University at Albany, State University of New York  
Department of Economics  
Ph.D. Preliminary Examination in Microeconomics, August 28, 2013

Instructions: Answer any three of the four numbered problems. Justify your answers whenever possible. Write your answer to each question in a separate bluebook. Write the number of the question AND NOTHING ELSE on the cover of the bluebook. In problem 4 you must also write on the exam sheet and turn it in with your bluebooks. No electronic devices may be used. The exam lasts 4 hours.

1. Consider an individual with preferences, $\succeq$, defined over two commodities, money, $m$, and TVs, $x$. While $m$ can be consumed in any quantity, $x$ is binary: she can either consume 0 or 1 unit. Her preferences are strictly monotonic in both commodities. Assume that for any bundle $(1, m)$ there is an equivalent amount of money, $\mu(m)$, such that $(1, m) \sim (0, \mu(m))$. The consumer is initially endowed with $M$ units of money and 0 units of $x$. The price of a TV at her present location is $p(0) > 0$.

a. Explain why $\mu(m) > m$.

b. Under what conditions would she choose to purchase a TV at her present location and when would she choose not to do so?

c. Generally, suppose the price of a TV at location $d$ is $p(d)$, where $d$ denotes the distance from 0. In addition, the cost of traveling distance $d$ is $cd$, where $c > 0$. Suppose the only options are (i) purchase a TV at 0, (ii) purchase a TV at $d$, or (iii) do not purchase a TV. Under what conditions would she choose to purchase a TV at $d$?

d. Next, suppose $p(0)$ is known, but in order to determine the price at location $d$ it would be necessary to travel to that location and thus incur the cost $cd$. Suppose the person's beliefs about $p(d)$ are that it is distributed on $[\bar{p}, \bar{p}]$ with density $f(p)$. Also assume she has the von Neumann - Morgenstern utility function $u(x, m) = 1000x + m$. Under what condition would it be worth traveling to $d$ in search of a lower price rather than purchasing a TV at location 0?

e. Again consider the case where the agent must travel to $d$ to discover $p(d)$ as in part d. Now suppose $d$ is an integer and that stores are located at each integer distance up to $d$. Assume the consumer is presently at $(d - 1)$ having traveled from 0 and having learned the price at each store along the way including $(d - 1)$. Her options at this point are to purchase a TV at any of the stores she has visited so far (without incurring any additional transportation cost) or to continue searching and go on to $d$. Determine the criterion for continuing her search. (Hint: would she continue if $p(d - 1) = \bar{p}$? If $p(d - 1) = \bar{p}$?)

f. How would your answer to part e change if she would have to incur an additional cost of $c(d - d')$ in order to return to the store at location $d'$?

2. Consider an economy with $I \geq 3$ consumers, $I$ firms, 2 private goods, and no technology for disposing of goods. Each consumer $i$ ($i = 1, \ldots, I$) owns firm $i$ and initially owns $e_i > 0$ units of money. Consumer $i$ gets utility $m_i + i\phi(x_i)$ from consuming $m_i$ units of money and $x_i \geq 0$ units of food, where $\phi' > 0$ and $\phi'' < 0$. Consumers can consume negative or positive amounts of money. Each firm $i$ produces 0 units of food using no inputs and produces $q_i > 0$ units of food using $K_i + c(q_i)$ units of money as input, where $c(0) = 0$, $c' > 0$, $c'' > 0$, and $0 \leq K_1 < K_2 < \cdots < K_I$.

a. Use the notation above to specify the set of feasible allocations in this economy.

b. Characterize the set of Pareto efficient (PE) allocations, being as specific as possible. Compare the output levels of the different firms in a given PE allocation. Explain why it
is possible that some firms do not produce in a PE allocation. What can be said about which firms they are?

c. Compare the levels of food consumption by different consumers in a given PE allocation. Compare the levels of food consumption that any given consumer gets in different PE allocations for the same economy. Explain how the different PE allocations differ from each other.

d. Define a competitive (Walrasian) equilibrium (CE) for this economy. Characterize a CE in which money is numeraire (assuming that such a CE exists). Be as specific as possible. For each firm \( i \) that produces in a CE, find an inequality that relates \( K_i \) to the output level and the marginal and total cost of firm \( i \) in equilibrium.

e. Is a CE allocation necessarily Pareto efficient in this economy?

f. Show that a CE allocation does not necessarily exist in this economy. Explain why CE might not exist. Use the notation above to specify an allocation that might plausibly arise if no CE exists. What can be said about how this allocation compares to PE allocations?

3. Consider the pricing problem faced by a monopolistic seller. There is a continuum of potential buyers of size 1. Each buyer demands at most one unit of the good. Buyers are of three types, \( i = 0, 1, 2 \). Each type \( i \) buyer values one unit of the good at \( V_i = \theta_i \sqrt{x} \), where \( x \geq 0 \) is the quality of the good and \( \theta_i \) is a parameter that describes the buyer's taste for quality. Assume \( 0 < \theta_0 < \theta_1 < \theta_2 \). Let \( n_i \) be the fraction of buyers of type \( i \). It costs the monopolist \( C(x) = c \cdot x \), where \( c > 0 \) is constant, to produce one unit of the good of quality \( x \). The monopolist's payoff is its expected profit. The buyers get payoffs equal to the value to them of what they buy minus what they pay.

a. Suppose the monopolist can directly observe buyer type and can offer contracts contingent on type. Characterize the profit maximizing set of contracts for the monopolist.

For the rest of the problem, suppose that types are not observable to the monopolist. The monopolist offers a menu of contracts of the form \( (p_i, x_i) \) where a type \( i \) contract is meant for type \( i \) buyers.

b. Formulate the monopolist's pricing problem with incentive and participation constraints, assuming each buyer has a reservation payoff equal to zero.

c. Consider the relaxed monopoly pricing problem (RP) in which only the following downward adjacent incentive constraints (DAIC) and a participation constraint (P0) for type \( i = 0 \) are imposed.

\[
\theta_2 \sqrt{x_2} - p_2 \geq \theta_2 \sqrt{x_1} - p_1, \quad \text{(DAIC2)}
\]

\[
\theta_1 \sqrt{x_1} - p_1 \geq \theta_1 \sqrt{x_0} - p_0, \quad \text{(DAIC1)}
\]

\[
\theta_0 \sqrt{x_0} - p_0 \geq 0 \quad \text{(P0)}
\]

Show that all these constraints bind in a solution to this relaxed problem.

d. Show that if the solution obtained in the relaxed problem (RP) satisfies monotonicity, i.e. \( x_2^* \geq x_1^* \geq x_0^* \), then all of the incentive constraints and participation constraints in the original problem are satisfied and the solution to the relaxed problem is also a solution to the original problem.

e. Solve the relaxed problem (RP). Compare the optimal quality levels \( (x_2^*, x_1^*, x_0^*) \) to the quality levels the monopolist would choose in part a. Discuss any differences.

f. Based on the solution to (RP) in part e, provide a sufficient condition on buyers' preferences such that the solution to (RP) in part e is indeed a solution to the original problem in part b. Interpret this condition. Is the monopoly better off when this condition holds than when a solution to (RP) is not a solution to the original problem in part b?
4. A monopoly firm $I$ knows that firm $E$ is considering entering its market and knows that the product design team at firm $E$ is either good (G) or bad (B). Firm $I$ initially believes that the design team is more likely good than bad. Firm $E$ knows the quality of its design team. If $E$ decides to enter the market, it can do so with either a high or low investment. If the investment is high and the design team is good, then $E$’s product quality is high. If the investment is low and the design team is bad, the product quality is low. The product quality is medium if either the investment is high and the design team is bad or else if the investment is low and the design team is good. Firm $I$ can see the quality of $E$’s product (high, medium, or low), but does not directly know the quality of $E$’s design team before deciding whether to fight or accommodate $E$ in the market.

If $E$ stays out of the market, its payoff is 0 and $I$’s is 5. The two tables below show the payoffs to $E$ and $I$, depending on their decisions, when $E$ enters the market. The left table shows the payoffs when $E$’s design team is good and right table shows the payoffs if the team is bad. The structure of this interaction is common knowledge to $E$ and $I$.

<table>
<thead>
<tr>
<th>Good Design:</th>
<th>$E \backslash I$</th>
<th>Fight</th>
<th>Accom.</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>0, -1</td>
<td>5, 0</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>-1, 2</td>
<td>3, 1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bad Design:</th>
<th>$E \backslash I$</th>
<th>Fight</th>
<th>Accom.</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>-3, 2</td>
<td>0, 3</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>-2, 4</td>
<td>1, 3</td>
<td></td>
</tr>
</tbody>
</table>

a. The interaction between $E$ and $I$ can be represented by a game with the tree above. Writing on this sheet, label the moves, the players that move them, and the payoffs in the tree above. (Turn in this sheet with your bluebook.) Explain briefly how you can tell which player moves in which information set.

b. Explain how you can tell that the decisions represented in the tables above are not the players’ pure strategies. List all of $E$’s pure strategies in the game.

c. How many pure strategies does $I$ have in the game? Give examples of two of them.

d. Does $E$ have any weakly dominated strategies in the game? If so find one.

e. Find every sequential equilibrium (SE) in which $I$ plays a pure strategy. Justify your answer. Explain what the outcomes SE are and discuss whether they are plausible. Is there an SE in which $I$ can detect for sure whether or not $E$’s design team is good?

f. Is there a Nash equilibrium (NE) in which $E$ chooses not to enter the market no matter what? If so, discuss the plausibility of this outcome. If not, explain why not.