ABIO320: Ecology

Problem Set E

1. Species 1 and 2 compete; community dynamics follows the Lotka-Volterra competition equations:

$$dN_i/dt = \frac{r_i N_i}{K_i}\left[K_i - N_i - \alpha_{ij} N_j\right] \quad i = 1,2;\ j = 1,2,\ j \neq i$$

N_i is a density, r_i is an intrinsic rate of increase ($r_i > 0$), K_i is a carrying capacity, and α_{ij} is the competitive effect of an individual of species j on the growth of species i ($0 < \alpha_{ij} \leq 1$). Suppose $K_1 = 1000$, $K_2 = 800$, $\alpha_{12} = 0.5$, and $\alpha_{21} = 0.1$. If $N_2 = 100$, what value of N_1 falls on the 0-isocline of species 1 (where $dN_1/dt = 0$)?

Write 0-isocline for dynamics of species 1.

$$dN_1/dt = 0 \Rightarrow K_1 - N_1 - \alpha_{12} N_2 = 0$$

Substitute known values.

$$0 = 1000 - N_1 - 0.5\,(100)$$

Solve. $N_1 = 1000 - 50 = 950$

2. What will be the outcome of competition between the 2 species in question 3? If necessary, consult the text.

Coexistence requires $1/\alpha_{21} > K_1/K_2 > \alpha_{12}$

Check for given parameter values.

$1/0.1 > 1000/800 > 0.5$ **Hence** $10 > 5/4 > 1/2$

Outcome: Coexistence

3. Species 1 and Species 2 compete for a common limiting resource. The two-species dynamics follows the Lotka-Volterra competition model (logistic competition). Species 1 has carrying capacity $K_1 = 100$; Species 2 has $K_2 = 150$. $\alpha_{12} = 0.5$. Suppose $\alpha_{21} = 1.5$. Analyze the 0-isoclines and predict the outcome of competition. Do the same for $\alpha_{21} = 0.5$.

Species 1 excludes Species 2 when $\alpha_{21} = 1.5$.

4. Species 1 and 2 compete for common resource(s). The dynamics has the form of Lotka-Volterra competition. N_i is the density of species i (i = 1, 2). We have

$$dN_1/dt = \frac{r_1 N_1}{50}\left(50 - N_1 - 0.5N_2\right)$$

$$dN_2/dt = \frac{r_2 N_2}{40}\left(40 - N_2 - 0.4N_1\right)$$

where r_i is the intrinsic rate of increase for species i. Show that the 0-isocline for sp. 1 is:

$$N_1 = 50 - 0.5N_2$$

Find the 0-isocline for sp. 2. In < 4 sentences, define the isoclines.

$$\boldsymbol{N_2 = K_2 - \alpha_{21} N_1}$$

Enter parameters in 0-isocline of Species 2.

$$\boldsymbol{N_2 = 40 - 0.4\, N_1}$$

5. Briefly explain three mechanisms of competitive coexistence.

See your class-notes.

6. Summarize the general invasion-analysis approach to interspecific competition, competitive exclusion and coexistence.

For 2 competing species coexistence requires mutual invisibility. Each species when rare, with the other resting at its carrying capacity, must be able to grow positively when rare. This will occur when each species responds more to intra-specific than to inter-specific competition. Otherwise, the outcome will be competitive exclusion.

7. Preemptive competition occurs when:

A) neither of two species can invade the other
B) species compete for space
C) one species overgrows the other
D) none of the above

8. Two species of grasshopper mouse are found in highly similar valleys among the Santa Rita Mts. of southern Arizona. In any single valley we find only one species. If the pattern of occurrence of the two species is a consequence of competition, we can predict:

A) neither species can self-regulate

B) neither species can invade the other

C) each species can invade the other

D) one must have a competitive refuge

9. Character displacement:

A) reduces inter-specific competition through "niche differentiation"

B) is a co-evolutionary response to inter-specific competition

C) implies that competing species will differ more in allopatry than in sympatry

D) A and B, but not C, are correct

E) A, B and C are correct

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10. Discriminate direct and indirect interactions between species. Explain “apparent competition” and “competitive mutualism.”

11. V(t) and E(t) are, respectively, prey density and predator density at time t. The densities have dynamics:

$$dV(t)/dt = V - 0.01\ VE$$

$$dE(t)/dt = 0.002\ VE - E$$

Find the positive equilibrium densities; recall that you first set both rates to zero.

Set prey dynamics to 0

$$\mathbf{0 = V - 0.1\, VE} \qquad \mathbf{Then:\ 0 = 1 - 0.1\, E}$$

Hence, predator equilibrium density is $\mathbf{E = 100}$

Set predator dynamics to 0

$$\mathbf{0 = 0.002\, VE - E} \qquad \mathbf{Then:\ \ 0 = 0.002\, V - 1}$$

Hence, prey equilibrium density is $\mathbf{V = 500}$

12. Graphically analyze the “paradox of enrichment.”

See class-notes and text.

13. Consider the predator-prey system with dynamics

$$dH/dt = rH - \alpha HP - cH^2$$
$$dP/dt = \alpha HP - mP$$

where H is the prey density, and P is the predator density. How, biologically, does this interaction differ from the Lotka-Voltera model? What happens in the absence of predation? What is the equilibrium where predator and prey coexist?

Prey dynamics includes a term representing self-regulation; logistic-type intraspecific competition: $-\,c\,H^2$

In the absence of predation, prey goes to carrying capacity, $^r/_c$

Let's begin with the predator dynamics, and set growth = 0

$$0 = \alpha\,HP - m\,P \qquad Then{:} \quad 0 = \alpha H - m$$

Positive equilibrium prey density is $H = {}^m/_\alpha$

Now set prey dynamics = 0

$$0 = r\,H - \alpha\,HP - cH^2 \qquad Then{:} \quad 0 = r - \alpha\,P - cH$$

Substitute equilibrium prey density and

$0 = r - \alpha\,P - c\,{}^m/_\alpha$ Then: $\alpha\,P = r - {}^{cm}/_\alpha$ and $P = {}^r/_\alpha - {}^{cm}/_{\alpha^2}$

14. Let S_t, I_t and R_t represent, respectively, the number of susceptible, infective, and removed individuals in a general epidemic. S_0 is the initial number of susceptibles, and $I_0 = 1$. Consider the dynamics:

$$dS/dt = -\,0.001\,S_tI_t$$

$$dI/dt = 0.001\,S_tI_t - 0.5\,I_t$$

$$dR/dt = 0.5\,I_t$$

Note that the "infection rate" appears in the first two equations, and that the removal rate appears in the second and third equations. Find the relative removal rate. If the initial population of susceptibles is $S_0 = 501$ individuals, do we expect the infection to advance to an epidemic?

For given form of SIR dynamics $R_0 = {}^{\beta\, S_0}\!/_{\gamma}$

Then $R_0 = {}^{0.001\,(501)}\!/_{0.5} = {}^{1002}\!/_{1000} > 1$

Epidemic