $\vec{\mathrm{E}}$, at a point in space is defined as the electric force acting on a positive test charge, *q_o*, placed at that point, divided by the test charge.

The direction of $\vec{\mathrm{E}}$ is that of the force on a positive test charge.

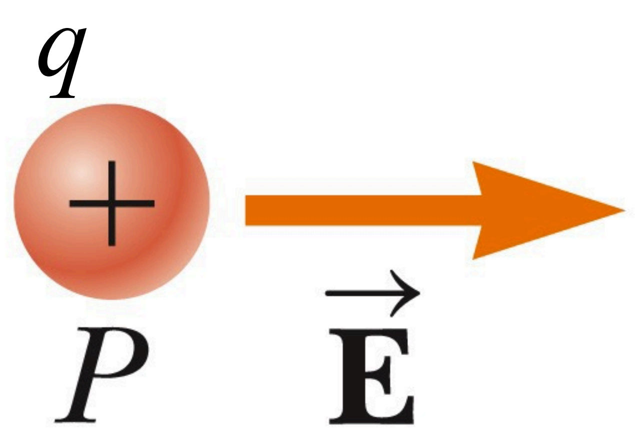
q_0 is the charge upon which the field acts.

Force and Electric Field

Corollary: if you are given the electric field at a point, how can you find the electric force?

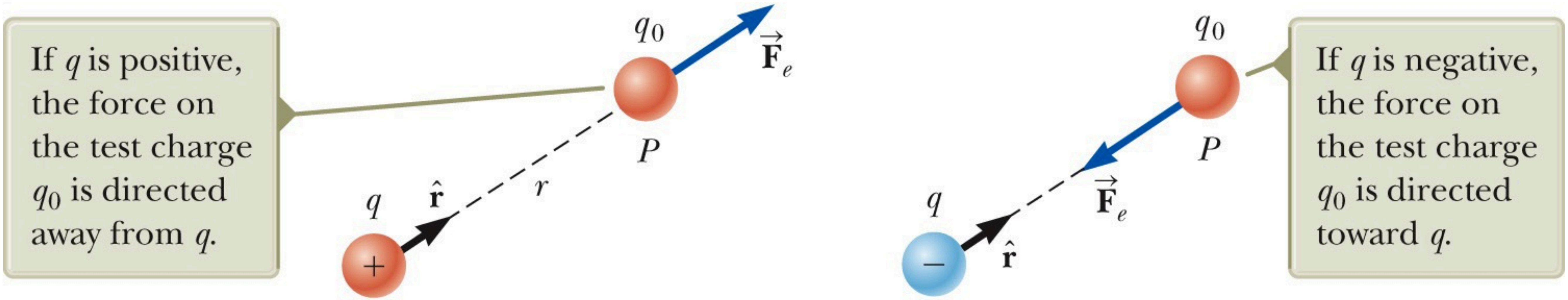
The electric force acting on a charge q placed in a field E is:

$$\boxed{\vec{F} = q\vec{E}}$$

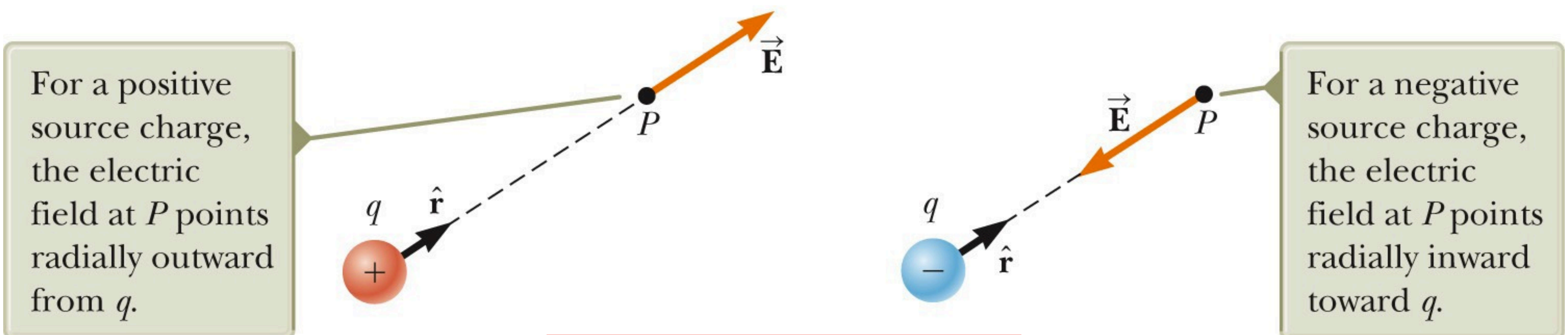


- This is valid for a **point charge only** = zero size
- For larger objects, the field may vary over the size of the object.
- **If *q* is positive, the force and the field are in the same direction.**
- **If *q* is negative, the force and the field are in opposite directions.**

Electric field, vector form



$$\boxed{\vec{F}_e = k_e \frac{q q_0}{r^2} \hat{r}}$$



$$\boxed{\vec{E} = \frac{\vec{F}_e}{q_0} = k_e \frac{q}{r^2} \hat{r}}$$

Example Problem 5

25. Four charged particles are at the corners of a square of side a as shown in Figure P23.25. Determine (a) the electric field at the location of charge q and (b) the total electric force exerted on q.

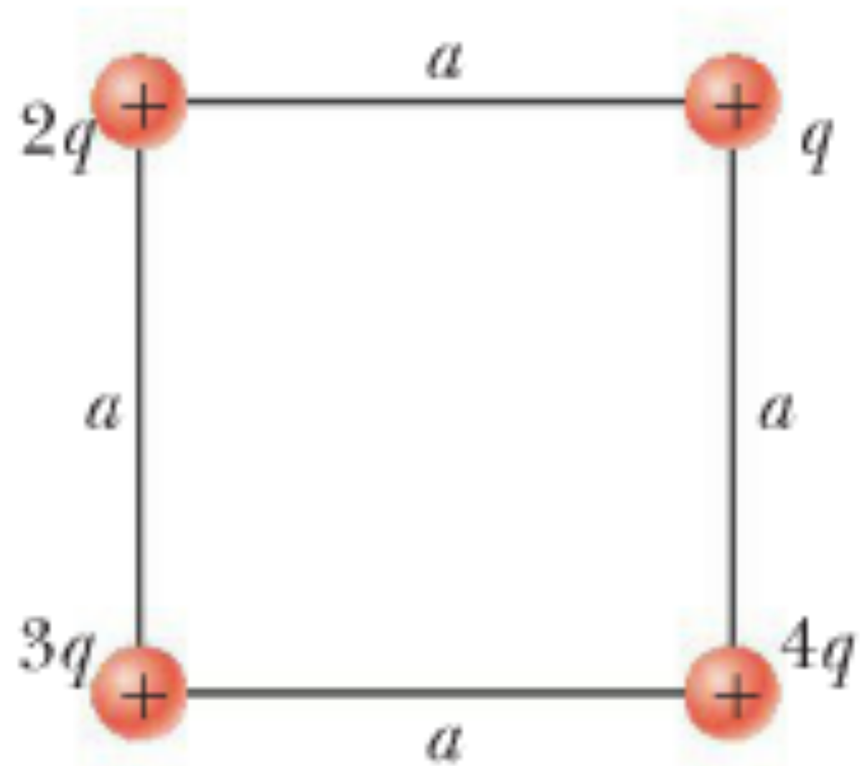


Figure P23.25

Example Problem 5 - Solution

The field at charge q is given by

$$\vec{\mathbf{E}} = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3$$

(a) Substituting for each of the charges gives

$$\vec{\mathbf{E}} = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$$

$$= \frac{k_e q}{a^2}\left[\left(2 + \frac{3}{2}\cos 45.0°\right)\hat{\mathbf{i}} + \left(\frac{3}{2}\sin 45.0° + 4\right)\hat{\mathbf{j}}\right]$$

$$= \boxed{\frac{k_e q}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

(b) The electric force on charge q is given by

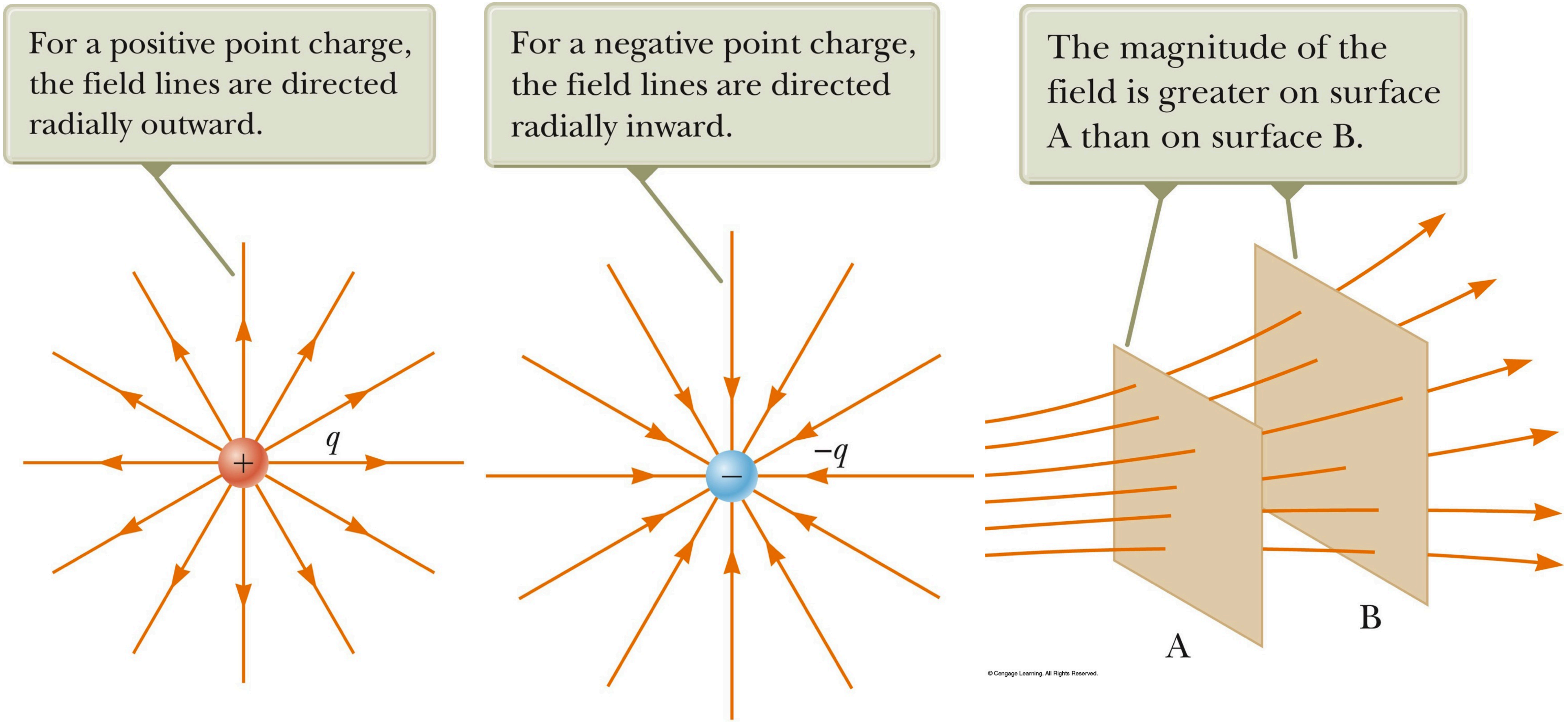
$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \boxed{\frac{k_e q^2}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

Electric field lines

Field lines give us a means of **representing the electric field pictorially**.

The electric field vector is tangent to the electric field line at each point.

The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

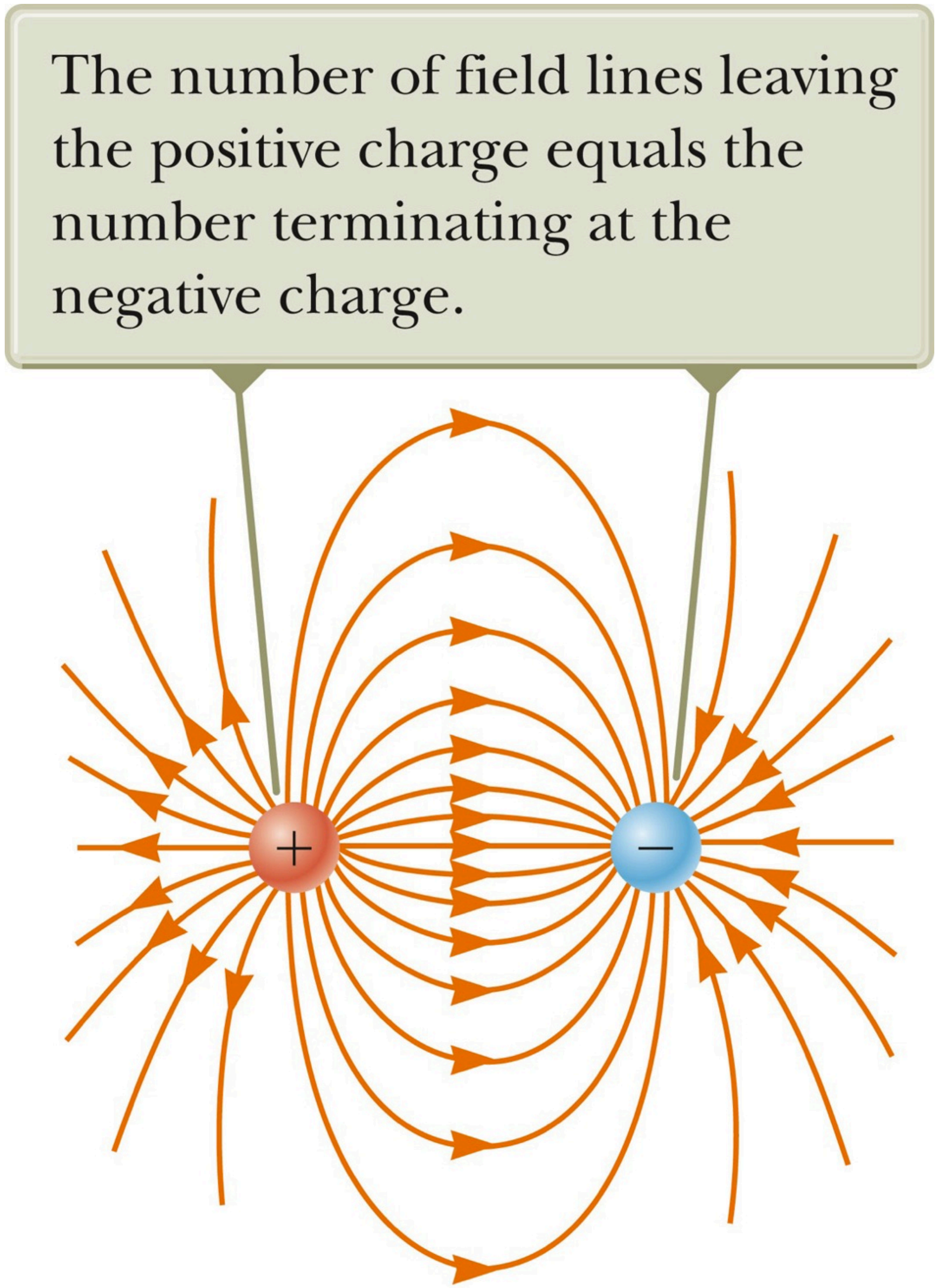


You can find field lines by imagining where a positive charge would go if you dropped it in the field.

Field Lines - Rules

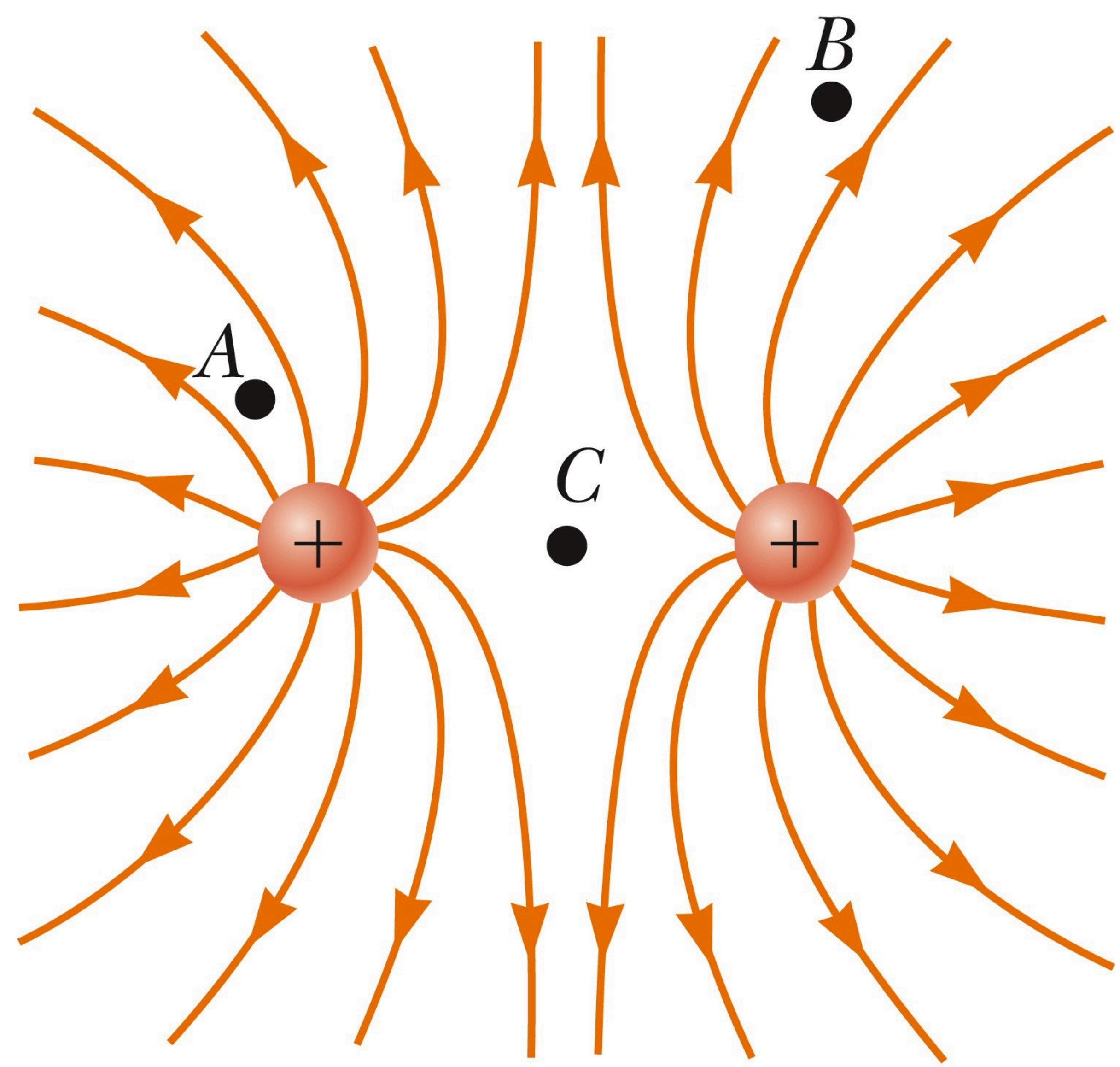
- **The lines must begin on a positive charge**
  - Field lines stretch infinitely away unless there is a negative charge in the vicinity, where the lines will terminate
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.
- Remember field lines are **not** material objects, they are a **pictorial representation** used to qualitatively describe the electric field.
- The field exists between the lines as well.
- Electric field lines are always perpendicular to the surface of a conductor in equilibrium
- The electric field vector is always tangent to the lines at any point

More field lines



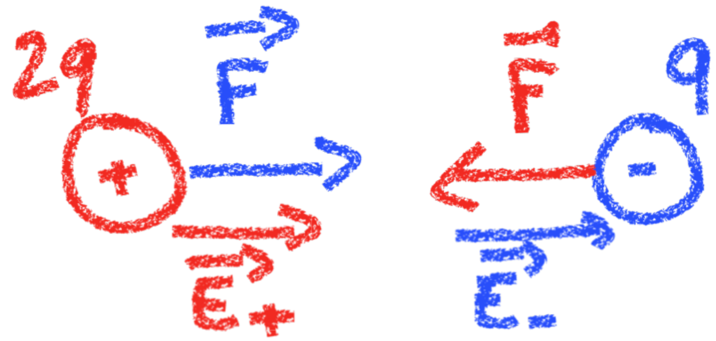
dipole

The same number of lines leave each charge since they are equal in magnitude.

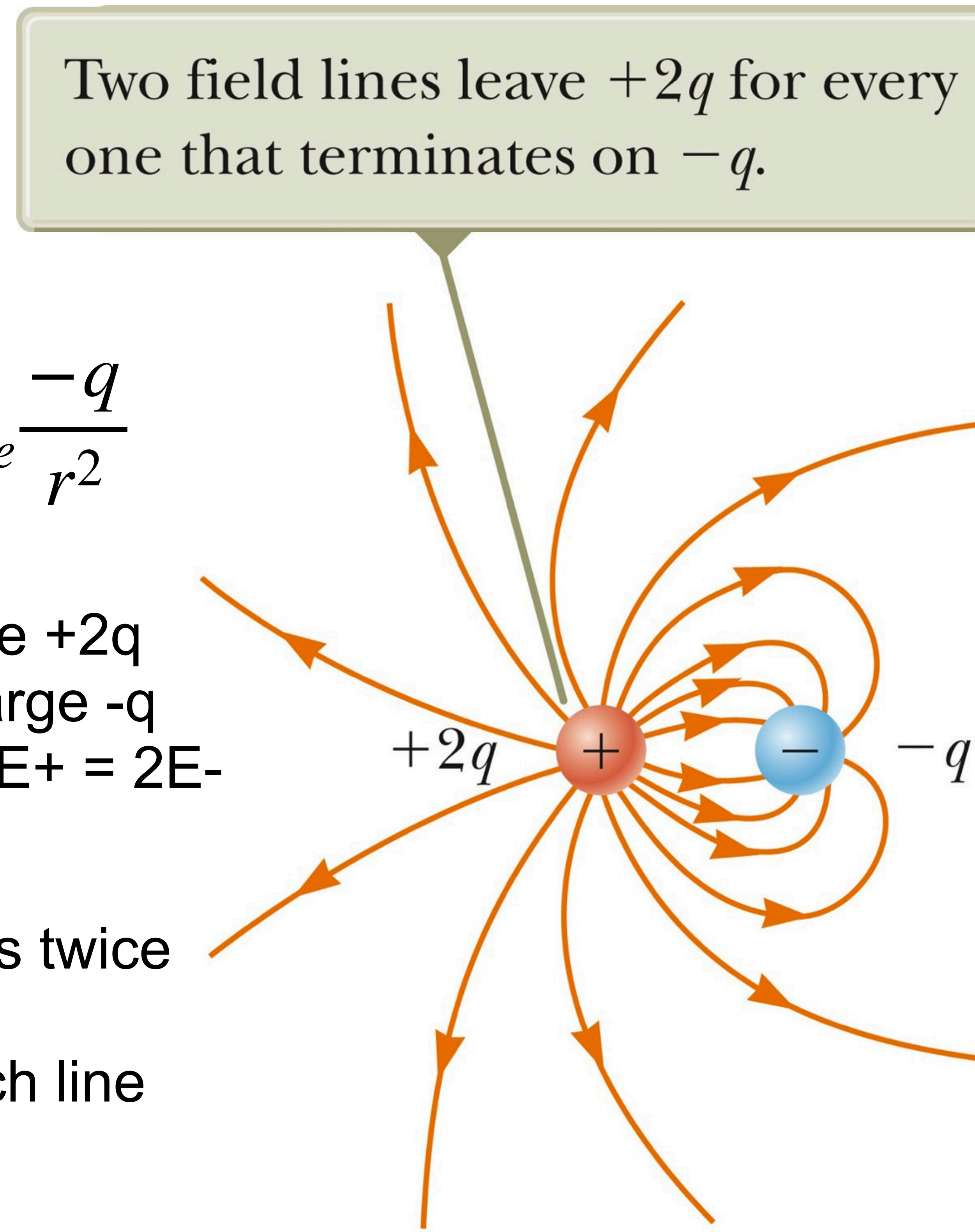


Since there are no negative charges available, the field lines end **infinitely** far away.

Field Lines - Unequal Charges



$$F = k_e \frac{q2q}{r^2}$$



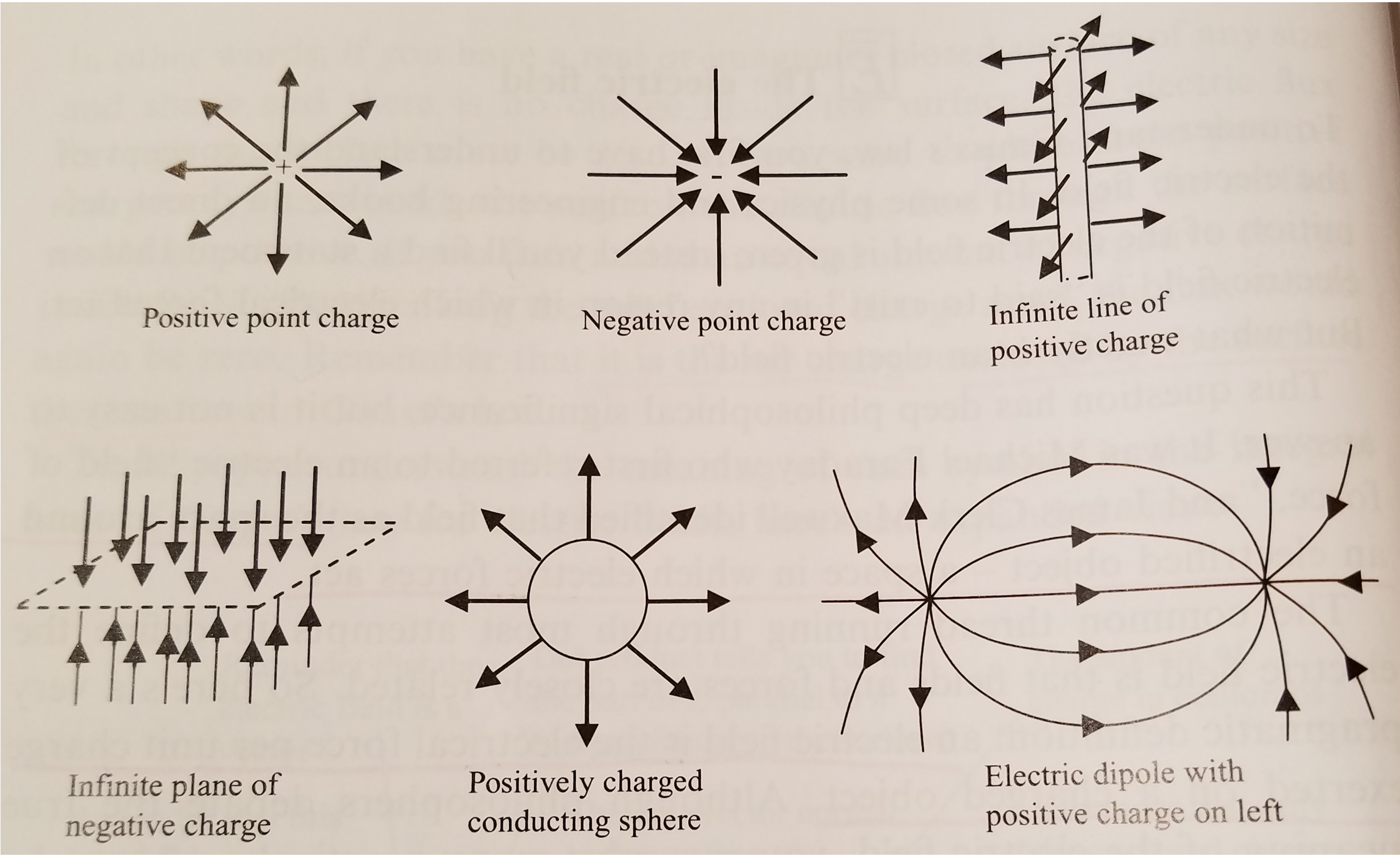
$$E_+ = \frac{F}{-q} = k_e \frac{2q}{r^2} \qquad E_- = \frac{F}{2q} = k_e \frac{-q}{r^2}$$

E- = field due to charge -q acting on charge +2q
E+ = field due to charge +2q acting on charge -q
More source charge means more field: E+ = 2E-

The field on -q is twice the field on 2q
The field that leaves the positive charges is twice as strong
Two lines leave the positive charge for each line that terminates on the negative charge.

At a great distance, the field would be approximately the same as that due to a single charge of +*q.*

Electric Field Lines - Examples



From: a student's guide to Maxwell's equations

Example Problem 6

Field lines are created by charges, but I said they are not real.
What is actually happening then? What “communicates” the electric force over distance?

Example Problem 6 - Solution

(virtual) photons i.e. photon particles, which are themselves neutral

Motion of Charged Particles

When a charged particle is placed in an electric field, it experiences an electrical force.

If this is the only force on the particle, it must be the net force.

The net force will cause the particle to accelerate according to Newton's second law.

$$\boxed{\vec{F} = q\vec{E} = m\vec{a}}$$

- If the field is uniform, then the acceleration is constant.
- The particle under constant acceleration model can be applied to the motion of the particle.
  - **The electric force causes a particle to move according to the models of forces and motion.**
- If the particle has a positive charge, its acceleration is in the direction of the field.
- If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Example: electron in uniform field

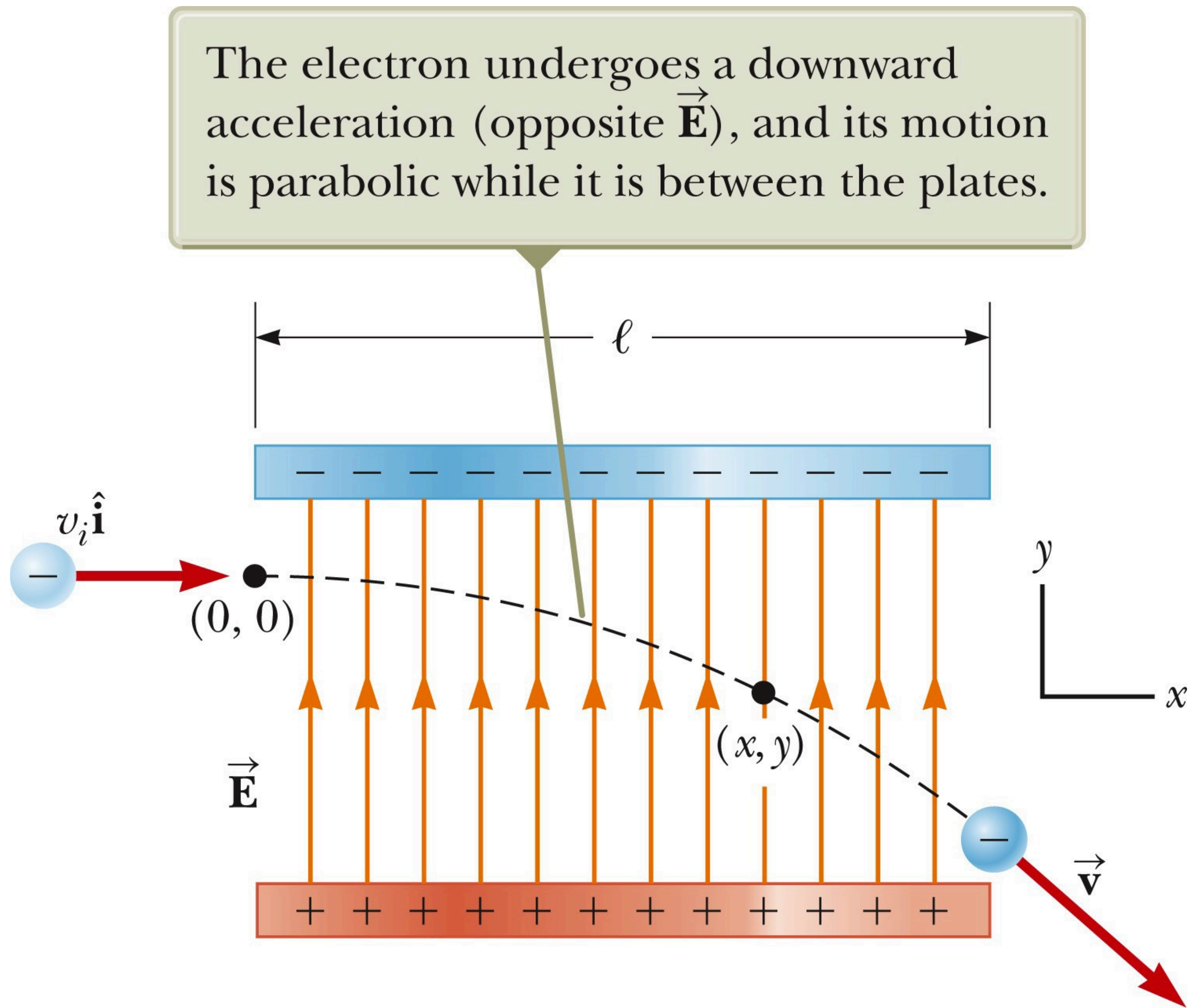
The electron is projected horizontally into a uniform electric field.

The electron undergoes a downward acceleration.

- **an electron has a negative charge, so its acceleration is opposite the direction of the field.**

$$\vec{F} = q\vec{E} = m\vec{a}$$

Its motion is parabolic while between the plates.



You can apply equations of motion like in physics 1!

Electric fields from multiple charges

At any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$\boxed{\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i}$$

E field - Continuous Charge Distribution

The distances between charges in a group of charges may be much smaller than the distance between the group and a point of interest.

→ the system of charges can be modeled as continuous.

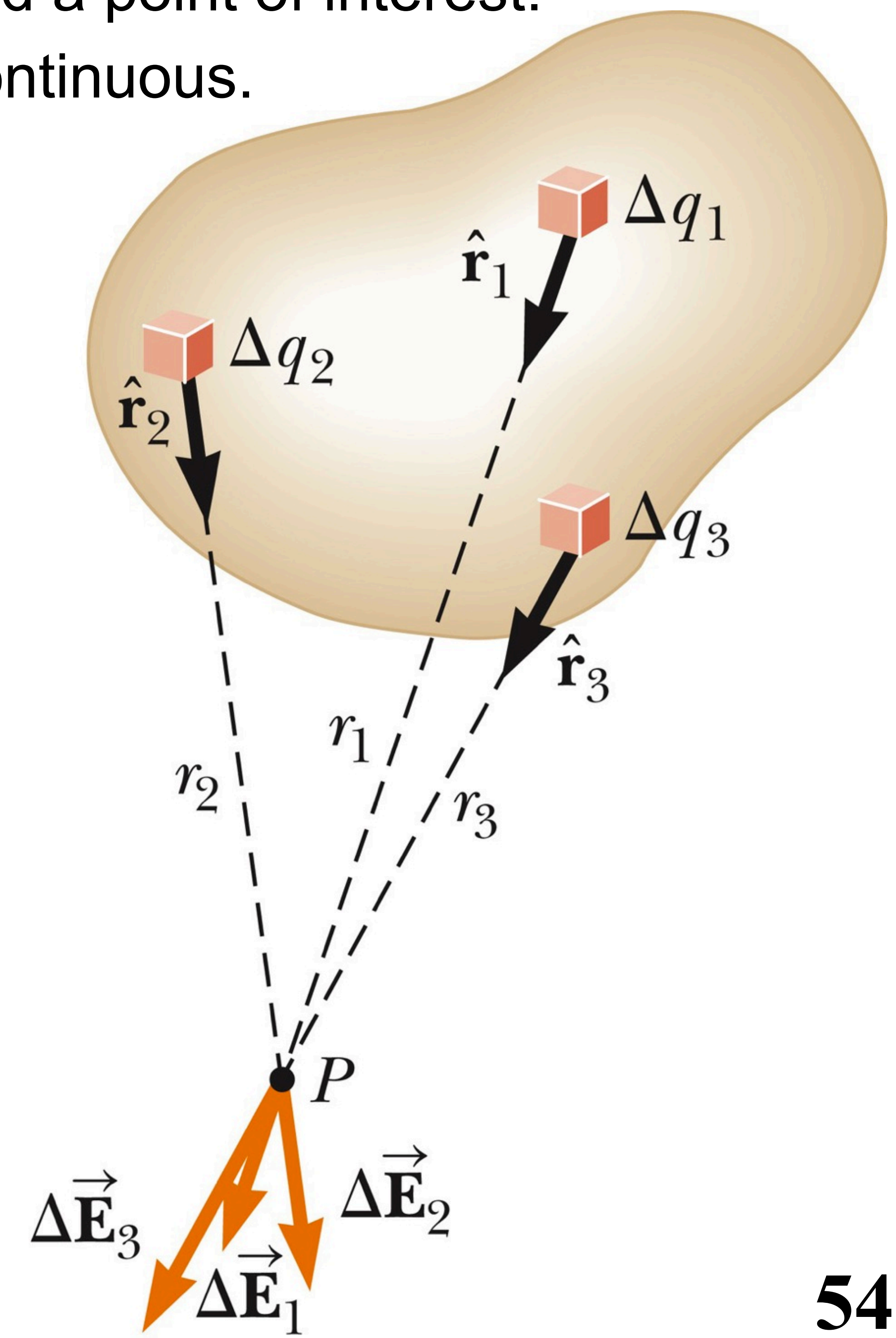
The system of closely spaced charges is equivalent to a total charge that is continuously distributed along some line, over some surface, or throughout some volume.

For the individual charge elements:

$$\boxed{\Delta\vec{E} = k_e \frac{\Delta q}{r^2}\hat{r}}$$

Because the charge distribution is continuous, the total electric field is:

$$\boxed{\vec{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}}$$



Charge densities

= how charge is uniformly distributed over a volume, surface or line

Q is the total charge

Volume charge density: when a charge is distributed evenly throughout a volume V

- $\boxed{\rho \equiv Q / V}$ with units C/m^3

Surface charge density: when a charge is distributed evenly over a surface area A

- $\boxed{\sigma \equiv Q / A}$ with units C/m^2

Linear charge density: when a charge is distributed along a line of length ℓ

- $\boxed{\lambda \equiv Q / \ell}$ with units C/m

Amount of charge in a small volume

If the charge is **nonuniformly distributed** over a volume, surface, or line, the **amount of charge, *dq***, is given by:

- For the volume: $dq = \rho \, dV$
- For the surface: $dq = \sigma \, dA$
- For the length element: $dq = \lambda \, d\ell$

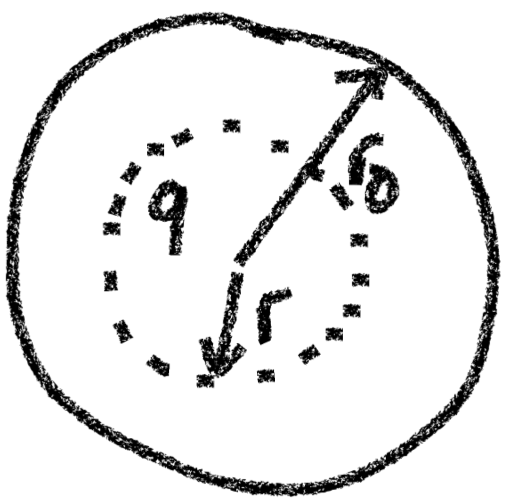
Electric field of a sphere

On the **outside** of a conducting or insulating sphere

$\times \vec{E} = ?$

$$E = k_e \frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

On the **inside** of an insulating sphere



$$Q = \rho V = \rho \frac{4}{3}\pi r_0^3$$

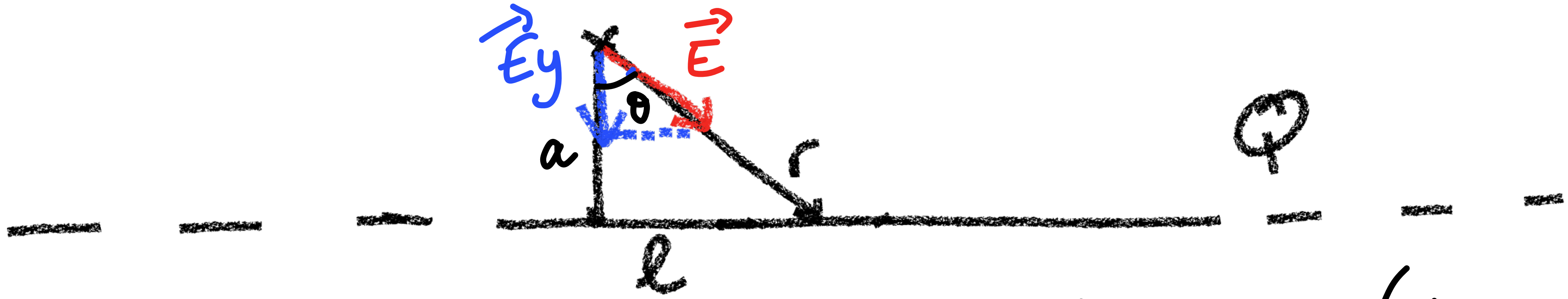
$$q = \rho V = \rho \frac{4}{3}\pi r^3 = \frac{Q}{r_0^3} r^3$$

$$E = k_e \frac{q}{r^2} = k_e \frac{Q}{r_0^3}\frac{r^3}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qr}{r_0^3}$$

The sphere has total charge Q.
q is the charge in the inside volume.

Electric field due to infinite line charge



$$dE = k_e \frac{dq}{\ell^2 + a^2} \qquad E_x = 0 \qquad E_y = \int dE_y = \int dE \cos\theta$$

$$\cos\theta = \frac{a}{\sqrt{\ell^2 + a^2}}$$

$$E_y = \int_{-\infty}^{+\infty} k_e \underbrace{\frac{dq}{\ell^2 + a^2}}_{dE} \underbrace{\frac{a}{\sqrt{\ell^2 + a^2}}}_{\cos\theta}$$

$$E_y = \int_{-\infty}^{+\infty} k_e a \frac{\lambda d\ell}{(\ell^2 + a^2)^{3/2}} = k_e a \lambda \int_{-\infty}^{+\infty} \frac{d\ell}{(\ell^2 + a^2)^{3/2}}$$

$$= k_e a \lambda \left[\frac{\ell}{d^2 \sqrt{\ell^2 + a^2}} \right]_{-\infty}^{+\infty}$$

$$= k_e \cancel{a} \lambda \frac{2}{a^{\cancel{2}}} = \frac{2 k_e \lambda}{a}$$

We will go back to this next class.
There are many ways to solve an E&M problem.
They are not all equal (this is the hard way).

Electric field equations for simple objects

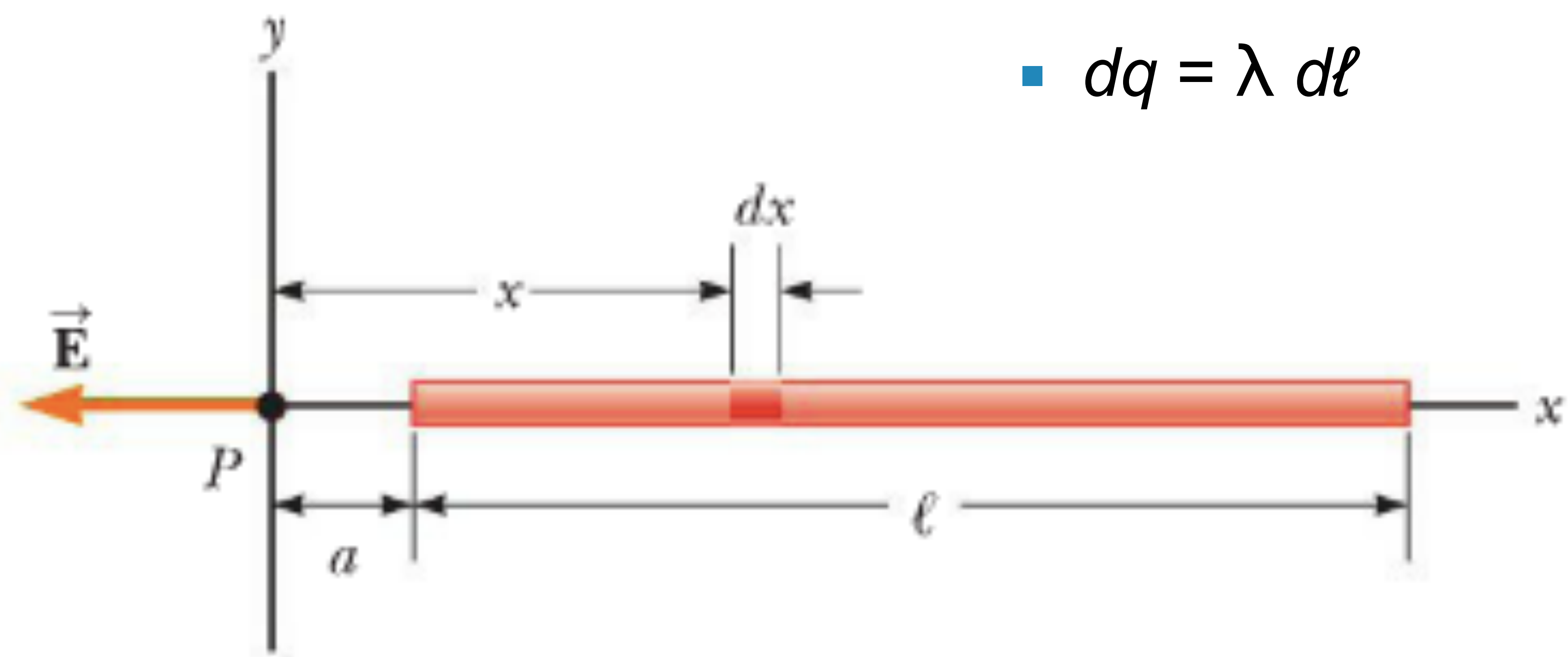
Point charge (charge $= q$)	$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ (at distance r from q)
Conducting sphere (charge $= Q$)	$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$ (outside, distance r from center) $\vec{E} = 0$ (inside)
Uniformly charged insulating sphere (charge $= Q$, radius $= r_0$)	$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$ (outside, distance r from center) $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{r_0^3}\hat{r}$ (inside, distance r from center)
Infinite line charge (linear charge density $= \lambda$)	$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r}$ (distance r from line)
Infinite flat plane (surface charge density $= \sigma$)	$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

From: a student's guide to Maxwell's equations

Electric field due to charge rod of length ℓ

- $dq = \lambda \, d\ell$

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda \, dx}{x^2}$$



$$E = \int_a^{\ell + a} k_e \lambda \frac{dx}{x^2}$$

$$E = k_e \lambda \int_a^{\ell + a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell + a}$$

$$(1) \quad E = k_e \frac{Q}{\ell} \left(\frac{1}{a} - \frac{1}{\ell + a} \right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

E field of a uniform ring of charge

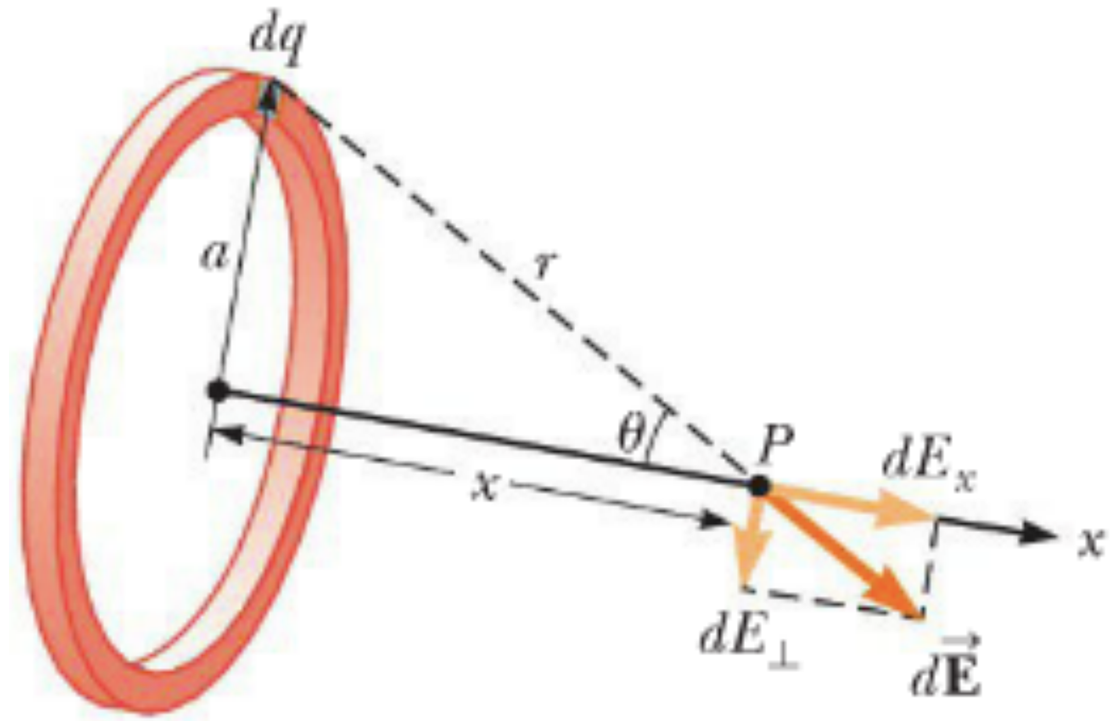
(1) $dE_x = k_e \frac{dq}{r^2} \cos\theta = k_e \frac{dq}{a^2 + x^2} \cos\theta$

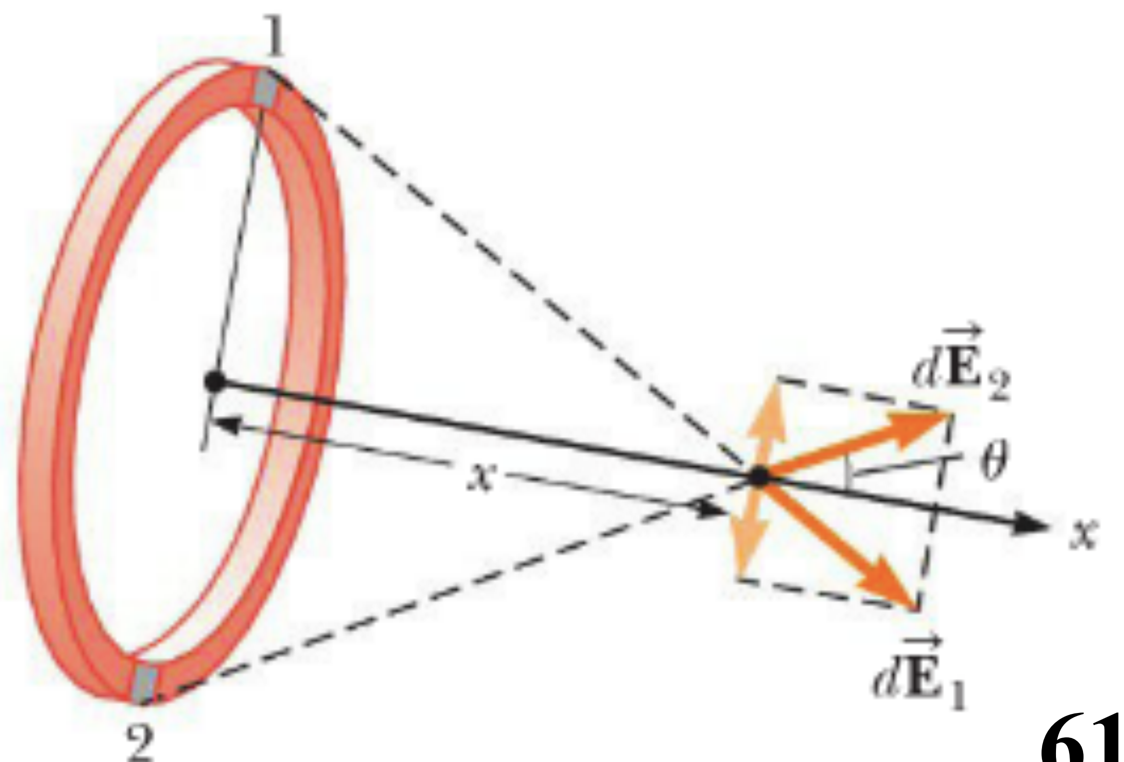
(2) $\cos\theta = \frac{x}{r} = \frac{x}{(a^2 + x^2)^{1/2}}$

$$dE_x = k_e \frac{dq}{a^2 + x^2} \left[\frac{x}{(a^2 + x^2)^{1/2}} \right] = \frac{k_e x}{(a^2 + x^2)^{3/2}} \, dq$$

$$E_x = \int \frac{k_e x}{(a^2 + x^2)^{3/2}} \, dq = \frac{k_e x}{(a^2 + x^2)^{3/2}} \int dq$$

(3) $E = \boxed{\frac{k_e x}{(a^2 + x^2)^{3/2}} \, Q}$





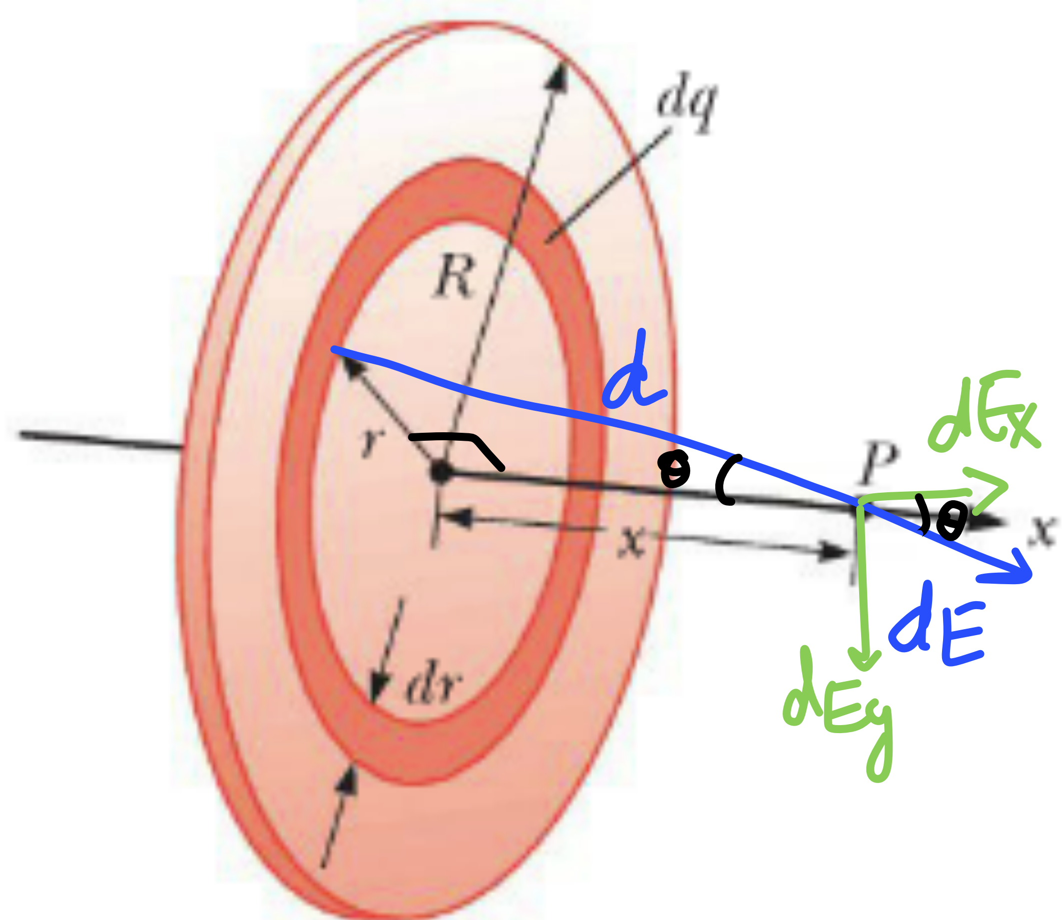
E field of uniformly charged disk

$$dE = k_e \frac{dq}{d^2} \qquad dE_x = dE \cos\theta$$

$$dE_x = k_e \frac{dq}{d^2}\cos\theta = k_e \frac{dq}{x^2 + r^2}\cos\theta$$

$$dq = \sigma\, dA = \sigma(2\pi r\, dr) = 2\pi\sigma r\, dr$$

$$dE_x = \frac{k_e x}{(r^2 + x^2)^{3/2}}(2\pi\sigma r\, dr)$$



$$\cos\theta = \frac{x}{d} = \frac{x}{r^2 + x^2}$$

$$\cos\theta = \frac{dE_x}{dE}$$

$$E_x = k_e x\pi\sigma \int_0^R \frac{2r\, dr}{(r^2 + x^2)^{3/2}}$$

$$= k_e x\pi\sigma \int_0^R (r^2 + x^2)^{-3/2} d(r^2)$$

$$= k_e x\pi\sigma \left[\frac{(r^2 + x^2)^{-1/2}}{-1/2}\right]_0^R = \boxed{2\pi k_e\sigma\left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]}$$

Example Problem 7

Why do we use continuous integration instead of discrete summation, when we know that individual protons and electrons exist?

Example Problem 7 - Solution

The distances between them are much smaller than the sizes of most typical objects

Example Problem 8

57. M A proton moves at 4.50×10^5 m/s in the horizontal direction. It enters a uniform vertical electric field with a magnitude of 9.60×10^3 N/C. Ignoring any gravitational effects, find (a) the time interval required for the proton to travel 5.00 cm horizontally, (b) its vertical displacement during the time interval in which it travels 5.00 cm horizontally, and (c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

Example Problem 8 - Solution

$\vec{\mathbf{E}}$ is directed along the y direction; therefore, $a_x = 0$ and $x = v_{xi}t$.

(a) $$t = \frac{x}{v_{xi}} = \frac{0.050\,0\text{ m}}{4.50\times10^{5}\text{ s}} = 1.11\times10^{-7}\text{ s} = \boxed{111\text{ ns}}$$

(b) $$a_y = \frac{qE}{m} = \frac{\left(1.602\times10^{-19}\text{ C}\right)\left(9.60\times10^{3}\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = 9.21\times10^{11}\text{ m/s}^2$$

$$y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2:$$

$$y_f = \frac{1}{2}\left(9.21\times10^{11}\text{ m/s}^2\right)\left(1.11\times10^{-7}\text{ s}\right)^2$$

$$= 5.68\times10^{-3}\text{ m} = \boxed{5.67\text{ mm}}$$

(c) $$v_x = 4.50\times10^{5}\text{ m/s}$$

$$v_{yf} = v_{yi} + a_y t = \left(9.21\times10^{11}\text{ m/s}^2\right)\left(1.11\times10^{-7}\text{ s}\right) = 1.02\times10^{5}\text{ m/s}$$

$$\vec{\mathbf{v}} = \boxed{\left(450\hat{\mathbf{i}} + 102\hat{\mathbf{j}}\right)\text{ km/s}}$$

Example Problem 9

79. Two hard rubber spheres, each of mass $m = 15.0$ g, are rubbed with fur on a dry day and are then suspended with two insulating strings of length $L = 5.00$ cm whose support points are a distance $d = 3.00$ cm from each other as shown in Figure P23.79. During the rubbing process, one sphere receives exactly twice the charge of the other. They are observed to hang at equilibrium, each at an angle of $\theta = 10.0°$ with the vertical. Find the amount of charge on each sphere.

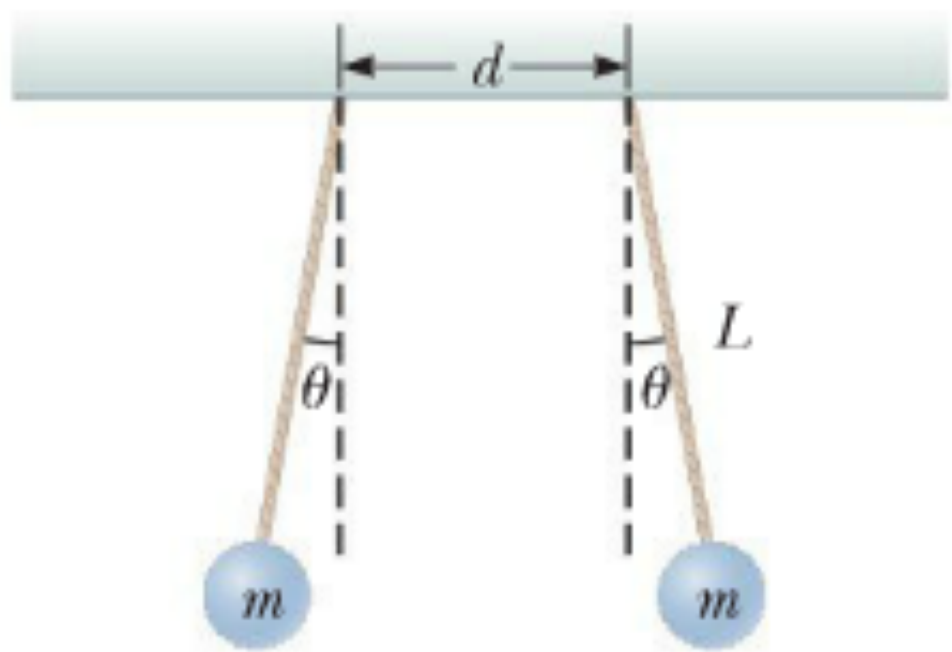


Figure P23.79

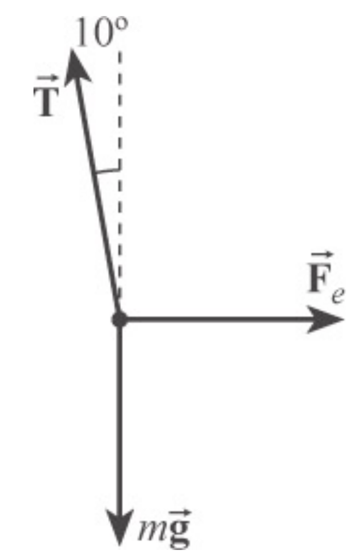
Example Problem 9 - Solution

The charges are q and $2q$. The magnitude of the repulsive force that one charge exerts on the other is

$$F_e = 2k_e \frac{q^2}{r^2}$$

From Figure P23.79 in the textbook, observe that the distance separating the two spheres is

$$r = d + 2L\sin 10°$$



ANS. FIG. P23.79

From the free-body diagram of one sphere given in ANS. FIG. P23.79, observe that

$$\sum F_y = 0 \;\Rightarrow\; T\cos 10° = mg \quad \text{or} \quad T = mg/\cos 10°$$

and

$$\sum F_x = 0 \;\Rightarrow\; F_e = T\sin 10° = \left(\frac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10°$$

Thus,

$$2k_e \frac{q^2}{r^2} = mg\tan 10° \quad \rightarrow \quad 2k_e \frac{q^2}{(d + 2L\sin 10°)^2} = mg\tan 10°$$

Example Problem 9 - Solution cont

$$q = \sqrt{\frac{mg\left(d + 2L\sin\theta\right)^2 \tan 10^\circ}{2k_e}}$$

$$= \sqrt{\frac{\left(0.015 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left[0.0300 \text{ m} + 2\left(0.0500 \text{ m}\right)\sin 10^\circ\right]^2 \tan 10^\circ}{2\left(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)}}$$

$$= 5.69\times 10^{-8} \text{ C}$$

giving $\boxed{1.14\times 10^{-7}\,\text{C on one sphere and } 5.69\times 10^{-8}\,\text{C on the other.}}$