1. Consider an economy with two consumers, 1 and 2. Each consumer consumes only grapes and wine and can use grapes as an input to produce wine. Grapes used as input cannot be consumed directly as grapes. Consumer 1 produces 1 unit of wine for each unit of grapes it uses as input. Consumer 2 produces 2 units of wine for each unit of grapes it uses as input. Consumer 1 initially owns 1 unit of grapes and no wine. Consumer 2 initially owns 1 unit of grapes and 2 units of wine. Consumer 1 gets utility $u_1(g_1, w_1) = g_1w_1$ from consuming $g_1 \geq 0$ units of grapes and $w_1 \geq 0$ units of wine. Consumer 2 gets utility $u_2(g_2, w_2) = g_2(w_2 - 1)$ from consuming $g_2 \geq 0$ units of grapes and $w_2 > 1$ units of wine, and gets utility $u_2(g_2, w_2) = 0$ otherwise.

a. Give a verbal explanation of the way the consumers’ preferences differ from each other.
b. Find the optimal production and consumption of each consumer separately, assuming that the consumers do not trade with each other.
c. Consider the private ownership economy consisting of the two consumers, each owning a competitive firm that uses that consumer’s wine-making technology. Prove that in a competitive (Walrasian) equilibrium neither good can have a price of 0.
d. Find every competitive equilibrium for this economy, letting grapes be numeraire. How many such equilibria are there? Compare the utility levels of the two consumers to what they receive in part b, when they do not trade.
e. Find a Pareto efficient (Pareto optimal) allocation for this economy that is not a competitive equilibrium allocation and that gives positive utility to consumer 1. (Try to find an allocation that is easy to characterize with little calculation.) Describe how this allocation differs from a competitive equilibrium allocation.
f. Is there any Pareto efficient allocation in which both firms produce positive quantities of wine? Is every competitive equilibrium allocation in this economy Pareto efficient?
g. In a competitive equilibrium in this economy, the consumer with the more productive technology supplies more input and the consumer who has more need of output for consumption gets more of the output. Is this property (more productive agents supply more input and agents more in need of a good get to consume more of it) a characteristic of all competitive equilibria in general equilibrium models? Explain and justify your answer by checking whether the property continues to hold in the economy above no matter what the consumers’ initial endowments are.

2. There are two types of consumer who have differing tastes. A firm faces a consumer. It is known that the probability of the consumer being of type $H$ is $\frac{1}{2}$ and being type $L$ is $\frac{1}{2}$. The $H$ type has utility function $U_H = 4\sqrt{q} - t$ and the $L$ type has utility function $U_L = \sqrt{q} - t$, where $q$ is the quality of a particular good the firm sells and $t$ is the monetary payment for the good. The firm can choose any level of quality $q \geq 0$ at a cost of $q$ dollars. There is no fixed cost. The reservation utility of both types is zero.

a. Suppose the firm can identify the type of the consumer before offering a two part tariff $(T, p)$; i.e., $t = T + pq$. What are the optimal two part tariffs and what is the corresponding ex-ante (before the firm learns the type of the consumer) expected profit?
b. From now on, assume that the firm cannot identify the types directly. What is the expected profit maximizing two-part tariff $(T, p)$ for the firm? What is the corresponding
expected profit?
c. Suppose now that the firm can offer any (non-linear) contract. What is an optimal contract and what is the corresponding expected profit of the firm? Compare your answer with your answer in part a.
d. Now, suppose there are two consumers whose types are drawn independently from the identical distribution in the above. The consumers know their types but not the other consumer’s type. The producer knows only their distributions. Suppose the producer produces \( q \) and auctions it off to the two consumers according to the ascending (English) auction. What is the expected revenue of the firm? What is the optimal \( q \) and corresponding profit of the firm?

3. The following is a three period model of a financial contract involving a lender (bank) and a borrower (firm). In the first period, the firm either builds a new facility at cost \( c > 0 \) or it does not. If the firm builds the new facility, its subsequent production function in value terms becomes \( 4\sqrt{L} \); otherwise it is \( 2\sqrt{L} \). Here, \( L \) is the amount of loan used in the production. In the second period, the bank offers a contract \((L, t)\) to the firm. A contract consists of \( L (\geq 0) \) the amount of loan and \( t (\geq 0) \), the gross amount the borrower has to repay. Assume that the firm does not default. In the third period, the firm either accepts the contract or rejects the offer. If a contract \((L, t)\) is offered and accepted, the bank receives the payoff of \( t - RL \), where \( R > 1 \), while the payoff to the firm is \( 4\sqrt{L} - t - c \) if it built the facility in the first period and \( 2\sqrt{L} - t \), if it did not. If the firm rejects the contract, the payoff to the bank is zero and the payoff to the firm is \( -c \) if the firm built the facility in the first period and is zero otherwise. Assume that the firm accepts the contract if accepting and rejecting the contract result in the same payoffs. Both the bank and the firm are risk neutral. Assume that \( \frac{2}{R} > c \).
a. Suppose the bank knows whether the firm built the new facility or not in the first period when it offers a contract. Draw the game tree of this game.
b. Find all sequential equilibria of the game in (a).
c. Suppose from now on that the firm’s possible investment in the first period is in software and the bank does not know whether or not the firm made the investment when it offers the contract. Draw the game tree of this game.
d. What is the optimal contract offer of the bank as a function of its beliefs? The bank believes that the firm invested in the software with probability \( p \).
e. Find all sequential equilibria of this game.

4. Consider a firm with the technology \( q = (x_1 x_2)^{\frac{1}{4}} \) that purchases inputs \( x_1 \) and \( x_2 \) in competitive factor markets at the prices \( w_1 = w_2 = 1 \). Initially, the firm faces the demand curve \( q(p) = 120 - p \) for its output.
a. Determine the firm’s optimal supply decision and profits.
b. Next, suppose the firm’s profits were taxed at the constant rate of 20%. How would this affect the firm’s optimal decisions? How much would the firm pay in taxes and what would be its post tax profits?
c. Suppose that, in addition to producing \( q \), the firm could also contribute money, \( y \), to charity and thereby reduce its overall tax rate. The rate would then become \( \tau(y) = \max\{(0.2 - 0.001y), 0\} \). Assuming the contribution is from pretax earnings, i.e., it can be treated as an additional cost, determine the optimal supply decision and charitable contribution of the firm. What are the firm’s profits? Compare the amount of taxes paid in part (b) to the amount of taxes and charity paid here. Also compare the firm’s post tax profits.
d. Suppose again that the tax rate is fixed at \( t = 0.2 \). Now, while charitable contributions do not affect the tax rate, they garner publicity for the firm and increase demand for its
product. Thus, the firm faces the demand \( q(p) = 120 - p + 2\sqrt{y} \). Again, assume the con-
tribution is from pretax earnings. Determine the firm’s optimal supply decision in this case. 
Also determine the amounts of taxes and charity paid and the post tax profits.
e. Assuming that tax revenues and charitable contributions are devoted to precisely the same 
uses, rank the cases (b), (c) and (d) in terms of the resources (tax revenues and charity) 
generated and post tax profits.