High School Math Problems
2015
Week 51
Problem and Solution

Simplify the expression

$$I(x) = \sqrt{9 + x^2 - 6x} + \sqrt{x^2 - 4x + 4}.$$

Solution:

$$\begin{aligned} I(x) &= \sqrt{9 + x^2 - 6x} + \sqrt{x^2 - 4x + 4} \\ &= \sqrt{(x-3)^2} + \sqrt{(x-2)^2} \\ &= |x-3| + |x-2| \end{aligned}$$

We now consider the following three cases:
Case 1: $x \in (-\infty, 2]$
In this case $x - 3 < 0$ and $x - 2 < 0$ and therefore $|x-3| = 3 - x$ and $|x-2| = 2 - x$. Therefore

$$I(x) = 3 - x + 2 - x = 5 - 2x.$$

Case 2: $x \in (2, 3]$
In this case $x - 3 < 0$ and $x - 2 > 0$ and therefore $|x-3| = 3 - x$ and $|x-2| = x - 2$. Therefore

$$I(x) = 3 - x + x - 2 = 1.$$

Case 3: $x \in (3, \infty)$
In this case $x - 3 > 0$ and $x - 2 > 0$ and therefore $|x-3| = x - 3$ and $|x-2| = x - 2$. Therefore

$$I(x) = x - 3 + x - 2 = 2x - 5.$$

Summarizing Cases 1, 2, and 3, we therefore have that

$$I(x) = \begin{cases} 5 - 2x & \text{for } x \in (-\infty, 2] \\ 1 & \text{for } x \in (2, 3] \\ 2x - 5 & \text{for } x \in (3, \infty) \end{cases}.$$