Instructions Answer 3 of the following 4 questions. Show all of your work. Write your answer to each question in a separate bluebook. On the cover of the bluebook, write the number of the question under “Section.” DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam lasts 3 hours.

[1] A coalition of \( n \) firms \((n > 2)\) engages in a joint venture. Firm \( i \) contributes input \( x_i \), and the total benefit resulting from the project is \( B(x) \), where \( x \) denotes the aggregate amount contributed. However, each firm only incurs the cost associated with its own contribution, \( C(x_i) \). Suppose \( B(0) = 0, B'(x) > 0 \) and \( B''(x) < 0 \), and that \( C(0) = 0, C'(x) > 0 \) and \( C''(x) > 0 \). Each firm receives an equal share of the benefit regardless of its contribution.

a. Formulate the decision problem facing firm \( i \) and characterize an optimal solution. Assume that the firms act simultaneously and independently.

b. State whether the following is True, False or Uncertain. Explain. “Since a firm can reap a benefit from the joint venture without incurring any of the cost, it is optimal to contribute zero.”

c. Discuss whether or not the structure described above is sufficient to ensure the existence of a best response by firm \( i \) to the contributions \( x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \) of the \( n - 1 \) other firms.

d. Discuss the efficiency of Nash equilibrium outcomes for this game.

e. Suppose the firms’ contributions affect their share of the benefit, i.e., firm \( i \) would receive a share \( f_i(x_1, \ldots, x_n) \geq 0 \), where \( \sum_i f_i(x_1, \ldots, x_n) = 1 \). Discuss conditions on \( f_i \) that would ensure the existence of a best response to \( x_{-i} \).

f. In contrast to the general expressions indicated above, suppose the benefit and cost functions were given explicitly by \( B(x) = \sum_i x_i \) and \( C(x_i) = x_i^2 \). Prove that each firm has a dominant strategy, and determine the (dominant strategy) equilibrium payoff for each firm.

g. Again assume \( B \) and \( C \) are as in part f., and assume any subcoalition would face the same benefit and cost schedule on its own. Prove that it would be advantageous for two of the firms to separate from the others and engage in a joint venture of their own. In light of the structure of this economy, discuss why this advantage occurs.

[2] Consider an economy with a continuum of workers and a continuum of firms. Workers care about their wage earnings but are also concerned about the conditions of their work environment. Specifically, individual \( i \) has preferences represented by the utility function

\[
    u_i(c, e) = c + \delta_i e,
\]

where \( c \) denotes consumption, \( e \) takes on the value 1 if the work environment is pleasant and \( -1 \) if it is unpleasant, and \( \delta_i \) measures the extent to which individual \( i \) is concerned about the work environment. Workers are thus characterized by their \( \delta_i \). For the entire population of workers, \( \delta \) is distributed uniformly on \([0, 1]\). Workers have no nonlabor income, and they each supply labor inelastically. Each firm hires one worker and produces output with value 1. In addition, firms can expend resources to ensure that the working environment is pleasant (i.e., that \( e = 1 \)), but this reduces the value of their output. Firms differ in their cost of providing a pleasant work environment. Specifically, if firm \( j \) were to do so, it would reduce the value of its output to \( \alpha_j \leq 1 \). Firms are thus characterized by their \( \alpha_j \); and among all firms, \( \alpha \) is distributed uniformly on \([0, 1]\). If a firm fails to expend any resources, then the environment is unpleasant. Finally, \( w_1 \) is the
wage paid to a worker in a pleasant environment (measured in units of output or consumption) and \( w_0 \) is the wage in an unpleasant environment.

a. What determines each worker’s choice of job?

b. Given \( w_0 - w_1 \), which workers choose to work in unpleasant environments?

c. What determines whether a given firm provides a pleasant environment or not?

d. Given \( w_0 - w_1 \), which firms invest in worker satisfaction?

e. Find the equilibrium \( w_0 - w_1 \) and the equilibrium proportion of pleasant jobs.

f. Which individuals end up in the unpleasant jobs and to which firms are they assigned? How does equilibrium utility vary with \( \delta \) and how do equilibrium profits vary with \( \alpha \)? Explain.

[3] Half of the workers in a particular population have high productivity and the rest have low productivity. High and low productivity types with education level \( e \) produce respectively 8+2\( e \) and 1+4\( e \) units of output per day no matter where they work. Workers in this population choose their (publicly observable and verifiable) education level \( e \in [0, 3] \) knowing their type. Then each worker enters a job market in which two firms simultaneously and independently offer wages to the worker. The worker accepts one firm’s offer or else rejects both firms and obtains utility 0. If the worker accepts a firm’s offer at the wage \( w \) then the worker obtains utility \( w - e^2 \) if its productivity is low and \( w - (1/2) e^2 \) if its productivity is high. If a firm hires the worker at a wage \( w \) then the firm obtains a payoff equal to the worker’s productivity minus \( w \). The above conditions are common knowledge among the workers and firms.

a. Suppose first that each worker’s type is publicly observable and verifiable. Find all combinations of education and wage for the different types of workers that can arise in subgame perfect Nash equilibrium. Justify your answer.

b. Evaluate the efficiency of the outcome allocations in part a. Are they Pareto efficient? Explain carefully what this means in the present model.

NOTE In the rest of the problem, assume that the firms cannot observe a worker’s type when they make their wage offers.

c. Find all possible education and wage combinations received by the different worker types in pure perfect Bayesian equilibrium (PBE). Show all your work. Use one or more diagrams to illustrate these education and wage combinations. NOTE that the workers cannot choose \( e \) greater than 3.

d. Are some of the equilibrium outcomes from part c more plausible than others? Explain.

e. Evaluate the efficiency of the equilibrium outcomes in part c. Are any outcomes Pareto efficient?

f. For each Pareto inefficient equilibrium outcome from part c, determine whether it would be possible for a social planner with the same information as the firms to achieve a Pareto improvement. What can be concluded about the efficiency of screening in a labor market like the one above? Justify your conclusions.

g. Suppose that before the workers choose their education levels the firms can simultaneously, independently and credibly commit themselves to offering particular wages for particular education levels. What can be said about subgame perfect Nash equilibrium outcomes of the resulting game played by a particular worker and the two firms? Be as specific as possible with the given information and interpret your conclusions. This question can be answered using the answer to part f. above.
Consider the following two-period model of a financial contract. In the first period, a lender offers a menu of contracts to a borrower. A contract consists of \( L \geq 0 \) the amount of loan and \( t \geq 0 \), the amount the borrower has to repay in the second period. Assume that the borrower does not default. The borrower either chooses an item in the menu or rejects the offer. If a contract \((L, t)\) is offered and accepted, the lender receives utility \( t - rL\), where \( r \) is the riskless alternative lending interest rate while the borrower invests the loan and produces \( \theta f(L) \), obtaining utility \( \theta f(L) - t \). Assume that \( f(0) = 0 \) and for \( L \) strictly positive, \( f'(L) > 0 \) and \( f''(L) < 0 \). Also, for any \( r > 0 \), there is \( L > 0 \) such that \( f'(L) = r \). The productivity parameter \( \theta \) can take value \( \theta_h \) with probability \( p \) and value \( \theta_l \) with probability \( 1 - p \), where \( \theta_h > \theta_l > 0 \) and \( 1 > p > 0 \). The borrower learns her type \( \theta \) before she obtains the loan. The borrower’s reservation utility is zero.

a. Describe the contracts lender would offer if he can identify the type of borrower before offering a contract. Explain why a contract offered by the lender and satisfying the participation constraint of the borrower yields non-negative utility for the lender.

b. Now, suppose that the lender does not learn the type of borrower when he makes the contract offer. Formulate the problem of the lender when he wants to maximize his expected utility subject to incentive and participation constraints. In the formulation, let \((L_h, t_h)\) be a contract aimed at type \( \theta_h \) and \((L_l, t_l)\) be a contract aimed at type \( \theta_l \).

c. Show that at an optimal contract for the lender, the participation constraint of the low productivity borrower is binding.

d. Show that at an optimal contract for the lender, the incentive constraint of the high productivity borrower is binding.

e. Show that the participation constraint of the high productivity borrower and the incentive constraint of the low productivity borrower are satisfied so long as the binding constraints in c and d are satisfied and \( L_h > L_l \).

f. Solve the problem of the lender and interpret the solution. Show in your solution, \( L_h > L_l \). Compare the solution with solutions in a. (you can assume that \( L_l > 0 \) at the optimum).