

ABIO320: Ecology

Problem Set B

1. A population with non-overlapping generations (*e.g.*, an annual plant) exhibits geometric growth. Initial population size is 400. A time $t = 2$, population size is $N_2 = 625$. What is the annual growth rate?

2. Consider a population of annual plants with dynamics $N_{t+1} = \beta N_t$, where N_t is the population density at time (generation) t , and $\beta > 0$. Given $N_1 = 100$, and $\Delta N_1 = 25$, find the initial population size N_0 . Suppose that $N_1 = 150$ and $\Delta N_1 = 30$; what is N_0 ?

3. A population of 600 individuals experiences spatial heterogeneity in its discrete-time growth rate. If $2/3$ of the population produces 1 individual per individual, and $1/3$ of the population produces 2 per individual, what will population size be next year?

$$N_t = 600: N_{t+1} = n_1(t) \lambda_1 + n_2(t) \lambda_2 = 400(1) + 200(2) = 800$$

4. Two annual-plant species occupy the same environment. Species A responds to temporal variation as if years were of 2 types. The annual reproductive rate λ_A takes 2 values, with differing probabilities: $\Pr[\lambda_A = 2/3] = 1/3$; $\Pr[\lambda_A = 6] = 2/3$. Species B responds to the same environment as if years were of 3 types. That is: $\Pr[\lambda_B = 1] = 1/6$; $\Pr[\lambda_B = 4] = 3/6$; $\Pr[\lambda_B = 8] = 1/3$. Which species has the greater geometric mean growth rate?

Species A

$$GM = (2/3)^{1/3} 6^{2/3} = (2/3 [36])^{1/3} = 24^{1/3} = 2 \cdot 3^{1/3}$$

Species B

$$GM = 1^{1/6} 4^{1/2} 8^{1/3} = 1 (2)(2) = 4 > 2 (3)^{1/3}$$

5. Two species with non-overlapping generations occupy the same environment. Warm years and cold years occur independently; each year is warm with probability 0.5. Species 1 produces 3.5 individuals per individual in a warm year, and 1.0 individual per individual in a cold year. Species 2 produces 8 individuals per individual in a warm year, and 0.25 individuals per individual in a cold year. Then:

- A) Species 1 has a lower arithmetic mean growth rate
- B) Species 1 has a lower geometric mean growth rate
- C) A & B are true
- D) neither A nor B is true

Species 1

$$GM = (3.5)^{1/2} 1^{1/2} = 3.5^{1/2} \quad AM = 4.5/2 = 2.25$$

Species 2

$$GM = 8^{1/2} (1/4)^{1/2} = 2^{1/2} \quad AM = 8.25/2 = 4.125$$

Sp 1: Lower AM, Larger GM

6. Global climate change will likely bring more extreme weather events, with impact at the year to year timescale. What might this imply for annual species with growth rates sensitive to temporal variation among generations?

7. In 2 of every three years, on average, the number of individuals produced per individual in a population with non-overlapping generations is 8. Otherwise, the mean number of individuals produced per individual is 1/8. Will the population persist in this time-varying environment?

$$GM = 8^{2/3} (1/8)^{1/3} = 2^2 (1/2) = 2 > 1$$

8. A population in a time-varying environment experiences growth rates of λ_1 and λ_2 (where $\lambda_2 > \lambda_1$) with equal probability each year. Suppose global climate change alters the environment so that the population now experiences growth rates of $(\lambda_1/2)$ and $2\lambda_2$; the first rate is halved and the second is doubled. We can conclude that global climate change has

- A) increased the arithmetic mean growth rate
- B) decreased the arithmetic mean growth rate
- C) increased the geometric mean growth rate
- D) A & C are true
- E) B & C are true

$$GM = (\lambda_1/2)^{1/2} (2\lambda_2)^{1/2} = (\lambda_1 \lambda_2)^{1/2} \text{ indicating no change.}$$

AM: new – old

$$\lambda_1/4 + \lambda_2 - \frac{1}{2} (\lambda_1 + \lambda_2) = \lambda_2/2 - \lambda_1/4 > 0$$

9. Consider a population with non-overlapping generations in a time-varying environment. The possible growth rates are [1/9, 1, 8]. The associated probabilities are [1/2, 1/6, 1/3]. Will this population persist (or go extinct) as time grows large?

$$GM = 1/3 (1) 2 = 2/3 < 1$$