3D Shape Registration using Regularized Medial Scaffolds

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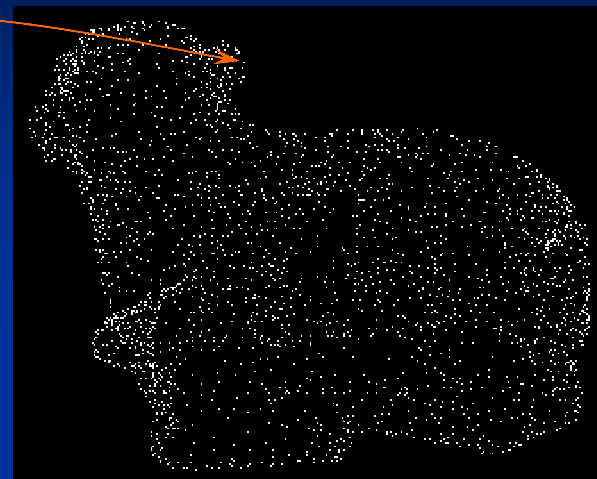
Outline

- Registration Background
- Medial Scaffold: Representation for 3D Shapes
- Graduated Assignment Graph Matching
- Results
- Conclusions

Registration: Defining Correspondence

Mesh with 20K points



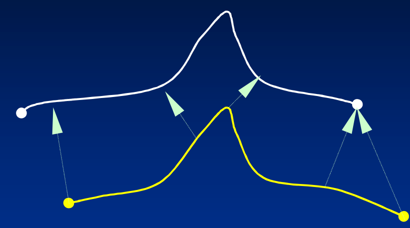
2K points

- Fundamental for processing scanned objects, modeling, matching, recognition, medical applications, etc.

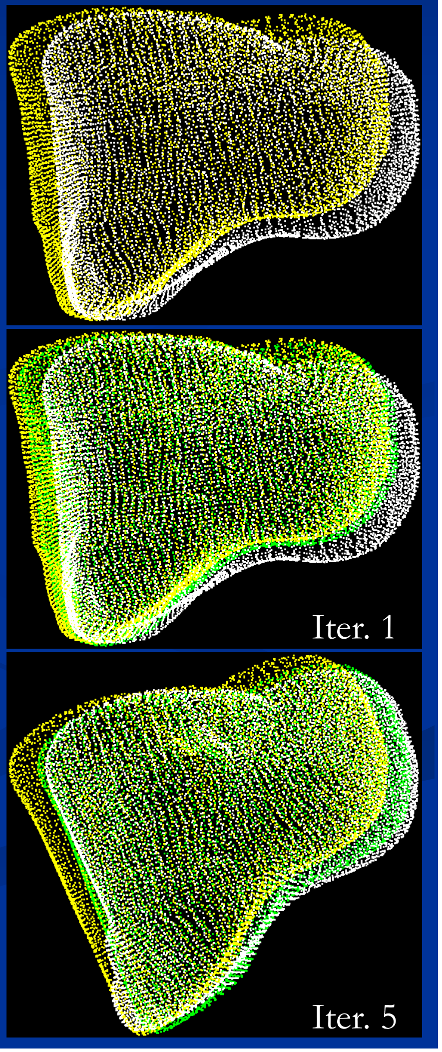
- Local Registration
  Initial position given. ICP and it's improvements
  Survey: [Campbell & Flynn CVIU'01], [3DIM'03]
- Global Registration
  Skeleton-based, Surface-feature based

More difficult.
Main focus of this talk.

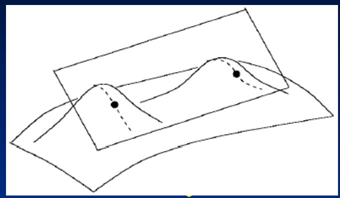
Local Registration: Iterative Closest Points (ICP)



- [Besl & McKay PAMI'92]
- Needs a good initial alignment
- Local search problems
  - Sensitive to local minimum, noise
  - May converge slowly
  - Lack of surface representation
- Improvements:
  - [Chen & Mendioni] accuracy: match closest point on the projected plane
  - Use color, *non-rigid* match to get better convergence, etc.

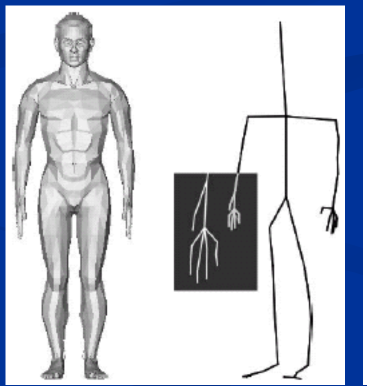


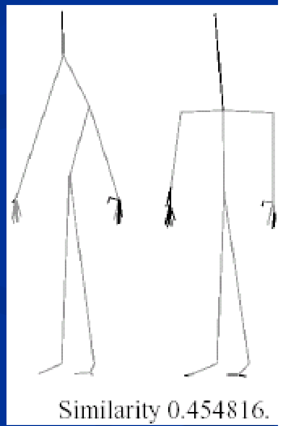
Global Registration



- Surface featured based
  - [Wyngaerd & Van Gool CVIU'02]: bitangent curve pairs as surface landmarks
  - [Allen *et. al.*'03]: straight lines as features in aligning architectural dataset

- Skeletal graph based
  - [Brennecke & Isenberg '04]:
    - Internal skeletal graph of a closed surface mesh, using an edge collapse algorithm
    - match *largest common subgraph*

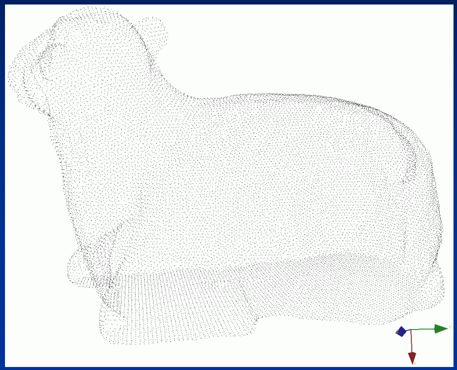




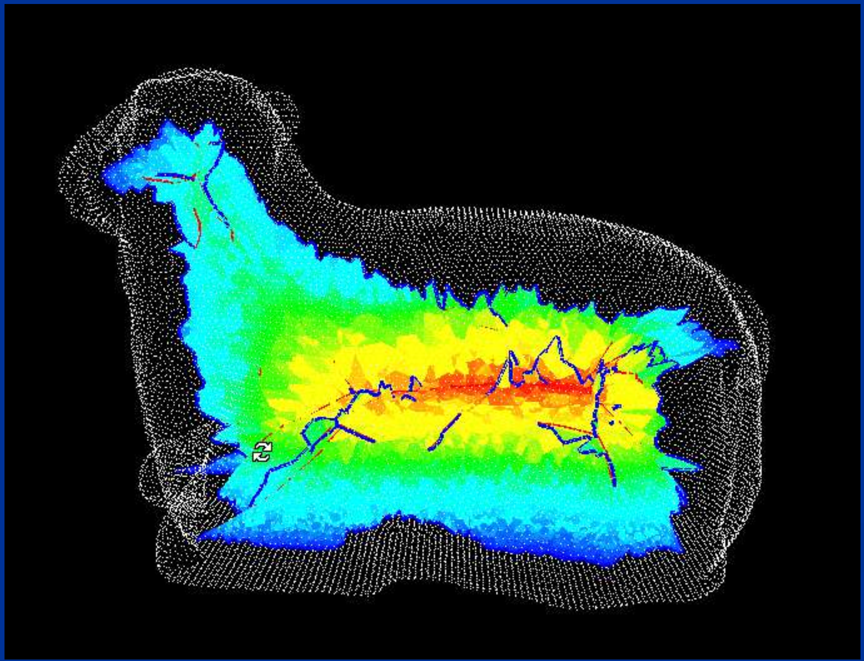
  - [Sundar *et. al.* '03]: Skeletal tree from thinning voxels via a distance transform, coarse-to-fine matching

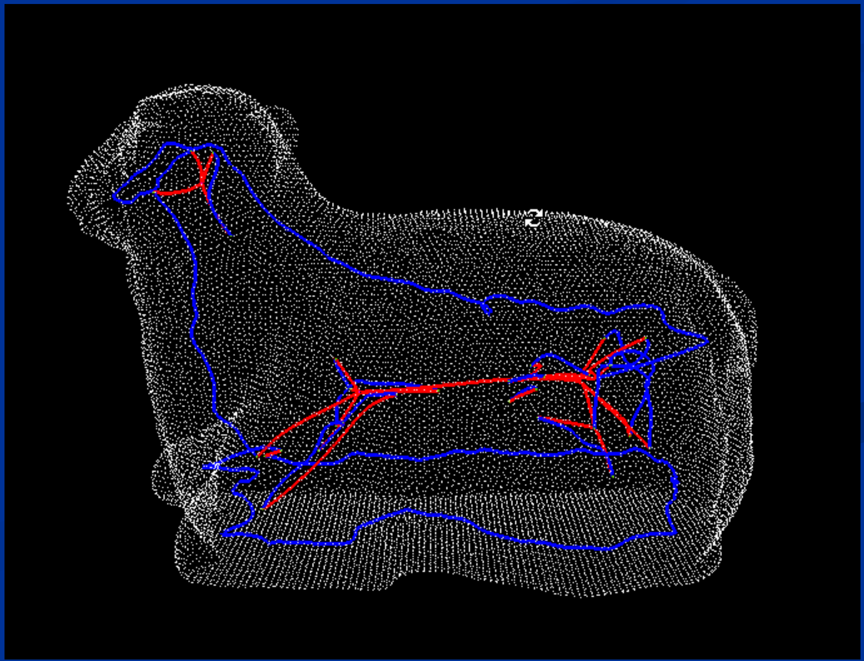
1. Skeletons over-simplified 2. Graph topology not handled well

Proposed: Match the Medial Scaffold



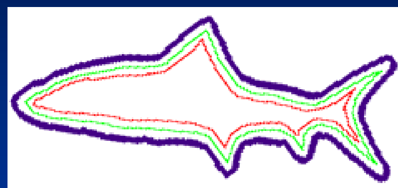
- Medial Scaffold: medial structure in the form of a 3D hypergraph

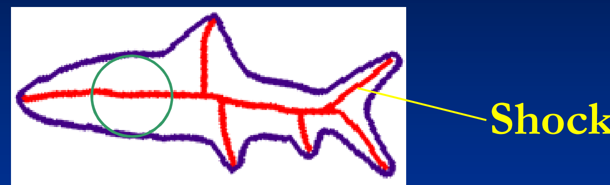


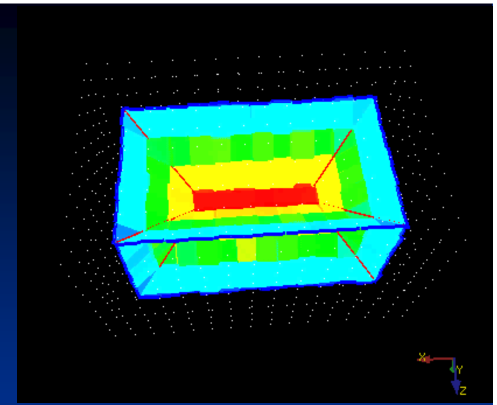


Medial Scaffold

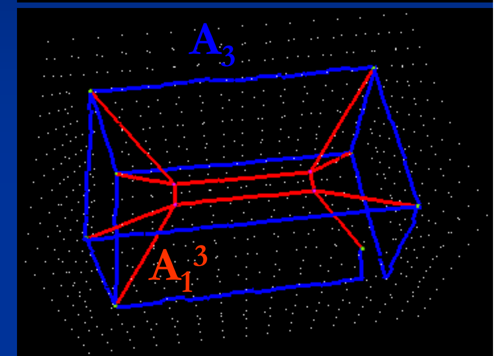
- Blum's medial axis (grassfire), wave propagation

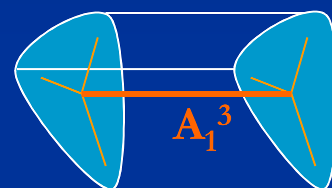


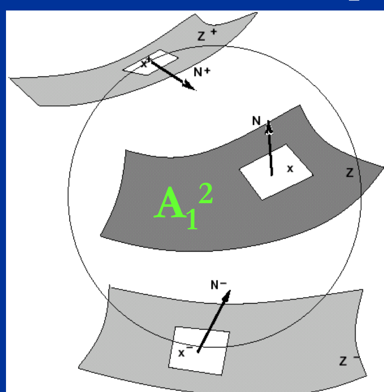


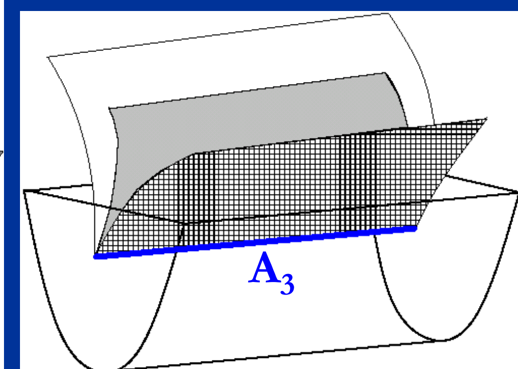


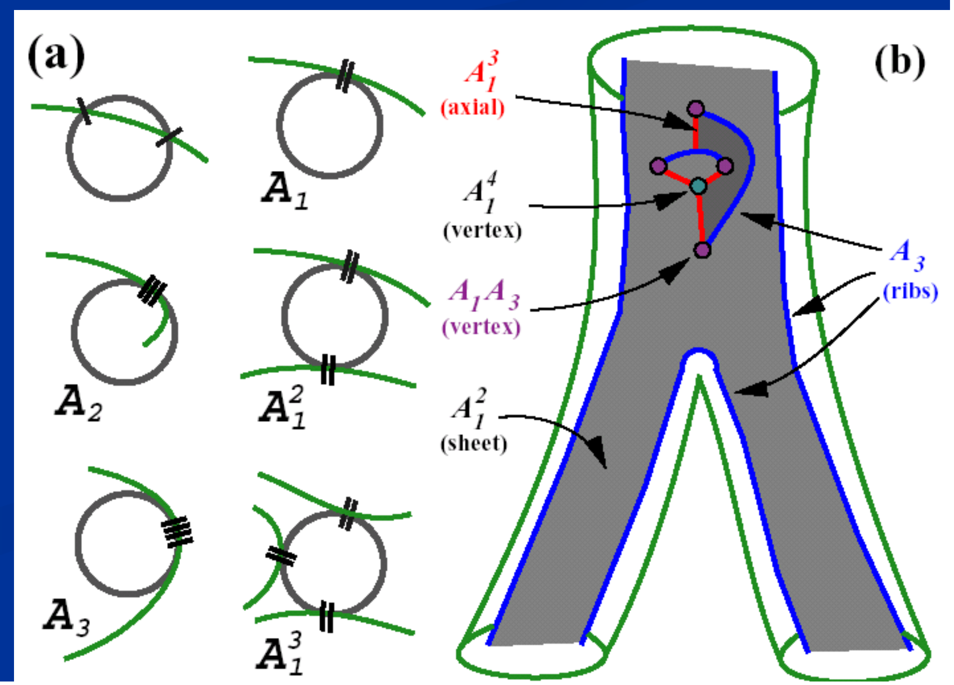
- 3D: Five types of points [Giblin & Kimia PAMI'04]:
  - Sheet: A_1^2









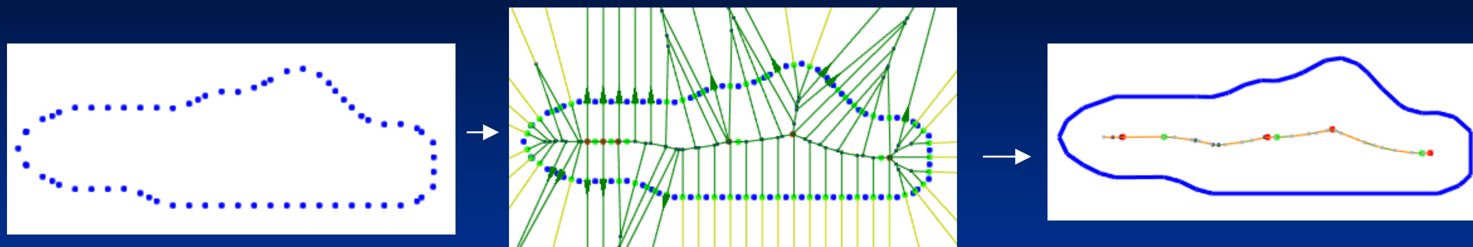


  - Links: A_1^3 (Axial), A_3 (Rib)
  - Nodes: A_1^4, A_1A_3

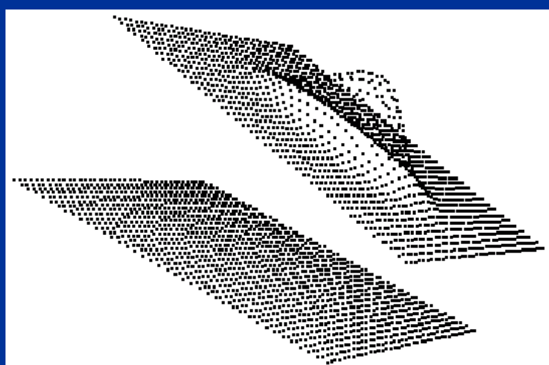
$\mathbf{A}_k^n$: contact at n distinct points, each with $k+1$ degree of contact

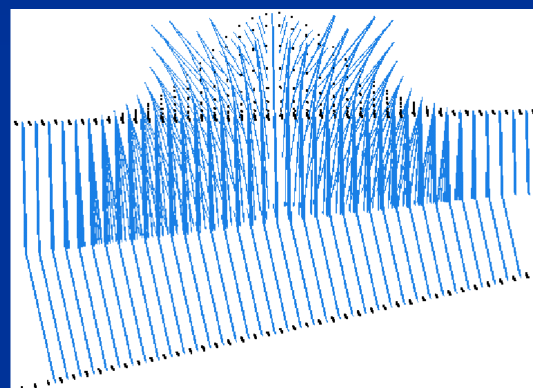
Compute the Medial Scaffold

2D

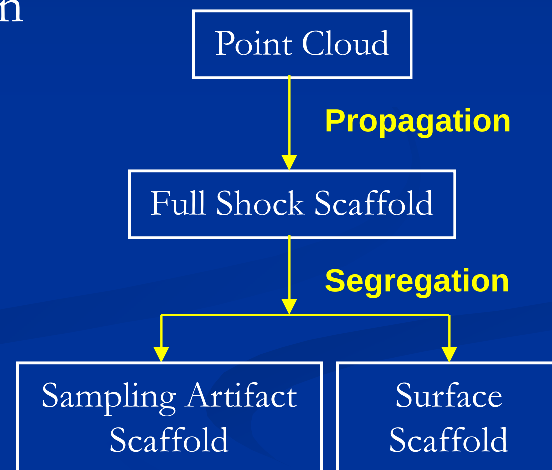


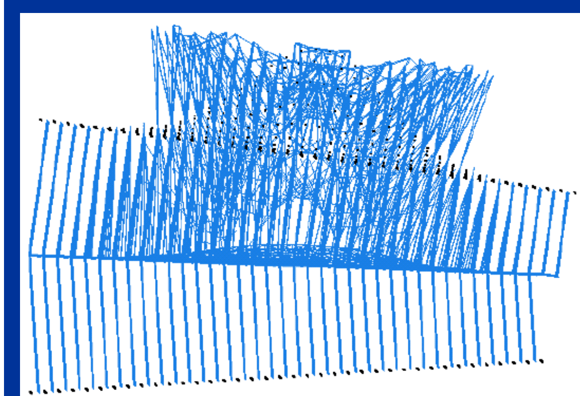
3D [Leymarie PhD]: Medial Scaffold Detection + Segregation



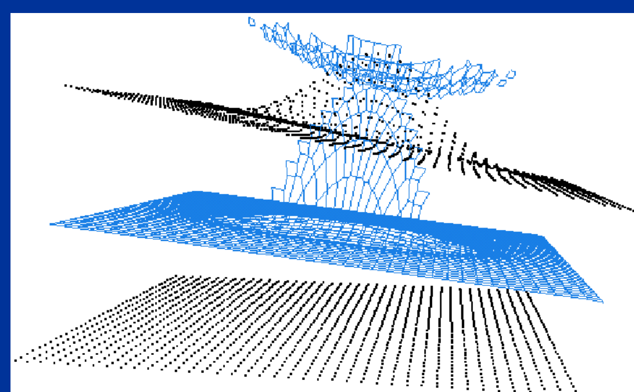


Sampling Artifact Scaffold

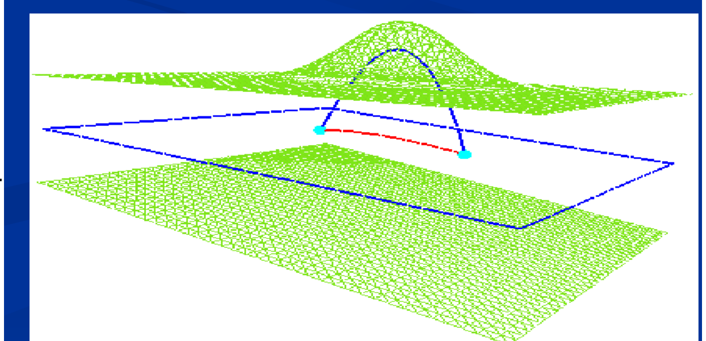




Full Scaffold

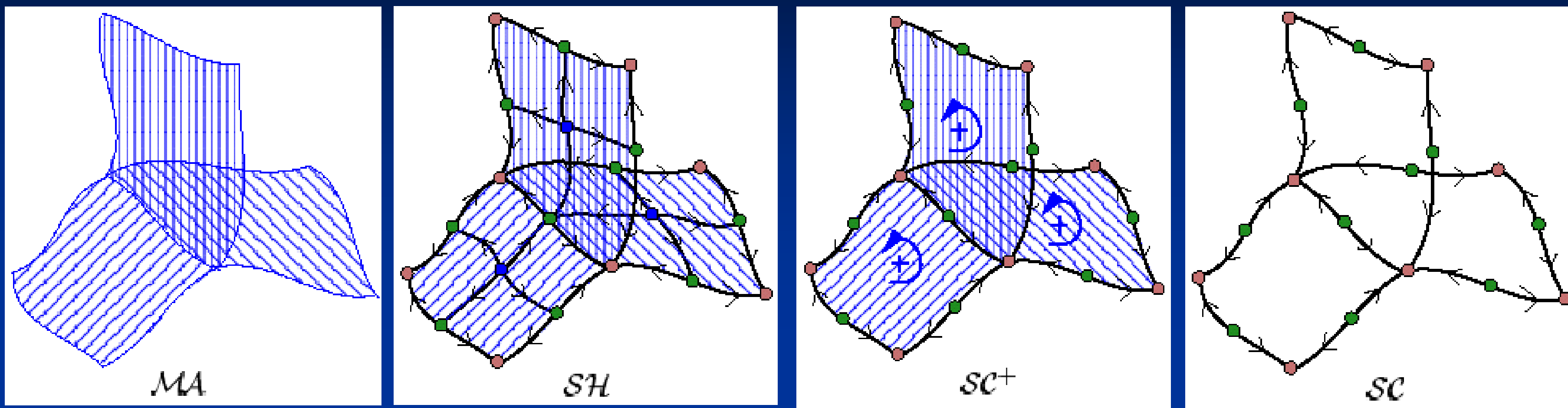


Surface Scaffold



Medial Scaffold

Medial Structure Hierarchy



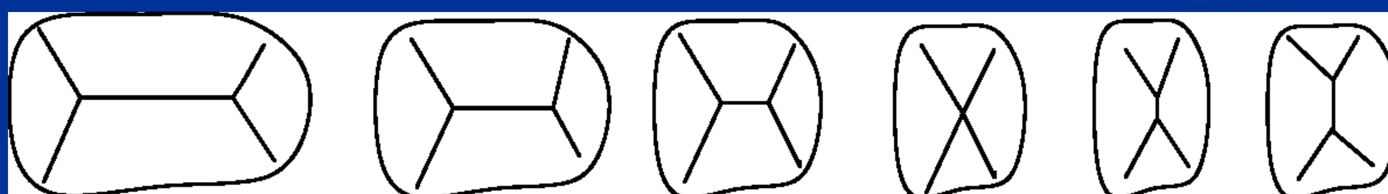
- Medial Axis (*MA*)
- Shock Hypergraph (*SH*)
- Shock Scaffold with Sheets (SC^+)
- Shock Scaffold (*SC*)

Only need to detect special nodes and links, while maintaining their connectivity.

Medial Structure Regularization

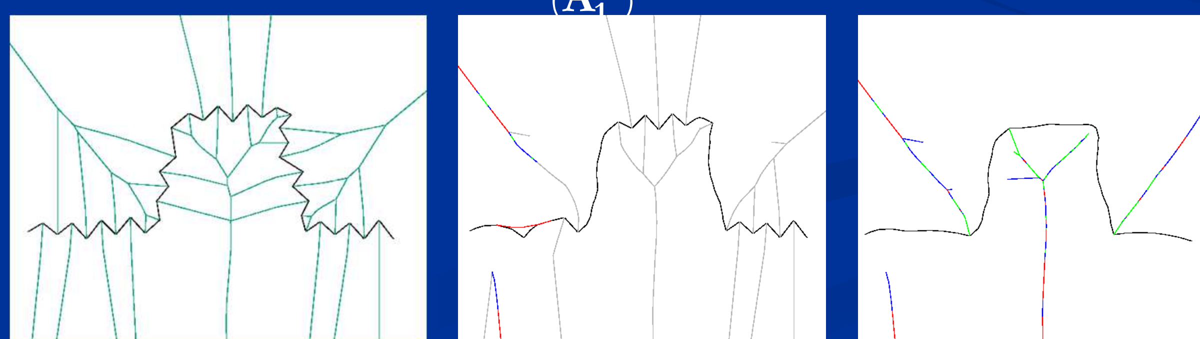
- Medial Axis is sensitive to noise & perturbations.
- Transitions: sudden changes in topology
- 2D examples:

The growth of an axis with small perturbations (**A_1A_3**)



The swapping of MA branches (**A_1^4**)

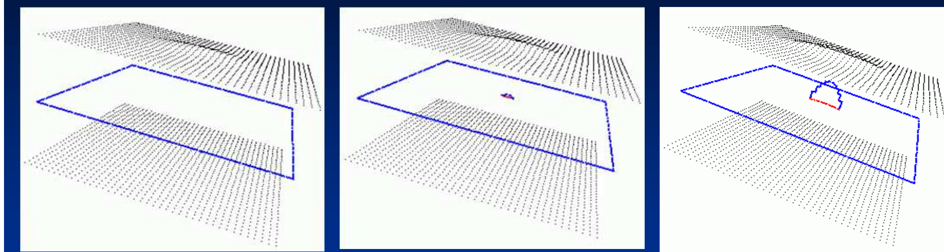
Pruning:



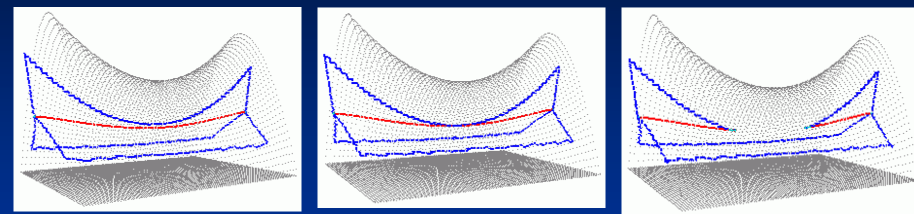
Smoothing/medial branch pruning

Seven Types of Transitions in 3D

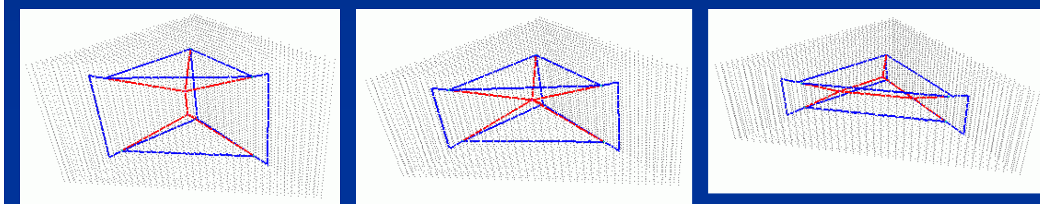
[Giblin & Kimia ECCV'02]



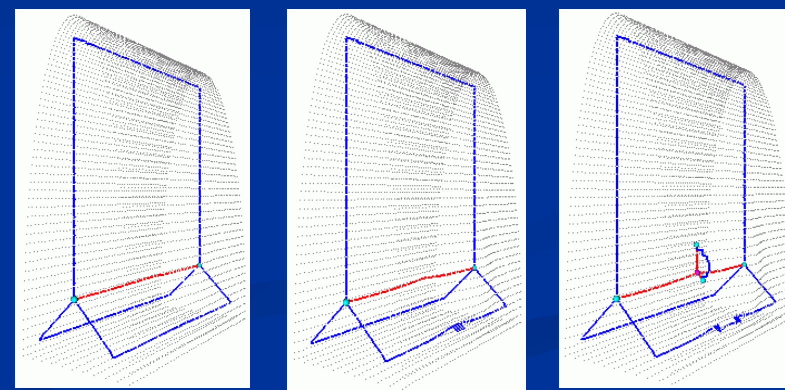
A_1A_3-I



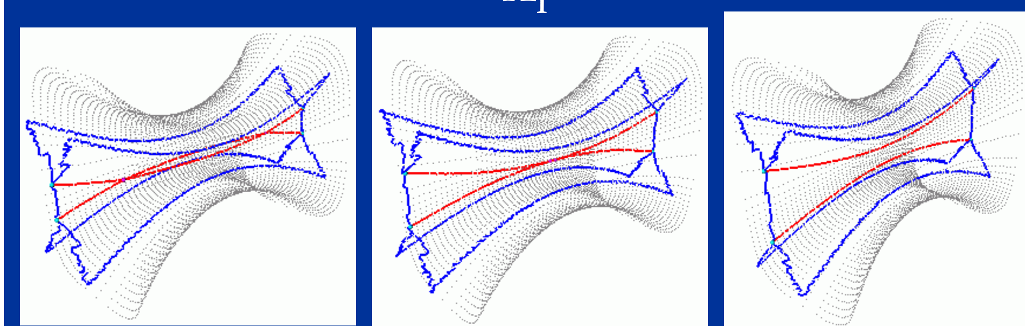
A_1A_3-II



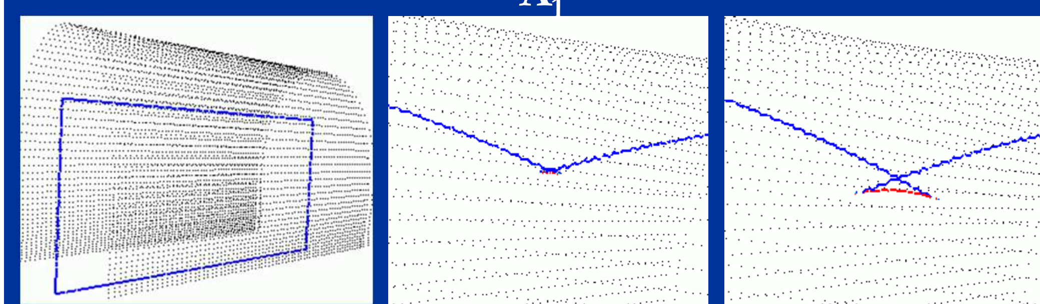
A_1^5



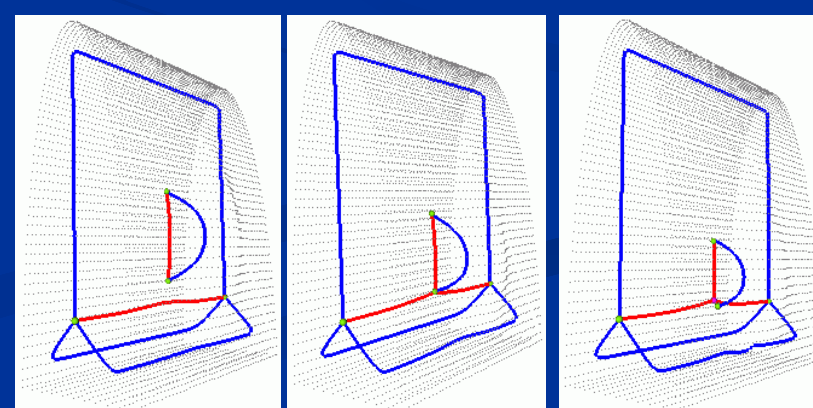
$A_1^2A_3$-I



A_1^4

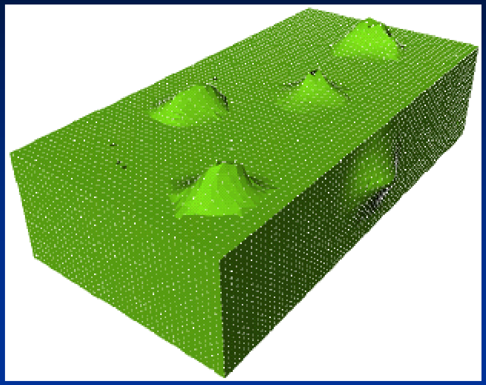
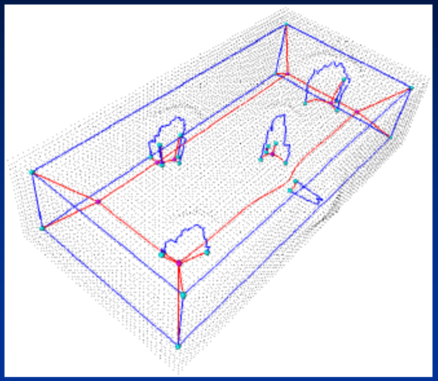
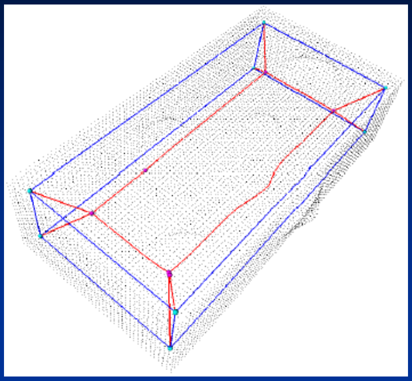


A_5

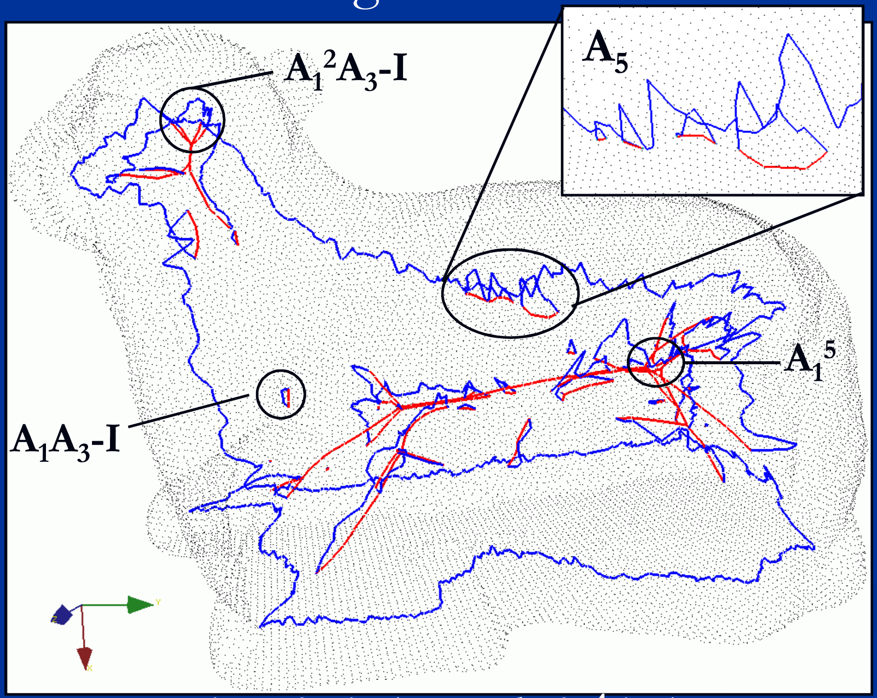


$A_1^2A_3$-II

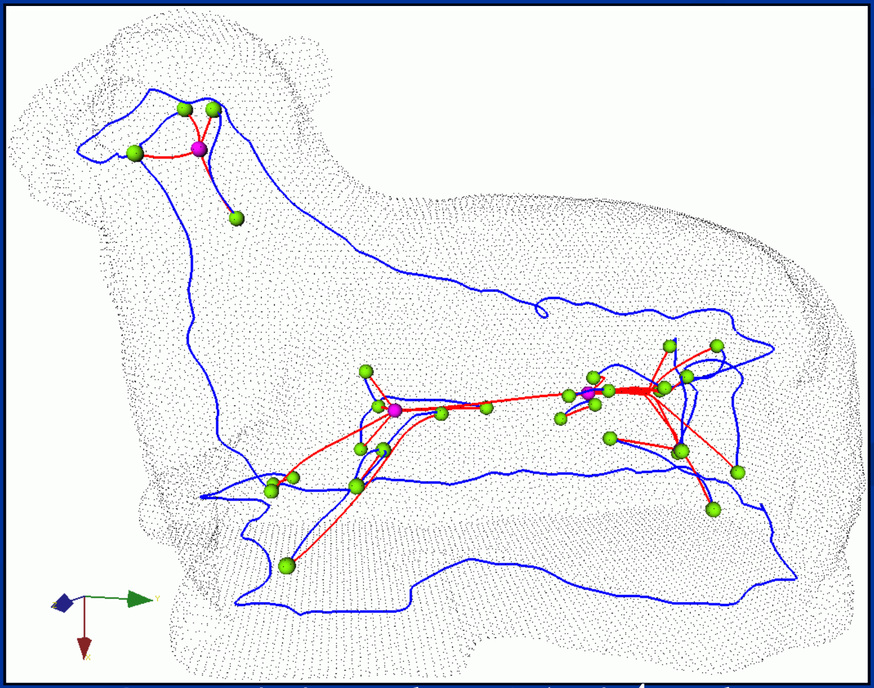
Scaffold Regularization [Leymarie *et. al.* ICPR'04]

- Transition removal, i.e. remove topological instability
- Smoothing



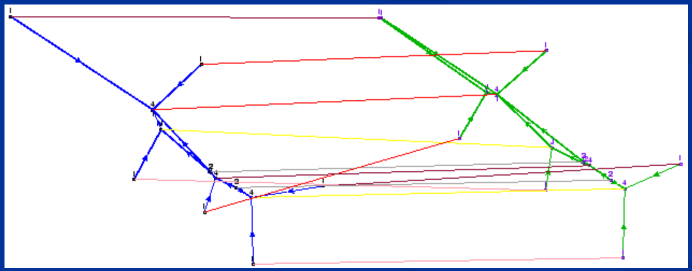
Blue: $\mathbf{A_3}$ links, Red: $\mathbf{A_1^4}$ links



Green: $\mathbf{A_1A_3}$ nodes, Pink: $\mathbf{A_1^4}$ nodes

Match Medial Scaffolds by Graph Matching

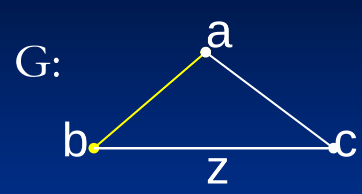
- Intractability
  - Weighted graph matching: **NP**-hard
  - One special case: Largest common subgraph: **NP**-complete
  - Only "good" sub-optimal solutions can be found
- Graduated Assignment [Gold & Rangarajan PAMI'96]
  - [Sharvit *et. al.* JVCIR'98] index 25-shape database by matching 2D shock graphs

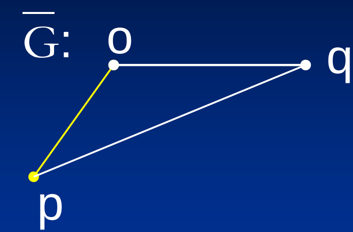


  - 3D hypergraph matching:
    - Additional dimension
    - Generally not a tree, might have isolated loops
    - No inside/outside: non-closed surfaces or surface patches

Graduated Assignment

Quadratic weighted graph matching





G, $\bar{G}$: 2 undirected graphs
I: # of nodes in G, $\bar{I}$: # of nodes in $\bar{G}$
$\{G_i\}$, $\{\bar{G}_{\bar{i}}\}$ nodes
$\{G_{ij}\}$, $\{\bar{G}_{\bar{i}\bar{j}}\}$ edges: adjacency matrices of graphs

The match matrix
$\mathbf{M}_{i\bar{i}} = 1$ if node i in G corresponds to node $\bar{i}$ in $\bar{G}$,
$= 0$ otherwise

Then objective function to maximize over the space of $\mathbf{M}$ is:

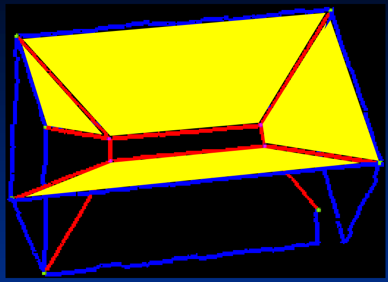
Cost of matching G_{ij} to $\bar{G}_{\bar{i}\bar{j}}$. If the nodes match, how similar the links are.

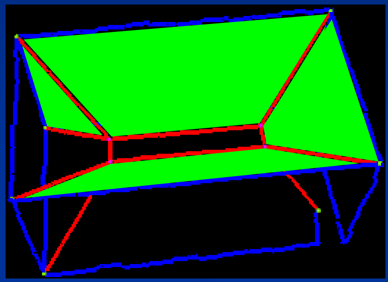
$$E(\mathbf{M}) = \sum_{i=1}^{I}\sum_{\bar{i}=1}^{\bar{I}}\sum_{j=1}^{I}\sum_{\bar{j}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}}\mathbf{M}_{j\bar{j}}L_{i\bar{i}j\bar{j}} + \alpha \sum_{i=1}^{I}\sum_{\bar{i}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}}N_{i\bar{i}}$$

Cost of matching G_i to $\bar{G}_{\bar{i}}$

$\mathbf{L}_{i\bar{i}j\bar{j}}$: link similarity between G_{ij} and $\bar{G}_{\bar{i}\bar{j}}$
$\mathbf{N}_{i\bar{i}}$: node similarity between G_i and $\bar{G}_{\bar{i}}$

Modified Graduated Assignment for 3D Medial Scaffold Matching





$$E(\mathbf{M}) = \alpha \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}} N_{i\bar{i}} + \beta \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} \sum_{j=1}^{I} \sum_{\bar{j}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}} \mathbf{M}_{j\bar{j}} L_{i\bar{i}j\bar{j}} + \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} \sum_{j=1}^{I} \sum_{\bar{j}=1}^{\bar{I}} \sum_{k=1}^{I} \sum_{\bar{k}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}} \mathbf{M}_{j\bar{j}} \mathbf{M}_{k\bar{k}} H_{i\bar{i}j\bar{j}k\bar{k}},$$

α, β: weights

Node cost: (radius)

$$N_{i\bar{i}}(G_i, \overline{G}_{\bar{i}}) = \begin{cases} 0, & \text{if } G_i \text{ and } \overline{G}_{\bar{i}} \text{ have different types,} \\ 1 - \left|\frac{r_i - r_{\bar{i}}}{max(R, \overline{R})}\right|, & \text{otherwise,} \end{cases}$$

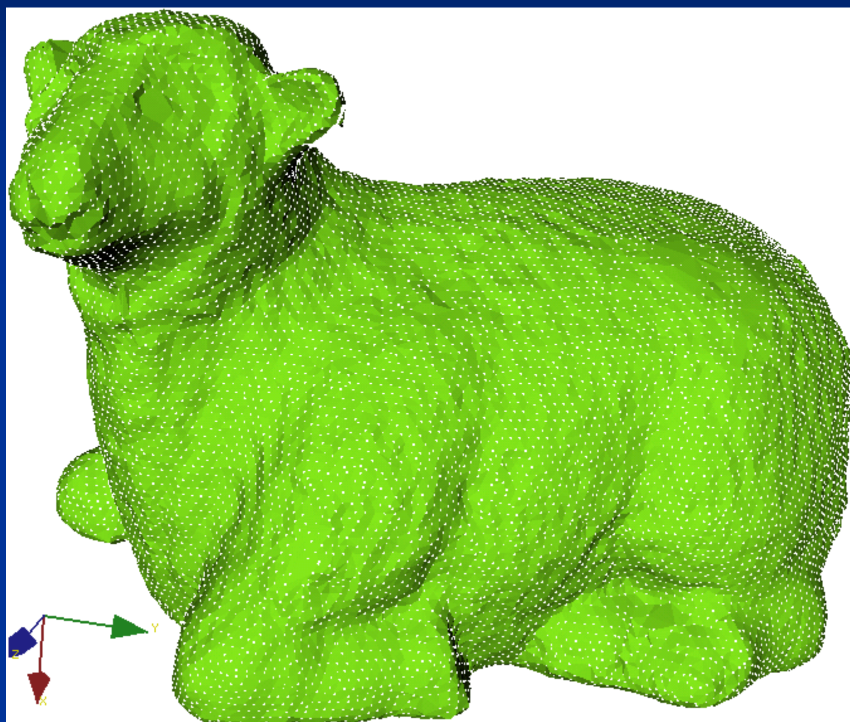
Link cost: (length)

$$L_{i\bar{i}j\bar{j}} = \begin{cases} 0, & \text{if any of links } ij \text{ and } \bar{i}\bar{j} \text{ are missing,} \\ 1 - \left|\frac{l_{ij} - l_{\bar{i}\bar{j}}}{max(L, \overline{L})}\right|, & \text{otherwise,} \end{cases}$$

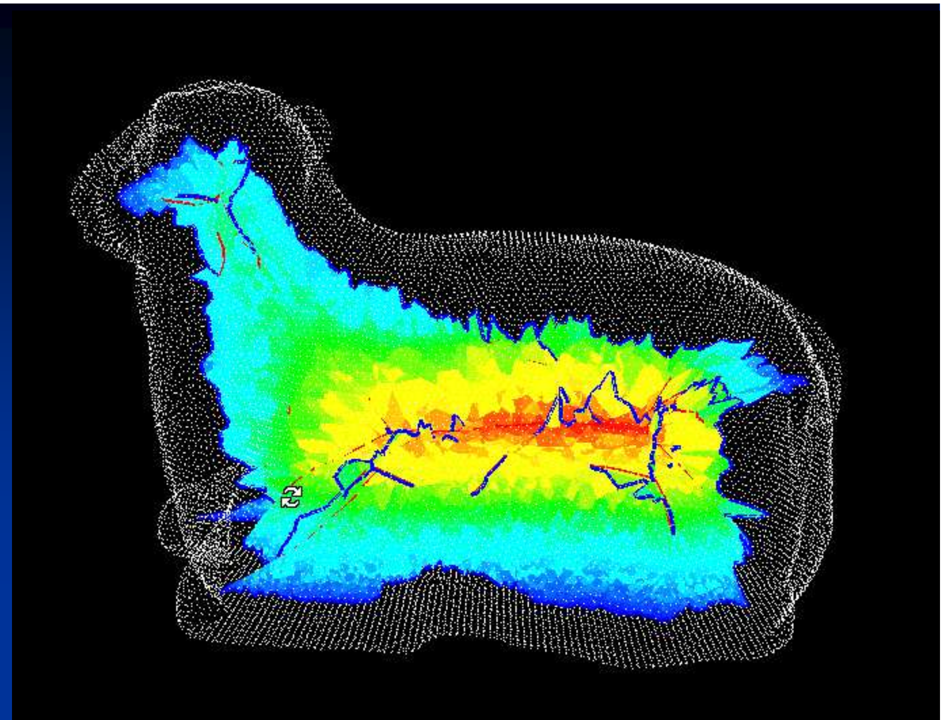
Sheet (hyperlink) cost: (corner angle)

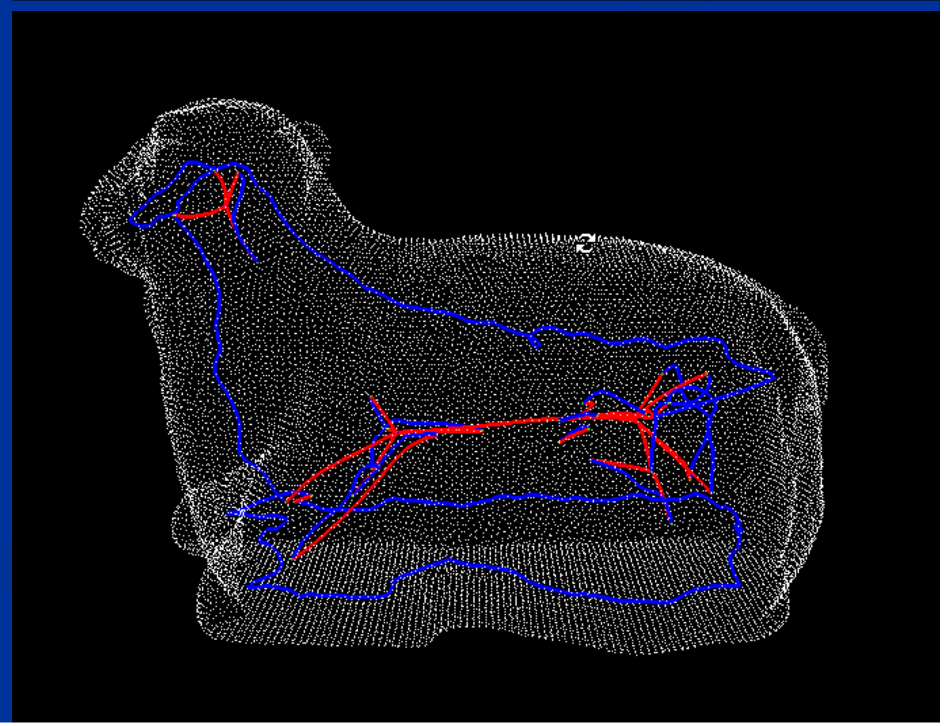
$$H_{i\bar{i}j\bar{j}k\bar{k}} = \begin{cases} 0, & \text{if any links } ij,\ jk,\ \bar{i}\bar{j},\ \bar{j}\bar{k} \text{ are missing,} \\ 1 - |\angle ijk - \angle \bar{i}\bar{j}\bar{k}|, & \text{otherwise,} \end{cases}$$

Results: Sheep



Sheep 20K points, after surface reconstruction

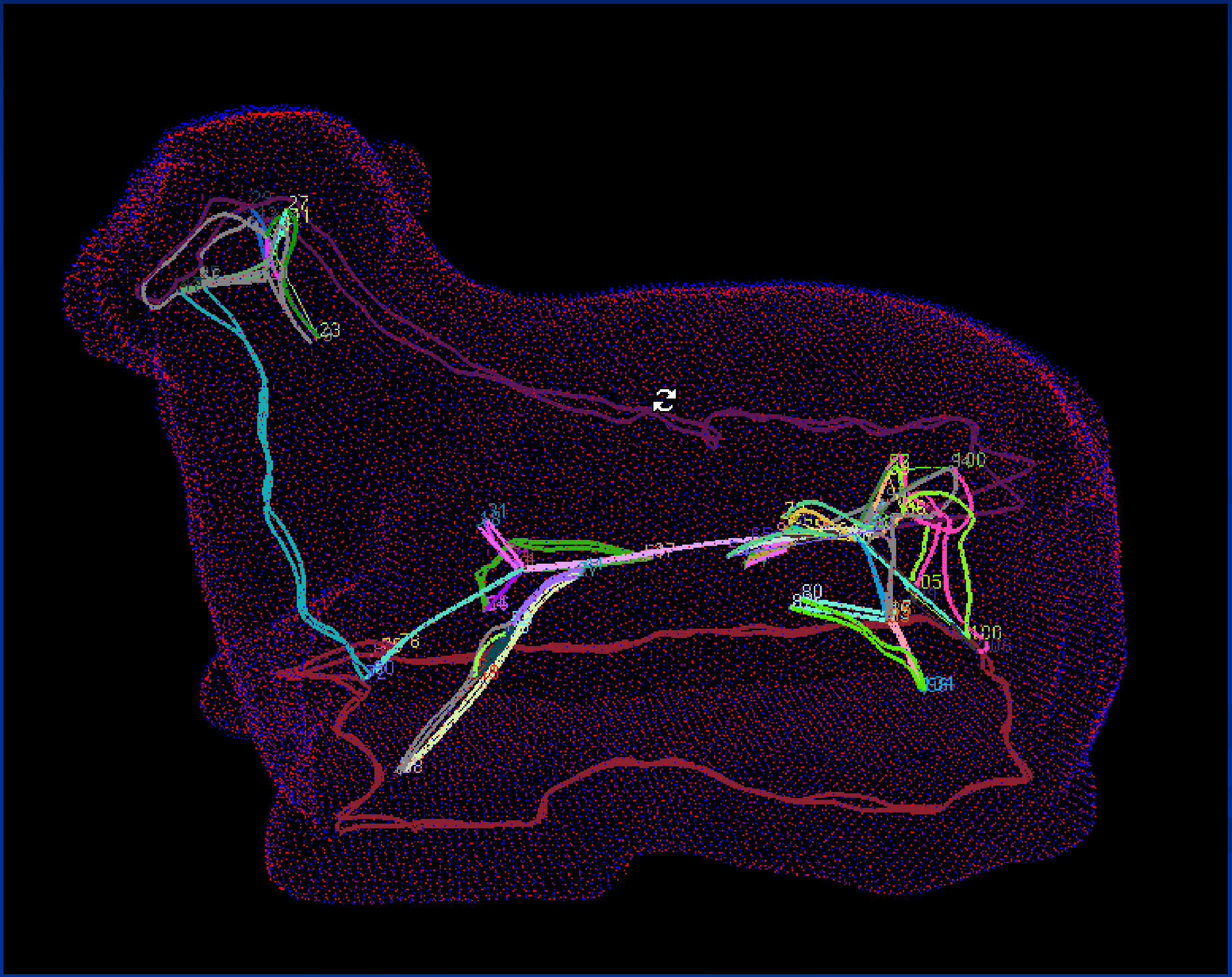




Result of Scaffold Graph Matching

Two scans of an object at the same resolution (20K points):

Colors to represent correct link matches; grays to represent miss matches.

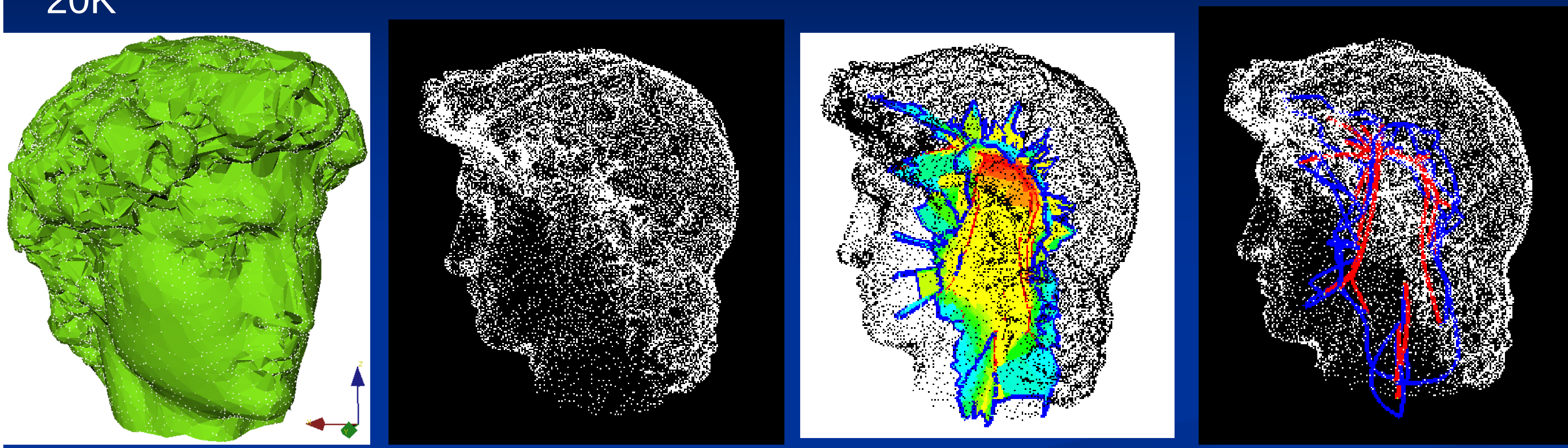


The scaffold matching is good enough that ICP is not required.

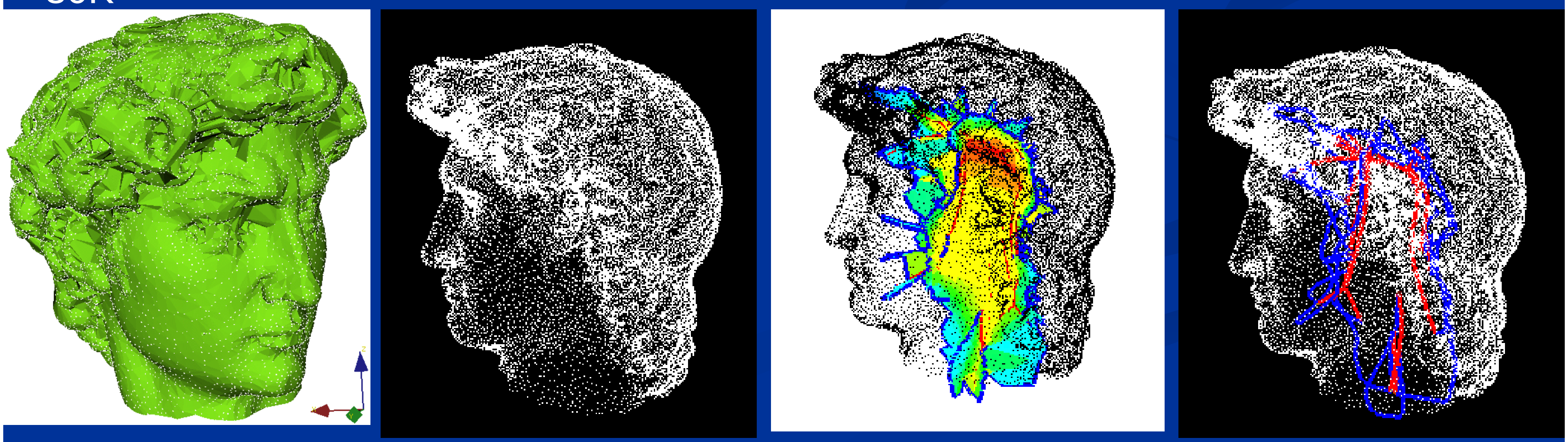
Results: David Head

Two sub-samples from the ground truth (42350 points)

20K

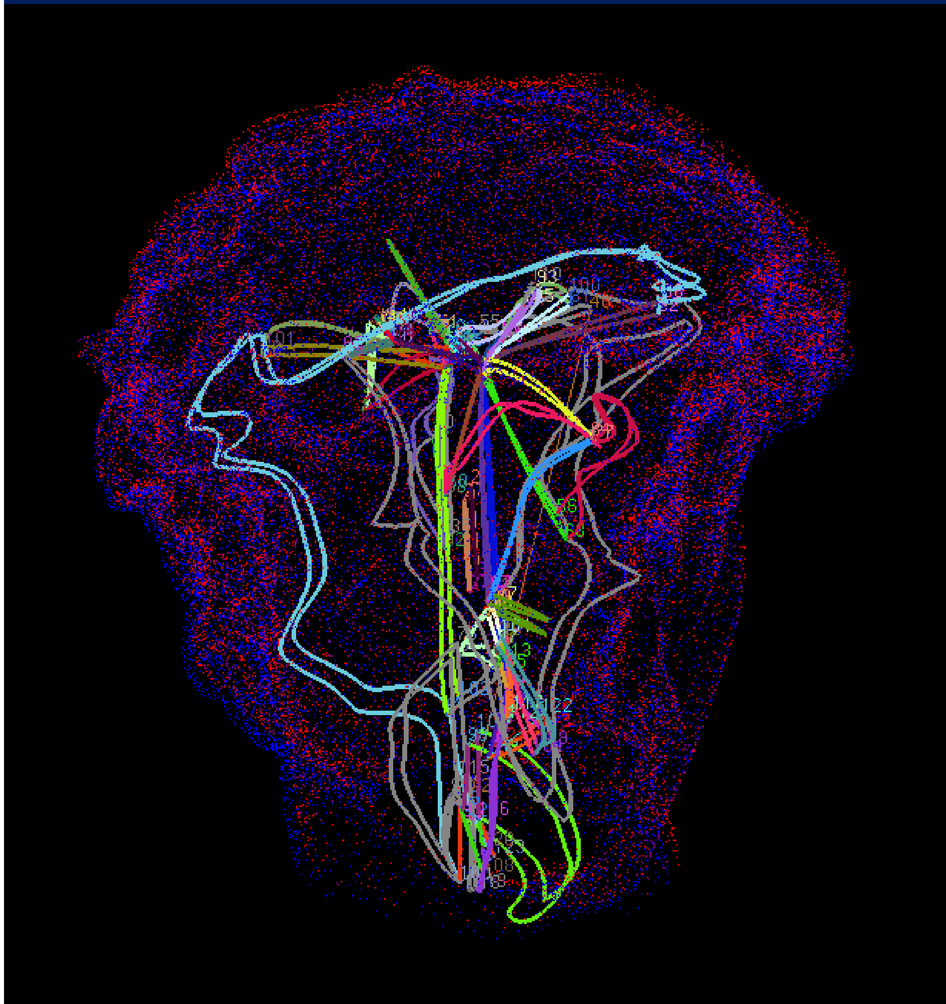


30K

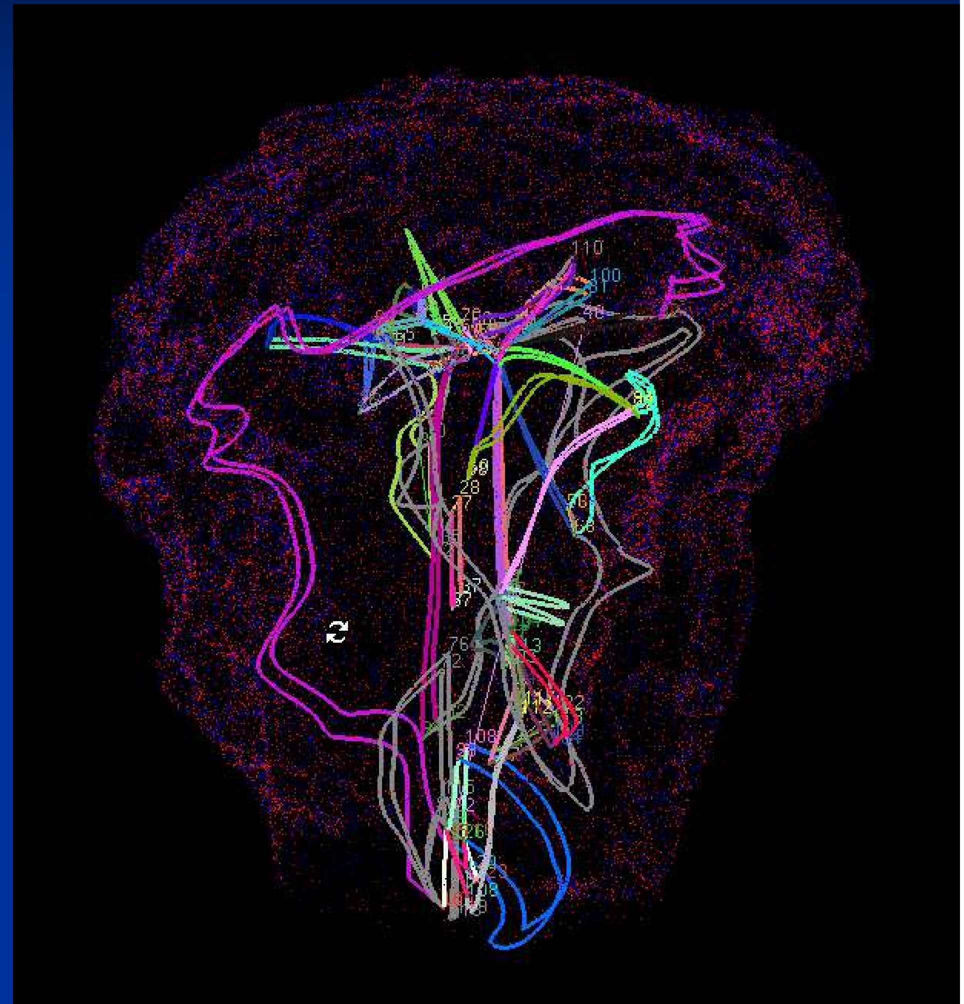


Matching Results

Scaffold matching result



Scaffold matching + ICP

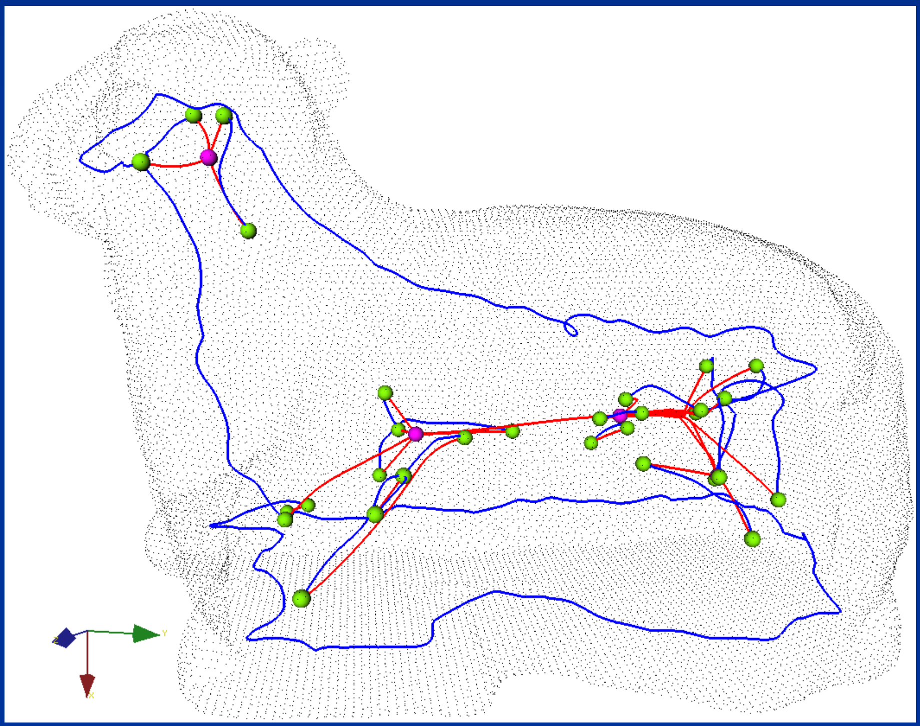


Validation against the ground truth: (object dimension = 69x69x76)

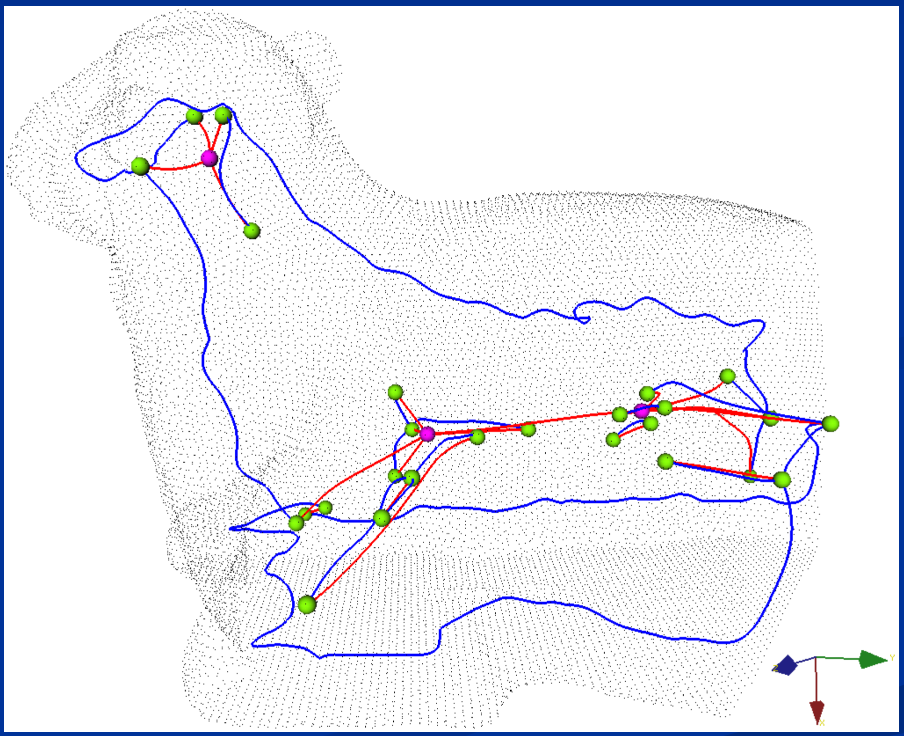
average sq dist 3.129372

average sq dist 0.000005

Partial Shape Matching: Sheep with the rear portion cut off

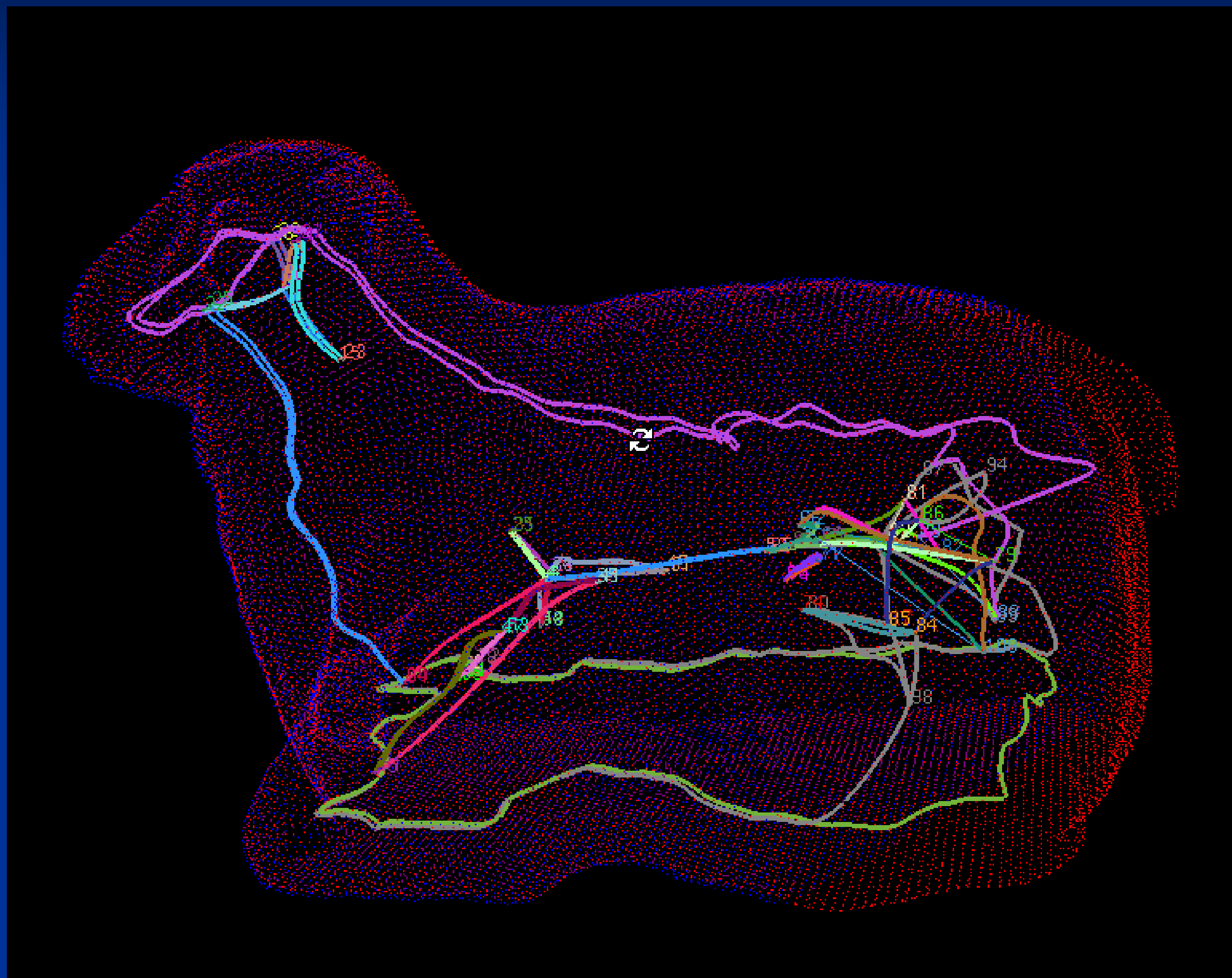


Sheep 1-20K scaffold

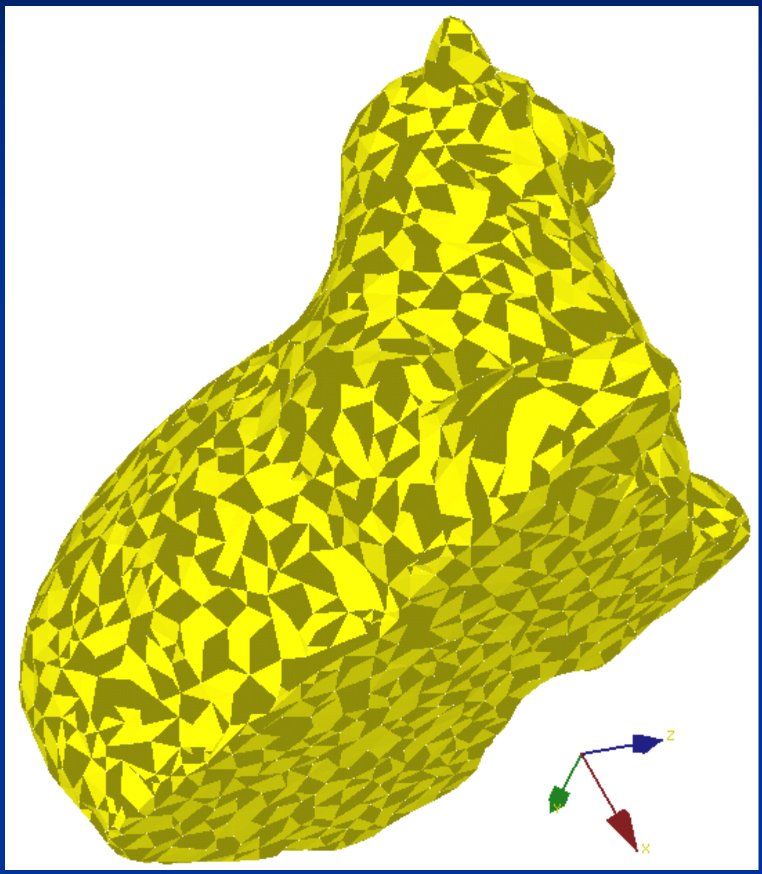


Sheep 1-20K with the rear portion cut

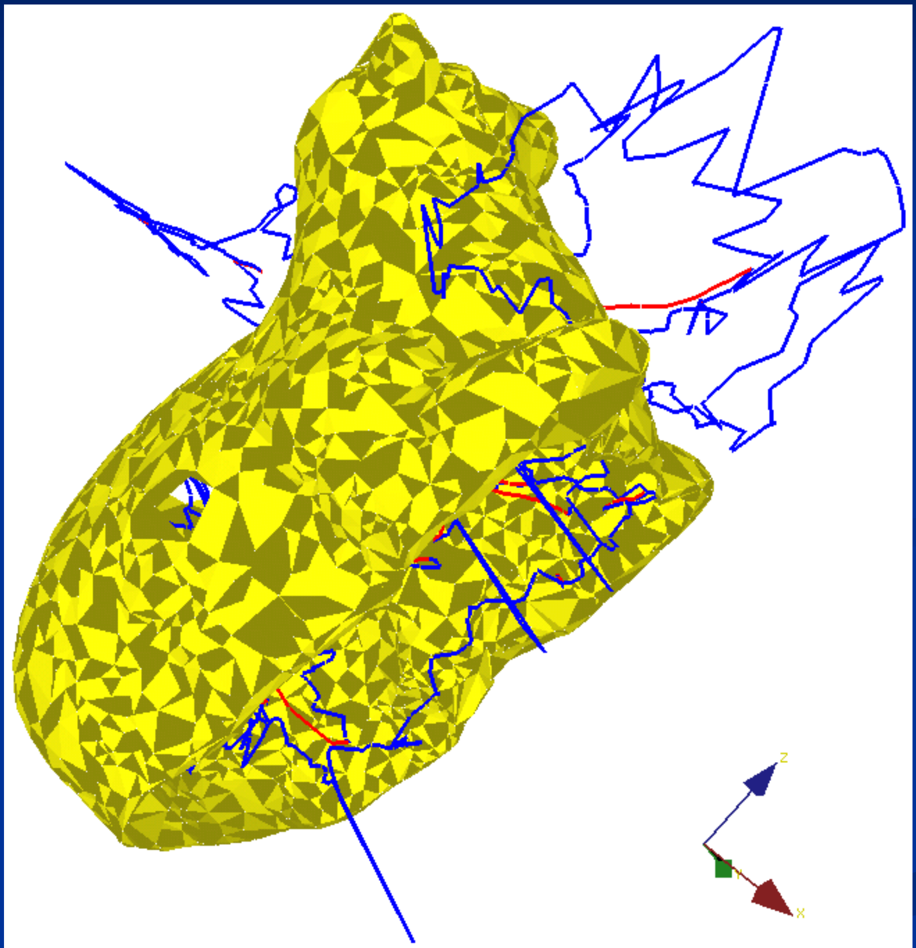
Partial Shape Matching Result



Partial Shape Matching (2nd example)



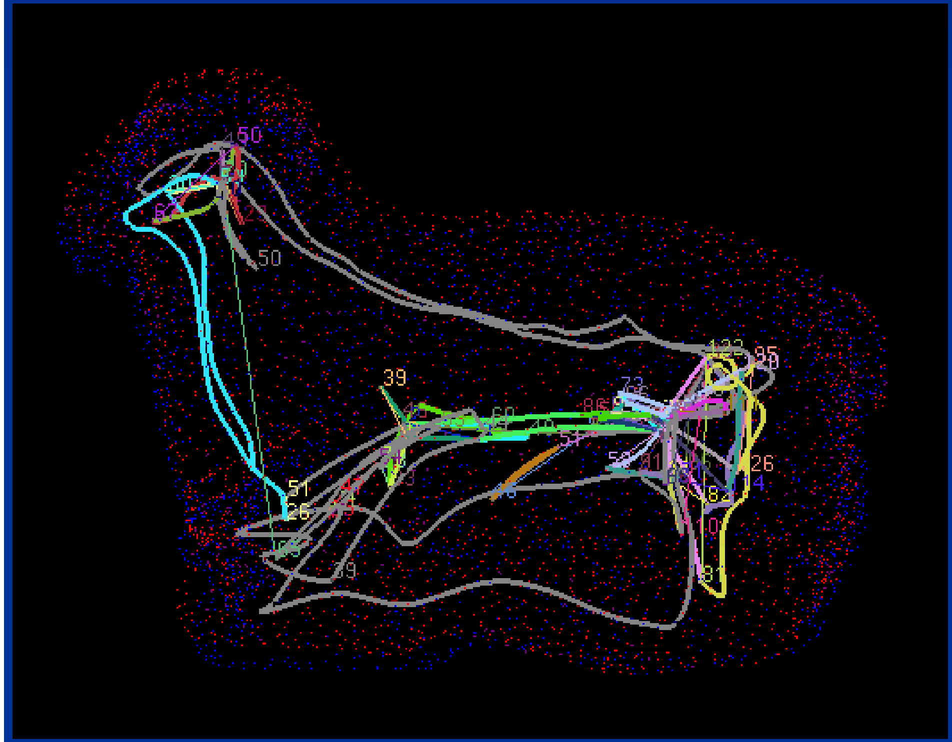
Sheep (2K points)



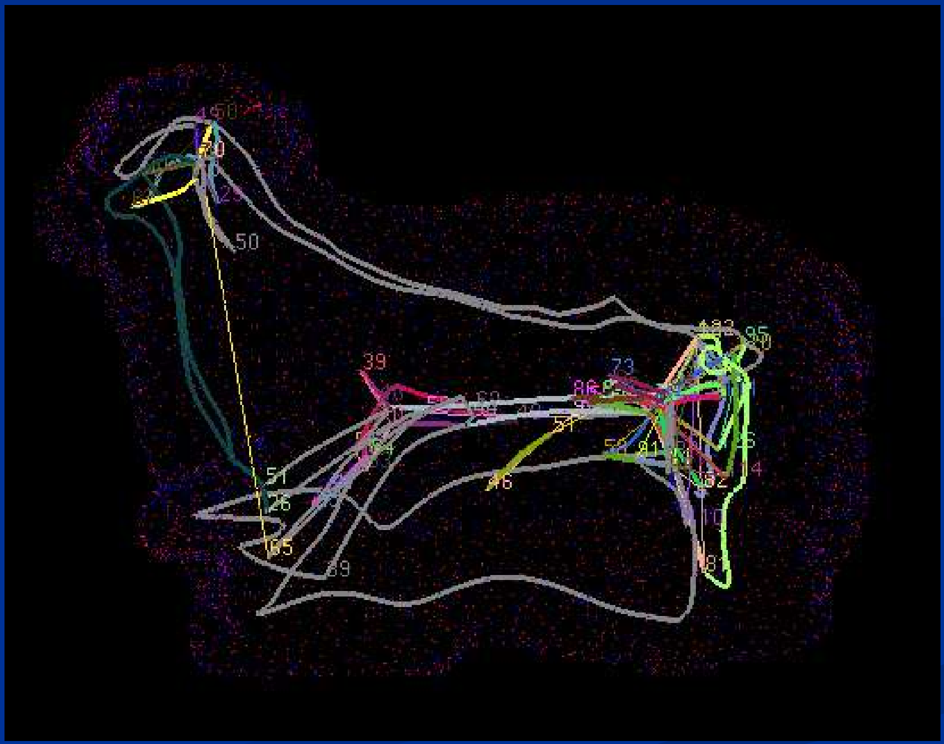
Another sheep of 2K points, but with no samples on the bottom

Partial Shape Matching (cont'd)

No match & Incorrect matches! Global registration still successes.

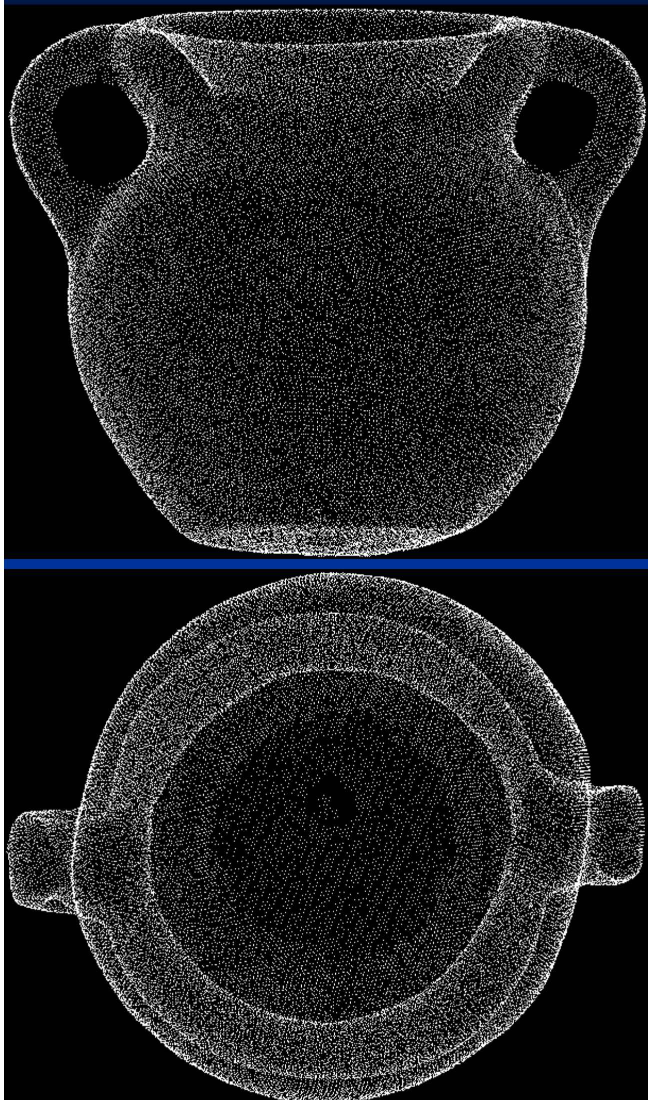


Result of scaffold matching

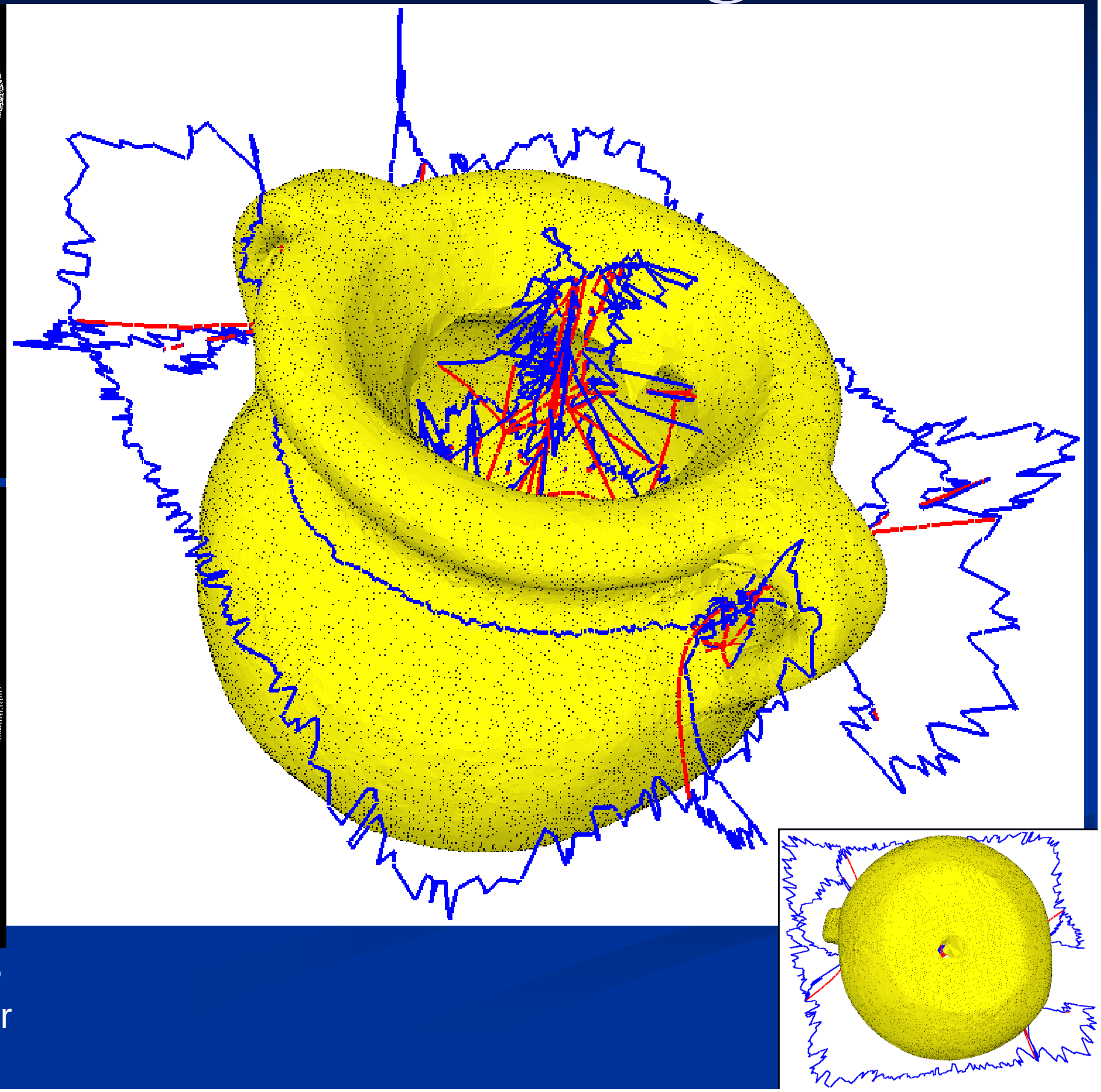


Result after ICP

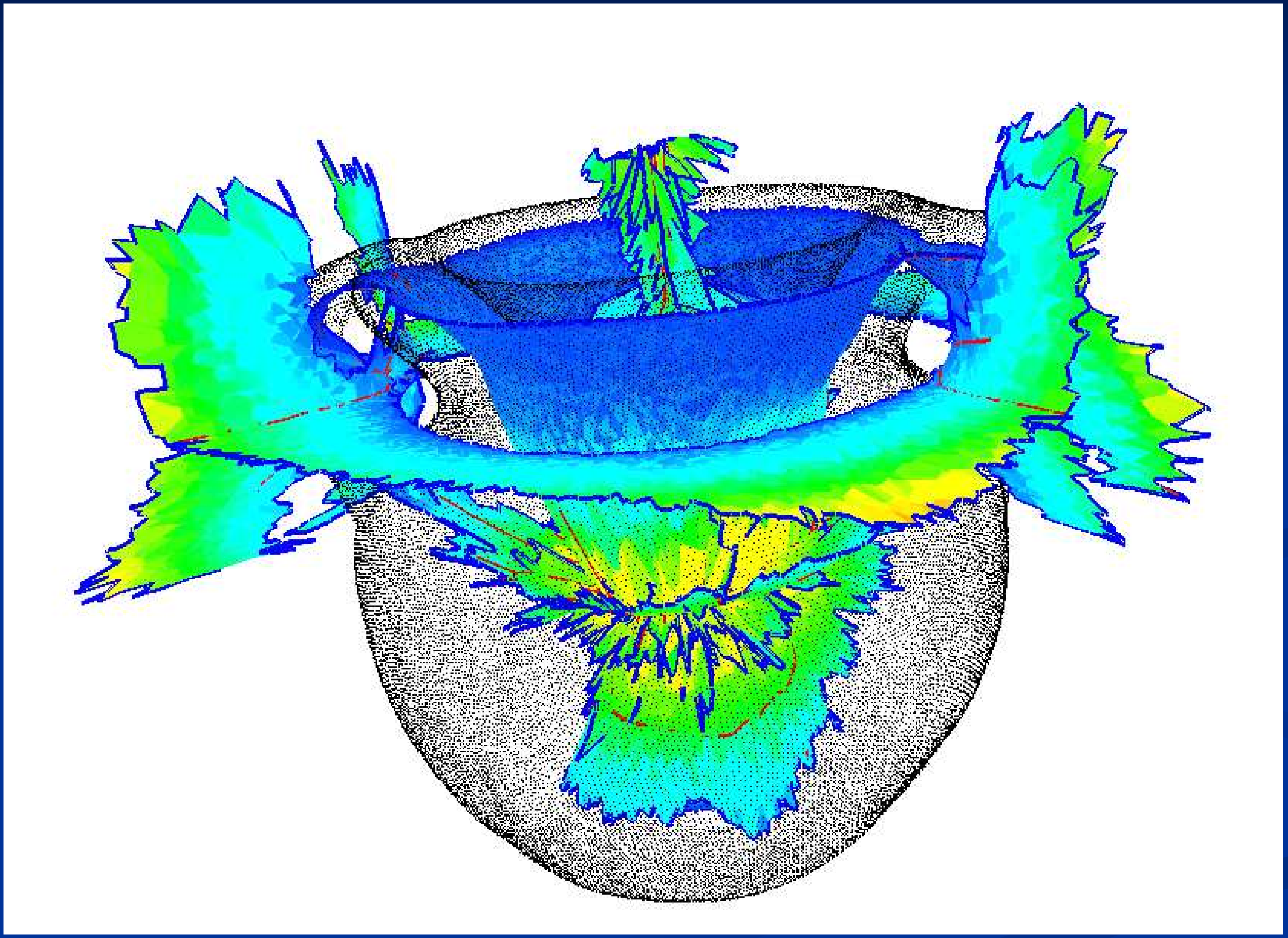
Non-closed Surface: Archaeological Pot



Two scans of the outside surface of a pot (50K and 40K). The inner surface of the pot is missing.

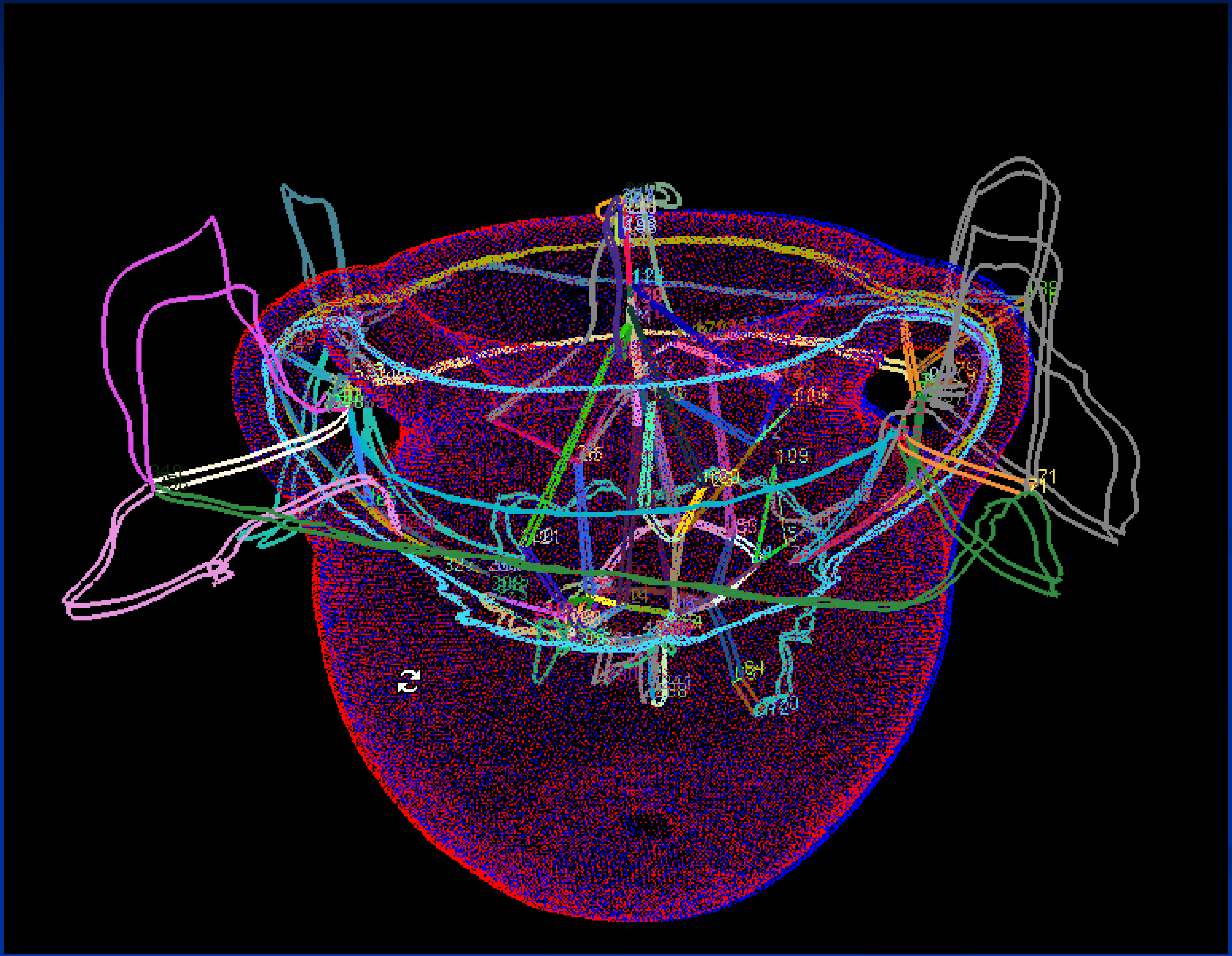


The Full Scaffold



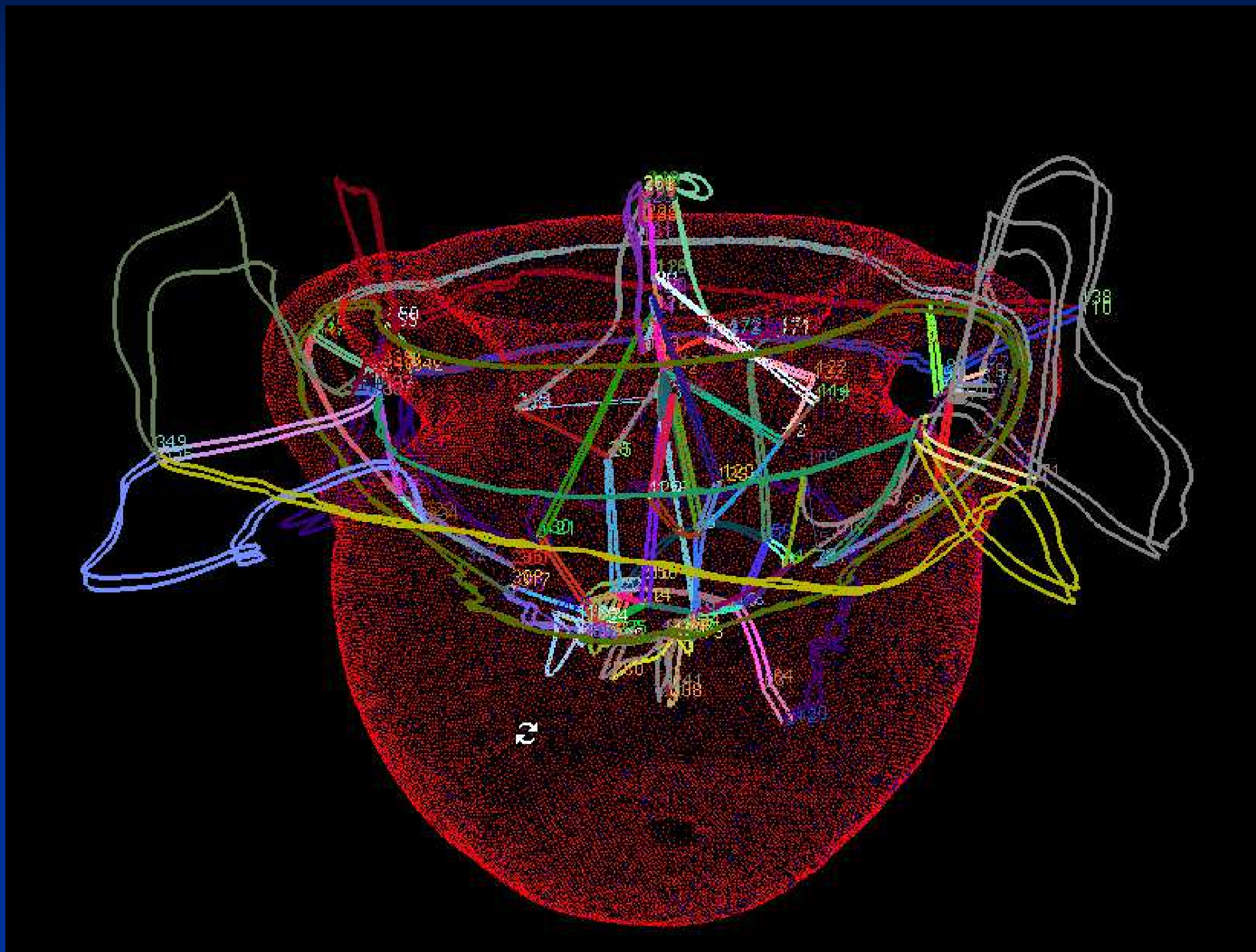
Both the inside and outside medial structures are connected together via shock sheets.

Alignment by Scaffold Matching



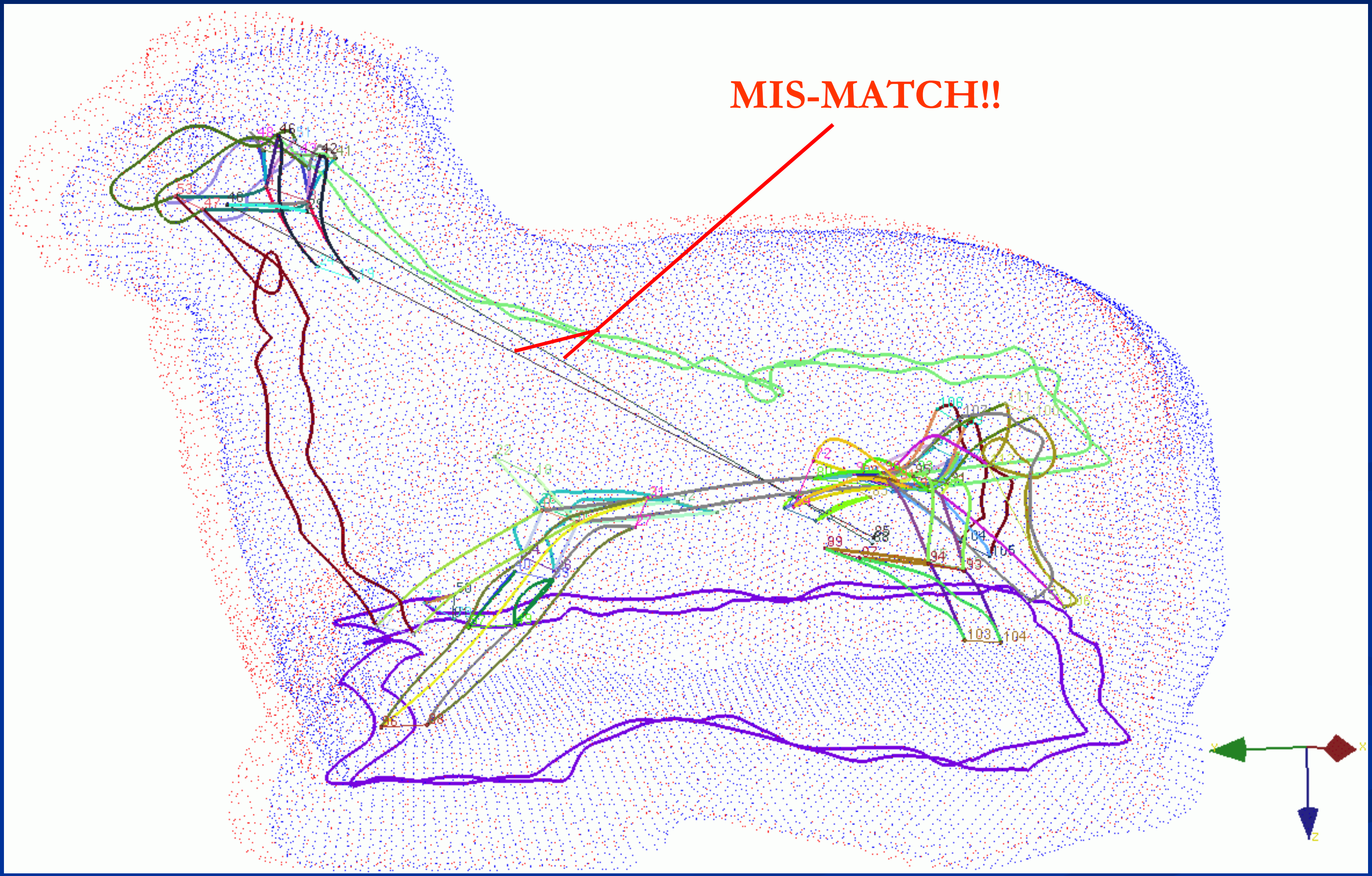
The scaffold matching result

Final Registration after ICP



Two Possible Reasons for Incorrect Matches

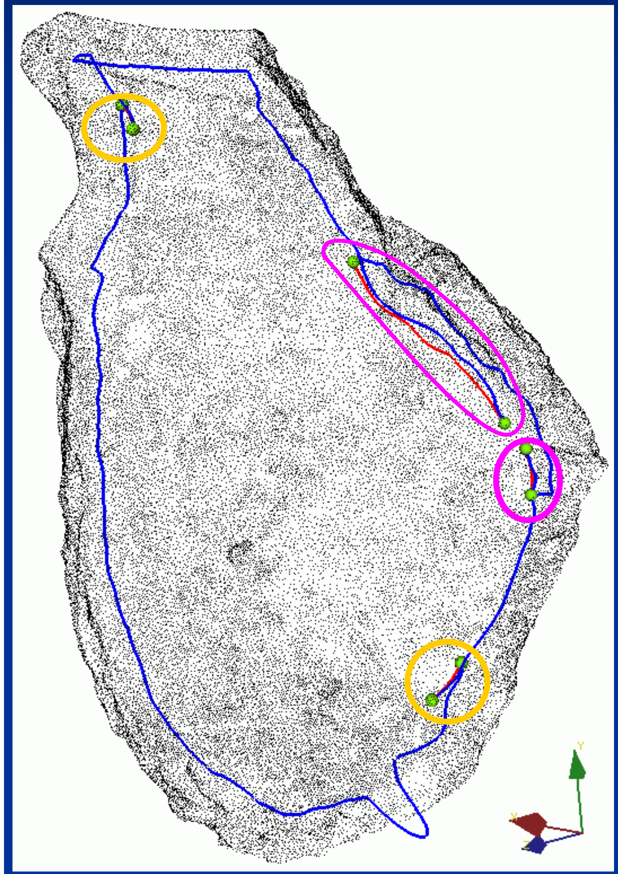
- Graduated assignment matching is not optimal.



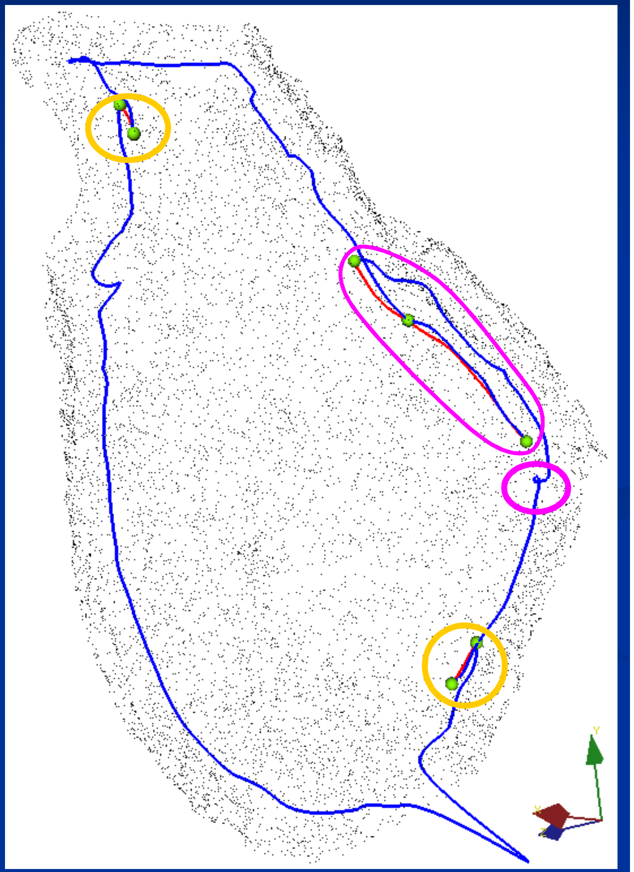
Typically this does not affect the overall registration if a sufficient number of nodes are correctly assigned.

Reasons for Incorrect Matches (cont'd)

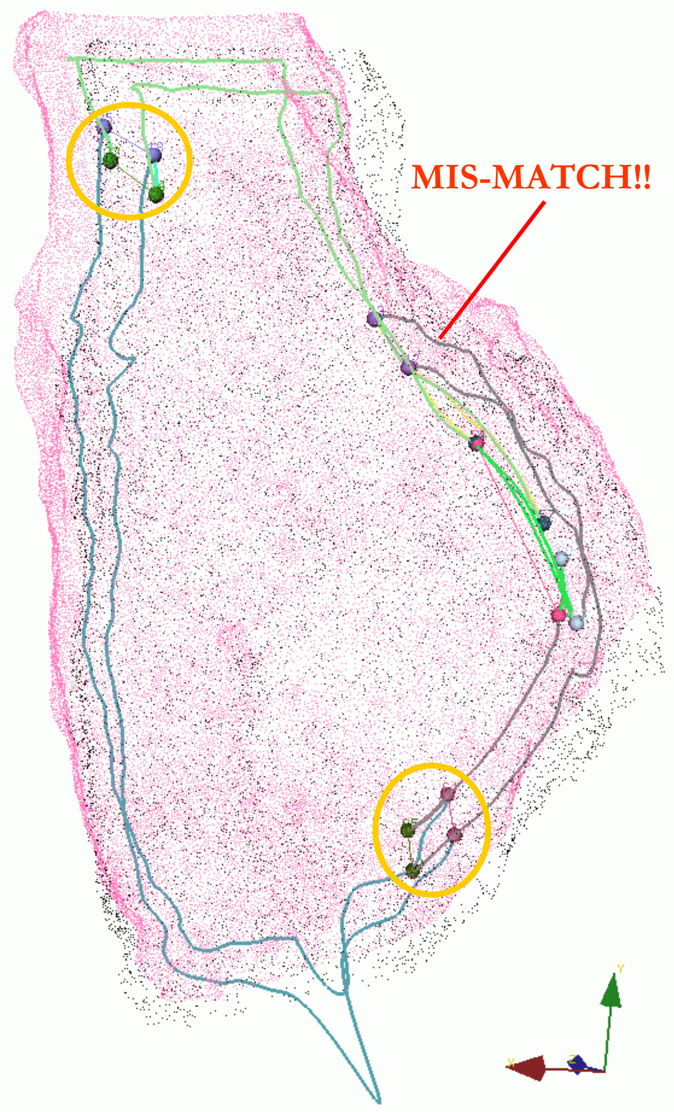
- Medial structure transitions are not completely handled.



Pot sherd 1 (50K)



Pot sherd 2 (10K)

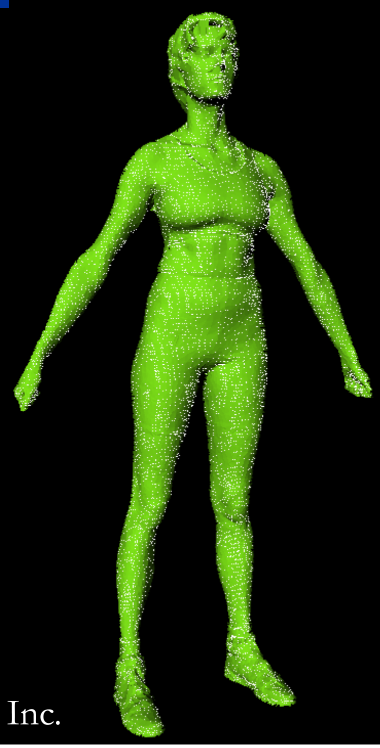


Result of shock matching

1. Only 8 shock vertices to match
2. Transitions not completely handled

Benefits of 3D Medial Scaffolds

- A global hierarchical structure is built-in.
- Scale is represented.
- Salient features are captured:
  - Generalized axes of elongated objects
  - *curvature extrema* and *ridges*
- The medial representation is *complete*. *Reconstruction* of the shape is always possible.
- Robust after regularization.
- Easy to handle shape deformations.



Data from Cyberware Inc.

Conclusions

Global Registration by Matching Medial Scaffolds

- Take input as point clouds or partial meshes.
- Robust to noise. Invariant under different resolutions & acquisition conditions.

\- Skeleton:

- Can be graphs with loops (not a tree).
- Contains *sheets*, *links*, *nodes*. Not over-simplified.
- Carefully Regularized.

\- Match:

- Nearly-optimal.
- Can be improved to do *fine* registration.
- Can be extended to register *non-rigid* objects.
- Can be extended to do *recognition*.

Thank You

Acknowledgements

- This material is based upon work supported by the National Science Foundation under Grants 0205477 and 0083231.