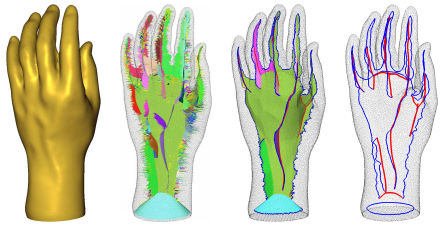
# Regularizing 3D Medial Axis Using Medial Scaffold Transforms

**Ming-Ching Chang** **Benjamin B. Kimia**

Laboratory for Engineering Man/Machine Systems (LEMS), Brown University, Providence RI, USA

## Overview



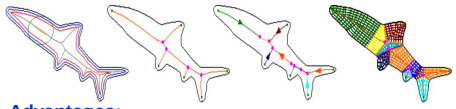
**Goal:** Find a *graph*-based **medial axis (MA)** to *qualitatively* represent 3D shapes.

- The 3D MA generally consists of medial sheets, curves, and isolated points and can be organized into a hypergraph form --- the **Medial Scaffold (MS)**.
- The *instability* of the 3D MA is classified into 7 generic cases (the *transitions*, sudden topological changes).
- A set of *transforms* is defined on the MS in a case-by-case analysis, to model the MA across all transitions.

The manipulation of the medial structure (coupled with the 3D shape) toward a nearby degenerate transition point is the basis of simplification of shape.

## Background and Motivation

The MA is with great promise as a universal model for shape:



**Advantages:**

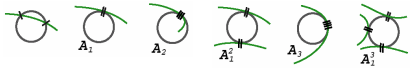
- Intuitive, qualitative description of the essence structure.
- Shape features: make explicit *curvature extrema* (*ridges*).
- Built-in hierarchy of scale: coarse-to-fine, parts made explicit.
- Complete: parameterize the whole embedded space of the shape, allow exact reconstruction.
- Generative: model deformation, generation of shapes.

**Difficulties:**

- Difficult to represent and compute.
- Instability: sensitive to small perturbations.

**Analysis of the MA:**

Notation $\mathbf{A}_k^n$: *k+1* degree of contact at *n* distinct points.



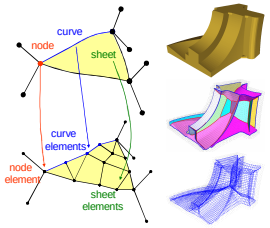

## The Medial Scaffold (MS): Dual-Scale Representation

Coarse-scale: hypergraph

- Vertex: $A_1^4$ or $A_1A_3$ node
- Link: $A_1^3$ or $A_3$ curve
- Hyperlink: $A_1^2$ sheet

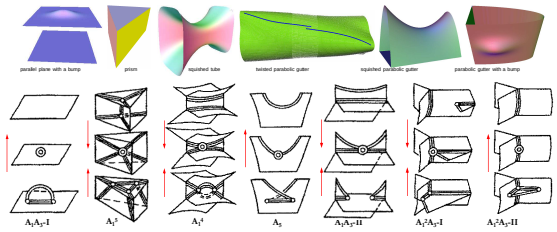
Fine-scale: (non-manifold) mesh

- Vertex: $A_1^4$ node element
- Edge: $A_1^3$ curve element
- Face: $A_1^2$ sheet element

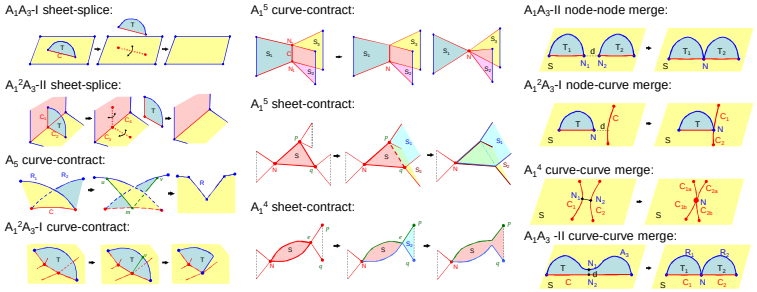



Captures *topology* + *geometry* + *dynamics* + *associated generators* of the MS and approximates the *true* MA of the underlying shape.

## Seven Generic MA Transitions

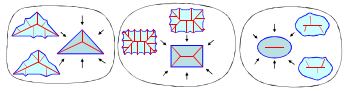



## Eleven MS Transforms (Splice, Contract, Merge)

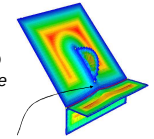



## The Regularization

Move toward a representative shape in each category.

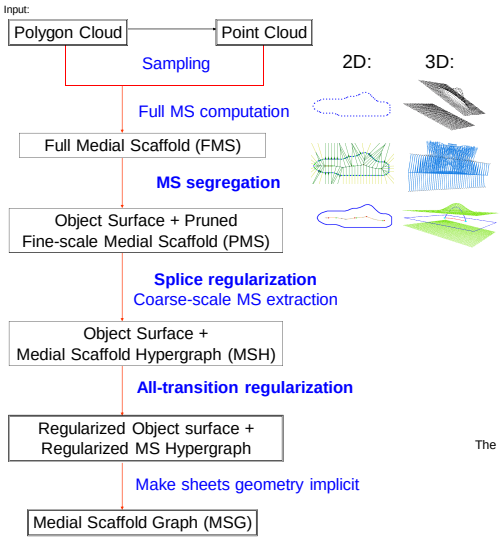


Need a geodesic distance transform to detect the next *merge* transform.

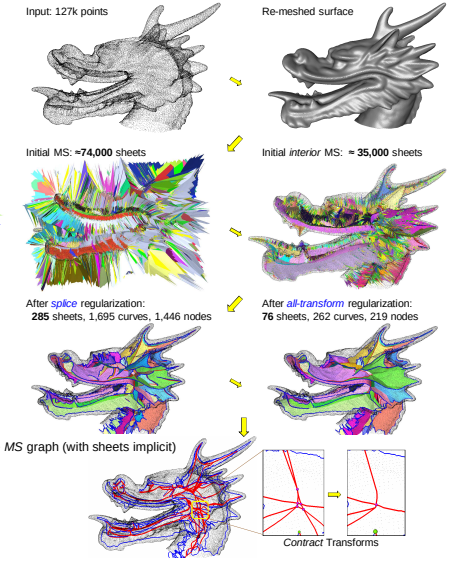


Next transition with lowest cost

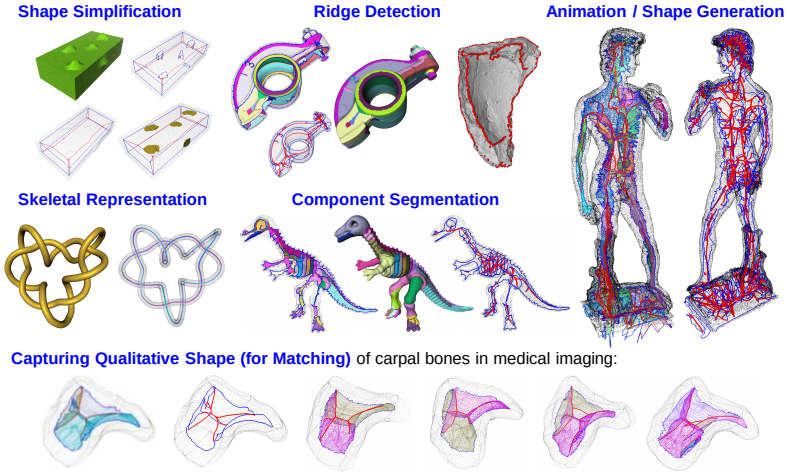
## Computational Scheme




## Result: the Stanford Dragon Head




## Applications

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