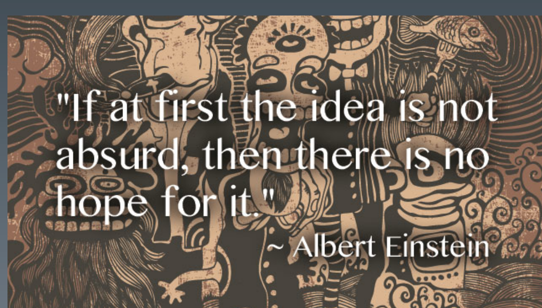
incredibly useful source during the preparation of today's lecture (Sean Carroll):
https://web.archive.org/web/20220128123936/https://www.discovermagazine.com/the-sciences/dark-energy-faq

Λ 

Star Wars joke: Force lightning from the Dark Side of the Force. Also, I work on DM myself, not DE.

Differential Equations Cont: **DE**



Feb. - March, 2024

Computational Physics 577/477

BG & Context

- The seminal discovery paper: http://www.stsci.edu/~ariess/documents/1998.pdf
- How to plot the scale factor as a function of time: https://arxiv.org/pdf/1407.6300.pdf
  - Ringermacher: cutting-edge research into the scale factor (also, gives you the inflection point)
- There is also Saul Perlmutter's parallel co-discovery, also from the late 1990s
- Coding assignment for homework, as usual (on related but unique topic)

- The discovery of dark energy, like inflation in the 1980s, solved a lot of cosmic mysteries cropping up towards the end of the 20th century (90s). I lived it. Elegant all-in-1 solution
  - Like stars in star clusters older than the entire universe in our galaxy ha
  - Large-scale structure: too much void
  - Matter sum so darn ***close*** to critical amount
  - Discrepant measurements of Hubble's constant H_0 (happening again??)

First, we need to review some cosmology: The FLRW Formula.

$$\underbrace{-c^2 \mathrm{d}\tau^2}_{\mathrm{d}s^2} = -c^2 \mathrm{d}t^2 + a(t)^2 \underbrace{\mathrm{d}\boldsymbol{\Sigma}^2}_{\mathrm{d}\ell^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2}$$

standard definition of space-time distance from flat SR: (x,y,z,-ct) with a(t) added

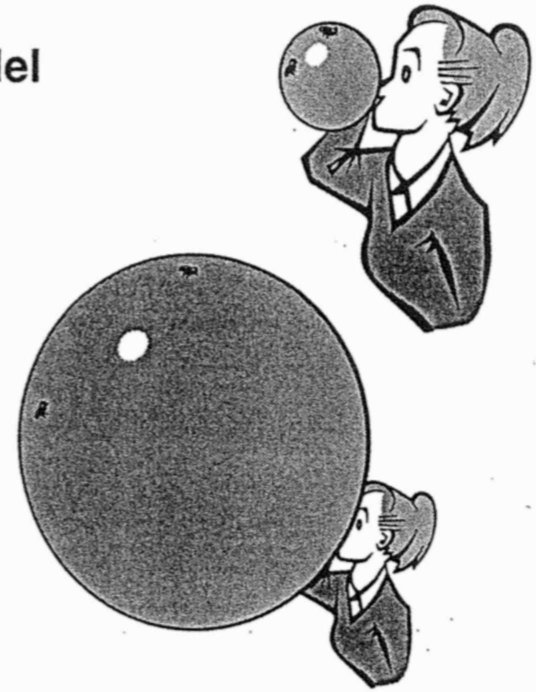
- The Friedman-Lemaitre-Robertson-Walker spacetime metric is a generic but exact solution to the Einstein equation in GR, representing a homogenous & isotropic universe, forced to either expand or contract
- a(t) encodes the rate at which such expansion, or contraction, is occurring

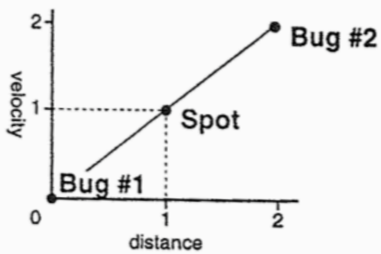
GR prediction of dynamic universe made Einstein uncomfortable. He liked concept of static, eternal cosmos.

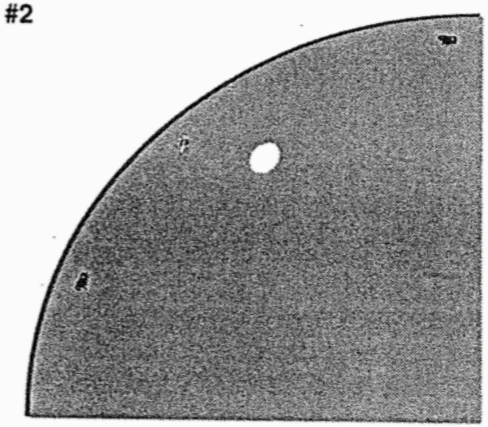
- Recall from cosmology, or consider for first time: objects do *NOT* move apart from each other in all directions *within* space as in the traditional sense of motion, not like like poles of magnets facing (it's not anti-gravity)
  - Instead, it is the "fabric" of the space-time continuum itself (the universe itself) that is stretching/elongating, with galaxies and galaxy clusters coming along for ride! Classic analogies include ants or dots on balloon or raisins in rising bread

Expanding Space Model

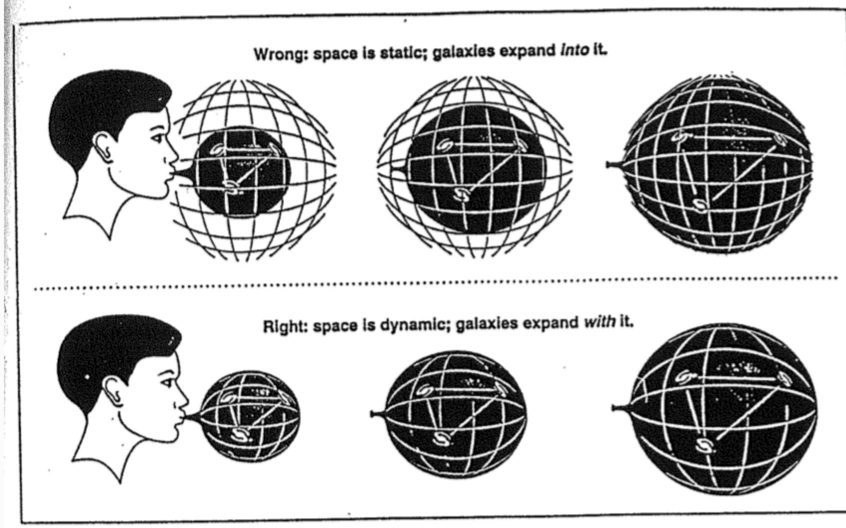
The apparent recessional velocity directly proportional to distance, for linear blowing







The distance between the bugs and the spot expands with the balloon, just as the distance between galaxies increases during the expansion of space. This does not necesssarily mean galaxies move *through* space; as space expands, it carries the galaxies with it. CHRISTOPHER SLYE



Galaxies expanding into space (*top*) versus galaxies expanding *with* space, not into it. SKY AND TELESCOPE

Figures taken from Chapter 3 "The Expanding Universe" of book *Wrinkles in Time* by Noble Prize winner George Smoot, co-discoverer of CMB anisotropy with COBE

https://www.youtube.com/watch?v=wYwkGoktdVA

Different Types of "Fluids" / Materials (Energy and/or Matter)

- Eqn. of state w = P / ρ, where P is pressure and ρ is the density (~~units~~)
  - w = 0 for non-relativistic (cold, slow) matter (magnetic monopoles too), diluted as length or distance cubed (volume) as universe expands
  - w = $^{+}$1/3 for relativistic (hot, fast) matter as well as radiation, which is diluted as length to the 4th power with expansion. Neutrinos and photons are included.
  - *w = $^{-}$1/3 for the intrinsic curvature of space-time (which itself can be + or -) Ignore not only because universe is most likely flat but because k term on next page already takes care of this component, so this is redundant*
  - w = -1 for the "cosmological constant," or the dark energy (energy of the vacuum of space itself?) No dilution! Density remains same during expansion
    - Note it is pressure (not density, impossibly?) that is negative
    - Maybe same equation of state for the inflation after the Big Bang
- Density as a function of time goes as 1 over 'a' to the 3*(1+w) power
  - Matter: a^-3 (until late 90s even we thought we lived in a flat matter-dominated universe)
  - Radiation: a^-4: Additional factor of 'a' comes from wavelength due to wave behavior
  - DE: a^0: Already above, but here worse: expansion doesn't conserve energy, in a sense

Solving for the Scale Factor

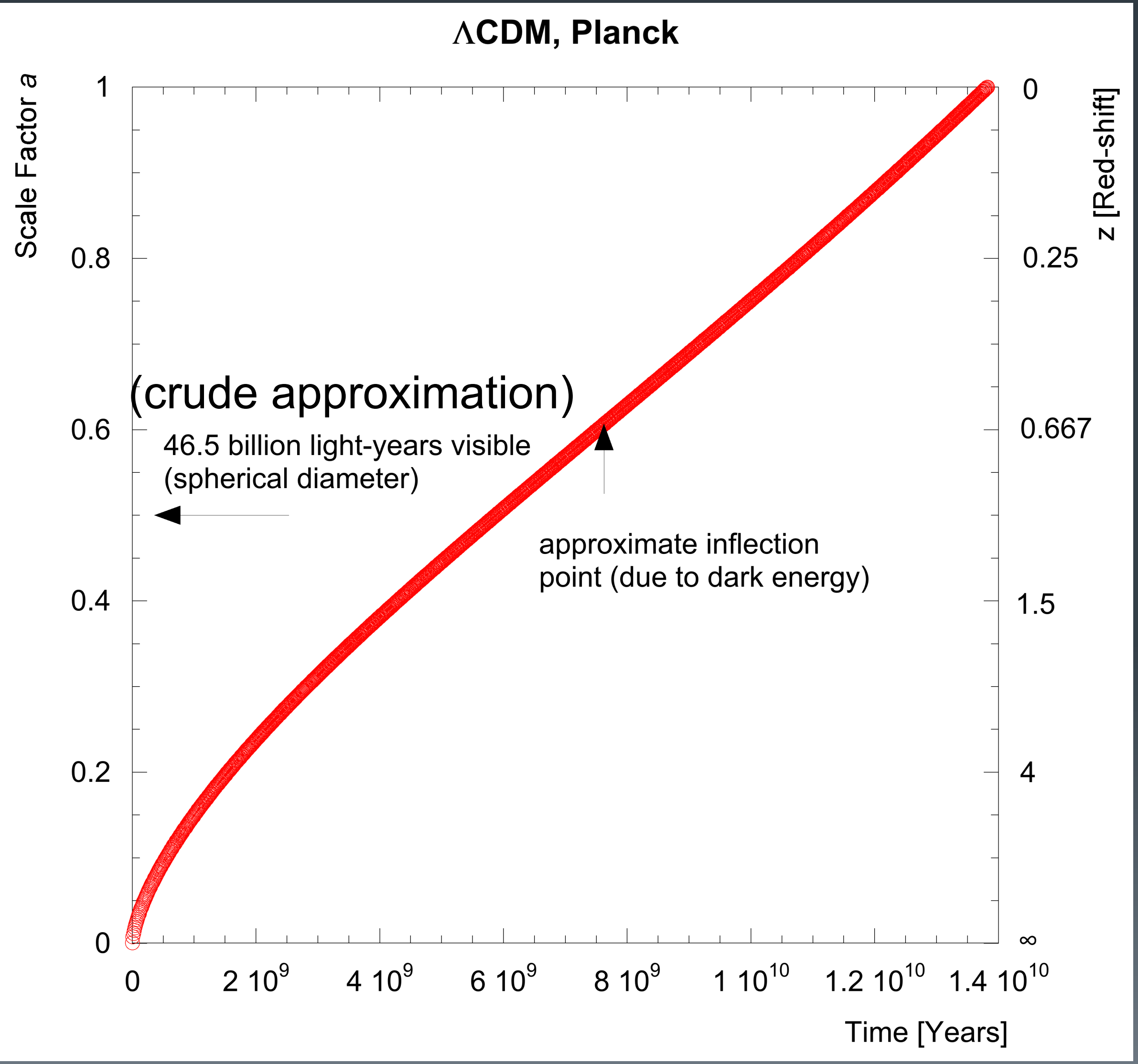
- $[(\dot{a} / a) * (1 / H_0)]^2 = \Omega_m^0 / a^3 + \Omega_r^0 / a^4 + \Omega_\Lambda^0 / a^0 + \Omega_k^0 / a^2$
  - A differential equation. 1st-order in a(t), but complicated (no general solution). We will do many examples of specific solvable cases on the board, however
  - Capital Greek-letter omegas (with superscript 0s) are the dimensionless values *today* (fractions) where “now” is defined as ‘a’ equal to 1.00
  - Fractions of what? Don’t have to sum to 1. Fractions of the “critical density”
  - Omega_k is simply 1 minus [sum of all other omegas]
  - Omega_Total = 1 implies FLAT universe (no net + or - curvature)
  - H_0 is value of Hubble’s constant today, traditionally quoted in units of km per second per Mpc. Convert into 1 over desired time unit above (e.g., years)
    - Experimental value is ~70 km/s/Mpc. ‘h’ is defined in cosmology as fraction of 100
- Easier if re-written da / dt or $\dot{a} = H_0 \sqrt{\Omega_m^0 / a + \Omega_r^0 / a^2 + \Omega_\Lambda^0 a^2 + \Omega_k^0}$
  - There are numerous other ways to re-write to make it easier to solve this
  - One easily gets Omega_m and Omega_Lambda from famous pie chart
  - They sum to ~1, and so you also know Omega_k as well. Omega_r is small
  - If you want to convert everything to SI: critical density $\rho_c = \frac{3H_0^2}{8\pi G}$ $\Omega_? = \frac{\rho_?}{\rho_c}$

Where ? = m, r, k, or lambda

Example Plot

Remember that you do not need to put all points in a plot!

- We are unable to measure ‘a’ itself
  - Our “rulers“ themselves are getting longer!
- We measure H_0 (directly possible) and the omegas, as well as the ‘z’
  - This is the Doppler Redshift (APHY 240)
  - $a = 1 / (1 + z)$ or $z = 1 / a – 1$
- Alternative x-axis: sometimes t = 1 is defined as today



How to Solve These Differential Equations Numerically (Brute Force)

- Necessary, when a closed-form analytical solution doesn't exist, or is simply too hard for mere mortals to derive

Wikipedia is not always "bad"! :0 Gets better every year too

- *Euler and Euler-Cromer*
  - http://tutorial.math.lamar.edu/Classes/DE/EulersMethod.aspx
    - Have you studied already the many examples on this webpage?
  - https://en.wikipedia.org/wiki/Semi-implicit_Euler_method (even better)
- **Runge-Kutta** (better still, but computationally more intensive)
  - http://lpsa.swarthmore.edu/NumInt/NumIntFourth.html (covering today)
  - https://en.wikipedia.org/wiki/Runge–Kutta_methods (more careful and full derivation that includes how to code up the RK formulae)
  - http://mathworld.wolfram.com/Runge-KuttaMethod.html (a concise summary from Wolfram's Math World)

- RK4 (order 4, but can be higher or lower) is the gold-standard default

  Also: what does the order mean here? We are going to find out

What You May Need (e.g., for ***HW***)

- Cosmological parameter values (~latest, final values of omegas): http://arxiv.org/pdf/1502.01589v2.pdf (look at the age too)
- Hyperphysics discussion of the Friedmann equation and Hubble parameter, to review: http://hyperphysics.phy-astr.gsu.edu/hbase/astro/fried.html
- Wikipedia
  - Scale factor with redshift definition: https://en.wikipedia.org/wiki/Scale_factor_%28cosmology%29
  - Equation of state in cosmology: https://en.wikipedia.org/wiki/Equation_of_state_%28cosmology%29
- For your further edification and knowledge
  - http://www-com.physik.hu-berlin.de/~fjeger/Cosmolect1-7.pdf (Friedmann eqn.'s hard-core full solutions with explanations, like ~~S~~HO)
- Hunt down more up-to-date numbers than 1st bullet above if you can

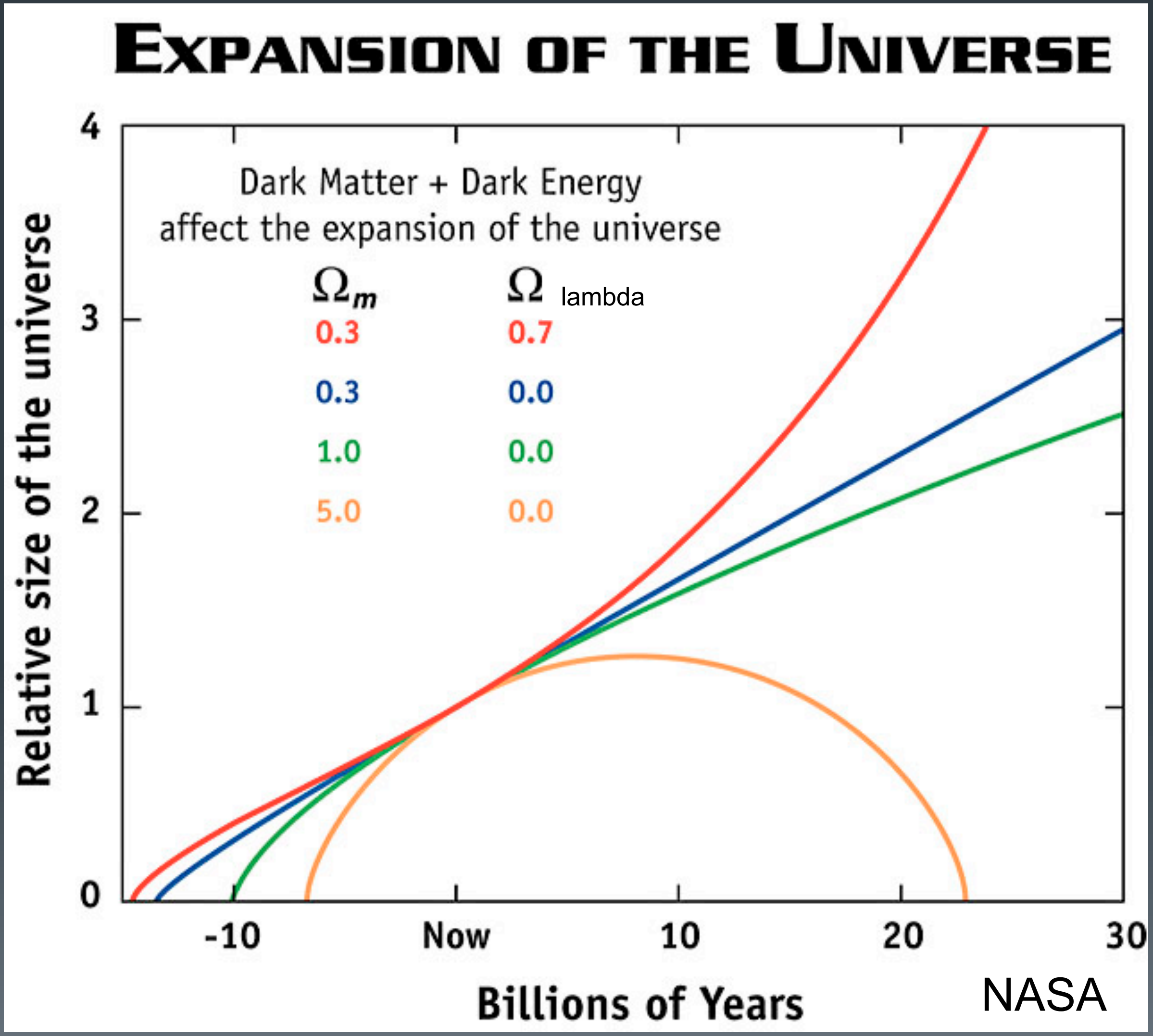
Current, and Older, Scenarios

and Equations of State ('*w*'), the Ratios of Pressure P to the density rho

For Matter:
w=0
p=-3(1+w)
=-3(1+0)
=-3

For DE:
w=-1
p=-3(1+w)
=-3(1-1)
= 0

For radiation:
w=+1/3
p=-3(1+w)
=-3 (1 + 1/3)
=-3(4/3)= -4



Curvature:
w=-1/3
p=-3(1+w)
=-3 (1 - 1/3)
=-3(2/3)=-2

Phantom DE:
is when w<-1
(1+w)<0 =>
3(1+w)<0
p=-3(1+w)>0

Dying DE:
is when w>-1
(1+w)>0
3(1+w)>0
p=-3(1+w)<0

Remember: $\Omega_x(t) = \Omega_x^0 * a(t) \wedge p$

'p' is called p or power in different places in my own code (functions).
Note that OmegaX_0 is defined as OmegaX(t=today, when a(t)=1.00000000000)

Other Possible Forms of Dark Energy, the Most Prevalent Component of our Universe

- w > -1 (but < -1/3 still) is weaker form of the dark energy (may be dynamic i.e. time-dependent not persistent: then called *quintessence*)
- w < -1 is speculative *phantom* dark energy, whose energy density counter-intuitively *grows* over time (no strong theory motivation)
  - The cosmic jerk: an accelerating acceleration! (3rd derivative)
- Spatial dependence of w? So-called chameleon dark energy. Non-uniform expansion in some directions? Experiments to detect it!

- $\Omega_\Lambda < 0$ => dark energy pulls the universe *inward* faster, not outward
  - Has nothing to do with negative sign of w, which tells one density(time)
  - This is called anti de Sitter space (de Sitter space is $\Omega_\Lambda > 0$, where we live)
  - Do not confuse negative pressure with negative mass/energy density. A negative w can be caused by either (and a positive w if both negative ha)
  - In different units/names: $\Lambda < 0$, or > 0

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho$$

Einstein inserted a fudge factor (the cosmological constant, lambda) that he later called his "greatest blunder," but now he has last laugh!

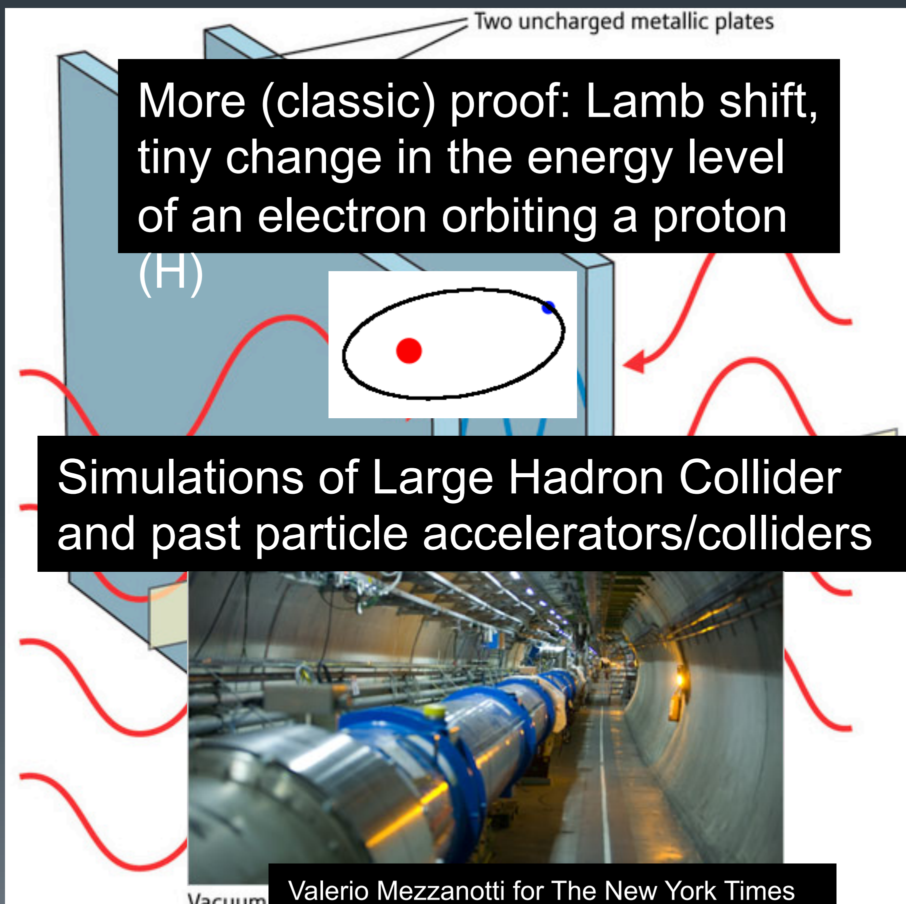
But, what IS this *Dark Energy*?

- Accelerates expansion. Related to inflation?? Repulsive behavior
- Majority component of matter or mass and energy in the cosmos today
  - But, probably only thing it shares with dark matter is the adjective dark in its name, a mask for our ignorance. Either one a new ether?
- Honestly, no one knows what the heck is going on here. Astronomers and physicists are still quite baffled
  - Even though Einstein "predicted," a cosmological constant, it was a mathematical term without physical meaning
- Quantum mechanics: zero-point energy? Would lead to the "constant"!
  - From QFT calculations we know vacuum is NOT "empty"
  - Space is teeming with submicroscopic (subatomic-level) quantum fluctuations: virtual particle-antiparticle pairs pop in and out of existence all of the time (but mathematical constructs only?)
- There is an important catch: measured dark energy is too small by a factor of 10^120!!!!! (value of lambda)
  - Less of a problem back when we thought it was zero, because one could imagine a symmetry which somehow canceled it perfectly (more palatable)

Symmetry: bosons and fermions?

Virtual Particles ARE a Reality (or, Handy)

http://muldoonshealthphysicspage.com/The_Atom.htm



More (classic) proof: Lamb shift, tiny change in the energy level of an electron orbiting a proton (H)

Simulations of Large Hadron Collider and past particle accelerators/colliders

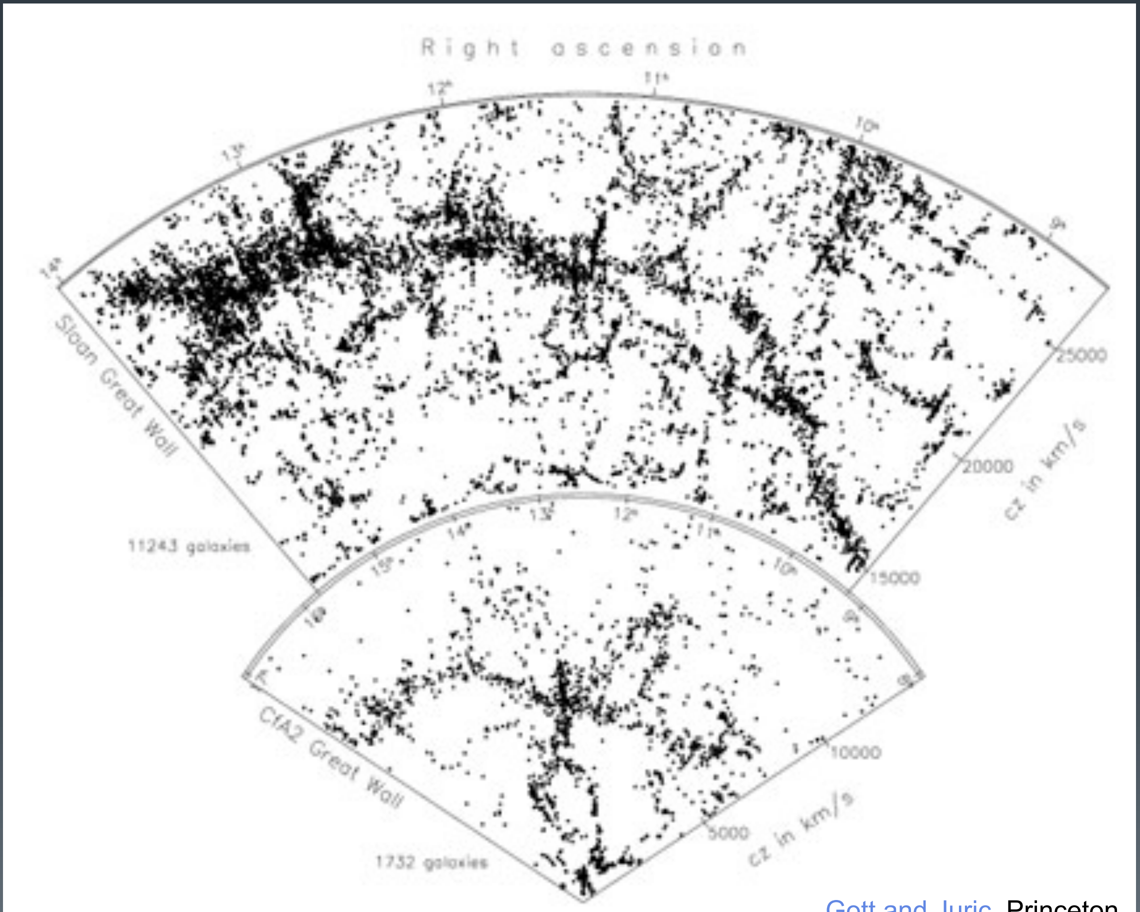
Valerio Mezzanotti for The New York Times

Copyright © 2008 Yampol'skii and Nori

- Can borrow a little energy from vacuum, and it doesn't even have to be a temporary loan (short distance scales)
  - *Virtual* particles have real effects (can carry a force)
  - t*Energy Heisenberg Uncertainty
- Casimir Effect: Plates initially uncharged develop calculable potential difference and then attract (or repel) each other (opposite, or same, charges)
  - Very small effect, difficult to measure, but we've done it, after subtraction of van der Waals etc.
- Live "off the mass shell" (<, >)
  - That means their E^2 does NOT equal m^2 + p^2 (c = 1 units)

Inhomogeneity: The Back Reaction

- We know that our Universe isn't as isotropic as we claim in our equations
  - Could the acceleration of the expansion be merely an illusion?
- Called the "back reaction"
  - There is a blog named after this!
- Unfortunately, the more data we get it appears cosmos is still smooth *on the largest scales*
  - Approximation of isotropy and homogeneity holds



Gott and Juric, Princeton

The Ultimate Fate of the Universe

Potential / Likely "Doomsday" Scenarios

- Big Crunch (positive curvature, closed universe), Big Chill a.k.a. Big Freeze (negative or zero / open or flat), and Big Rip (phantom energy)
  - Crunch does not necessarily lead to re-birth (singularity?)
- Dark energy or expanding universe in general can overcome local binding (matter held together with mutual attractive forces, gravity and/or others like EM, strong)
  - A cosmological constant screws up simple geometry-fate concordance: a technically closed universe, positively curved, can still expand forever
- If the universe expands too fast, the light horizon recedes (the visible universe shrinks instead of growing). We may be living in a special time (fund astronomers and telescopes today, before it is too late! :-)
  - Expansion of space is why the spherical radius of the visible universe is not directly related simply to the age of the universe
- If it contracts, no center of collapse. It's like expansion except reversed

In *your* plots, remember that you do NOT need to plot every point.

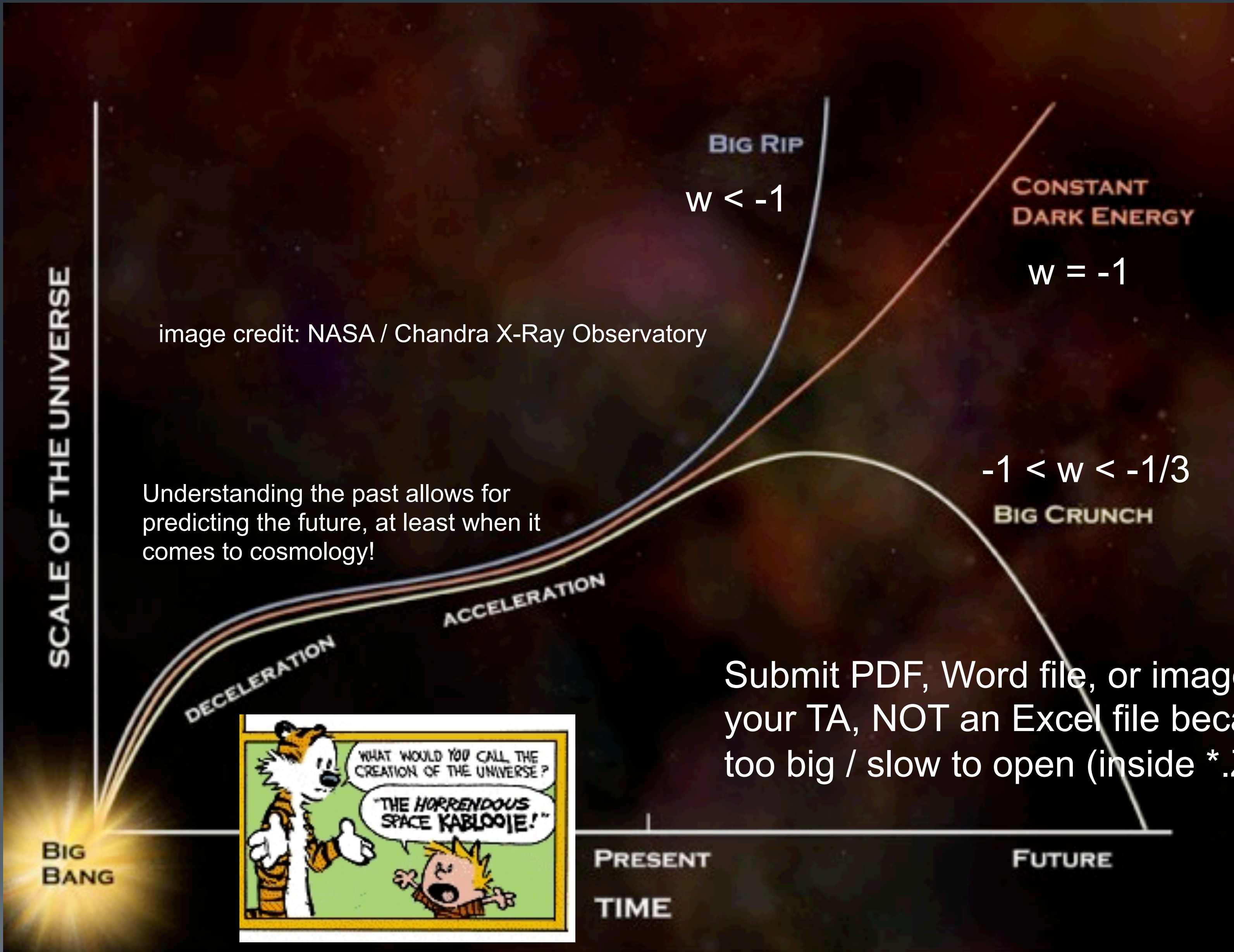


image credit: NASA / Chandra X-Ray Observatory

Understanding the past allows for predicting the future, at least when it comes to cosmology!

Submit PDF, Word file, or image to your TA, NOT an Excel file because too big / slow to open (inside *.ZIP)

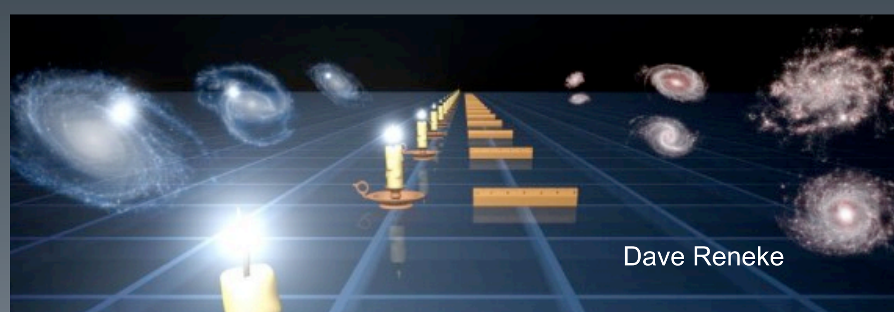
> Space is big. You just won't believe how vastly, hugely, mind- bogglingly big it is. I mean, you may think it's a long way down the road to the chemist's, but that's just peanuts to space.
>
> **Douglas Adams**, *The Hitchhiker's Guide to the Galaxy*
>
> *English humorist & science fiction novelist (1952 - 2001)*

chemist?!

How do we know all this? Standard candles!

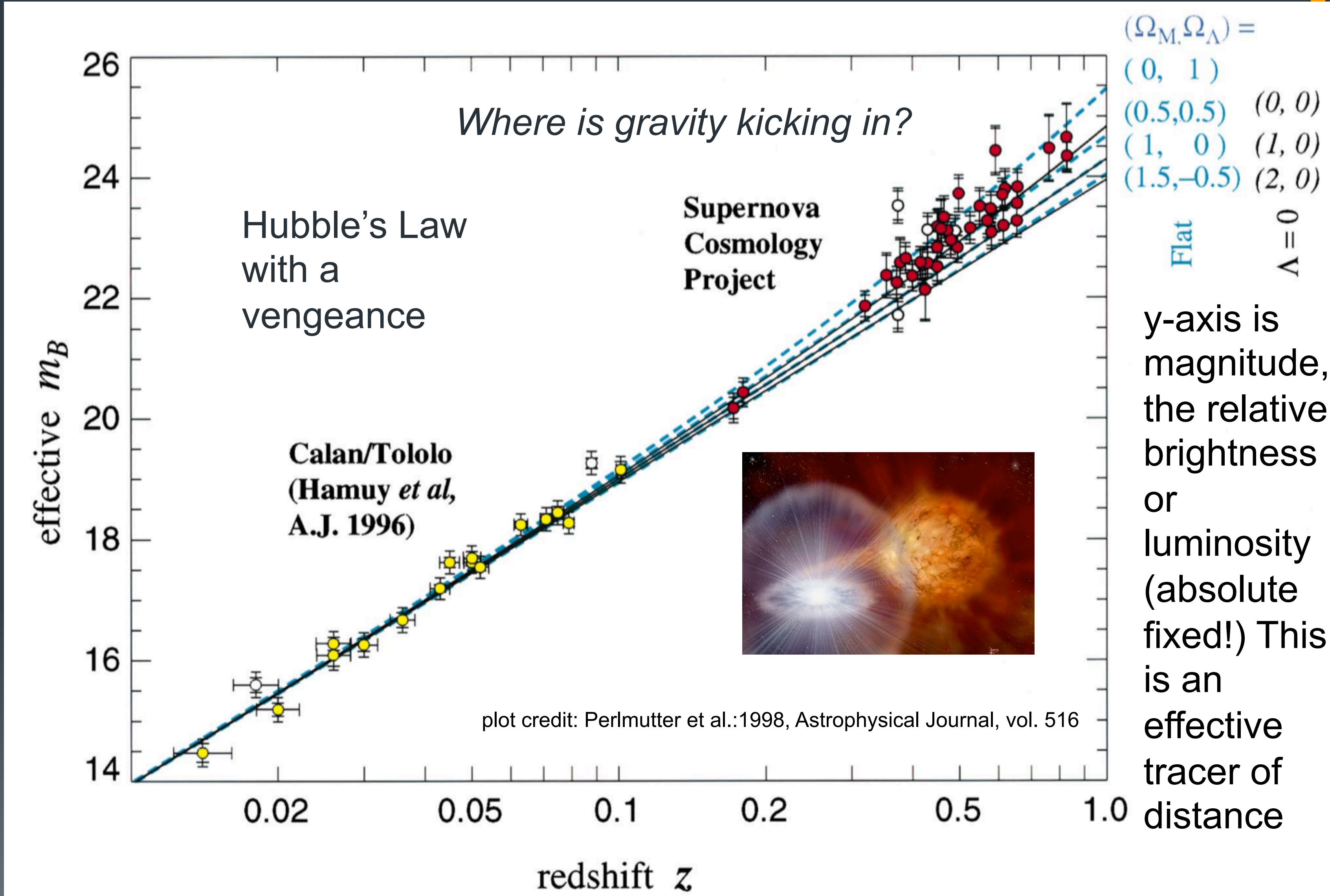
- The Type Ia supernova is the last and latest famous standard candle. Its brightness lets us go the furthest out in space. Best one we study
  - Billions of light-years or parsecs (so, Gpc)
- At peak intensity a supernova explosion can outshine an entire galaxy full of stars!
  - Billions of times the solar luminosity, at peak
- Nuclear astrophysics (^{56}Ni in white dwarfs) behind phenomenon leads to a *consistent* absolute brightness and light curve as a function of time
- Most famous for clueing us in that Universe is not only getting bigger, but getting bigger faster (expanding and accelerating in its expansion)

Note: standard ruler from BAO! Now, gravitational waves may be standard *sirens*!



BAO = Baryon Acoustic Oscillations

instead of seeing a turn-down (would be true of a standard closed, flat, or even open universe) shocked to see the opposite!



y-axis is magnitude, the relative brightness or luminosity (absolute fixed!) This is an effective tracer of distance

To 1st-order, galaxies appear to be receding from us at $v = c * z = H_0 * d$

Running My Example Code

- $[(\dot{a}/a) * (1/H_0)]^2 =$
- $\Omega_m^0 / a^3 + \Omega_r^0 / a^4 + \Omega_\Lambda^0 a^{-3(1+w)} + \Omega_k^0 / a^2$
- da / dt or $\dot{a} =$
- $H_0 \sqrt{\Omega_m^0 / a + \Omega_r^0 / a^2 + \Omega_\Lambda^0 a^{2+p} + \Omega_k^0}$
  - Where ‘p’ is defined as " -3*(1+w) "

- Decided against wave eqn. (or L-J Xe): example directly helps you out
- Determining in the default scenario (real universe, ΛCDM) these values
  - The correct age of the universe. There are numerous websites we can check
  - The dark-energy inflection point. Getting both right. May mean playing with sig figs on our omegas, and/or with initial (t0, a0) and of course our step ‘dt’
- Time permitting we may explore the classic or *traditional* matter-dominated cosmos
  - flat, open, closed (zero, negative, positive curvatures) Playing with Hubble
  - seeing turn-around time for closed case

HW6: Cosmo / GR. Due Thurs 3/07

- Forget Euler, Euler-Cromer and similar methods. Go with RK4 approach
  - OPTIONAL: May need to up precision, double to long double vars (again)
- Make these plots (combinations fine as always) using default numbers
  - a(t) as well as the first, second, and third derivatives of the scale factor *a* for TWO POINTS (0.5 points each). Those include having right approach in code
  - All omegas as a function of time, z, & scale factor (1 plot different axes okay) TWO POINTS. Interpretations matter (in your report)
  - Learn some physics here and don't just generate a lot of plots blindly
- Verify code is working for the 4 simplest cases, showing 'a' but with fits: did them on board for you (only 1 omega non-0 at a time) 0.25 pts. each
- Lastly, you must also do the following (flexibility on exacts #s)
  - Phantom DE: cause a Big Rip and find its time scale i.e. tMax. Use w but also omega-lambda. May take LONG t (consider log t). 2 + 3 POINTS (code + rep)
- BONUS: on next page. In addition to all of the above just *1* more thing

The Extra Credit Challenge!

- Try out decreasing DE
  - Cause Crunch and find turn-around t but w/ lambda CDM and within errors on known values
  - Again, may take LONG time, and you need to be clever: work smarter not harder. Sqrt degenerate.
- (For the main assignment, you don't have to stay w/ realistic values!)