University at Albany, State University of New York  
Department of Economics  
Ph.D. Preliminary Examination in Microeconomics, June 15, 2016

Answer any three of the following four numbered problems. Justify your answers whenever possible. Write your answer to each numbered problem in a separate bluebook. Write the number of the problem AND NOTHING ELSE on the cover of the bluebook. No electronic devices may be used. The exam lasts 4 hours.

1. Consider a monopolist who produces output $Q$ under constant returns to scale, with marginal cost of 10. The monopolist produces during $T > 2$ periods. Demand for its product in each period is $D(P) = 100 - 2P$, for $P \leq 50$. However, the monopolist does not know the demand at the outset. It only knows that the demand is linear and stationary.

a. One recourse is for the monopolist to sample the market by announcing prices $P_1$ and $P_2$ in periods 1 and 2 and producing whatever quantities $Q_1$ and $Q_2$ consumers demand at these prices. It would then determine demand and behave optimally in subsequent periods. Show how this procedure works for the sample prices $P_1 = 20$ and $P_2 = 40$ and determine the optimal supply and pricing strategy for each period $t > 2$. (Assume there is no discounting of future profits.)

b. An alternative recourse would be for the monopolist to hire a market research firm at a cost of $s$ to determine the exact demand it faces. What is the value of knowing the exact demand prior to the commencement of operations (i.e., prior to period 1) rather than having to announce the sample prices $P_1 = 20$ and $P_2 = 40$ in periods 1 and 2? Would the firm be willing to pay an amount $s$ up to this value? Is it capable of making this determination ex ante? Discuss.

c. Instead of announcing specific prices $P_1 = 20$ and $P_2 = 40$, suppose the monopolist were to draw prices randomly from a uniform distribution on the interval $[10, 50]$. Explain how to determine the firm’s expected profit in the single period when it sets this randomly chosen price. Is the firm capable of performing this calculation ex ante? Explain.

d. Alternatively, suppose the firm knows that its demand is of the form $D(P) = mP + b$. The firm believes that $m$ and $b$ are independently distributed; $m$ has density function $f$ on $[-1, -4]$ and $b$ has density $g$ on $[80, 200]$. In this case, write an expression for the firm’s expected single-period profit from announcing price $P$. Write an expression for its expected profit if it were to announce an optimal $P$. Is it capable of evaluating these?

e. Discuss how the sampling procedure described above involving one observation per period might be generalized beyond the linear case to allow for an arbitrary demand curve. What considerations might be relevant in deciding whether to hire a market research firm to determine the firm’s demand?

2. Consider a pure exchange economy with two consumers and two goods. When consumer $i$ consumes $x_{1i} \geq 0$ units of good 1, consumer 1 gets utility $x_{11} + \phi(x_{21})$ and consumer 2 gets utility $\phi(x_{12}) + x_{22} - \theta x_{21}^2$, where $\theta \geq 0$ and $\phi' > 0$, $\phi'' < 0$. Consumer 1 initially owns $e_1 > 0$ units of good 1 and consumer 2 owns $e_2 > 0$ units of good 2.

a. Describe in English the consumers’ preferences and demand functions, being as specific and complete as possible. For what goods might real consumers have preferences similar to those in the model?

b. What is the set of feasible allocations in this economy?
c. Characterize the set of interior Pareto efficient (PE) allocations (in which both consumers consume positive amounts of both goods). In what way does this set depend on \( \theta \)? Starting from a PE allocation, does a small transfer of good 1 from consumer 1 to consumer 2 leave the allocation Pareto efficient?
d. Let \( \theta = 0 \). Characterize a competitive (Walrasian) equilibrium (CE) for this economy. Is the allocation Pareto efficient?
e. Let \( \theta = 1/2 \). Characterize a CE when consumer 2 acts as if the equilibrium consumption of good 2 by consumer 1 is fixed, unaffected by actions of consumer 2.
f. Under what, if any, condition is the CE allocation in part e Pareto efficient?
g. Is it possible that in the CE allocation in part e consumer 1 consumes inefficiently little of good 2, given consumer 1’s consumption of good 1? Consider the case in which \( \phi(t) = \ln t \). Explain and interpret your answer.

3. A government devises an optimal tax schedule for a population, half of whom are of type \( H \), half of type \( L \). A type \( H \) agent produces \( q_H = 2e_H \) units of output (income) when it provides \( e_H \in [0, 1] \) units of effort. A type \( L \) agent produces \( q_L = (3/2)e_L \) units of output when it provides \( e_L \in [0, 1] \) units of effort. Outputs produced by different agents are physically homogenous, hence indistinguishable, and (except in part a, below) the government cannot directly observe the type of any particular agent or its effort level. The government observes agents’ incomes (outputs) and assigns a tax \( t \) to each income level. The government knows that if a type \( i \) agent produces \( q_i \) units of output, providing \( e_i \) units of effort and paying \( t_i \leq q_i - e_i \) in tax, the agent gets utility \( (q_i - t_i - e_i)^{1/2} \).

(a) For this part of the problem only, consider the case of perfect information, where the government can observe the effort level of each agent and tax accordingly. Explain why, under an optimal tax schedule, the chosen \((q_i, e_i, t_i)\) for types \( i = H, L \) maximize average utility subject only to the constraint that net tax revenue from these choices is nonnegative. Compute the solution to this optimization problem.

(b) From now on, we return to the case in which the government cannot observe agents’ types and effort levels. Explain why, under an optimal tax schedule, the chosen \((q_i, e_i, t_i)\) for types \( i = H, L \) maximize average utility subject to the tax revenue constraint and to an incentive constraint for each type, which specifies that that type gets at least the utility it would get by producing as much output as the other type chooses. Write this constrained optimization problem using the notation above.

c. Show that if the incentive constraint for type \( L \) is omitted from the optimization problem in part b, then the incentive constraint for type \( H \) binds at a solution.

d. Solve the optimization problem in part b with the incentive constraint for type \( L \) omitted.

e. Show that at the solution to the optimization problem in part d, the incentive constraint for type \( L \) in part b is satisfied. What can be concluded about the allocation and the utilities of the different types under an optimal tax schedule in part b? Be as specific as possible.
4. Candidates for a job with a single employer, $E$, are either of type $H$ or $L$. Knowing their own type, they choose whether or not to take a training course. An untrained candidate (who has not completed the course) produces output valued at 2 units working for $E$. A type $H$ candidate who is trained (has completed the course) produces a value of 6 units working for $E$. A trained type $L$ produces 5 units working for $E$. Untrained workers can get jobs with other firms paying 1 unit. Trained workers of type $H$ [respectively, type $L$] can get jobs with other firms paying 4 units [respectively, 2 units]. Training costs type $H$ a value of 1 unit and costs type $L$ 2 units, a cost that must be fully paid before the course is completed. The interaction between the candidate and $E$ can be represented as a game in which $E$'s payoff equals the value of the candidate's output minus the wage $E$ pays if the candidate accepts $E$'s offer and equals 0 otherwise. An offered wage can be any real number. The candidate's payoff is its wage (from $E$ or else from another firm) minus its training cost (no cost if it does not take the training course). In the game, the candidate first chooses whether to get training, then $E$ offers a wage that can depend on whether the candidate is trained. The candidate accepts $E$'s offer or else rejects it and works for another firm. The interaction described above is common knowledge.

a. Why might it make sense to model a type $L$ candidate as having a higher cost of training than a type $H$ candidate even if all students pay the same price for training courses?

b. Suppose that the job candidate is of type $L$ and $E$ knows it. Explain what a pure strategy for $E$ is. How many pure strategies does $E$ have? Give an example of a pure strategy of the type $L$ candidate. Be sure to specify or describe it completely.

c. For the game in part b with a type $L$ candidate, find a pure subgame perfect Nash equilibrium (SPE) of the subgame after the candidate gets training.

d. Find a pure SPE for the entire game in part b, where $E$ knows that the candidate is type $L$. Interpret the outcome. Does the outcome maximize total surplus over all allocations that are feasible with a type $L$ candidate?

In the remaining parts of the problem, assume that $E$ initially does not know the candidate's type and believes it is $H$ with probability $\lambda$, where $1/2 < \lambda < 1$.

e. Draw a game tree representing the interaction between $E$ and the job candidate.

f. In the game described above, how many pure strategies does the employer $E$ have? Give an example of one.

g. In the game described above, how many pure strategies does the job candidate have? Give an example of one.

h. Show that there is a weak sequential equilibrium (WSE, also called WPBE) in which neither type candidate gets training. What can be said about the plausibility of $E$'s beliefs in such an equilibrium?

i. Show that there is a pure WSE in which both candidate types get training. Compare the efficiency of the allocation for type $L$ in this WSE to that in the SPE of part d. Use the comparison to describe the impact of $E$'s uncertainty about the candidate's type.