Instructions: Answer any three of the four numbered problems. Justify your answers whenever possible. Write your answer to each problem in a separate bluebook. Write the number of the problem AND NOTHING ELSE on the cover of the bluebook. You may not use any electronic devices. The exam lasts 4 hours.

1. In practice, firms often price their products by “marking up” a fixed percentage over (average) cost. To investigate the consequences of markup pricing, consider a single firm that faces the demand $Q = 90 - P$, for $P \leq 90$. The firm’s TOTAL cost function is $C(Q) = 20Q$.

a. If the firm marks its prices up 50% over average cost, how much would it produce? What price would it charge? And what would be its profits?

b. In contrast, determine the firm’s profit maximizing price and output decisions and its maximal profits.

c. Given that the firm can make more money by behaving as in part b rather than as in part a, give one reason why it could choose a markup price.

d. In this example, would a 50% markup lead to a more or less efficient outcome than the profit maximizing rule in part b? How you define efficiency? Explain.

e. Is there a general relationship between markup pricing and market efficiency when production exhibits constant returns to scale?

f. Is there a general relationship between markup pricing and market efficiency under decreasing returns to scale?

g. For the firm in part a, derive conditions under which an increase in average cost would increase profits.

h. For a profit maximizing firm with the cost curve $C(Q) = aQ^2$, is it possible that an increase in $a$ (hence, marginal cost) would increase profits? Explain.

i. Is it possible that either a profit maximizing firm or a firm using markup pricing would choose not to operate in this market?

2. First, consider an economy with one consumer and two commodities, labor ($L$) and food ($y$). Food can be produced from labor according to the production function $y = L(4 - L)$. The consumer is endowed with 4 units of labor and has preferences represented by $u(y, L) = y(4 - L)$.

a. Characterize the set of Pareto efficient allocations for this economy.

b. Suppose the consumer owns the technology but it is operated independently as a profit maximizing enterprise. Compute a competitive equilibrium, taking food as numeraire.

Next, suppose there are two consumers, $A$ and $B$, each of whom is endowed with 4 units of labor and has the utility function $u_i(y_i, L_i) = y_i(4 - L_i)$, where $y_i$ and $L_i$ denote the quantities of food and labor of agent $i$.

c. Characterize the interior Pareto efficient allocations in this case. How do they differ from each other?

d. Is it possible that at an efficient allocation, an agent might work ($L_i > 0$) but consume no food ($y_i = 0$)? Explain.
e. Suppose that here, too, resources were allocated via the Walrasian mechanism with each consumer owning half of the firm. Prove that the equilibrium wage would be lower than in the equilibrium in part b. Explain why it is.
f. Under these circumstances, could there be an asymmetric Walrasian equilibrium in which the consumers get different consumption bundles? Explain.
g. Explain what is meant by the statement, “the decreasing portion of the production function is economically irrelevant.”
h. Suppose that instead of \( L = L_1 + L_2 \) it were the case that \( L = L_1 + \alpha L_2 \), for \( 0 < \alpha < 1 \). Discuss as thoroughly as possible how this would affect the analysis and properties of a competitive equilibrium.

3. A single monopolistic seller sells products to a continuum of buyers of size 1. Buyers are of one of two types \( i = 1, 2 \). Every type 2 buyer receives utility equal to

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16q - \frac{q^2}{2} - t
\]

if he purchases \( q \) units of the good for a total payment of \( t \) dollars. Every type 1 buyer receives utility equal to

\[
12q - \frac{q^2}{2} - t
\]

if he purchases \( q \) units of the good for a total payment of \( t \) dollars. It costs the seller a constant average cost of \$4\ per unit to produce the good. Let \( n_i \) be the fraction of buyers of type \( i \), so \( n_1 + n_2 = 1 \). Each buyer receives a reservation utility equal to zero if the buyer does not purchase anything from the monopolistic seller.

a. Suppose the seller can observe each buyer’s type and can force each type \( i \) to choose between a contract \((q_i, t_i)\) or else buying nothing. What is the seller’s profit-maximizing (i.e. optimal) contract \((q_i, t_i)\) for each type \( i \)?

For the rest of the problem, suppose that buyer types are not observable to the monopolist. The monopolist can offer a menu of two contracts \( \{(q_1, t_1), (q_2, t_2)\} \) to buyers. If a buyer selects contract \((q_i, t_i)\) then the buyer is entitled to receive \( q_i \) units of the good by paying the amount \( t_i \) to the seller regardless of the buyer’s true type. For parts b through d, assume \( n_1 = n_2 = 1/2 \).

b. Formulate the monopolist’s profit-maximizing problem: write down the objective function and all the constraints including incentive constraints (ICs) and individual rationality constraints (IRs) for both types. Call this the original problem (OP).

c. Consider the seller’s problem but without the type 2 buyer’s IR constraint (IR2) and without the type 1 buyer’s IC constraint (IC1). Call this the relaxed problem (RP). Find the optimal set of contracts \((q_i, t_i)\) that maximize the seller’s profits in this relaxed problem.

d. Show that the solution in part c also satisfies the additional constraints, IR2, IC1, in the original problem. Argue that based on this result the optimal contracts found in RP in fact are also optimal for the original problem OP.

e. Explain how the distribution of the two types \((n_1, n_2)\) affects the optimal contracts.
4. Two producers can grow food for a consumer who cares only about food and money. Producer $j$ ($j = 1, 2$) can plant $q_j > 0$ units of seed at a cost of $(1/3) + q_j$ (in units of money) and then produce $q_j$ units of food from the seed at no additional cost. If a producer plants no seed, it has no cost. The agents' interaction can be described as a game in which the producers independently plant $q_j \geq 0$ units and pay any cost of planting. Next, they independently choose food prices $p_j \geq 0$ (in terms of money) and then the consumer chooses to buy $c_j \in [0, q_j]$ units of food from each producer $j$. The payoff of producer $j$ is its revenue $p_j c_j$ minus its cost. (It can borrow to pay for seed at no interest and repay its loan from its revenue.) The consumer's payoff is $3c - (1/2)c^2 - p_1 c_1 - p_2 c_2$, where $c = c_1 + c_2$. All this information is common knowledge.

a. How many pure strategies does the consumer have? Give an example of one.

b. Show that in the consumer's best response to $(q_1, q_2, p_1, p_2)$, if $p_1 < p_2$ and $q_1 + p_1 \leq 3$, then $c_1 = q_1$.

In parts c and d, consider only subgames that follow the producers' choices of amounts to plant: $q_1 > 0$ and $q_2 > 0$.

c. Show that there is no pure subgame perfect equilibrium (SPE) of this subgame in which $p_1 < p_2$ and $c_2 > 0$. Hint: Use part b and consider deviation by producer 1.

d. Use parts b and c to show that this subgame has no pure SPE of this subgame in which $0 < p_1 < p_2$. Hint: Consider deviation by producer 2.

In the remaining parts of the problem, consider the whole game.

e. The whole game has a pure subgame perfect equilibrium (SPE) in which the quantities $q_j$ are quantities the producers would choose if they acted as Cournot duopolists selling to the consumer. Find the outcome prices and levels of $q_j$ and $c_j$ and explain why they can arise in SPE.

f. Find a SPE of the whole game in which $q_2 = 0$. Explain why it is SPE and compare the total surplus (the sum of the three agents' payoffs) to that in the SPE of part f.