

Electric Flux & Gauss' law

Chapter 23(b)

HW02 is up on WebAssign, due Thursday 02/01

Introduction

Gauss' Law can be used as **an alternative procedure for calculating electric fields.**

It is convenient for calculating the electric field of **highly symmetric charge distributions.**

Gauss' Law is important in understanding and verifying the properties of conductors in electrostatic equilibrium.

To use Gauss' Law, you must understand the concept of **electric flux.**

Electric Flux

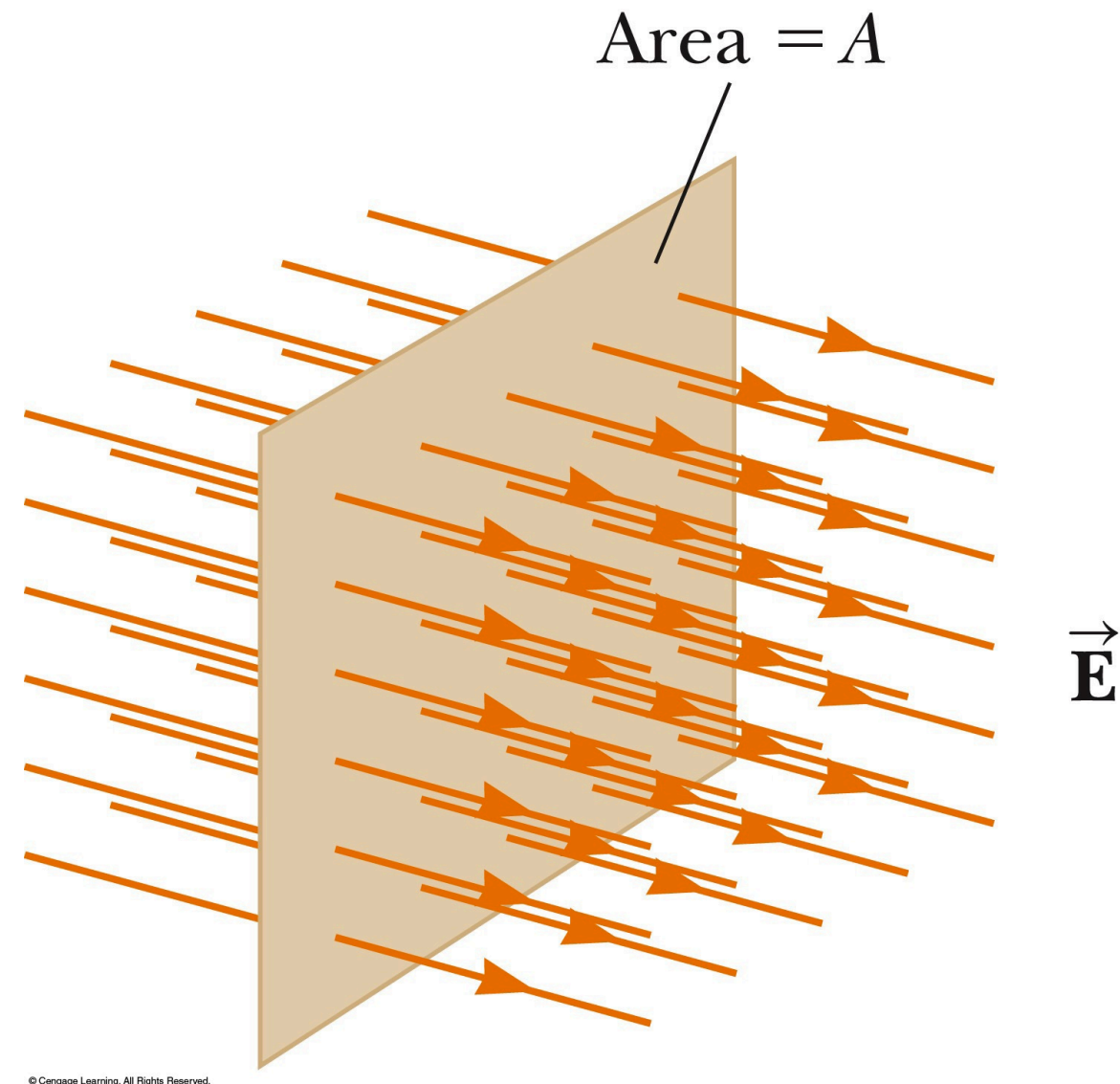
Electric flux is the product of the magnitude of the electric field and the surface area, A , perpendicular to the field.

$$\Phi_E = EA$$

Units: $\text{N} \cdot \text{m}^2 / \text{C}$

The electric flux can be thought of as how much field is going through a surface at any time.

Analogy: how much air is going through a window



Electric Flux - Non \perp surface

The electric flux is **proportional to the number of electric field lines penetrating some surface**.

The field lines may make some angle θ with the perpendicular to the surface.

Then $\Phi_E = EA \cos \theta$

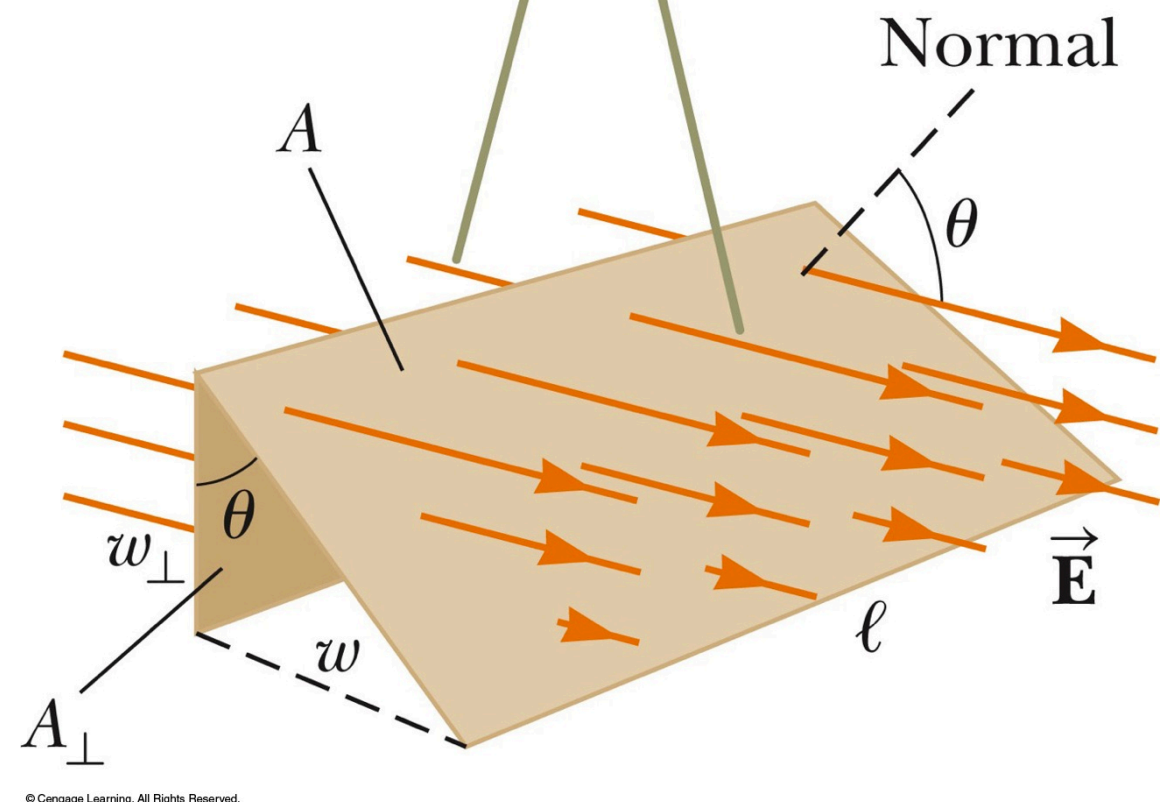
The flux is a **maximum** when the surface is **perpendicular** to the field.

- $\theta = 0^\circ$

The flux is **zero** when the surface is **parallel** to the field.

- $\theta = 90^\circ$

The number of field lines that go through the area A_\perp is the same as the number that go through area A .



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If the field varies over the surface, $\Phi = EA \cos \theta$ is valid for only a small element of the area.

Electric flux for solid object

In the more general case, look at a small area element.

$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \vec{E}_i \cdot \Delta\vec{A}_i$$

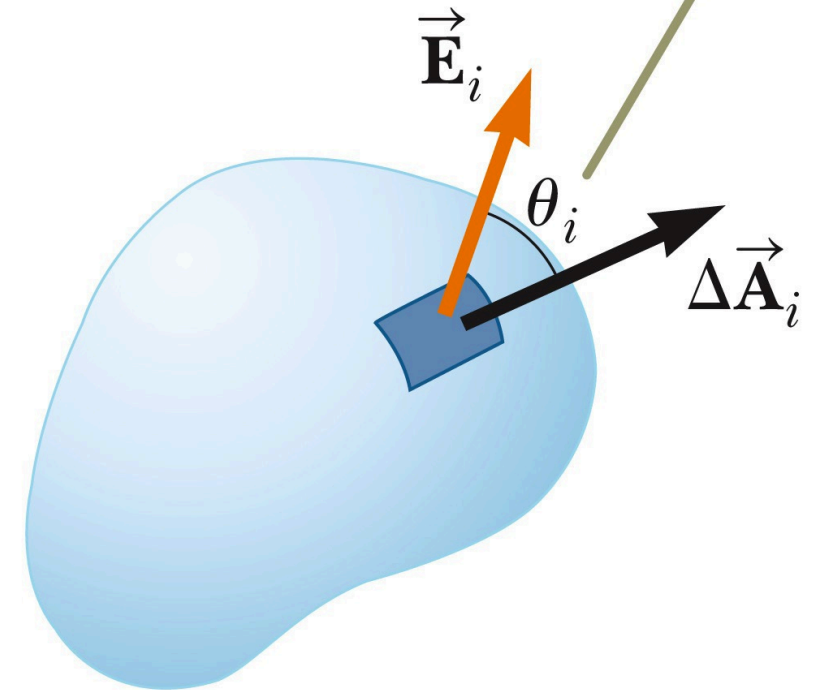
(dot product)

The electric field makes an angle θ_i with the vector $\Delta\vec{A}_i$, defined as being normal to the surface element.

When the size of the elements is infinitely small (so it approaches zero) you can rewrite as:

$$\Phi_E = \lim_{\Delta A_i \rightarrow 0} \sum \vec{E}_i \cdot \Delta\vec{A}_i$$

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A} \quad \text{(surface integral)}$$



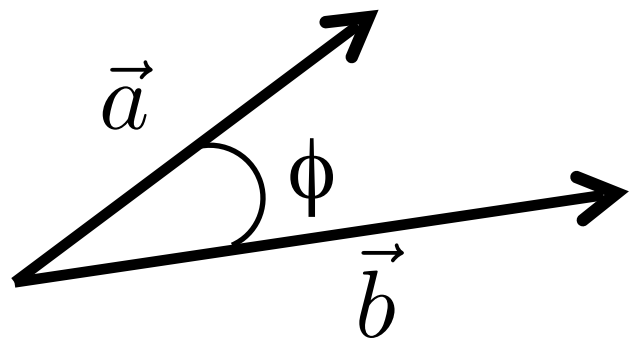
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The surface integral means the integral must be evaluated over the surface in question.

In general, the value of the flux will depend both on the field pattern and on the surface.

Scalar Product (or dot product)

Starts with two vectors and **gives a scalar** that is independent of coordinate system.



$$\vec{a} \cdot \vec{b} = ab \cos \phi$$

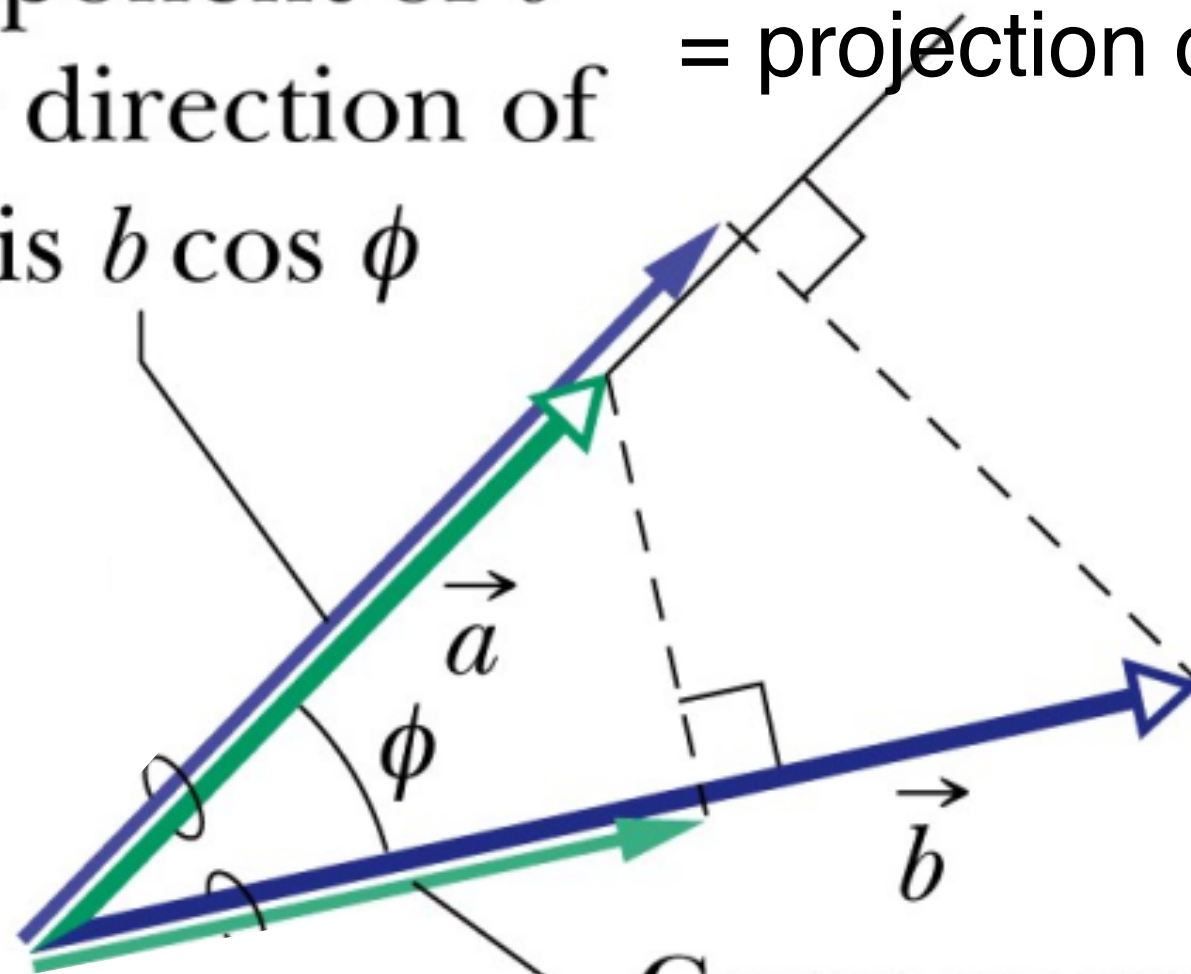
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab \cos \phi$$

A dot product gives a scalar = only a magnitude

Scalar Product

Component of \vec{b}
along direction of
 \vec{a} is $b \cos \phi$

= projection of b on a



Component of \vec{a}
along direction of
 \vec{b} is $a \cos \phi$

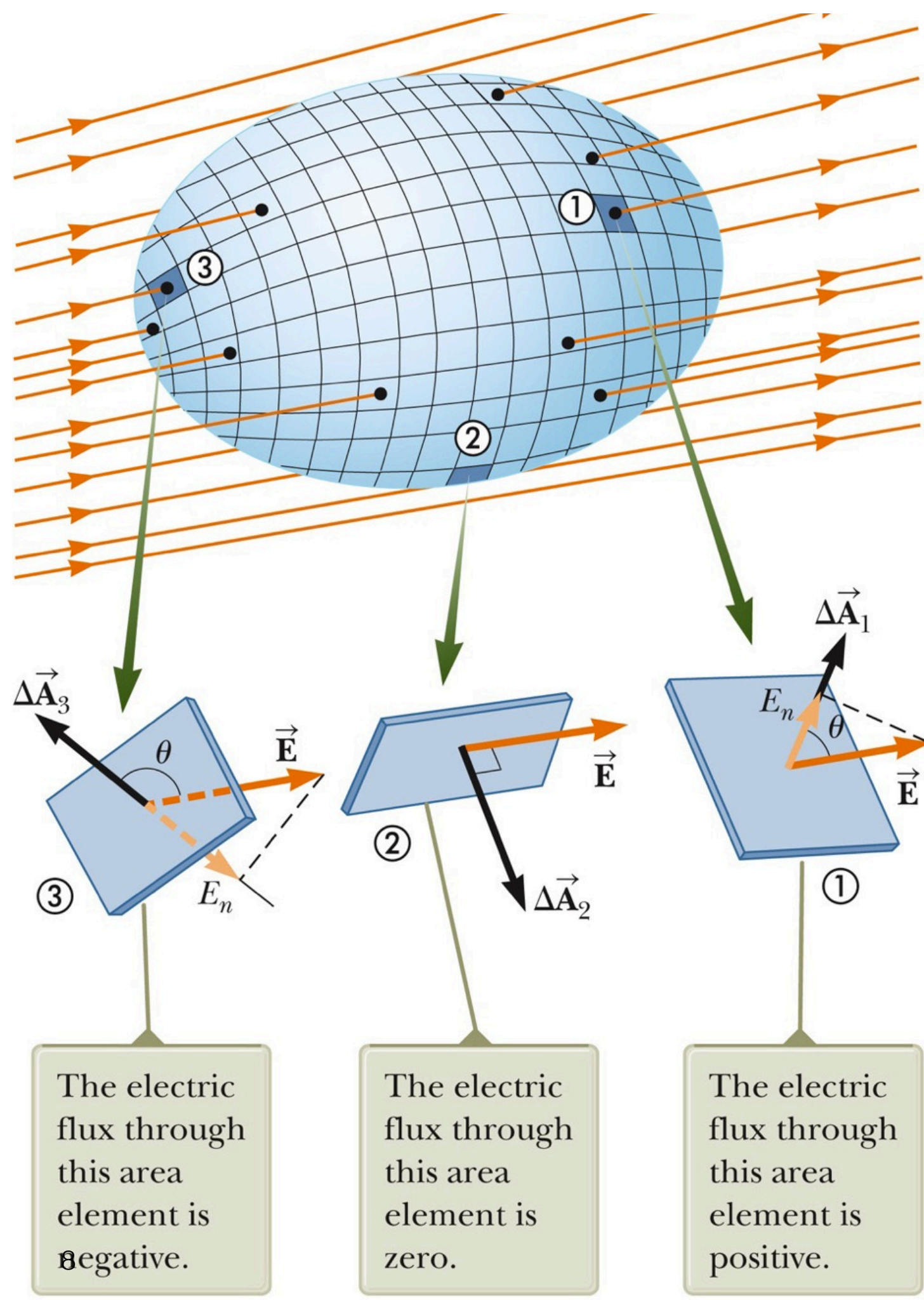
= projection of a on b

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

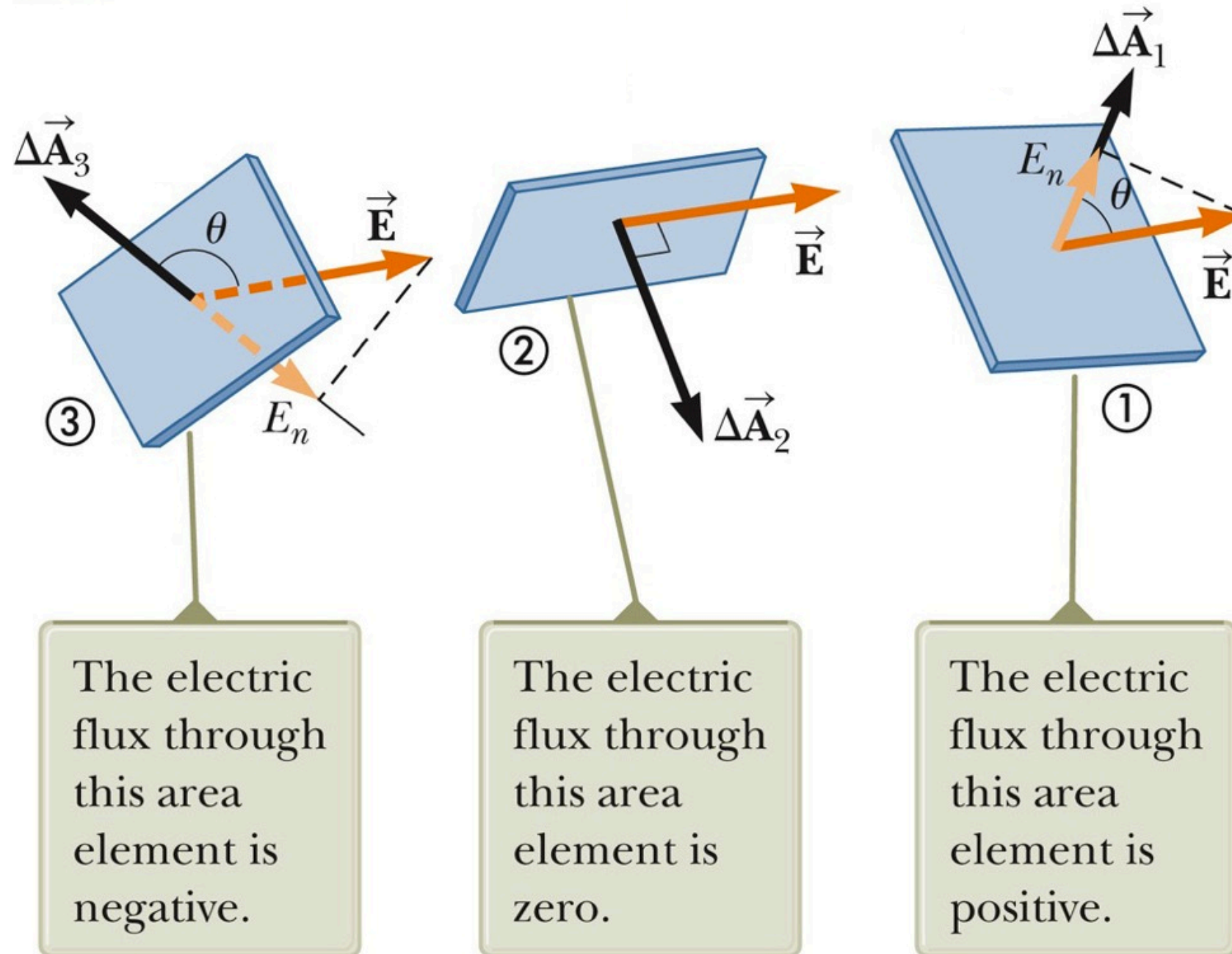
Electric Flux - Closed Surface

The vectors $\Delta\vec{A}$ point in different directions.

- At each point, they are **perpendicular to the surface**.
- By convention, they point outward.
- These are **normal vectors to the surface**: they give you the orientation of the surface
- They give you the sign of the flux



Electric Flux - Closed Surface



At (1), the field lines are crossing the surface from the inside to the outside; $\theta < 90^\circ$, Φ is positive.

At (2), the field lines graze surface; $\theta = 90^\circ$, $\Phi = 0$

At (3), the field lines are crossing the surface from the outside to the inside; $180^\circ > \theta > 90^\circ$, Φ is negative.

Electric Flux - Closed Surface

The *net* flux through the surface is proportional to the net number of lines leaving the surface.

- This net number of lines is the **number of lines leaving the surface minus the number entering the surface**.
- The integral is over a closed surface (that's what the circle around the integral means)
- E_n is the component of the field perpendicular to the surface

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E_n dA$$

Remember: If \vec{E} and $d\vec{A}$ are perpendicular, the flux is 0
which is the same as saying if \vec{E} is parallel to the surface, the flux is 0.

Example: Flux through a cube

Use the geometry to find which surfaces are parallel or perpendicular to the field lines.

The field lines pass through two surfaces perpendicularly (1 and 2) and are parallel to the other four surfaces (3,4,5,6).

For faces 3,4,5,6 : flux = 0

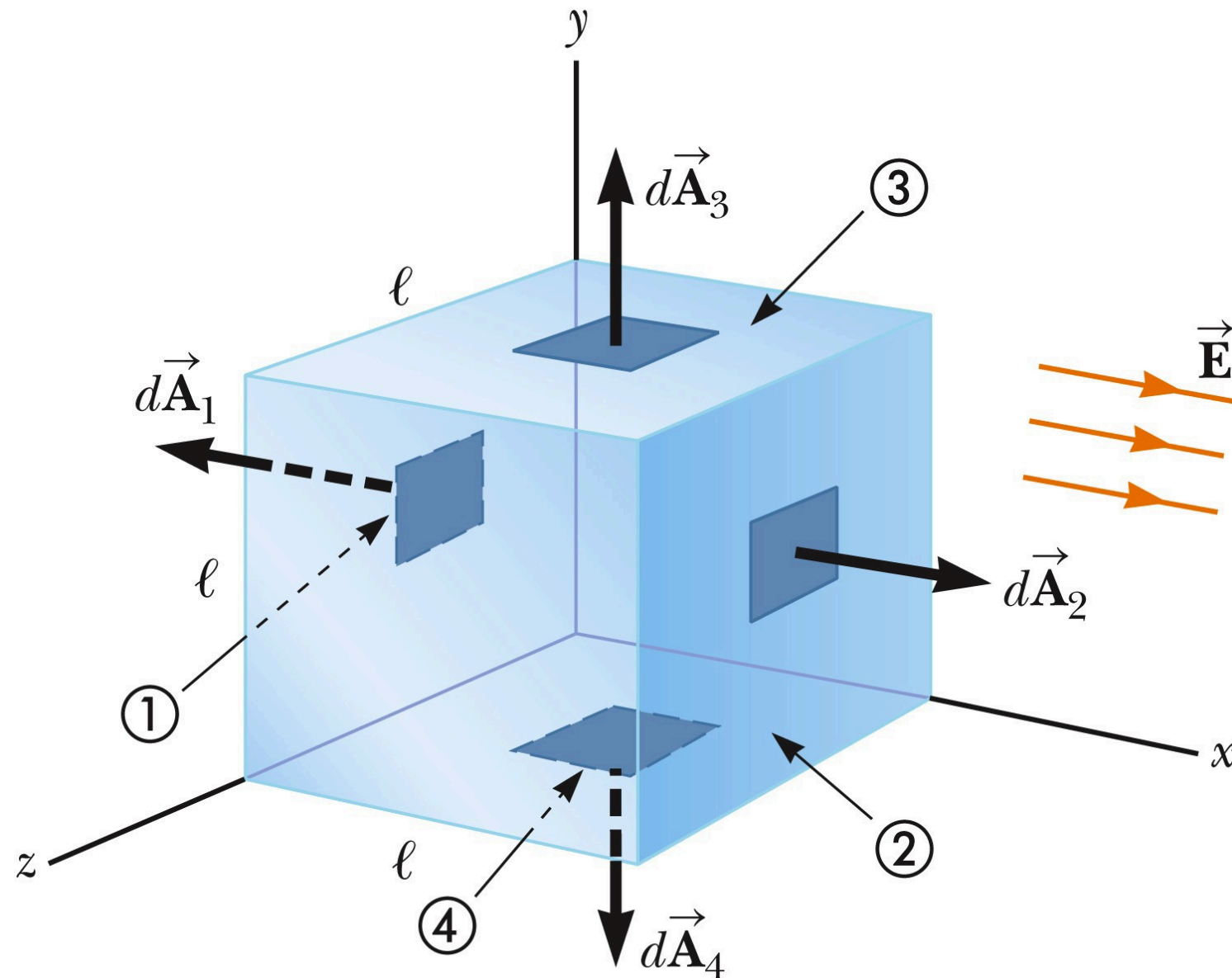
For face 2:

$$\Phi = EdA \cos(0) = E\ell\ell = E\ell^2$$

For face 1:

$$\Phi = EdA \cos(180) = -E\ell\ell = -E\ell^2$$

Therefore, $\Phi_{\text{net}} = 0$



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$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E_n dA$$

Example Problem 1

What is a flux capacitor?

Example Problem 1: Solution

A fictional, paradoxical device!

Example Problem 2

Does flux have any other meaning?

Example Problem 2: Solution

Yes! Particles, radiation, dark matter; also liquids like water

Example Problem 3

4. Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.80 \times 10^4 \text{ N/C}$ as shown in Figure P24.4. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.

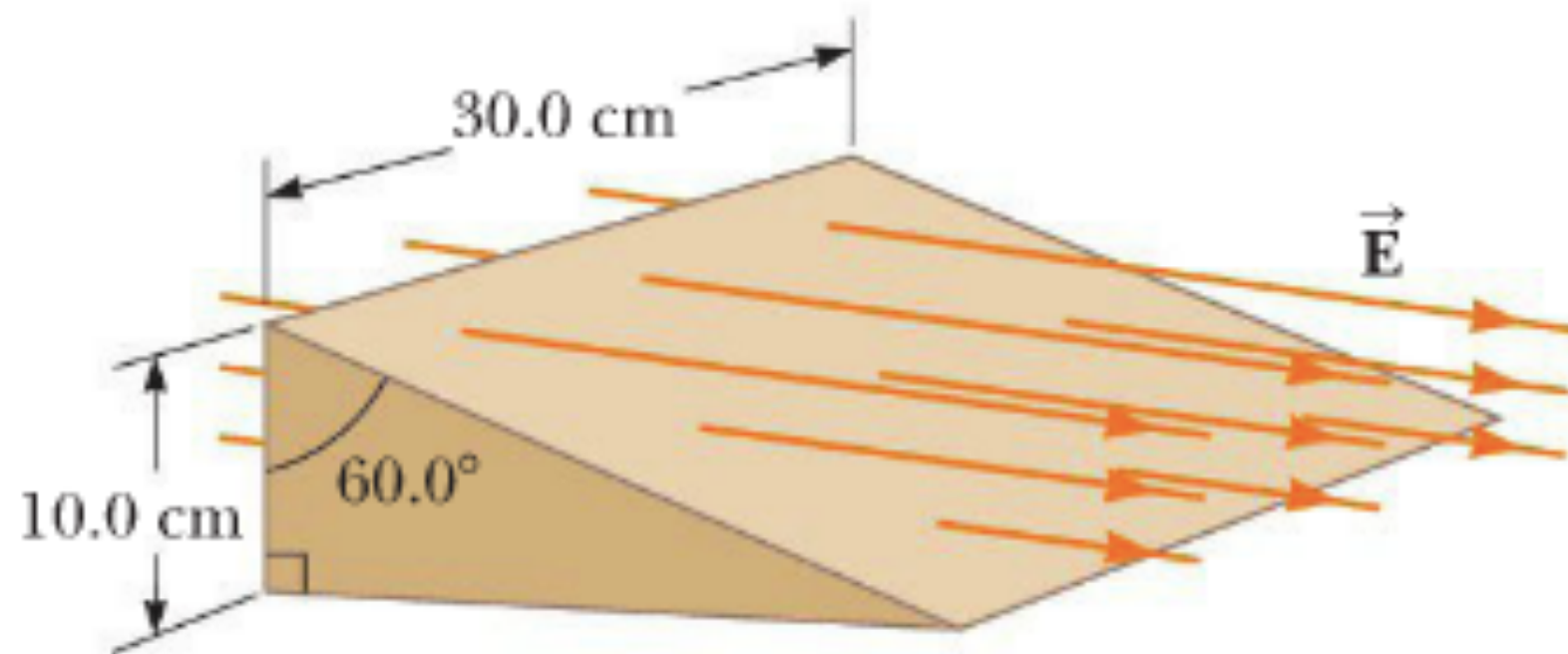


Figure P24.4

Example Problem 3: Solution

- (a) For the vertical rectangular surface, the area (shown as A' in ANS FIG. P24.4) is

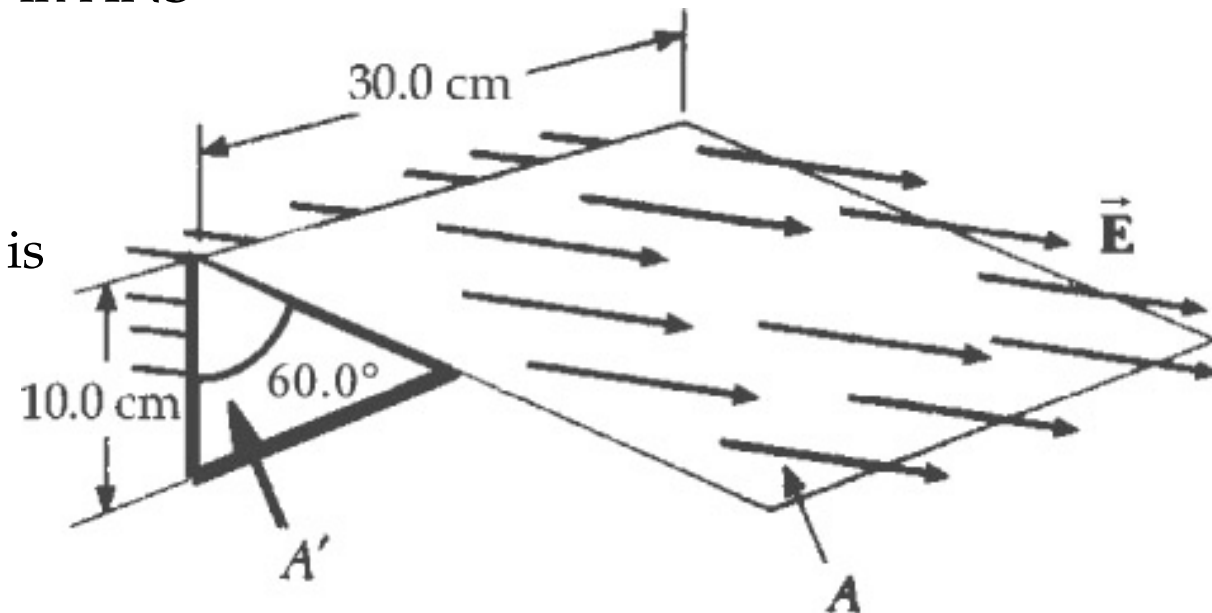
$$A' = (10.0 \text{ cm})(30.0 \text{ cm}) = 300 \text{ cm}^2 = 0.0300 \text{ m}^2$$

Since the electric field is perpendicular to the surface and is directed inward, $\theta = 180^\circ$ and

$$\Phi_{E,A'} = EA' \cos \theta$$

$$\Phi_{E,A'} = (7.80 \times 10^4 \text{ N/C})(0.0300 \text{ m}^2) \cos 180^\circ$$

$$\Phi_{E,A'} = \boxed{-2.34 \text{ kN} \cdot \text{m}^2 / \text{C}}$$



- (b) To find the area A of the slanted surface, we note that the side for which dimensions are not given has length $(10.0 \text{ cm}) = w \cos 60.0^\circ$, so that

$$\begin{aligned} A &= (30.0 \text{ cm})(w) = (30.0 \text{ cm}) \left(\frac{10.0 \text{ cm}}{\cos 60.0^\circ} \right) = 600 \text{ cm}^2 \\ &= 0.0600 \text{ m}^2 \end{aligned}$$

The flux through this surface is then

$$\begin{aligned} \Phi_{E,A} &= EA \cos \theta = (7.80 \times 10^4)(A) \cos 60.0^\circ \\ &= (7.80 \times 10^4 \text{ N/C})(0.0600 \text{ m}^2) \cos 60.0^\circ \\ &= \boxed{+2.34 \text{ kN} \cdot \text{m}^2 / \text{C}} \end{aligned}$$

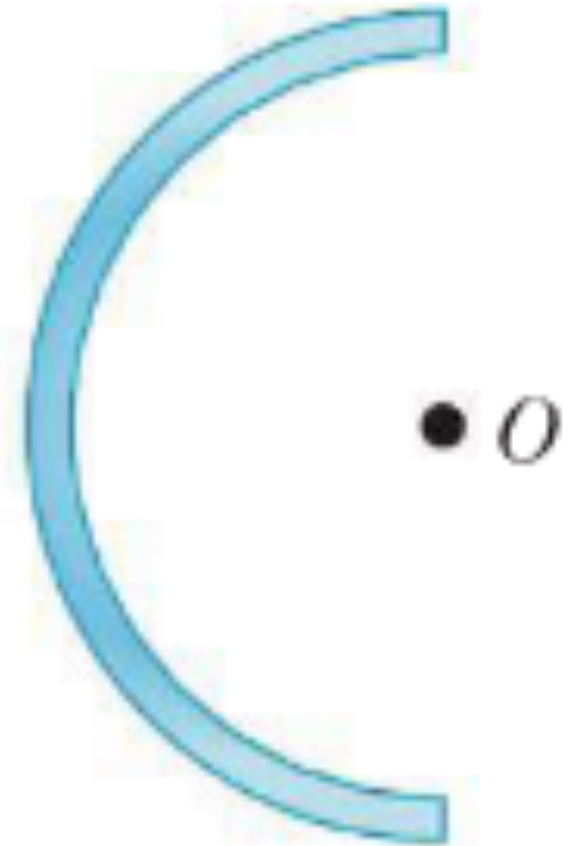
Example Problem 3: Solution

- (c) The bottom and the two triangular sides all lie *parallel* to \vec{E} , so $\Phi_E = 0$ for each of these. Thus,

$$\Phi_{E, \text{ total}} = -2.34 \text{ kN} \cdot \text{m}^2 / C + 2.34 \text{ kN} \cdot \text{m}^2 / C + 0 + 0 + 0 = \boxed{0}$$

Example Problem 4

45. A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure P23.45. The rod has a total charge of $-7.50 \mu\text{C}$. Find (a) the magnitude and (b) the direction of the electric field at O , the center of the semicircle.

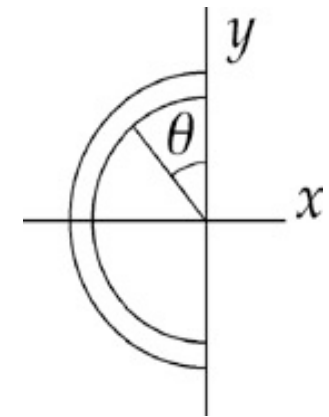


Example Problem 4: Solution

Due to symmetry, $E_y = \int dE_y = 0$, and

$$E_x = -\int dE \sin \theta = -k_e \int \frac{dq \sin \theta}{r^2} \text{ where } dq = \lambda ds = \lambda r d\theta; \text{ the}$$

component E_x is negative because charge $q = -7.50 \mu\text{C}$, causing the net electric field to be directed to the left.



ANS. FIG.
P23.45

$$E_x = -\frac{k_e \lambda}{r} \int_0^\pi \sin \theta d\theta = -\frac{k_e \lambda}{r} (-\cos \theta) \Big|_0^\pi = -\frac{2k_e \lambda}{r}$$

where $\lambda = \frac{|q|}{L}$ and $r = \frac{L}{\pi}$. Thus,

$$E_x = -\frac{2k_e |q| \pi}{L^2} = -\frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(7.50 \times 10^{-6} \text{ C})\pi}{(0.140 \text{ m})^2}$$

$$E_x = -2.16 \times 10^7 \text{ N/C}$$

(a) magnitude $E = \boxed{2.16 \times 10^7 \text{ N/C}}$

(b) $\boxed{\text{to the left}}$

Electric field equations for simple objects

Point charge (charge = q)

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \text{ (at distance } r \text{ from } q)$$

Conducting sphere (charge = Q)

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \text{ (outside, distance } r \text{ from center)}$$

$$\vec{E} = 0 \text{ (inside)}$$

Uniformly charged insulating sphere (charge = Q , radius = r_0)

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \text{ (outside, distance } r \text{ from center)}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Qr}{r_0^3} \hat{r} \text{ (inside, distance } r \text{ from center)}$$

Infinite line charge (linear charge density = λ)

$$\vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} \hat{r} \text{ (distance } r \text{ from line)}$$

Infinite flat plane (surface charge density = σ)

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

Gauss's law

Gauss's law is an expression of the **general relationship between the net electric flux through a closed surface and the charge enclosed by the surface.**

$$\Phi \propto q$$

- The closed surface is often called a *gaussian surface*.

Gauss's law is of fundamental importance in the study of electric fields.

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Karl Friedrich Gauss
1777 – 1855

Gauss's law

A positive point charge, q , is located at the center of a sphere of radius r .

The magnitude of the electric field everywhere **on the surface of the sphere is**

$$E = k_e q / r^2$$

The field lines are directed radially outward and are perpendicular to the surface at every point.

The area of a sphere is $A_{\text{sphere}} = 4\pi r^2$

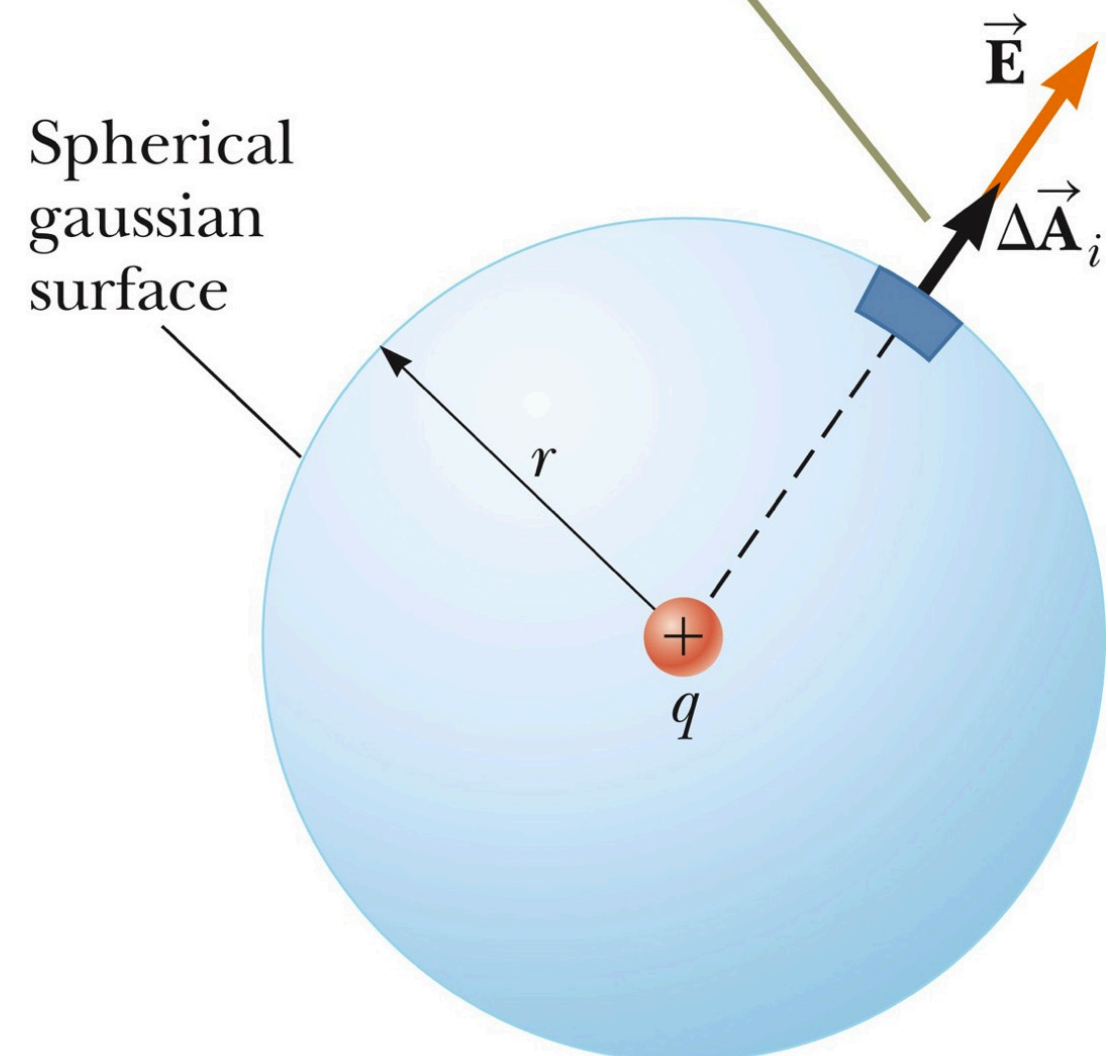
The electric flux is:

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E_n dA$$

$$\Phi_E = E_n A$$

$$\Phi_E = 4\pi k_e q = \frac{q}{\epsilon_0}$$

When the charge is at the center of the sphere, the electric field is everywhere normal to the surface and constant in magnitude.



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$$\blacksquare k_e = 8.9876 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 = 1/(4\pi\epsilon_0)$$

Gauss's Law

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

This is our first Maxwell's equation.

- Q is the net charge inside the surface.
- E is the *total electric field* and may have contributions from charges both inside and outside of the surface.

The net flux through any closed surface surrounding a point charge, q , is given by q/ϵ_0 and is independent of the shape of that surface.

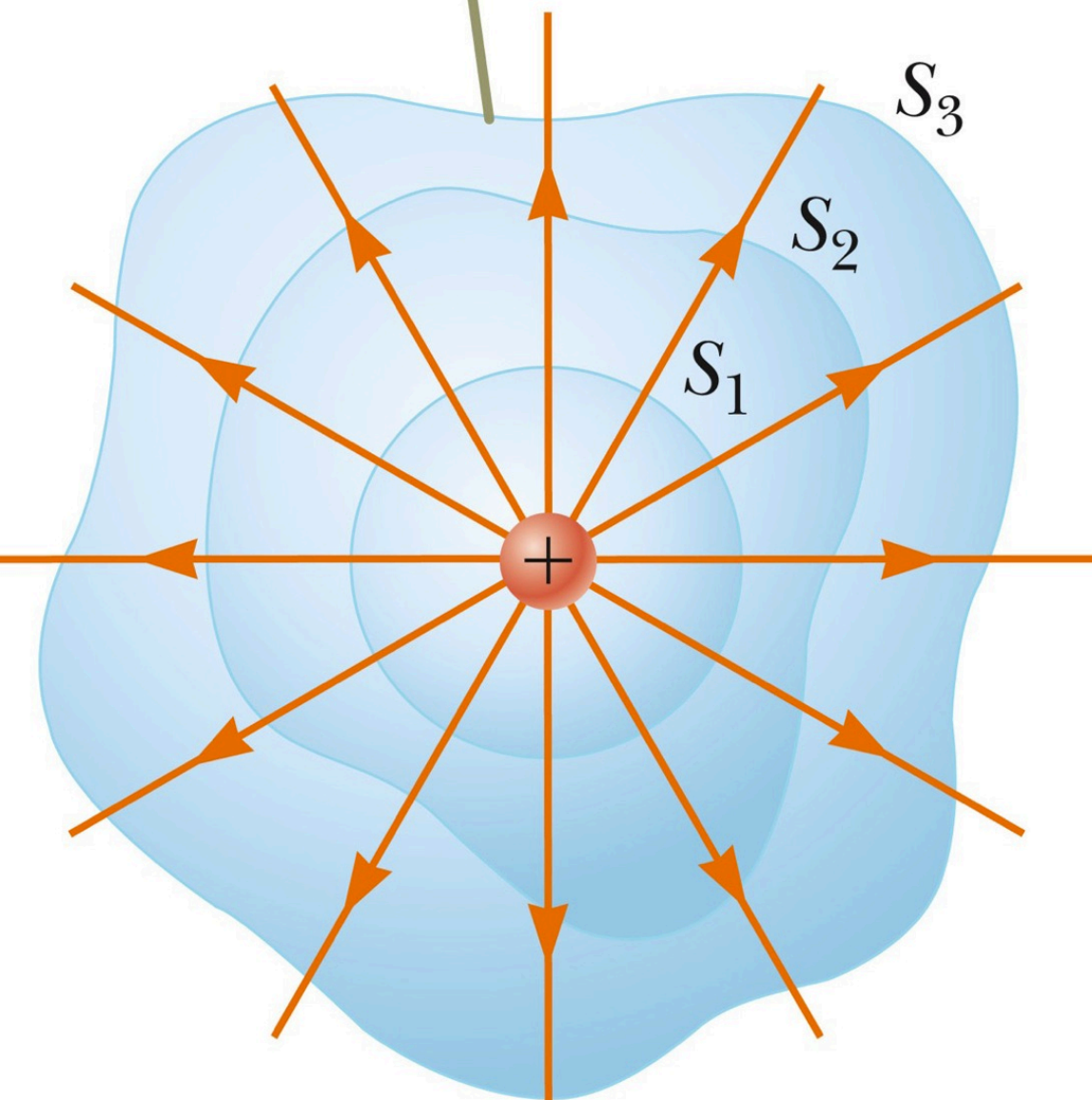
If there is no charge within the closed surface, there is no flux .

Although Gauss's law can, in theory, be solved to find E for any charge configuration, in practice it is limited to symmetric situations.

Gaussian Surface

A closed surface where you can apply Gauss's law.

The net electric flux is the same through all surfaces.



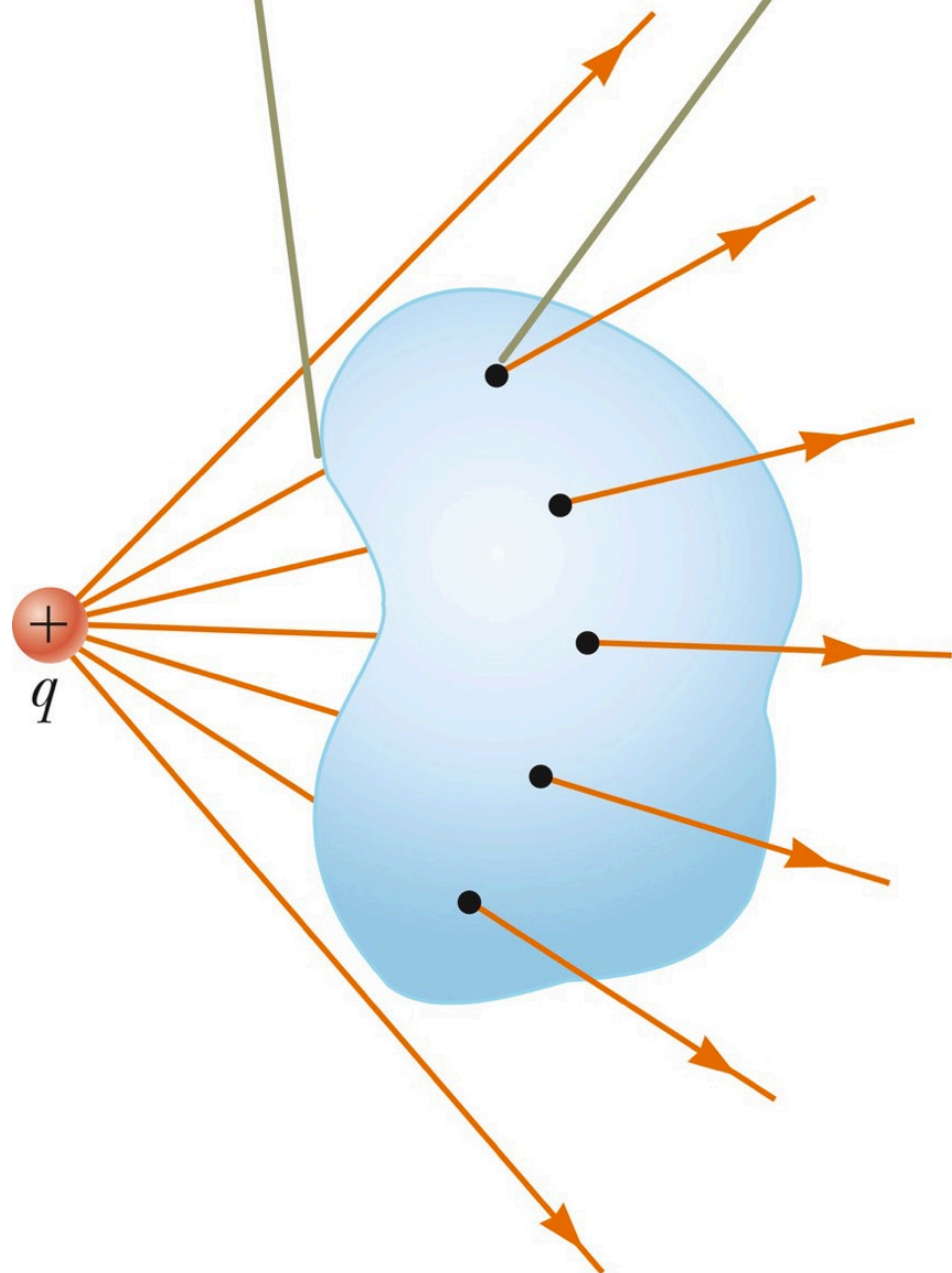
The charge is **inside** the surface.

The shape of the surface doesn't matter.

The net flux through any closed surface surrounding a point charge q is given by q/ϵ_0 .

Gaussian Surface

The number of field lines entering the surface equals the number leaving the surface.



The charge is **outside** the closed surface with an arbitrary shape.

Any field line entering the surface leaves at another point.

The electric flux through a closed surface that surrounds no charge is **zero**.

Flux due to several charges

Since the electric field due to many charges is the vector sum of the electric fields produced by the individual charges, the flux through any closed surface can be expressed as

$$\oint \vec{E} \cdot d\vec{A} = \oint (\vec{E}_1 + \vec{E}_2) \cdot d\vec{A}$$

Remember the dot product is **distributive**

Applying Gauss's law

To use Gauss's law, you want to choose a gaussian surface over which **the surface integral can be simplified and the electric field determined.**

Take advantage of **symmetry**.

Remember, the gaussian surface is a surface **you choose**, it does not have to coincide with a real surface.

Try to choose a surface that satisfies one or more of these conditions:

- The value of the electric field can be argued from symmetry to be constant over the surface.
- The dot product of **\mathbf{E} and $d\mathbf{A}$** can be expressed as a simple algebraic product EdA because **\mathbf{E} and $d\mathbf{A}$** are parallel.
- The dot product is 0 because **\mathbf{E} and $d\mathbf{A}$** are perpendicular.
- The field is zero over the portion of the surface.

If the charge distribution does not have sufficient symmetry such that a gaussian surface that satisfies these conditions can be found, Gauss' law is not useful for determining the electric field for that charge distribution.

Electric field of a sphere

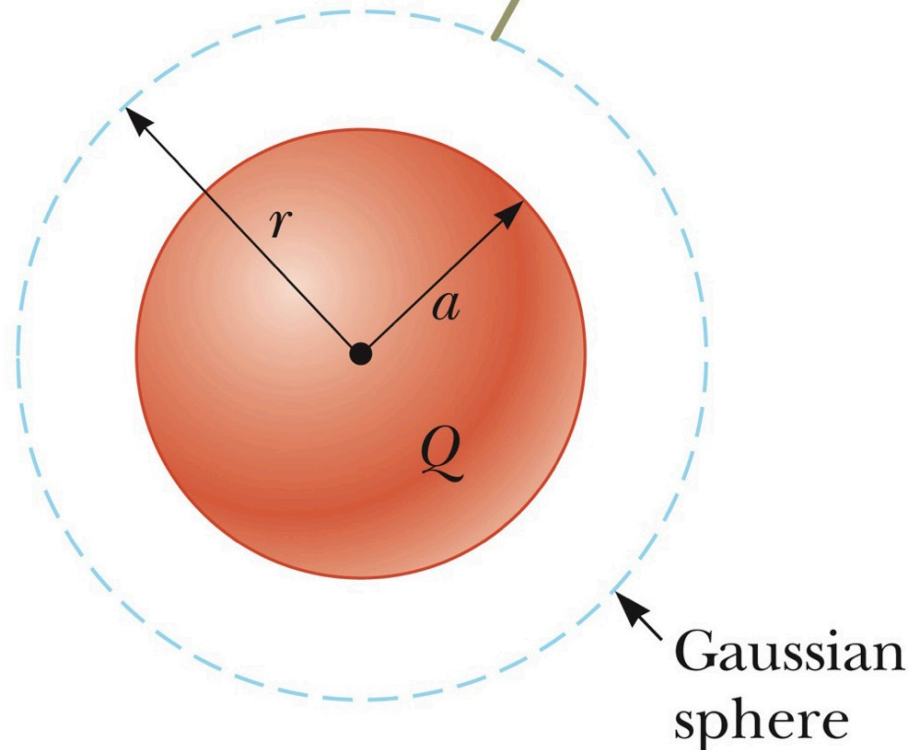
On the **outside** of a conducting or insulating sphere (without Gauss's law)



$$\vec{E} = ?$$

$$E = k_e \frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

For points outside the sphere, a large, spherical gaussian surface is drawn concentric with the sphere.



Select a sphere as the gaussian surface.
For $r > a$: (with Gauss's law)

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E_n dA = \frac{q_{enc}}{\epsilon_0}$$

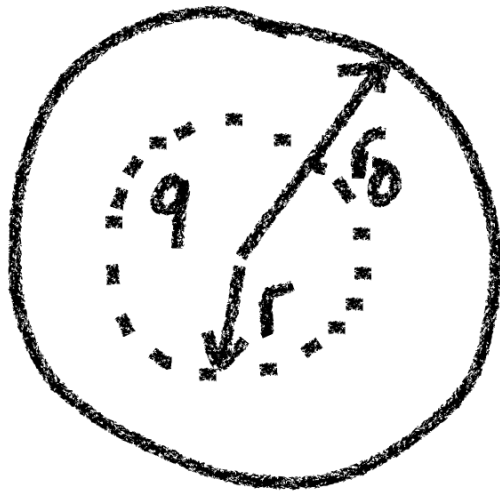
The area of a sphere is $A_{\text{sphere}} = 4\pi r^2$

$$E \times 4\pi r^2 = \frac{q_{enc}}{\epsilon_0}$$

$$E = \frac{q_{enc}}{4\pi r^2 \epsilon_0}$$

Electric field of a sphere

On the **inside** of an insulating sphere (this is what we did before **without Gauss's law**)



$$Q = \rho V = \rho \frac{4\pi r_0^3}{3}$$

$$q = \rho V = \rho \frac{4\pi r^3}{3} = \frac{Q}{r_0^3} r^3$$

$$E = k_e \frac{q}{r^2} = k_e \frac{Q}{r_0^3} \frac{r^3}{r^2}$$

The sphere has total charge Q .
 q is the charge in the inside volume.

$$= \frac{1}{4\pi\epsilon_0} \frac{Qr}{r_0^3}$$

Akin to selecting a sphere as the gaussian surface at r . (**with Gauss's law**)

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E_n dA = \frac{q_{enc}}{\epsilon_0} \quad q_{enc} = \frac{Qr^3}{r_0^3}$$

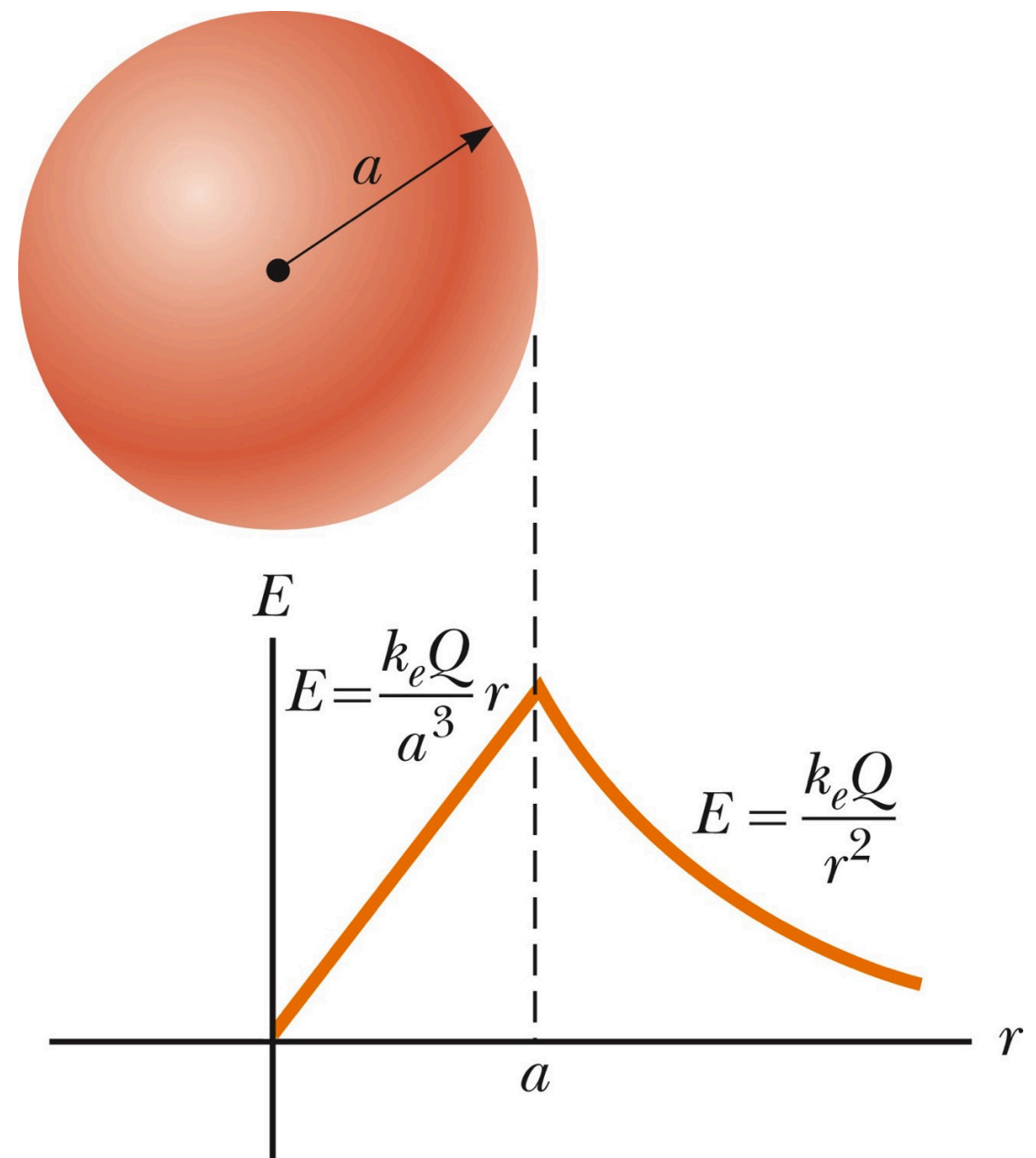
$$EA = \frac{q_{enc}}{\epsilon_0} \quad E = \frac{q_{enc}}{4\pi r^2 \epsilon_0} = \frac{Qr^3}{r_0^3} \frac{1}{4\pi r^2 \epsilon_0} = \frac{Qr}{4\pi\epsilon_0 r_0^3}$$

E field due to spherically symmetric charge distribution - Summary

Inside the sphere, E varies linearly with r

- $E \rightarrow 0$ as $r \rightarrow 0$

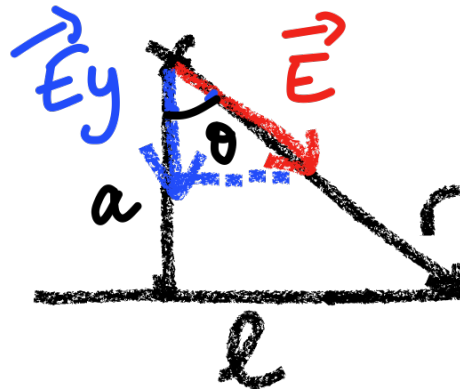
The field outside the sphere is equivalent to that of a point charge located at the center of the sphere.



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Electric field on an infinite line charge

$$\vec{E} = k_e \int \frac{dq}{r^2} \hat{r}$$



(without Gauss's law)

$$dE = k_e \frac{dq}{l^2 + a^2}$$

$$E_x = 0$$

$$E_y = \int dE_y = \int dE \cos \theta$$

$$E_y = \int_{-\infty}^{+\infty} k_e \underbrace{\frac{dq}{l^2 + a^2}}_{dE} \underbrace{\frac{a}{\sqrt{l^2 + a^2}}}_{\cos \theta}$$

$$\cos \theta = \frac{a}{\sqrt{l^2 + a^2}}$$

$$E_y = \int_{-\infty}^{+\infty} k_e a \frac{\lambda dl}{(l^2 + a^2)^{3/2}} = k_e a \lambda \int_{-\infty}^{+\infty} \frac{dl}{(l^2 + a^2)^{3/2}}$$

$$= k_e a \lambda \left[\frac{l}{a^2 \sqrt{l^2 + a^2}} \right]_{-\infty}^{+\infty}$$

$$= k_e \lambda \frac{2}{a}$$

$$= \frac{2k_e \lambda}{a}$$

We will go back to this next week.

There are many ways to solve an E&M problem.

They are not all equal (this is the hard way).

Remember: no net field component along x because each E_x cancels out.

Electric field on an infinite line charge

- Select a gaussian surface as a cylinder with radius of r and a length of ℓ .
- \vec{E} is constant in magnitude and perpendicular to the surface at every point on the curved part of the surface.
- Use Gauss's law to find the field.

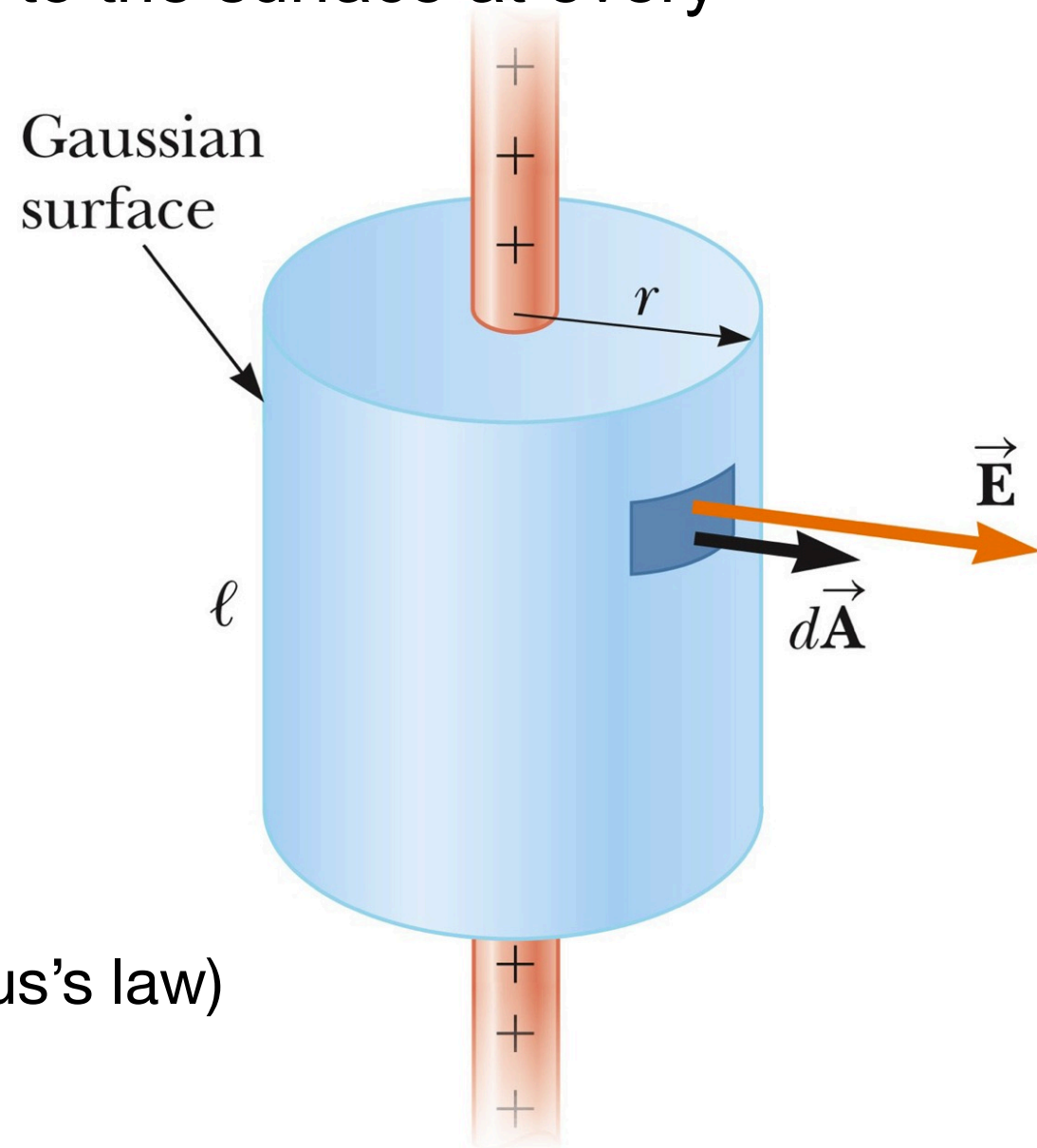
$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E_n dA = \frac{q_{enc}}{\epsilon_0}$$

$$E \times 2\pi r \ell = \frac{q_{enc}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$E = 2k_e \frac{\lambda}{r}$$

(with Gauss's law)



The field through the ends of the cylinder is 0 since the field is parallel to these surfaces (which is the same conclusion as saying E_x cancel out).

Field due to a plane of charge

\vec{E} must be perpendicular to the plane and must have the same magnitude at all points equidistant from the plane.

Choose a small cylinder whose axis is perpendicular to the plane for the gaussian surface.

\vec{E} is parallel to the curved surface and there is no contribution to the surface area from this curved part of the cylinder.

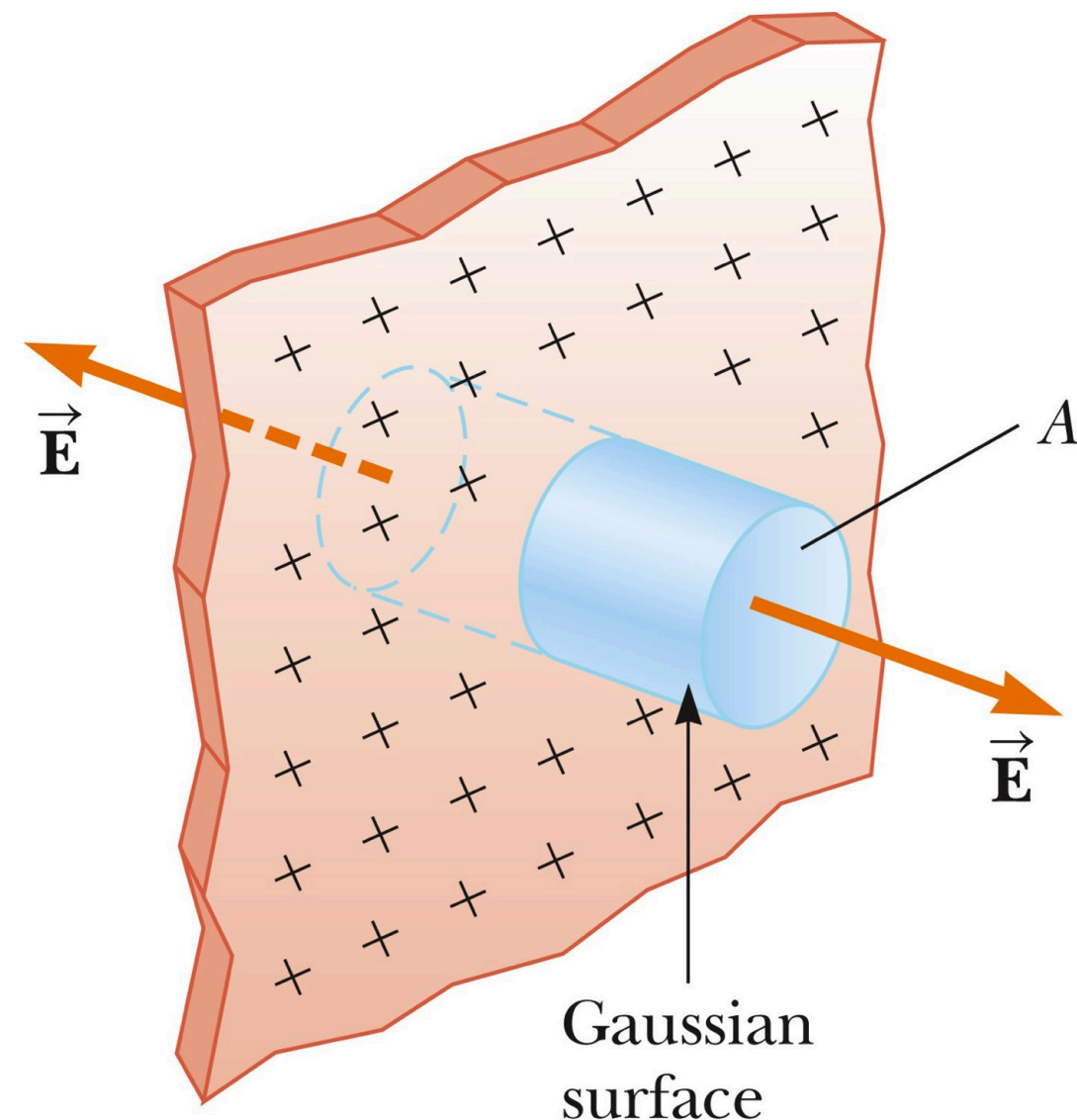
The flux through each end of the cylinder is EA and so **the total flux is $2EA$.**

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E_n dA = \frac{q_{enc}}{\epsilon_0}$$

The total charge in the surface is $q = \sigma A$.
Applying Gauss's law:

$$\Phi_E = 2EA = \frac{\sigma A}{\epsilon_0} \text{ and } \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Note, this does not depend on r .
Therefore, **the field is uniform everywhere.**



Properties of conductors

- When there is no net motion of charge within a conductor, the conductor is said to be in **electrostatic equilibrium**.
- **The electric field is zero everywhere inside the conductor.**
 - Whether the conductor is solid or hollow
- If the conductor is isolated and carries a charge, **the charge resides on its surface.**
- **The electric field at a point just outside a charged conductor is perpendicular to the surface and has a magnitude of σ/ϵ_0 .**
 - σ is the surface charge density at that point.
- On an irregularly shaped conductor, **the surface charge density is greatest at locations where the radius of curvature is the smallest.**

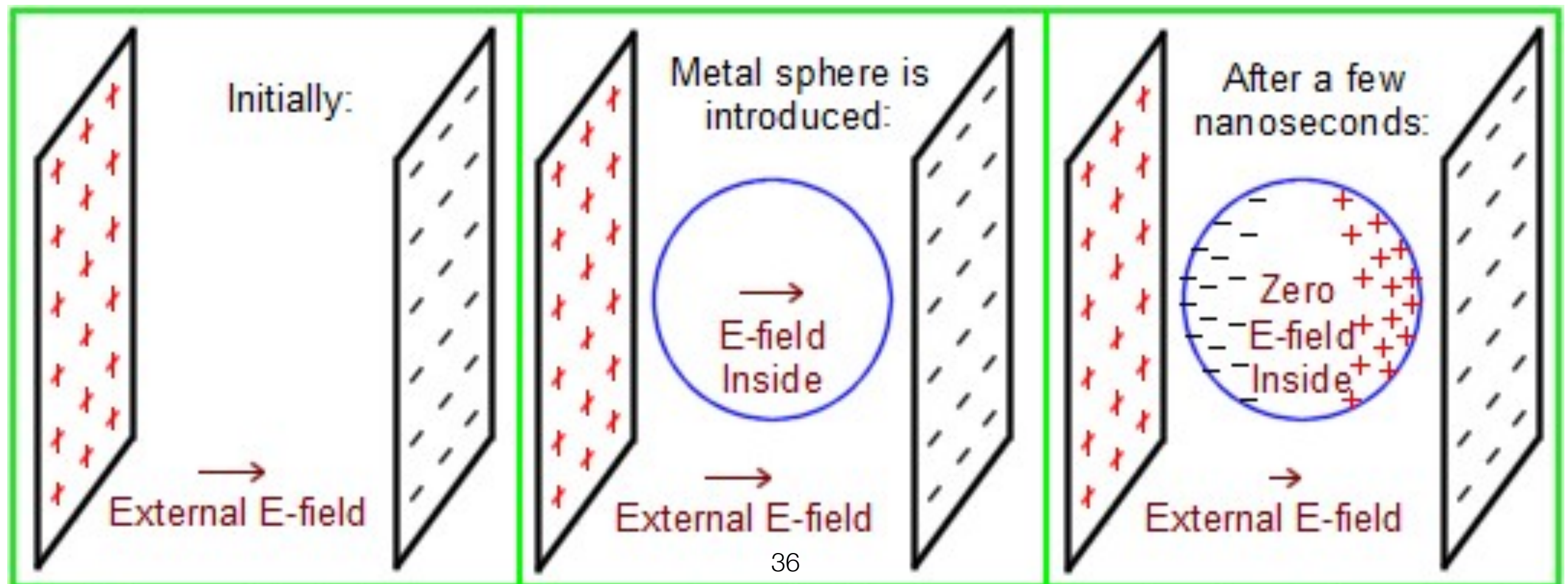
E field inside conductor = 0

Remember a conductor contains electrons that can move freely.

Take a conductor, before the external field is applied, free electrons are uniformly distributed throughout the conductor (therefore there is no net field, if there was a field, the electrons would move).

Place the conductor in an electric field: the charges rearrange themselves and follow the electric field lines. They move as far apart as they can from each other, which means they settle **on** the surface.

“The charges move in a conductor so as to kill the external field.”



Field's magnitude and direction for conductors

Choose a cylinder as the gaussian surface.
The field must be perpendicular to the surface.

- If there were a parallel component to \mathbf{E} , charges would experience a force and accelerate along the surface and it would not be in equilibrium.

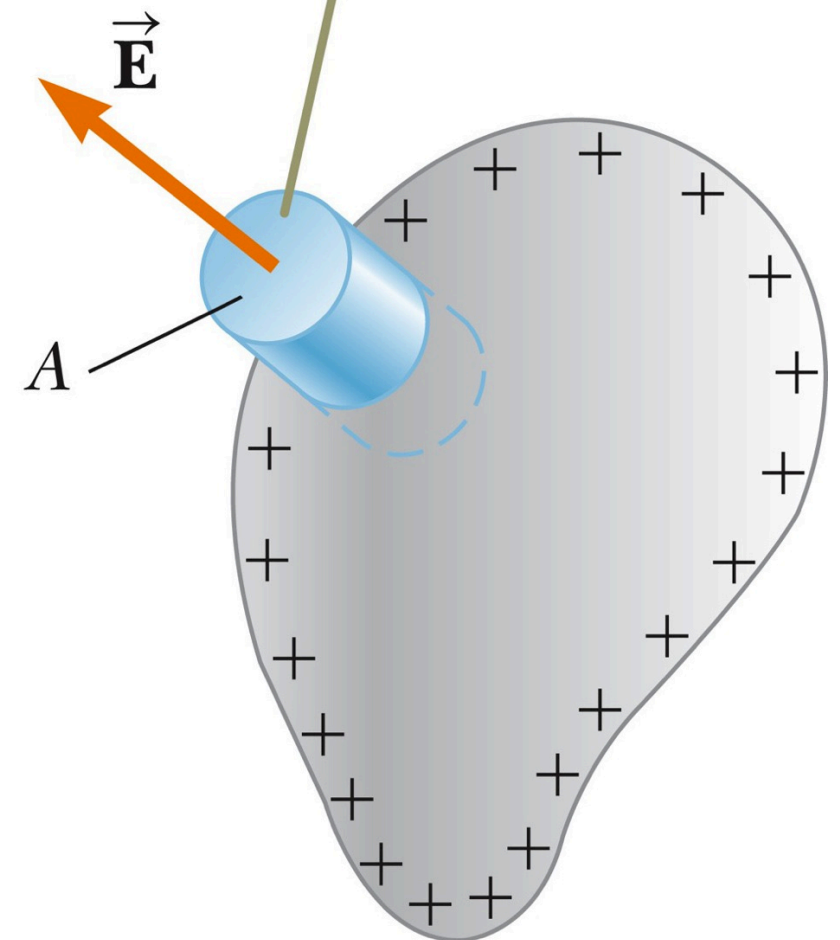
The net flux through the gaussian surface is through only the flat face outside the conductor.

- The field here is perpendicular to the surface.

Applying Gauss's law:

$$\Phi_E = EA = \frac{\sigma A}{\epsilon_0} \text{ and } E = \frac{\sigma}{\epsilon_0}$$

The flux through the gaussian surface is EA .



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Sphere and shell example

A charged sphere with charge Q is surrounded by a conducting shell with total charge $-2Q$.

Find the electric field in regions 1, 2, 3, and 4.

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E_n dA = \frac{q_{enc}}{\epsilon_0}$$

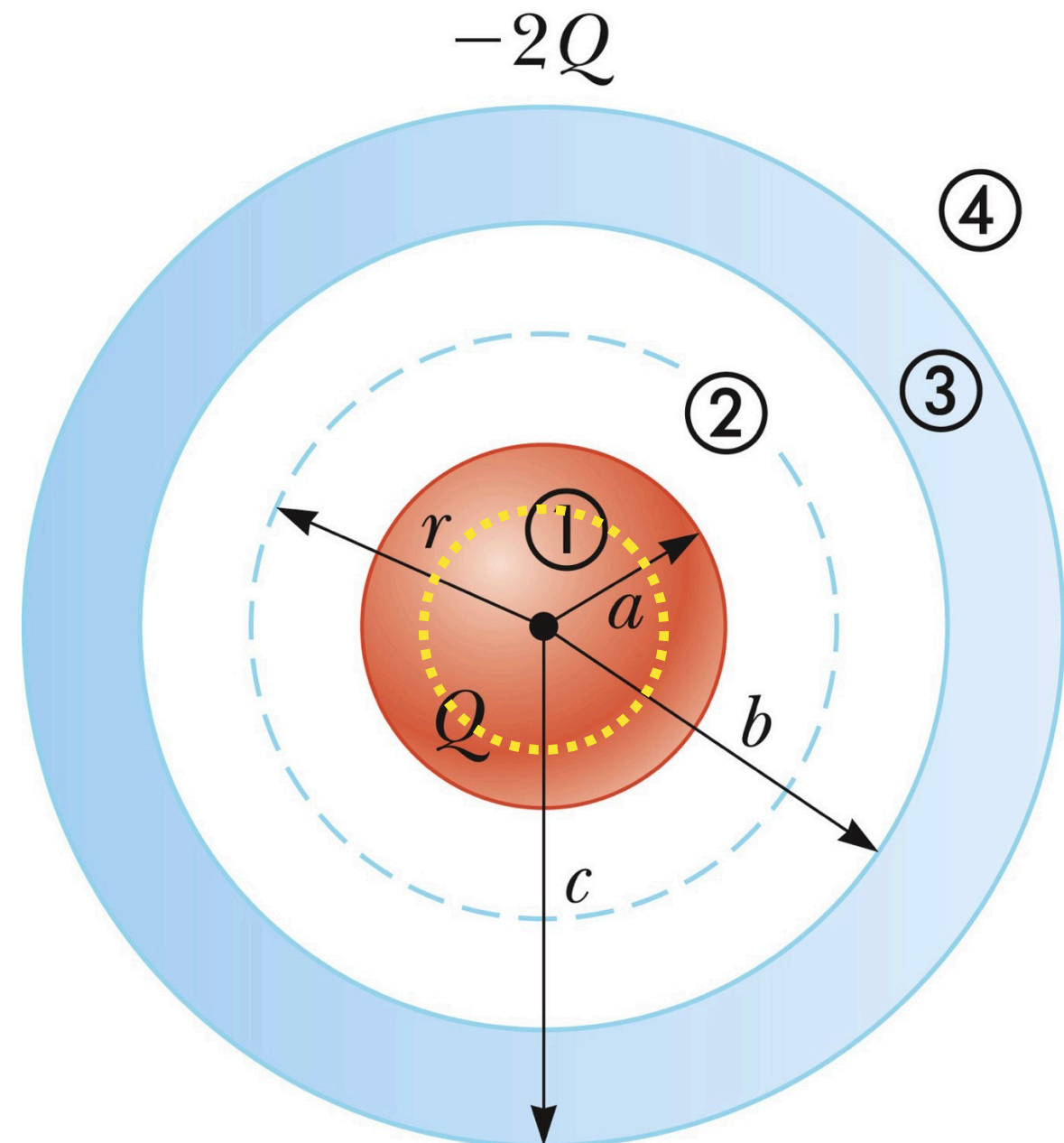
Region 1: gaussian surface dotted yellow (see slide 29). Charge enclosed is Q .

$$E_1 = k_e \frac{Q}{a^3} r \quad (\text{for } r < a) \quad \text{insulating sphere}$$

$$E_1 = 0 \quad \text{conducting sphere}$$

Region 2: gaussian surface is dashed blue. Charge enclosed is Q (see slide 29)

$$E_2 = k_e \frac{Q}{r^2} \quad (\text{for } a < r < b)$$



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Sphere and shell example

A charged sphere with charge Q is surrounded by a conducting shell with charge $-2Q$.

Find the electric field in regions 1, 2, 3, and 4.

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E_n dA = \frac{q_{enc}}{\epsilon_0}$$

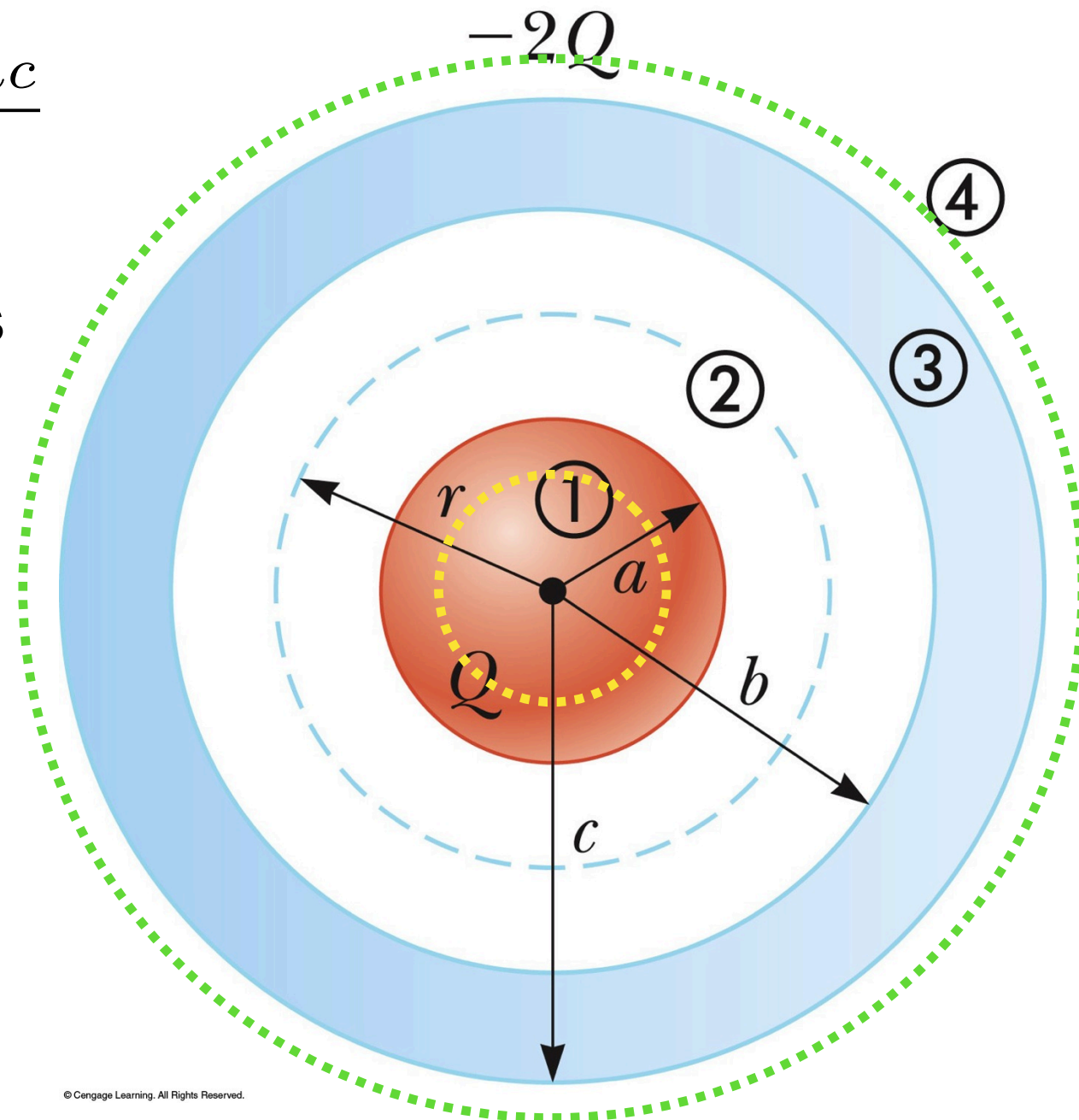
Region 3: Charge enclosed in this region is 0.

$$E_3 = 0 \quad (\text{for } b < r < c)$$

Region 4: gaussian surface is dashed green. Charge enclosed is $Q - 2Q = -Q$

$$EA = \frac{-Q}{\epsilon_0} \quad E = \frac{-Q}{4\pi r^2 \epsilon_0}$$

$$E_4 = -k_e \frac{Q}{r^2} \quad (\text{for } r > c)$$



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Example Problem 5

What else was Gauss known for?

Example Problem 5: Solution

The Gaussian distribution: the bell curve, or “normal” distribution

Example Problem 6

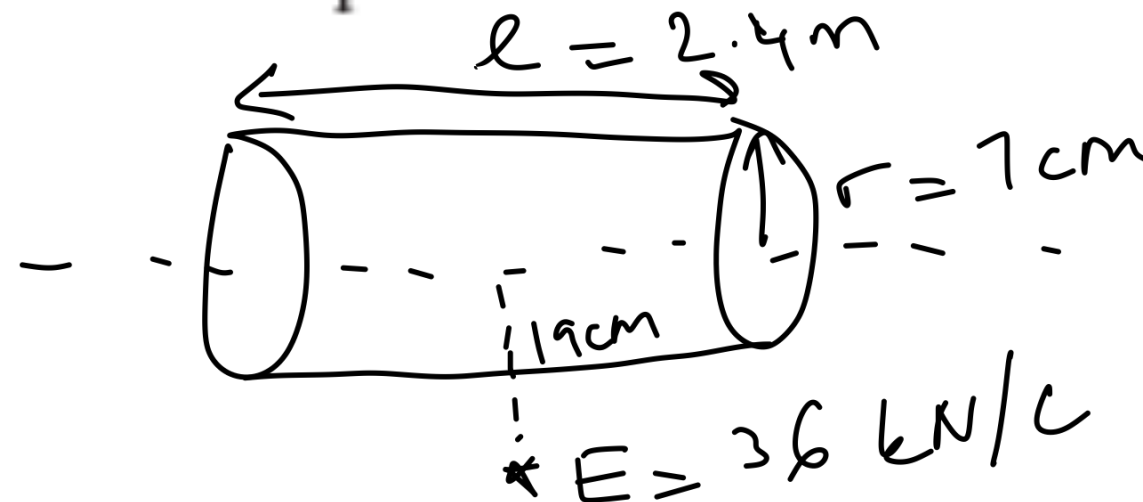
Calculus review: what is the power rule for integration?

Example Problem 6: Solution

$$\int x^n dx = [x^{(n+1)} / (n+1)] + C \text{ (where } n \neq -1)$$

Example Problem 7

- 34.** A cylindrical shell of radius 7.00 cm and length 2.40 m has its charge uniformly distributed on its curved surface. The magnitude of the electric field at a point 19.0 cm radially outward from its axis (measured from the midpoint of the shell) is 36.0 kN/C. Find (a) the net charge on the shell and (b) the electric field at a point 4.00 cm from the axis, measured radially outward from the midpoint of the shell.



Example Problem 7 : solution

- (a) The electric field is given by

$$E = \frac{2k_e \lambda}{r} = \frac{2k_e (Q / \ell)}{r} \quad \text{see slide 33}$$

Solving for the charge Q gives

$$Q = \frac{Er\ell}{2k_e} = \frac{(3.60 \times 10^4 \text{ N/C})(0.190 \text{ m})(2.40 \text{ m})}{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C})} =$$

$$Q = +9.13 \times 10^{-7} \text{ C} = \boxed{+913 \text{ nC}}$$

- (b) Since the charge is uniformly distributed on the surface of the cylindrical shell, a gaussian surface in the shape of a cylinder of 4.00 cm in radius encloses no charge, and $\vec{E} = \boxed{0}$.

Example Problem 8

- 56.** Two infinite, nonconducting sheets of charge are parallel to each other as shown in Figure P24.56. The sheet on the left has a uniform surface charge density σ , and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets. (d) **What If?** Find the electric fields in all three regions if both sheets have *positive* uniform surface charge densities of value σ .

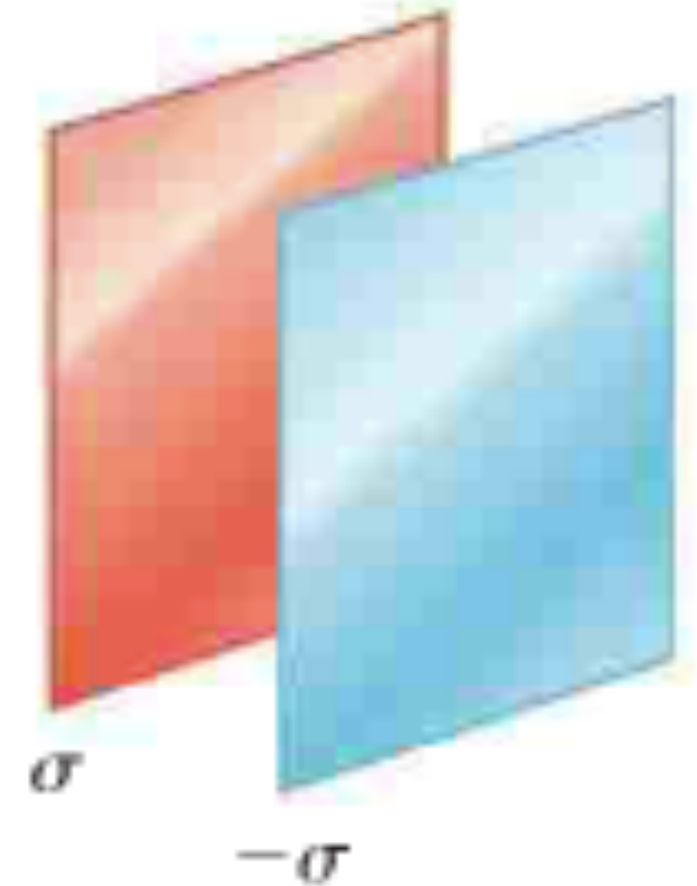


Figure P24.56

Example Problem 8 : solution

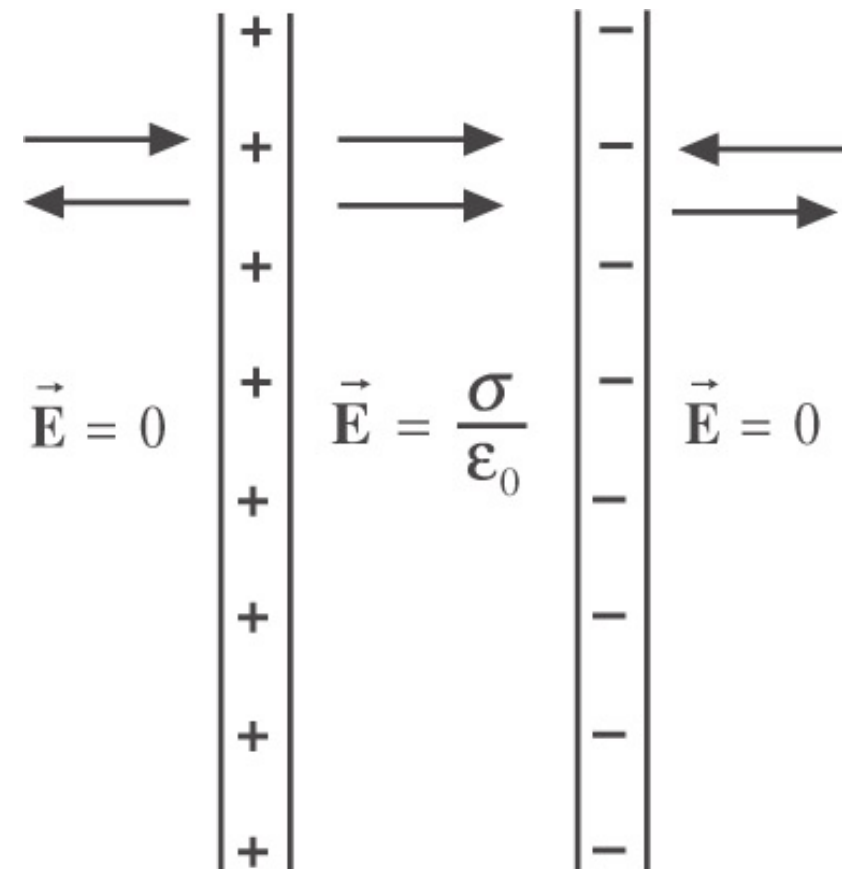
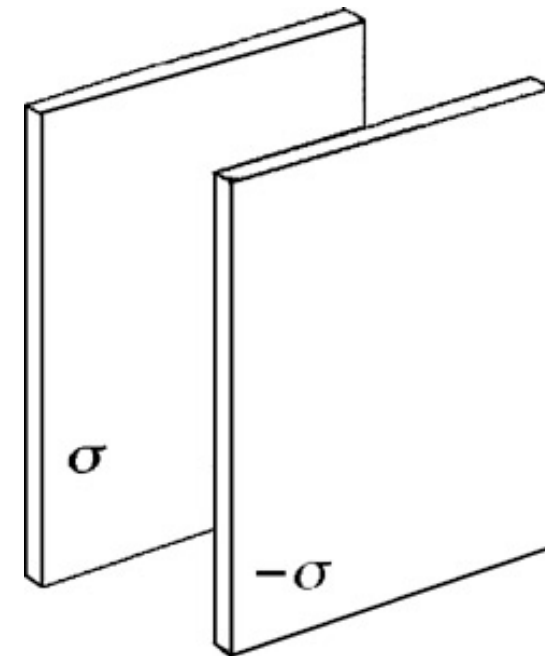
Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by the textbook equation

$$|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0} \quad \text{see slide 34}$$

- (a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\vec{E} = \boxed{0}$.

- (b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\vec{E} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$



ANS. FIG. P24.56(a-c)

Example Problem 8 : solution

- (c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\vec{E} = \boxed{0}$.

- (d) Now, both sheets are positively charged. We find that

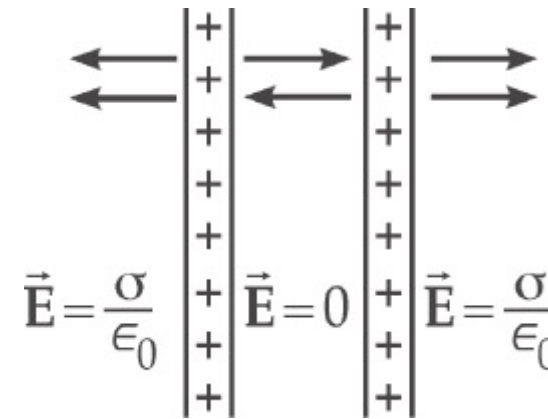
- (1) To the left of both sheets, both fields are directed toward the left:

$$\vec{E} = \boxed{2 \frac{\sigma}{\epsilon_0} \text{ to the left}}$$

- (2) Between the sheets, the fields cancel because they are opposite to each other: $\vec{E} = \boxed{0}$.

- (3) To the right of both sheets, both fields are directed toward the right:

$$\vec{E} = \boxed{2 \frac{\sigma}{\epsilon_0} \text{ to the right}}$$



ANS. FIG. P24.56(d)

Example Problem 9

58

An insulating solid sphere of radius a has a uniform volume charge density and carries a total positive charge Q . A spherical gaussian surface of radius r , which shares a common center with the insulating sphere, is inflated starting from $r = 0$. (a) Find an expression for the electric flux passing through the surface of the gaussian sphere as a function of r for $r < a$. (b) Find an expression for the electric flux for $r > a$. (c) Plot the flux versus r .

Example Problem 9 : solution

The charge density is determined by $Q = \frac{4}{3}\pi a^3 \rho$. Solving gives

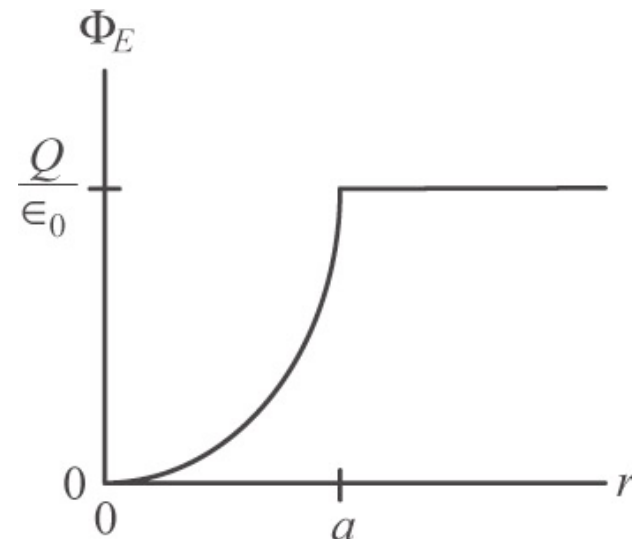
$$\rho = \frac{3Q}{4\pi a^3} \quad \rho = \frac{Q}{V}$$

(a) The flux is that created by the enclosed charge within radius r :

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{4\pi r^3 \rho}{3\epsilon_0} = \frac{4\pi r^3 3Q}{3\epsilon_0 4\pi a^3} = \boxed{\frac{Qr^3}{\epsilon_0 a^3}}$$

(b) $\Phi_E = \boxed{\frac{Q}{\epsilon_0}}$. Note that the answers to parts (a) and (b) agree at $r = a$.

(c) ANS. FIG. P24.58(c) plots the flux vs. r .



ANS. FIG. P24.58(c) ⁵⁰