UNIVERSITY AT ALBANY
State University of New York

College of Arts and Sciences
Department of Mathematics and Statistics

Colloquium

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The Rise, Fall and Rebirth of the Muckenhoupt-Wheeden Conjectures

Friday, November 7, 2014
3:00 p.m. in ES-143
(tea & coffee at 2:30 p.m. in ES-152)

Abstract. In the 1970s, Muckenhoupt, Wheeden and others began the systematic study of weighted norm inequalities. In the so-called one-weight case, they showed that the correct condition for many weak and strong-type inequalities for maximal operators or singular integrals is that $w \in A_p$:

$$[w]_{A_p} = \sup_Q \left(\fint_Q w(x)\,dx \right) \left(\fint_Q w(x)^{1-p'}\,dx \right)^{p-1} < \infty.$$

When the single weight w is replaced by a pair of weights (u, v) the analogous two-weight A_p condition is necessary but not sufficient. Muckenhoupt and Wheeden made a series of conjectures for sufficient conditions for singular integrals to satisfy two-weight, weak and strong (p, p) inequalities. Though very influential, almost no direct progress was made on these conjectures until very recently.

Since 2005 the Muckenhoupt-Wheeden conjectures became the subject of renewed interest because they were shown to be closely connected to the important A_2 conjecture regarding sharp constants in one-weight norm inequalities. In 2010 the A_2 conjecture was proved, and soon thereafter all of the original conjectures were shown to be false. But at the same time a number of related conjectures were proved and new problems were raised.

In this talk I will discuss the history of the Muckenhoupt-Wheeden conjectures, sketch their connection to the A_2 conjecture, and then describe recent joint work with Chema Martell, Kabe Moen, and Carlos Pérez on generalizations of the Muckenhoupt-Wheeden conjectures.