High School Math Problems
2015
Week 4
Problem and Solution

Solve the equation

$$\frac{2}{x^2 - x} + \frac{x+1}{x-1} = \frac{2x+3}{x}.$$

Solution:
The domain of this equation is

$$D = \{x \in \mathbb{R} : x \neq 0, 1\}.$$

Taking a common denominator, we obtain the new equation

$$\begin{aligned} 2 + (x+1)\,x &= (2x+3)\,(x-1) \\ 2x^2 + x - 3 - 2 - x^2 - x &= 0 \\ x^2 - 5 &= 0 \\ x^2 &= 5 \\ x &= \pm\sqrt{5} \end{aligned}$$

Since both $\pm\sqrt{5} \in D$, it follows that the solution to the given equation is

$$x = \pm\sqrt{5}.$$