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Colloquium

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Cubic Column Relations in Truncated Moment Problems

Friday, October 15, 2010
3:00 p.m. in ES-143
(Tea at 2:30 p.m. in ES-152)

ABSTRACT. Inverse problems naturally occur in many branches of science and mathematics. An inverse problem entails finding the values of one or more parameters using the values obtained from observed data. A typical example of an inverse problem is the inversion of the Radon transform. Here a function (for example of two variables) is deduced from its integrals along all possible lines. This problem is intimately connected with image reconstruction for X-ray computerized tomography.

Moment problems are a special class of inverse problems. While the classical theory of moments dates back to the beginning of the 20th century, the systematic study of *truncated* moment problems began only a few years ago. In this talk we will first survey the elementary theory of truncated moment problems, and then focus on those problems with cubic column relations.

For a degree $2n$ real d-dimensional multisequence $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{i \in Z_+^d, |i| \leq 2n}$ to have a representing measure μ, it is necessary for the *associated moment matrix* $M(n)$ to be positive semidefinite, and for the *algebraic variety* associated to β, V_β, to satisfy rank $M(n) \leq$ card V_β as well as the following *consistency condition*: if a polynomial $p(x) \equiv \sum_{|i| \leq 2n} a_i x^i$ vanishes on V_β, then $p(\beta) := \sum_{|i| \leq 2n} a_i \beta_i = 0$. In previous joint work with L. Fialkow and M. Möller, we proved that for the *extremal* case (rank $M(n) =$ card V_β), positivity and consistency are sufficient for the existence of a (unique, rank $M(n)$-atomic) representing measure.

In recent joint work with Seonguk Yoo we consider cubic column relations in $M(3)$ of the form (in complex notation) $Z^3 = itZ + u\bar{Z}$, where u and t are real numbers. For (u, t) in the interior of a real cone, we prove that the algebraic variety V_β consists of exactly 7 points, and we then apply the above mentioned solution of the extremal moment problem to obtain a necessary and sufficient condition for the existence of a representing measure. This requires a new representation theorem for sextic polynomials in Z and $\bar{Z}$ which vanish in the 7-point set V_β. Our proof of this representation theorem relies on two successive applications of the Fundamental Theorem of Linear Algebra.