1. Consider an exchange economy with two goods, $x$ and $y$, and two agents, 1 and 2. Each agent is initially endowed with 5 units of $x$ and 5 units of $y$. Agent 2 has preferences for $x$ and $y$ represented by the utility function $u_2(x_2, y_2) = x_2 y_2$. Agent 1 also likes $x$ and $y$, but, in particular, it derives utility from consuming more $x$ than agent 2. Agent 1’s preferences are represented by the utility function $u_1(x_1, y_1, x_2) = (x_1 - x_2)y_1$. Thus, while it does not choose $x_2$, it is affected by it.

   a. Assuming the agents were able to trade at competitively determined prices, set up and solve each agent’s maximization problem and determine it’s demands for $x$ and $y$.

   b. Find a competitive equilibrium.

   c. Explain how the process of determining a competitive equilibrium in this case differs from the usual case in which agents only care about their own consumption.

   d. Set up and solve the Social Planner’s Problem for determining the Pareto efficient allocations.

   e. Verify that the equilibrium allocation in part b is inefficient. Explain why.

   f. Compare the above outcome in b to the case in which agent 1 had “standard” preferences and only cared about its own consumption, say, $u_1(x_1, y_1) = x_1 y_1$. What would be the equilibrium allocation in that case? Does the fact that 1 compares its $x$ consumption to 2’s increase or decrease the amount 1 demands of good $x$?

   g. Argue that if there were “mutual one-upmanship,” that is, if each tried to consume more than the other (say, $u_i(x_i, y_i, x_j) = (x_i - x_j)y_i$, for $i, j = 1, 2, i \neq j$), then no competitive equilibrium would exist in such an economy.
A firm, $E$, is considering entering a market with a single incumbent firm, $I$. If $E$ enters, firm $I$ can either fight or share the market. Before $E$ enters, $I$ thinks that with probability $p$, $E$’s technology is better than $I$’s and with probability $1-p$, $E$’s technology is worse (where $0 < p < 1$). When it considers entering, $E$ knows if its technology is better or worse than $I$’s (the result of past choices that it cannot change). If $E$ does not enter, then it gets 0 profit and $I$ gets a profit of 10. If $E$ enters with worse technology and $I$ fights, then $E$ gets profit of $-4$ and $I$ gets 5. If, instead, $I$ shares the market, then $E$ gets profit of $-1$ and $I$ gets 4. If $E$ enters with better technology and $I$ fights, then $E$ gets profit of $-1$ and $I$ gets $-3$. If $E$ enters with better technology and $I$ shares the market, then $E$ gets profit of 4 and $I$ gets 1. Each firm seeks to maximize its expected profit. The information above is common knowledge. The firms’ interaction can be represented by the game tree in Figure 1.

![Game Tree](image)

**Figure 1**

a. At each node in the tree in Figure 1, write a label for the player who moves at that node. Label each move in the tree and write the firms’ payoffs at the appropriate places.
b. How many pure strategies does firm $E$ have? Give an example of one of them.
c. Find the set of Nash equilibria (NE) in which $I$ chooses a pure strategy. Describe the NE strategies. Does this set of NE depend on the value of $p$? If so, how? If not, show that it does not.

In the remaining parts of the problem, assume that $p = 1/2$.
d. Find every sequential equilibrium (SE) in which firm $I$ chooses a pure strategy.
e. If an equilibrium existed in which $I$ chose a totally mixed strategy, how could this behavior by $I$ be interpreted as part of its interaction with firm $E$?
f. Does the game have a sequential equilibrium (SE) in which $I$ chooses a totally mixed strategy? If so, find one. If not, show that no such SE exists.
g. Compare the SE’s you have found. Are any of them more plausible than the others as representations of outcomes of the interaction between rational firms with the characteristics of $E$ and $I$? Describe and explain the relevant differences among these SE’s.
3. Five medical students $i = 1, \ldots, 5$ are candidates to be placed in residencies with four hospitals, $j = 1, \ldots, 4$. No student wants more than one residency position and no hospital wants to hire more than one student. The tables below show the students’ rankings of hospital residencies and the hospitals rankings of the students. Each student $i$ [respectively, hospital $j$] ranks the hospitals [resp. students] as listed in the row beginning with $i$ [resp. $j$], with more preferred listed first, reading left to right. For example, student 3 ranks hospital 4 first, then hospital 3, then hospital 1, then 2. Student 5 ranks 1 first, then hospital 2, then 4, and prefers no residency to a residency at hospital 3. Each hospital would rather fill its residency with any student instead of leaving the position empty.

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a. Define a matching of students to hospital residencies. Note that no hospital can be matched with more than one student, so there must be at least one student without a residency.
b. Define a stable matching in this environment.
c. Find the outcome of the student-proposing deferred acceptance algorithm (SDA) applied to the preference rankings above. Show all your work.
d. Is the outcome matching in part c stable? Show that your answer is correct.
e. Is there a group of students who could each get a more preferred outcome than the one in part c if, for the SDA, they all reported different from their true preferences listed above? Assume that the agents outside the group report their preferences listed above. Justify your answer. You do not need to prove it is correct.
f. Is there a hospital that could get a more preferred outcome than the one in part c by reporting different preferences to be used in the SDA when all other agents report their true preferences listed above? Show that your answer is correct.
g. Is there a procedure that would take reported rankings by students and hospitals and determine a stable matching such that every hospital would have an incentive to report its true ranking listed above when the students report their true preferences? Explain.
h. What economic reason might explain why, in the absence of a formal organized matching procedure, hospitals typically offer residencies to students rather than students proposing to hospitals?
4. A monopolistic seller sells products to a continuum of buyers of size 1. There are two types of buyers, \( i = 1, 2 \). Every type 2 buyer receives utility equal to
\[ 16q_2 - (q_2)^2 - t_2 \]
if it purchases \( q_2 \) units of the good for a total payment of \( t_2 \) dollars. Every type 1 buyer receives utility equal to
\[ 12q_1 - (q_1)^2 - t_1 \]
if it purchases \( q_1 \) units of the good for a total payment of \( t_1 \) dollars. It costs the seller a constant average cost of $2 to produce one unit of the good. Let \( n_i \) be the fraction of buyers of type \( i \), so \( n_1 + n_2 = 1 \). Each buyer receives a reservation utility equal to zero if the buyer does not purchase anything from the seller.

(a) First suppose the seller can observe each buyer’s type and can offer each type-\( i \) buyer a contract \((q_i, t_i)\). Each type \( i \) buyer can either choose the contract \((q_i, t_i)\) or walk away. What is the seller’s profit-maximizing (i.e. optimal) contract \((q_i, t_i)\) for each type \( i \)?

For the rest of the problem, suppose buyer types are not observable to the seller. The seller can offer a menu of two contracts \{\((q_1, t_1), (q_2, t_2)\)\} to all buyers. Each buyer may choose one of the contracts or walk away. If a buyer selects contract \((q_i, t_i)\) then the buyer is entitled to receive \( q_i \) units of the good by paying the total price \( t_i \) to the seller regardless of the buyer’s true type. For parts (b) through (e), assume \( n_1 = n_2 = 1/2 \).

(b) Formulate the seller’s profit-maximizing problem: write down the seller’s total profits as the objective function and all the constraints including incentive constraints (ICs) and individual rationality constraints (IRs) for both types. Call this the original problem (OP).

(c) Next consider the seller’s profit optimization problem but without type 2 buyer’s IR constraint (IR2) and without type 1 buyer’s IC constraint (IC1). Call this the relaxed problem (RP) because the constraints of the original problem have been relaxed. With only constraints (IC2) and (IR1), find the optimal menu of contracts \{\((q_1, t_1), (q_2, t_2)\)\} that maximize the seller’s profits in this relaxed problem.

(d) Show that the solution in part (c) also satisfies the additional constraints, IR2, IC1, in the original problem OP in (b).

(e) Argue that based on part (d), the optimal contracts found in (c) are, in fact, also optimal for the original problem OP in (b).

(f) Now suppose the fraction of type 2 buyers goes up to 3/4 and the fraction of type 1 buyers goes down to 1/4. What does the optimal menu of contracts look like now for the seller? Give an economic explanation of the comparison.