Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.

1. The emphasis on the unemployment rate conceals what is really going on in the aggregate labor market. A better measure is the Labor Market Participation Rate.

2. It is better for the government to spend tax revenues on actual goods and services than to distribute them to households in the form of financial transfers.

3. The expected drop in future productivity definitely leads to a decrease in investment.

4. The fact that many people play the lottery means that, contrary to the usual assumptions made on utility functions, people are risk loving.

5. The fact that employment growth lags output growth is evidence of search frictions in labor markets.

6. Tighten environmental regulation on firm production, e.g. forcing firms to reduce coal usage, has an ambiguous effect on the real interest rate.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Diamond "style" OG model with concerned parenting

Time: discrete, infinite horizon, \( t = 1, 2, ... \)

Demography: A mass \( N_t \equiv N \) for all \( t \) of newborns enter in period \( t \) (i.e. no population growth). Each person is born into a household or dynasty. Everyone lives for 2 periods except for the first generation of old people who live for one.

Preferences: for the generations born in and after period 1,

\[
U(c_{1,t}, c_{2,t+1}, s_{t+1}) = u_1(c_{1,t}) + \beta u_2(c_{2,t+1}) + \beta u_s(s_{t+1})
\]

where \( c_{i,t} \) is consumption in period \( t \) and stage \( i \) of life and \( s_{t+1} \) is the amount of old age consumption good given up in the process of providing human capital \( h \) for the young person in their household. For the initial old generation \( U(c_{2,1}) = u_2(c_{2,1}) + u_s(s_1) \). The utility functions, \( u_1(.) \) \( i = 1, 2, s \) are all twice differentiable and concave. The function \( u_s(.) \) represents the extent to which a parent cares about the quality of their child’s future skill set.

Productive technology: The production function available to firms is \( F(H, N) \) where \( H \) is the human capital stock and \( N \) is the number of workers employed. \( F(.,.) \) is a constant-returns-to-scale standard neoclassical production function which satisfies the Inada conditions. (You may find it convenient to use the implied per young person production function, \( f(h) \) where \( h \) is the human capital stock per worker.) Foregone consumption (or saving) by the old is converted one-to-one into the human capital of their child. Children cannot create human capital on their own account. So, the only way they can accumulate human capital is if their parents spend the required resources on their behalf.

Endowments: Everyone has one unit of labor services when young. (The old survive by renting their human capital.) The initial old share an endowment, \( H_1 \) of capital so they have \( h_1 \) units each.

Institutions: There are competitive markets every period for labor and human capital. (You can think of a single representative firm which takes wages and interest rates as given.)

(a) Write down and solve the problem faced by the individuals born in period \( t \).
(b) Write down and solve the representative firm’s problem in period \( t \).
(c) Write down the market clearing condition for human capital and define a competitive equilibrium.
(d) Solve for an equation that characterizes the dynamics of the human capital stock.

Now suppose \( u_1(c) = u_2(c) = \ln(c) \), \( u_s(s) = Bs \) where \( B \) is a constant and \( f(h) = Ah^2 \) where \( A \) is a constant.
(e) Obtain any steady state value(s) of human capital per worker, \( h^* \), and characterize the dynamical properties.
(f) Draw the phase diagram

(g) Now, without doing the work, state how the model analysis would differ if each dynasty started out with a different initial human capital endowment say distributed uniformly on \([0, (\alpha A)^2]\)?

8. Consider the following variant of the Lucas tree model. The preferences of the representative consumer are

\[ E_0 \left( \sum_{t=0}^{\infty} \beta^t \theta_t \ln(c_t) \right), \quad 0 < \beta < 1, \]

where \(\theta_t\) is a preference shifter, \(c_t\) denotes consumption, and \(E_t(\cdot)\) denotes expectations conditional on time-\(t\) information. \(\theta_t \in \{\theta_l, \theta_h\}, \theta_l < \theta_h\) is an i.i.d. shock with probabilities \(Pr(\theta_l) = 1/2\), and \(Pr(\theta_h) = 1/2\).

Output is produced by an infinite-lived tree: each period, the tree produces \(d_t\) units of non-storable output. Output is a non-negative, exogenous random variable governed by a Markov process with the stationary transition density \(f(d', d)\). The economy starts off with each consumer owning one tree apiece.

Let \(p_t = p(d_t, \theta_t)\) be the price at time \(t\) of all future profits from a tree, which we will call a unit of “ideal stock”. Let \(q(d', d_t, \theta_t)\) be the kernel used to price the one-step-ahead contingent claim that delivers one unit of the consumption good when \(d_{t+1} = d'\) (regardless of the realization of \(\theta_{t+1}\)). Finally, let \(\omega_t\) denote the consumer’s financial resources, which she allocates between ideal stock, contingent claims, and consumption, let \(s_t\) denote the number of trees that the consumer owns at the beginning of time \(t\), and let \(y_{t+1}(d')\) denote the amount of contingent claims that consumers purchase at the end of time \(t\) that will pay back one unit of consumption goods if the next period realization is \(d'\).

(a) Define a competitive equilibrium.

(b) Write down the consumer’s problem in a recursive form (Bellman’s equation and constraints).

(c) Find the first order conditions. Intuitively explain the meaning of the left-hand-side and the right-hand-side of two Euler equations.

(d) Find the equilibrium price for ideal stock, \(p(d_t, \theta_t)\). Note that for a LEDE of

\[ E_t(x_{t+1}) = \lambda x_t + z_t \]

the backward solution is:

\[ x_t = \sum_{j=0}^{\infty} \lambda^j z_{t-1-j} + b_t, \]

and the forward solution is:

\[ x_t = -\frac{1}{\lambda} E_t \left( \sum_{j=0}^{\infty} \left( \frac{1}{\lambda} \right)^j z_{t+j} \right) + b_t, \]

where \(x_t\) is the variable of interest, \(z_t\) is a forcing term, and \(b_t\) is a bubble term.
(e) How does the price of ideal stock changes with respect to the level of current dividend $d_t$ and with respect to current preference shock $\theta_t$. Provide an intuitive explanation.

9. One-shot DMP model (inspired by Albrecht, Navaro and Vroman [2010])

**Time:** One period

**Demography:** A unit mass continuum of workers and a larger mass of firms who can create individual vacancies. Workers are ex ante heterogenous, they are indexed by $y \sim F$ on $[0, 1]$ (which will be their individual productivity level). $F(.)$ is a continuous distribution with density $f(.)$.

**Preferences:** Both firms and workers are risk neutral.

**Productive technology:** An employed worker of type $y$ produces $y$ units of the consumption good. If a worker does not find employment they will receive $b > 0$ units of the consumption good. Creating a vacancy costs $a$ units of the consumption good.

**Matching technology:** All workers begin the period unemployed. The probability that a worker meets a firm is $m(v)$ where $v$ is the measure of vacancies created. The function $m(.)$ is twice differentiable, strictly increasing and strictly concave. We have $m(0) = 0$, $m'(0) = 1$ and $\lim_{v \to \infty} m(v) = 1$. Let $\eta(v) = m'(v)v/m(v)$, the elasticity of $m(v)$. Then, assume further that $\eta(v) < 1$.

**Institutions:** Employed workers of type $y$ receive the wage $w = \beta y$.

(a) What is the cut-off (or reservation) value, $y^*$, of worker productivity above which matches will form and below which it will not?

(b) What is the value, $V$, to creating a vacancy?

(c) Given that firms will not create any more vacancies once $V = 0$, define a free-entry equilibrium for this simple economy and obtain an equation to characterize the equilibrium.

(d) How does the equilibrium mass of vacancies depend on $\beta$? Derive a condition on $\beta$, $b$ and $f(.)$, under which an increase in $\beta$ increases $v$.

(e) Explain why the relationship obtained in part d is generally ambiguous.

10. Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

\[
\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,
\]

\[
Y_t = L_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{PRF}
\]

where: $\Pi_t$ is profit; $L_t$ is labor; $W_t$ is the real wage; and $Y_t$ is output.

The preferences of the representative household over consumption, $C_t$, Government expenditure, $G_t$, and labor are given by

\[
E_0 \left( \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\nu} \left( (C_t + \xi G_t) - \frac{\chi}{1+\gamma} L_t^{1+\gamma} \right)^{1-\nu} \right),
\]
0 < \beta < 1, \quad \nu > 0, \quad 0 < \xi < 1, \quad \gamma > 0, \quad \chi > 0.

Households receive labor income and profits from firms, and pay labor income taxes to the government. Households can store their assets $K_t$, and earn zero returns on the stored assets. As usual, assume that assets held at the beginning of period $t + 1$, $K_{t+1}$, are chosen in period $t$. Note that capital is used only as a storage device, and not as a factor of production. Households face the usual initial, non-negativity and No-Ponzi-Game conditions.

The government collects taxes from labor income and maintains a balanced budget rule:

$$\tau_t W_t L_t = G_t.$$  \hspace{1cm} (BB)

Taxes are driven by government spending, which follows an AR(1) process around the log of its steady state value:

$$\widehat{g}_t \equiv \ln \left( \frac{G_t}{G_{ss}} \right) = \phi \widehat{g}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1,$$  \hspace{1cm} (TS)

where $\{\varepsilon_t\}$ is an exogenous i.i.d. process, and $G_{ss} > 0$ is steady state government spending. Individual consumers and producers are sufficiently small to take $\tau_t$ as well as $G_t$ as given.

(a) Define a competitive equilibrium

(b) Write down the firm’s problem and find the first order conditions that maximize profits

(c) Write down the household problem in a recursive form (Bellman’s equation and constraints), and find the first order conditions that maximize household utility.

(d) Let lower-case letters with carats “^” denote deviations of logged variables around their steady state values, and letters with subscript $ss$ denote steady state values. Log linearize the wage equation (the FOC of the firm), Euler equation, and the equations that characterizes the labor-leisure trade-off. Note that to analyze the next question, you had better to substitute $\tau_t$ and $W_t$ as functions of $L_t$ and $G_t$.

(e) Suppose the economy is hit by a temporal expansionary fiscal policy shock ($\varepsilon_t > 0$). Under what conditions, would labor hours increase, and under what conditions, would labor hours decrease? Provide an intuitive explanation.