University at Albany, State University of New York

Department of Mathematics and Statistics

Colloquium

Alfonso Montes Rodríguez

University of Sevilla, Spain

Uniqueness Sets for the Klein-Gordon Equation and the Solution of a Conjecture of Salem

Friday, November 12, 2010
4:30 p.m. in ES-143
(tea & coffee at 4:00 p.m. in ES-152)

Abstract. Let (Γ, Λ) be a pair where Γ is a curve contained in $\mathbb{C}$ and Λ is a set also contained in $\mathbb{C}$. We say that (Γ, Λ) is a Heisenberg uniqueness pair if each bounded Borel measure μ supported in Γ, which is absolutely continuous with respect to the arc length, and whose Fourier transform $\widehat{\mu}$ vanishes on $\Lambda \subset \mathbb{C}$, must be automatically the zero measure. We prove that if Γ is the hyperbola $x_1 x_2 = 1$, and Λ is the lattice-cross $\Lambda = (\alpha\mathbb{Z} \times \{0\}) \cup (\{0\} \times \beta\mathbb{Z})$, then (Γ, Λ) is Heisenberg uniqueness pair if and only if $\alpha\beta < 1$; in this situation $\widehat{\mu}$ solves the Klein-Gordon equation. As a consequence we solve a problem about density of linear algebras generated by two inner functions posed by Stessin and Matheron. As for continuous singular measures, the result fails to be true. In order to prove this, we have to deal with the Minkowski measure, which is usually studied in Number Theory. Indeed, we prove that the Minkowski measure is a Rajchman measure, that is, the Fourier transform of the Minkowski measure vanishes at ∞. Thus, in particular, answering a conjecture by Salem in 1942, a result that has some interesting applications in Number Theory.

Joint work with Hakan Hedenmalm, Royal Institute of Technology, Stockholm (Sweden).