ABIO320: Ecology

Problem Set C

1. A population grows according to a logistic model where:

$$dN/dt = 0.2\ N - 0.0002\ N^2$$

What equation describes the individual's contribution to population growth, as a function of density N_t? That is, find (dN/dt)/N. For the given logistic (*i.e.*, r = 0.2), show that the carrying capacity is 1000.

2. A density-dependent population grows according to $dN_t/dt = 0.4\ N_t - 0.002\ (N_t)^2$, where N_t is the population size at continuous time t. The carrying capacity is

A) 50 B) 80 C) 200 D) 800 E) need further information

3. Using a diagram, indicate how population densities should vary in an ecosystem organized by "bottom-up" biotic interactions. Use a simple linear food chain with four (4) trophic levels as your model ecosystem. Briefly outline an experiment that would test the effects suggested by your diagram in the previous problem. You may envision a laboratory "microcosm" or a field test.

4. A population grows in continuous time according to the logistic model. The rate of population growth per individual

A) declines more slowly with population size (N) after the carrying capacity (K) is increased
B) at first increases, and then decreases, as population size increases
C) increases with an increase in the intrinsic rate *r* for all N < K
D) A & C are true E) B & C are true

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5. In discrete time, a population with density-dependent dynamics grows according to:

$$x_{t+1} = 2\ x_t\ (1 - x_t)$$

where x_t is proportional density. At dynamic equilibrium $x^* = x_{t+1} = x_t$. Find the equilibrium x^*.

6. At time t a population has size N_t, where $0 \leq N_t \leq N_{maximum}$. Let $x_t = N_t/N_{maximum}$, so that $0 \leq x_t \leq 1$. In discrete time x_t has dynamics:

$$x_{t+1} = 2.5\, x_t\, (1 - x_t)$$

At positive equilibrium $x_{t+1} = x_t > 0$. (a) Find the positive equilibrium. (b) What will happen (briefly) if $x_{t+1} = 0.5\, x_t\, (1 - x_t)$?

7. Consider one form of the discrete-time logistic model:

$$N_{t+1} \;=\; N_t\, e^{r\left(1 - N_t/K\right)}$$

Show that $N = K$ at positive equilibrium. Next, consider a population with the following discrete-time dynamics

$$N_{t+1} = N_t\left[(1+r) - \frac{r}{K} N_t\right],$$

where $r, K > 0$. Again, show that if $N_t = K$, the population size has reached positive equilibrium.