1. (a) Higher \( \delta_i \) implies a willingness to give up more consumption and leisure in return for living farther from the polluter.

(b) The problem refers to consumers choosing their residential locations, and the only residential cost mentioned is commuting time. If there is no other residential cost, then consumer \( i \) chooses \( a, c \) and \( l \) to maximize \( a^{\delta_i} cd \) subject to \( l = 1 - a - c \). This is the optimization problem for a Cobb-Douglas utility function with prices and income (or wealth) equal to 1. The solution is \( a = \delta_i/(2 + \delta_i), c = l = 1/(2 + \delta_i) \).

(c) Total output equals total consumption, \( \int_0^1 [1/(2 + \delta)] d\delta = \ln(3/2) \), since the \( \delta_i \) values are uniformly distributed on \([0, 1]\).

(d) For a consumer to be better off, the value of its total consumption \( a + c + l \) would have to increase. But feasibility implies that some other consumer’s total consumption and utility would have to fall. So no Pareto improvement is possible and the allocation is Pareto efficient. Another way to see this is to note that the allocation is the same as in a competitive equilibrium in an economy with no externality and with locally nonsatiable utilities, in which \( a \) represents the quantity of another good produced using \( a \) units of time. The external effect of manufacturing in the original problem has no effect on efficiency since the level of the externality is fixed.

(e), (f) The solution to (b) implies that each consumer chooses \( c = l = 1/2 \) and \( a = 0 \), and resides at the manufacturing location. A consumer obtains more consumption and leisure, and also more utility \((= 1/4)\), than if it had \( \delta_i > 0 \). If \( \delta_i = \delta \), then the logarithm of the consumer’s utility in (b) is \( \delta \ln \delta - (2 + \delta) \ln(2 + \delta) \), with derivative \( \ln \delta - \ln(2 + \delta) < 0 \). So the consumer obtains higher utility if \( \delta_i = 0 \) than if \( \delta_i > 0 \). As in the answer to (d), this does not indicate that the outcome in (b) is inefficient. The consumers cannot change their utility functions. The allocation in (b) is Pareto efficient, meaning efficient among feasible alternatives.

(g) Consumer \( i \) chooses \( a, L \) and \( l \) to maximize \( a^{\delta_i} f(L, a) l \) subject to \( l = 1 - a - L \). The same argument in the answer to (d) applies and the resulting allocation is Pareto efficient. The health effect does not introduce any new externality since the location and labor choice of a given consumer has no effect on other consumers.

2(a) The firm assigns an easy job since the expected profit \( y_{eu} - w_e \) is greater than from a difficult job, \( y_{du} - w_d \).

(b) The game tree starts with two branches corresponding to the worker’s choice of being skilled or unskilled. Each branch is followed by two branches corresponding to the firm’s choice of a difficult or easy job. The worker’s payoff from job type \( i \) is \( w_i - c \) if the worker is skilled and \( w_i \) otherwise. The firm’s payoff is \( y_{ij} - w_i \), where \( j = s \) if the worker is skilled and \( j = u \) otherwise.

In SPNE, as in part (a), the firm assigns an easy job when the worker is unskilled. When the worker is skilled, the firm assigns a difficult job only if \( y_{ds} - w_d \geq y_{es} - w_e \). Not investing is a weakly dominant strategy for the worker if \( w_e \geq w_d - c \), and is strictly dominant if this inequality is strict. Suppose that \( w_e \geq w_d - c \). Then there are SPNE with the worker not investing, and with a skilled worker getting a difficult [respectively, easy] job assignment if \( y_{ds} - w_d \geq y_{es} - w_e \). Suppose instead that \( w_e \leq w_d - c \). If \( y_{ds} - w_d \geq y_{es} - w_e \), then there is a SPNE in which the skilled worker is assigned a difficult job and the worker invests. If \( y_{ds} - w_d \leq y_{es} - w_e \), then there is a SPNE in which the firm assigns an easy job to the skilled worker and the worker does not invest (since the payoff is \( w_e - c \) from investing). These are the only possible SPNE.

Note that in describing SPNE it is necessary to give the moves chosen in all the information sets.

(c) The answer to (b) implies that such a SPNE exists if and only if \( y_{ds} - y_{es} \geq w_d - w_e \geq c \).

(d) If in equilibrium, after investing, the worker is assigned an easy job, then the worker will not invest. If in equilibrium the worker does not invest, then the best outcome for the firm is with \( w_e = 0 \), and the firm’s payoff is \( y_{eu} \). Suppose instead that the worker invests and is assigned a difficult job in a SPNE. The payoffs are \( w_d - c \geq 0 \) for the worker and \( y_{du} - w_d \) for the firm. The best payoff for the firm is attained when \( w_d = c \), and if \( w_e = 0 \). If these wages are announced, then there is an SPNE continuation in which the worker does invest and is assigned a difficult job. The firm’s payoff is then \( y_{ds} - c > y_{eu} \), so in SPNE, the firm chooses \( w_d = c \) and \( w_e \leq 0 \) and assigns a difficult job if the worker is skilled and an easy job if the worker is unskilled. The rest of the SPNE is described in the answer to (b).

3. (a) \( L \) is strictly dominated by \( H \).
(b) After L is removed, h is strictly dominated by m. No other strictly dominated strategies remain, so the remaining pairs are (H, m), (H, l), (M, m), (M, l).

c) If a player plays a particular pure strategy with positive probability in Nash equilibrium (NE) then that strategy must be a best response to the (possibly mixed) equilibrium strategy of the other player. It follows that no player plays a strictly dominated strategy with positive probability in NE. But then no player puts positive probability on a strategy that is removed in the iterated elimination of parts (a) and (b) of this problem. If player 1 plays H with positive probability in NE, then player 2 plays m for sure. But then M is a best response for player 1. Therefore, every NE has M played for sure. Any probability distribution with 0 probability for h is a best response for player 2, but for M to be a best response for player 1, the probability of m must be at least 1/2. These are all the possible NE, and the only pure NE is (M, m).

d) A pure strategy for player 1 in this case is a response to any price set by player 2. An example is H if 2 plays l or m and M if 2 plays h.

e) The game tree starts with three branches corresponding to the choices h, m or l by player 2. At the end of each of these branches are three branches corresponding to H, M and L played by player 1. In a subgame perfect Nash equilibrium (SPNE), player 1 responds to h and m with M and responds to l with H. There are two pure SPNEs with player 2 playing either of its best responses, m or l; and in mixed SPNE, player 2 chooses any probability distribution with 0 probability for h. The payoff to player 2 is 150, the same as in the NE of the original game. But the payoff to player 1 in the original game is at least 125, whereas in the the game of part (d), player 2 can impose a payoff as low as 110 on player 1 by choosing l. Moving first gives player 2 more power in this sense.

4. (a) In a feasible allocation, each consumer consumes a positive amount of good 1. If one consumer consumes none of good 2, then reducing that consumer’s consumption of good 1 does not change that consumer’s utility, but raises the other consumer’s utility, so the allocation is Pareto inefficient. Therefore every Pareto efficient allocation is interior (each consumer consumes both goods), so the consumers’ marginal rates of substitution must be equal. This equality is sufficient for Pareto efficiency, since the utility functions are quasiconcave. Therefore in the notation in the problem, a feasible allocation is Pareto efficient allocation if and only if $x_{21}/(2x_{11}) = 2x_{22}/x_{12} = 2(1-x_{21})/(1-x_{11})$, which requires $x_{21} = 4x_{11}/(3x_{11}+1)$. The same restriction can be obtained by maximizing the utility of consumer 1 over feasible allocations giving consumer 2 an arbitrary utility $\bar{u}_2$.

(b) The consumption ratio for consumer 1 is $x_{21}/x_{11} = 4/(3x_{11}+1)$. The consumption ratio of consumer 2 is $x_{22}/x_{12} = (1-x_{21})/(1-x_{11}) = 1/(3x_{11}+1)$, less than the ratio for consumer 1. The comparison does not depend on $a$ or $b$. [Note: Both ratios are decreasing functions of $x_{11}$ even though the aggregate ratio of good 2 to good 1 is fixed at 1. The reason this is possible is that a change from one efficient allocation to another with higher $x_{11}$ raises the consumption of both goods for the consumer with the higher consumption ratio.]

Pareto efficient allocations in exchange economies depend only on the preferences and the total endowment, not on the way the endowment is divided. Consumer 1 has a stronger preference for good 2 and so receives a higher ratio of good 2 to good 1 than consumer 2 does in efficient allocations.

c) Yes. An example of such a price vector is the gradient of consumer 1’s utility at that point: $((x_{21})^2, 2x_{11}x_{21})$. If this is the price vector $p$ and the consumer’s wealth is $p \cdot (x_{11}, x_{21})$, then the price ratio $p_1/p_2$ equals consumer 1’s marginal rate of substitution at $x_1 = (x_{11}, x_{21})$, and consumer 1 demands $x_1$. Consumer 2 has the same marginal rate of substitution at $x_2 = (1-x_{11}, 1-x_{21})$, and demands $x_2$ at price vector $p$ if given the wealth $p \cdot x_2$. These wealth levels add up to $p \cdot (1,1)$, so $(p, x_1, x_2)$ is a price equilibrium with transfers.

d) In a competitive equilibrium with price vector $p = (p_1, p_2)$, the Cobb-Douglas demands for good 1 by consumers 1 and 2 are $(ap_1bp_2)/(3p_1)$ and $2(1-a)p_1+(1-b)p_2)/(3p_1)$, and total demand equals total supply of 1. So $p_2/p_1 = (1+a)/(2-b)$. This determines $x_{11} = (ap_1+bp_2)/(3p_1) = (a+b)p_2/p_1)/3 = (2a+b)/(3(2-b))$, and the budget identity and feasibility determine the rest of the allocation.

e) A competitive equilibrium allocation is Pareto efficient in this economy. If a consumer gets more of good 1 in efficient allocation A than in efficient allocation B, then that consumer also gets more of good 2 in allocation A. Since consumer 1’s competitive equilibrium consumption of good 1 is higher when either a or b is raised, consumer 1’s utility is also an increasing function of a and b.

The actual utility is $x_{11}(x_{21})^2 = \left(\frac{2a+b}{3(2-b)}\right)^2 \left[\frac{2b}{a+1} + b\right]^2$.

(f) No. This follows from the answer to (e).