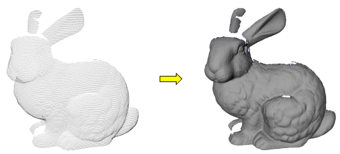
# Surface Reconstruction by Transforming the Medial Scaffold

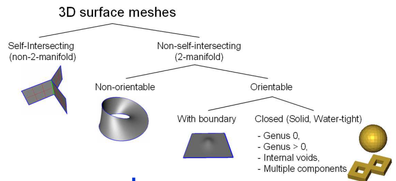

**Ming-Ching Chang** — Brown University, Providence RI, USA
**Frederic F. Leymarie** — Goldsmiths College, University of London, UK
**Benjamin B. Kimia** — Brown University, Providence RI, USA


## Overview



**Problem:** Reconstruct a surface mesh from *unorganized* points, with a *'minimal'* set of assumptions: the samples are nearby a possible surface.

**Goal:** *general* approach applicable to surfaces with various topologies, without assuming knowing surface normals, smoothness, sampling conditions, and able to handle large datasets (millions of points).




**Previous approaches:**

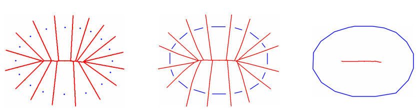
- Implicit distance functions.
- Propagation based methods.
- Voronoi / Delaunay geometric constructs.

## Our Approach

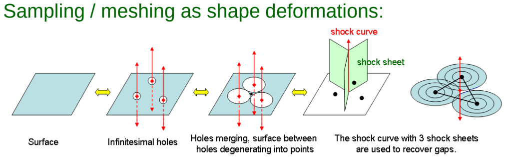
Work on the shape itself to *recover* the sampling process.

**Key ideas:**

- Relate the sampled shape with the underlying shape by a sequence of shape deformations (growing from samples).
- Represent shapes by their *medial* representations: the shock graphs in 2D, the medial scaffolds in 3D.
- Recover the mesh connectivity (on the gaps) by using shock *transitions* across different shock topologies.



Sampling / meshing as shape deformations:



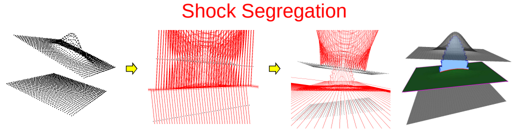

**Shocks:** medial axis points endowed with *dynamics* of flow.
**Gap Transform:** removal of a shock curve and creation of its dual (Delaunay) triangle.

**Medial Scaffold:** classification of shock points into *five* general types and organized into a hyper-graph form.

### Surface meshing and symmetry computation:

- The medial scaffold of a point cloud represents both the symmetries due to sampling and the original object symmetries.
- Rank order medial scaffold transitions (edits), *i.e.*, **gap transforms**, to *segregate* the two types and to simulate the recovery of the sampling process.
- The result is the meshed surface together with its *organized* medial axis (as medial scaffold).

Shock Segregation



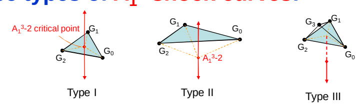
## Greedy Meshing Algorithm

### Ranking shock curves (which represent candidate triangles):

Assign a cost for each shock curve reflecting:

- Likelihood that it represents a surface patch
- Consistency in the local context
- Allowable local surface topology.

### Three types of $A_1^3$ shock curves:

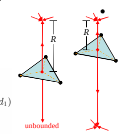



### Cost of an isolated triangle:

$$\rho_1 = \begin{cases} \frac{P}{R} \cdot \frac{1}{C^2}, & \text{if } D < d_{\max} \\ \infty, & \text{if } D \geq d_{\max} \end{cases}$$

Triangle geometry:

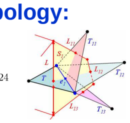
$$\begin{aligned} D &= \max(d_1, d_2, d_3) \\ P &= d_1 + d_2 + d_3 \\ m &= (d_1+d_2-d_3)(d_3+d_1-d_2)(d_2+d_3-d_1) \\ A &= \sqrt{(P \cdot m)}/16 \\ C &= 4\sqrt{3} \cdot A/(d_1^2+d_2^2+d_3^2), \end{aligned}$$



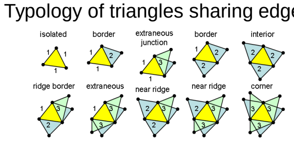

Favor *compact* triangles with large shock radius *R*.

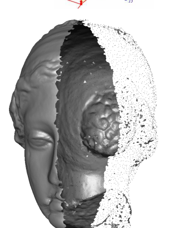
### Cost reflecting local context & topology:

$$\rho_2 = \frac{d}{R} \cdot \frac{1}{C^2} \cdot f(\theta), \quad f(\theta) = [\exp^{\theta} - 1]^2 - 1 \begin{cases} \theta = 0, f(\theta) = -1 \\ \theta = 40°, f(\theta) \simeq 0 \\ \theta = 80°, f(\theta) \simeq 8.24 \end{cases}$$

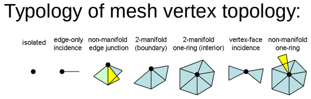


Typology of triangles sharing edges:



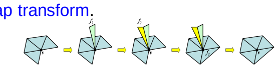



Typology of mesh vertex topology:




### Strategy in handling errors:

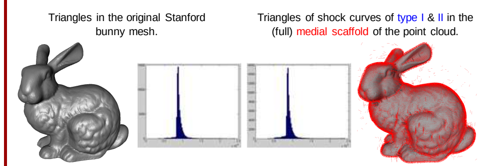
- Multi-pass greedy iterations
  - First construct low-cost triangles without ambiguities.
- Postpone ambiguous decisions
  - Delay *related* candidate shock curves with similar ranks, until additional *supportive context* is available.
  - Delay potential topology violations.
- Error recovery
  - For each gap transform, re-evaluate cost of both related *neighboring (already built)* & *candidate* triangles.
  - If the cost of any existing triangle exceeds the top candidate, *undo* its gap transform.



### Estimate the sampling scale:

The maximum expected triangle size $d_{max}$ can be estimated from shock radius distribution analysis.

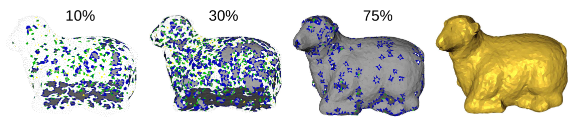
Distribution of the $A_1^3$-2 radii of all the shock curves:




The median of the distribution ($d_{med}$) approximates its peak.

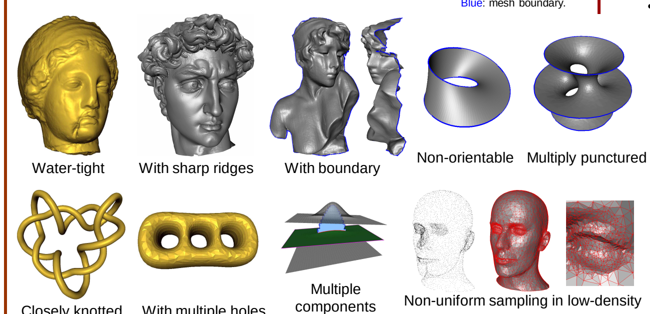
## Results

Visualization of the greedy meshing process:




### Meshing surfaces with various types:

Gold: water-tight surface.
Blue: mesh boundary.




## Extensions

### Re-mesh a partial mesh:

Assign high priority to existing triangles and let candidates compete in the greedy algorithm.

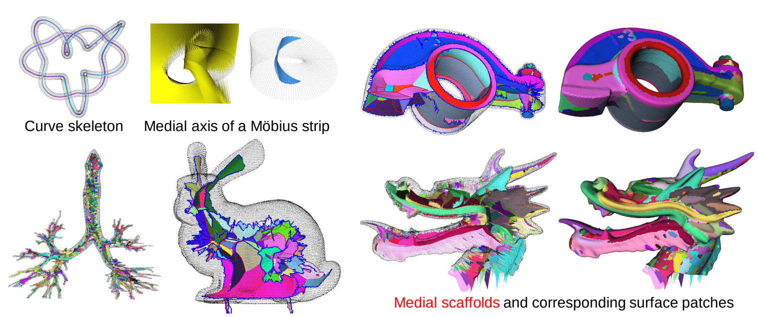
### Handle Large Datasets:

- Divide input into buckets and mesh the surfaces in each bucket.
- Stitch the surfaces by applying the same algorithm again.

Meshing Stanford Asian Dragon (3.6M points) in buckets.

## Medial Axis Computation & Regularization

**Applications:** Shape Analysis, Reconstruction, Segmentation, Manipulation.




Meshing Stanford Thai Statue (5M points) in buckets.


Dataset are courtesy of Cyberware, Stanford, MPII, Stony Brook, Columbia Univ., H. Hoppe.