Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.

1. The U.S. will likely have to increase its military activities in the Middle East. This should cause output to rise.

2. Suppose that the data shows that times of high consumer and firm borrowing are also times of high economic growth. We could then conclude that financial constraints are an important determinant of economic activity.

3. What Macroeconomic models miss is that consumers do not want to borrow during recessions.

4. Policies used to bring us out of one recession sow the seeds of the next.

5. During a liquidity trap, it is possible that increases in money supply can cause deflation.

6. We observe that during the Great Recession, housing prices drop substantially. Meanwhile, mortgage interest rate also drop. Therefore, a drop in interest rates causes declines in housing price.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Consider an economy populated by a large number of yeoman farmers. Each farmer's preferences over consumption and leisure are:

\[ E_t \left( \sum_{j=0}^{\infty} \beta^j \left[ \ln (c_{t+j}) + \chi \ln (1 - \ell_{t+j}) \right] \right), \quad 0 < \beta < 1, \quad \chi > 0. \]

Let \( I_t \) denote the overall state of the economy, a Markov process with the stationary transition density \( f (I', I) \). Let \( q (I', I_t) \) be the kernel used to price the one-step-ahead contingent claim that delivers one unit of the consumption good when \( I_{t+1} = I' \), and let \( z (I') \) denote the quantity of such claims. The farmer’s resources evolve according to

\[ z (I_{t+1}) q (I_{t+1}, I_t) dI_{t+1} + c_t = d_t \ell_t + z (I_t), \quad \text{(FBC)} \]

where \( d_t > 0 \) is an exogenous shifter. Output is not storable—**there is no capital**.

(a) Letting \( z_t \) denote realized contingent claims, we write the consumer’s problem as a Lagrangean:

\[ V(z_t, I_t) = \min_{\lambda_t \geq 0} \max_{c_t \geq 0, \ell_t \in [0,1], z(\cdot)} \lambda_t \left( z_t + d_t \ell_t - \int z (I_{t+1}) q (I_{t+1}, I_t) dI_{t+1} - c_t \right) \]

\[ + \beta \int V (z (I_{t+1}), d(I_{t+1})) \times f (I_{t+1}, I_t) dI_{t+1}, \]

The FOC for an interior solution are:

\[ \frac{1}{c_t} = \lambda_t, \quad \text{(FOC1)} \]

\[ \frac{\chi}{1 - \ell_t} = \lambda_t d_t, \quad \text{(FOC2)} \]

\[ \lambda_t q (I', I_t) = \beta \frac{\partial V (z (I_{t+1}), d(I_{t+1}))}{\partial z_{t+1}} f (I', I_t), \quad \forall I'. \quad \text{(FOC3)} \]

Since (following Benveniste-Scheinkman),

\[ \frac{\partial V [t]}{\partial z_t} = \lambda_t, \]

the Euler equation for contingent claims is

\[ q (I_{t+1}, I_t) = \beta \frac{c_t}{c_{t+1}(I_{t+1})} f (I_{t+1}, I_t), \quad \forall I_{t+1}. \quad \text{(EE)} \]

Combining equations (FOC1) and (FOC2) yields the labor allocation condition:

\[ \chi \frac{1}{1 - \ell_t} = d_t \frac{1}{c_t}. \quad \text{(LL)} \]
(b) Now we assume there are two types of farms, occurring in equal proportions. Half the farms have productivity level $d_{1t}$, while the remainder have productivity level $d_{2t}$.

i. Let $c_1$ ($c_2$) and $\ell_1$ ($\ell_2$) denote the consumption and labor of a type-1 (type-2) farmer. Given equal weights – consistent with equal initial wealth – the social planner’s problem can be written as

$$\max_{\{c_{1t}, c_{2t}, \ell_{1t}, \ell_{2t}\}} E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_{1t}) + \chi \ln(1 - \ell_{1t}) + \ln(c_{2t}) + \chi \ln(1 - \ell_{2t}) \right] \right),$$

s.t. $c_{1t} + c_{2t} = d_{1t} \ell_{1t} + d_{2t} \ell_{2t}$, \hspace{1cm} (RC)

$0 \leq \ell_{1t} \leq 1; \hspace{0.5cm} 0 \leq \ell_{2t} \leq 1.$

ii. With no storage, the social planner’s problem boils down to a series of static problems. Dropping the time subscript, the first-order conditions for any period are

$$\frac{1}{c_1} = \mu,$$
$$\frac{1}{c_2} = \mu,$$
$$\chi \frac{1}{1 - \ell_1} = \mu d_1,$$
$$\chi \frac{1}{1 - \ell_2} = \mu d_2,$$

where $\mu$ denotes the multiplier on the resource constraint. Eliminating the multiplier yields

$$c_1 = c_2, \hspace{1cm} (SP1)$$
$$\chi c_1 = d_1 (1 - \ell_1), \hspace{1cm} (SP2)$$
$$\chi c_2 = d_2 (1 - \ell_2). \hspace{1cm} (SP3)$$

These three equations, along with the resource constraint (RC), characterize the solution to the social planner’s problem.

(c) For any variable $x$, let $x_t = \frac{1}{2}(x_{1t} + x_{2t})$ denote per capita averages taken across the two types of farms.

i. It follows from equation (RC) that

$$2y_t = y_{1t} + y_{2t} = d_{1t} \ell_{1t} + d_{2t} \ell_{2t} = c_{1t} + c_{2t} = 2c_t.$$

Next, combine equations (SP2) - (SP3):

$$2c_t = \frac{1}{\chi} \left[ d_{1t} (1 - \ell_{1t}) + d_{2t} (1 - \ell_{2t}) \right] = \frac{1}{\chi} \left[ d_{1t} + d_{2t} - (d_{1t} \ell_{1t} + d_{2t} \ell_{2t}) \right],$$

so that

$$d_{1t} \ell_{1t} + d_{2t} \ell_{2t} = \frac{1}{\chi} \left[ d_{1t} + d_{2t} - (d_{1t} \ell_{1t} + d_{2t} \ell_{2t}) \right] = \frac{1}{1 + \chi} \left[ d_{1t} + d_{2t} \right].$$
We thus have
\[ y_t = \frac{1}{1 + \chi} \left( \frac{d_{1t} + d_{2t}}{2} \right) \equiv d_t \bar{\ell}. \] (PRF)

ii. Unless the distribution of \( d_{t+1} \) conditional on \( I_t \) can be reduced to the distribution of \( d_{t+1} \) conditional on \( d_t \), \( d_t \) is not the complete state vector. Even though \( d_t \) is sufficient to determine output and consumption, we may need additional elements of \( I_t \) to predict \( d_{t+1} \). Given that \( d_{t+1} \) is the average of two variables, this is likely to be the case.

(d) We finish by considering the equilibrium in more detail.

   i. It follows from equations (RC), (SP1) and (PRF) that \( c_{1t} = c_{2t} = c_t = d_t \bar{\ell} \).

   ii. Inserting this result in equation (EE) yields
   \[ q (I_{t+1}, I_t) = \beta \frac{d_t}{d_{t+1}(I_{t+1})} f (I_{t+1}, I_t), \forall I_{t+1}. \] (EE′)

   iii. In the model, consumption depends on only the aggregate shock \( d_t \): consumption is completely insured against idiosyncratic productivity shocks. Farmers are able to do this in the model because they can buy contingent claims that depend on both \( d_{1t} \) and \( d_{2t} \). However, in the real world, idiosyncratic productivity is hard to observe, and insurance holders might be able to falsely claim bad productivity and avoid work. As a result, this sort of insurance is rarely offered, at least in its fullest form.
8. Diamond OG model with immigration policy

We will investigate the possibility that the government has control over the rate of population growth, \( n \in [0, \bar{n}] \) through controlling the amount of immigration.

**Time:** discrete, infinite horizon, \( t = 1, 2, \ldots \)

**Demography:** A mass \( N_t = N_0(1 + n)^t \) of newborns enter in period \( t \). Of these, \( N_{t-1} \) are native born and the rest are immigrants so that \( n \in [0, \bar{n}] \) is the rate of immigration. Everyone lives for 2 periods except for the first generation of old people who live for one.

**Preferences:** for the generations born in and after period 1;

\[
U(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1})
\]

where \( c_{i,t} \) is consumption in period \( t \) and stage \( i \) of life. For the initial old generation \( \bar{U}(c_{2,1}) = \ln(c_{2,1}) \).

**Productive technology:** The production function available to firms is \( F(K, N) = K^\alpha N^{1-\alpha} \) where \( K \) is the capital stock and \( N \) is the number of workers employed. You may find it convenient to use the implied per young person production function, \( f(k) = k^\alpha \) where \( k \) is the capital stock per worker.

**Endowments:** Everyone has one unit of labor services when young. The initial old share an endowment, \( K_1 \) of capital so they have \( (1 + n)k_1 \) units each.

**Institutions:** There are competitive markets every period for labor and capital. (You can think of a single collectively owned firm which takes wages and interest rates as given.)

(a) Write down and solve the problem faced by the individuals born (or immigrating) in period \( t \).

\[
\max_{s_{t+1}} \{ \ln(w_t - s_{t+1}) + \beta \ln(R_{t+1} s_{t+1}) \}
\]

FOC:

\[
-\frac{1}{w_t - s_{t+1}} + \frac{\beta R_{t+1}}{R_{t+1} s_{t+1}} = 0.
\]

So

\[
s_{t+1} = \frac{\beta w_t}{1 + \beta}
\]

(b) Write down and solve the representative firm’s problem in period \( t \).

\[
\max_{k_t} \{ k_t^\alpha - w_t - R_t k_t \}
\]

\[
R_t = \alpha k_t^{\alpha-1}
\]

\[
w_t = (1 - \alpha) k_t^\alpha
\]
(c) Write down the market clearing condition for capital and define a competitive equilibrium.

\[(1 + n)k_t = s_t\]

**Definition 1** A competitive equilibrium is an allocation, \(\{c_{1t}, c_{2t}, k_t\}\) and a sequence of prices, \(\{w_t, R_{t+1}\}\), such that given prices the allocation solves the households’ and firms’ problems and, markets clear.

(d) Solve for an equation that characterizes the dynamics of the capital stock.

\[(1 + n)k_{t+1} = \frac{\beta (1 - \alpha) k_t^\alpha}{1 + \beta}\]

(e) Solve for the steady state(s) and obtain their dynamical properties.

There are 2 steady states at \(k = 0\) and

\[k = \bar{k} = \left[ \frac{(1 - \alpha) \beta}{(1 + n)(1 + \beta)} \right] \frac{1}{1 - \alpha}.\]

Expressing the dynamical system as \(k_{t+1} = g(k_t)\) we need to obtain the derivative of \(g(.)\) at each steady state. So

\[g'(k) = \frac{\alpha \beta (1 - \alpha) k_t^{\alpha - 1}}{(1 + n)(1 + \beta)}\]

and

\[g'(0) = \lim_{k \to 0} g'(k) = \infty.\]

The steady state at 0 has monotone dynamics and is unstable.

\[g'(\bar{k}) = \alpha.\]

The steady state at \(\bar{k}\) has monotone dynamics and is stable.

(f) The government will use immigration policy to set the value for \(n\) (which is taken as given by the population). The government will maximize the utility of a representative household in a representative generation, \(U(c_{1t}, c_{2t+1})\) taking as given the capital stock per worker, \(k_t\). Use the above results to write down an equation for this utility in terms of \(k_t\) and \(n\).

\[c_{1t} = w_t - s_{t+1} = \left(1 - \frac{\beta}{1 + \beta}\right) w_t = \frac{(1 - \alpha) k_t^\alpha}{1 + \beta}\]

and

\[c_{2t+1} = R_{t+1} s_{t+1} = \frac{\alpha k_{t+1}^{\alpha - 1} \beta (1 - \alpha) k_t^\alpha}{1 + \beta}\]
substituting for $k_{t+1}$ implies

$$c_{2t+1} = \frac{\alpha(1 - \alpha)\beta \left[ \frac{\beta(1-\alpha)k_1^\alpha}{(1+n)(1+\beta)} \right]^\alpha \alpha k_t^\alpha}{1 + \beta}$$

As $U(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1})$, it takes the form of

$$A + B \ln k_t + (1 - \alpha) \ln(1 + n)$$

where $A$, $B$ and $C$ are constants.

(g) Is there an optimal level of population growth? Explain your answer.

As higher $n$ means higher utility for the household, the optimal $n$ is $\bar{n}$. With log utility the income and substitution effects from a change in the interest rate exactly offset each other. Taking $k_t$ as given, the wage is unaffected by $n$ but the interest rate is monotonically increasing in $n$ making second period consumption higher.

(I’m not expecting this part I was just curious as what was going on) In fact, though, $k_t$ will depend only $k_1$ and $n$. Thus

$$k_t = \left( \frac{\beta(1 - \alpha)}{(1 + n)(1 + \beta)} \right)^{\sum_{i=0}^t \alpha_i} k_1 \frac{1}{\sum_{i=1}^{t+1} \alpha_i}$$

for $t$ high enough this converges to

$$\lim_{t \to \infty} \left( \frac{\beta(1 - \alpha)}{(1 + n)(1 + \beta)} \right)^{\frac{1}{1 - \alpha}} k_1^{\frac{1}{1 - \alpha}}$$

which would mean $U(c_{1,t}, c_{2,t+1})$ takes the form

$$D + \frac{(1 - \alpha)^2 - 1}{1 - \alpha} \ln(1 + n)$$

With a high enough weight on future generation’s welfare optimal policy puts $n = 0$. This is consistent with the fact that $\bar{k}$ is decreasing in $n$. 

7
9. We are considering a version of the Sidrauski model with endogenous labor supply. The economy is populated by a large number of identical yeoman farmers. Each farmer derives utility from consumption, leisure and real balances, according to:

\[
E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) + \eta \ln(m_t) - \chi \frac{1}{1 + \gamma} \ell_t^{1+\gamma} \right] \right),
\]

\[0 < \beta < 1, \quad \eta > 0, \quad \gamma > 0, \quad \chi > 0.
\]

The farmer has access to the production technology

\[y_t = A_t \ell_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (PRF)\]

Productivity, \(A_t\), follows an AR(1) process in logs:

\[\hat{a}_t \equiv \ln(A_t/A_{ss}) = \phi \hat{a}_{t-1} + \epsilon_t, \quad 0 \leq \phi < 1. \quad (TS)\]

Farmers receive lump-sum transfers, \(S_t = P_t s_t\), from the government. They hold two assets: money, and capital, \(k_t\). Capital earns a real return of \((1+r)\), with \(\beta (1 + r) = 1\), and is used only as a storage device.

The nominal money supply \(M_t\) evolves according to

\[M_t = (1 + \theta_t)M_{t-1}, \quad \theta > -1. \quad (MS)\]

In equilibrium, lump-sum transfers equal seigniorage:

\[s_t = \frac{M_t - M_{t-1}}{P_t}. \quad (GBC)\]

(a) In nominal terms, the farmer’s budget constraint is

\[M_t + P_t c_t + P_t k_{t+1} = P_t (s_t + A_t \ell_t^{1-\alpha}) + M_{t-1} + (1 + r) P_t k_t.\]

Dividing both sides of this equation by \(P_t\), we get

\[m_t + k_{t+1} + c_t = s_t + A_t \ell_t^{1-\alpha} + \frac{m_{t-1}}{1 + \pi_t} + (1 + r) k_t. \quad (FBC)\]

Write the yeoman farmer’s problem as a Lagrangean:

\[E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) + \eta \ln(m_t) - \chi \frac{1}{1 + \gamma} \ell_t^{1+\gamma} \right. \right.
\]

\[\left. + \mu_t \left( s_t + A_t \ell_t^{1-\alpha} + \frac{m_{t-1}}{1 + \pi_t} + (1 + r) k_t - m_t - k_{t+1} - c_t \right) \right] \bigg). \]

The first-order conditions are

\[\mu_t = \frac{1}{c_t}, \quad \mu_t \ell_t^{1-\alpha} = \chi \ell_t^{\gamma},\quad \mu_t = \eta \frac{1}{m_t} + \beta E_t \left( \frac{1}{1 + \mu_{t+1}} \right),\quad \mu_t = \beta (1 + r) E_t \left( \mu_{t+1} \right) \quad \text{(FOC1, FOC2, FOC3, FOC4)}\]
(b) Combining equations (FOC1) and (FOC2) yields the time allocation condition
\[
\frac{1 - \alpha}{\chi} A_t \frac{1}{c_t} = \ell^\gamma + \alpha. \tag{LL}
\]
Combining equations (FOC1), (FOC3) and (FOC4) yields the Euler equations for capital and money

\[
\frac{1}{c_t} = E_t \left( \frac{1}{c_{t+1}} \right), \tag{EEK}
\]
\[
\frac{1}{c_t} = \eta \frac{1}{m_t} + \beta E_t \left( \frac{1}{c_{t+1}} \cdot \frac{1}{1 + \pi_{t+1}} \right). \tag{EEM}
\]
Finally combining equations (FBC) and (GBC) yields the resource constraint
\[
k_{t+1} = (1 + r)k_t + A_t \ell_t^{1-\alpha} - c_t. \tag{RC}
\]
(c) Let lower-case letters with carats "\(^\cdot\)" denote deviations of logged variables around their steady state values. It follows from equation (LL) that
\[
\exp\left( \hat{\ell}_t \right) \equiv \frac{\ell_t}{\ell_{ss}} = \left( \frac{1 - \alpha}{\chi} \right)^\zeta A_t^{\zeta} c_t^{-\zeta} = \exp\left( \zeta \hat{a}_t \right) \exp\left( -\zeta \hat{c}_t \right),
\]

with \( \gamma \equiv 1/(\gamma + \alpha) > 0 \). Logging both sides yields
\[
\hat{\ell}_t = \zeta (\hat{a}_t - \hat{c}_t). \tag{LL'}
\]
Proceeding similarly, it follows from equation (PRF') that
\[
\hat{y}_t = a_t + (1 - \alpha) \hat{\ell}_t = (1 + \lambda) \hat{a}_t - \lambda \hat{c}_t, \tag{PRF'}
\]
with \( \lambda \equiv (1 - \alpha)\zeta \).
(d) Rewrite equation (EEK) as
\[
\frac{c_{ss}}{c_t} = E_t \left( \frac{c_{ss}}{c_{t+1}} \right). \tag{EEK'}
\]
Logging both sides yields
\[
\hat{c}_t \approx - \ln \left( E_t \left( \frac{c_{ss}}{c_{t+1}} \right) \right) \approx E_t (\hat{c}_{t+1}),
\]
with the approximation in the last step holding when the variance of \( 1/c_{t+1} \) is small. Next, rewrite equation (EEM) as
\[
\exp(- \ln c_t) = \eta \exp(- \ln m_t) + \beta E_t \left( \exp(- \ln c_{t+1}) \cdot \exp(- \ln[1 + \pi_{t+1}]) \right). \]
Implicitly differentiate this equation:

\[-\exp(-\ln c_t)d\ln c_t = -\eta \exp(-\ln m_t)\ln m_t + \beta E_t\left(-\exp(-\ln c_{t+1}) \cdot \exp(-\ln[1+\pi_{t+1}]) \times [d\ln c_{t+1} + d\ln(1+\pi_{t+1})]\right)\]

so that

\[-\frac{1}{c_{ss}} \hat{c}_t \approx -\eta \frac{1}{m_{ss}} \hat{m}_t + \beta E_t \left(-\frac{1}{c_{ss}} \cdot \frac{1}{1+\pi_{ss}} (\hat{c}_{t+1} + \hat{\pi}_{t+1})\right)\]

where \(\hat{\pi}_t = \ln\left(\frac{1 + \pi_t}{1 + \pi_{ss}}\right)\) denotes inflation deviations. Using the steady-state version of equation (EEM), \(\beta/(1+\pi_{ss}) = 1 - \eta c_{ss}/m_{ss}\), and inserting equation (EEK) this simplifies to

\[\hat{c}_t \approx \eta \frac{c_{ss}}{m_{ss}} \hat{m}_t + \beta E_t \left(\frac{1}{1+\pi_{ss}} (\hat{c}_{t+1} + \hat{\pi}_{t+1})\right)\]

yielding

\[\hat{m}_t = \hat{c}_t - \omega E_t (\hat{\pi}_{t+1})\],

\[\omega \equiv \left(\frac{\eta c_{ss}}{m_{ss}}\right)^{-1} \left(1 - \frac{\eta c_{ss}}{m_{ss}}\right) = \left(\frac{\eta c_{ss}}{m_{ss}}\right)^{-1} \beta \frac{1}{1+\pi_{ss}} > 0.\]

(e) Holdings of real balances are increasing in consumption because agents prefer to evenly allocate their resources between consumption and real balances. Real balances are decreasing in expected inflation because higher inflation increases the opportunity cost of holding money.

10. One-sided search with benefit expiry and re-entitlement

Time: Discrete, infinite horizon.

Demography: A single infinite lived worker.

Preferences: The worker is risk neutral (i.e. \(u(x) = x\)) and discounts the future at the rate \(r\).

Endowments: The endowments of the worker depend on his state within the labor market. There are 4 possible states determined by employment and eligibility status for benefits.

- When unemployed and eligible for benefits he receives flow income \(b\) per period, with probability \(\alpha\) he receives a job offer and with probability \(\gamma\) he becomes ineligible for benefits. (Note: getting a job and becoming ineligible are mutually exclusive events. Assume \(\alpha + \gamma < 1\).)
• When unemployed and ineligible for benefits he receives flow income $s < b$ per period ($s$ is called subsistence allowance). He continues to receive job offers with probability $\alpha$ per period.

• All jobs pay the same wage, $w > b$. A worker who gets hired while eligible for benefits becomes employed and eligible for benefits. He receives flow income $w$ per period and with probability $\lambda$ he loses his job becoming unemployed and eligible for benefits.

• A worker who gets hired while ineligible for benefits becomes employed and ineligible for benefits. He receives flow income $w$ per period, with probability $\lambda$ he loses his job becoming unemployed and ineligible for benefits, and with probability $\gamma$ he becomes eligible for benefits. (Note: losing a job and becoming eligible for benefits are mutually exclusive events. Assume $\lambda + \gamma < 1$.)

• The key is that you have to have a job to become eligible for benefits and you have to be unemployed to become ineligible.

a. Write down the flow asset value (flow Bellman) equations for the worker.

Unemployed and eligible

$$rV_u^e = b + \alpha(V_u^e - V_u^e) + \gamma(V_u^i - V_u^e)$$

Employed and eligible

$$rV_e^e = w + \lambda(V_u^e - V_e^e)$$

Unemployed and ineligible

$$rV_u^i = s + \alpha(V_e^i - V_u^i)$$

Employed and ineligible

$$rV_e^i = w + \lambda(V_u^i - V_e^i) + \gamma(V_e^i - V_e^e)$$

b. Show that if the capital gain associated with gaining eligibility when employed exceeds the capital loss associated with losing eligibility when unemployed, then the gain from employment over unemployment is larger for the ineligible than the eligible.

Differencing $rV_e^e - rV_u^e$ implies

$$(r + \alpha + \lambda)(V_e^e - V_u^e) = w - b - \gamma(V_u^i - V_u^e)$$

Differencing $rV_i^i - rV_u^i$ implies

$$(r + \alpha + \lambda)(V_i^i - V_u^i) = w - s + \gamma(V_e^i - V_i^i)$$

As $s < b$, $V_e^e - V_e^i > -(V_u^i - V_u^e)$ implies $V_e^i - V_u^i > V_e^e - V_u^e$.

b. Show that if the capital gain associated with gaining eligibility when employed exceeds the capital loss associated with losing eligibility when unemployed, then the gain from employment over unemployment is larger for the ineligible than the eligible.

Differencing $rV_e^e - rV_u^e$ implies

$$(r + \alpha + \lambda)(V_e^e - V_u^e) = w - b - \gamma(V_u^i - V_u^e)$$

Differencing $rV_i^i - rV_u^i$ implies

$$(r + \alpha + \lambda)(V_i^i - V_u^i) = w - s + \gamma(V_e^i - V_i^i)$$

As $s < b$, $V_e^e - V_e^i > -(V_u^i - V_u^e)$ implies $V_e^i - V_u^i > V_e^e - V_u^e$.

c. Draw a flow diagram showing the population movements between states when there is a continuum (mass 1) of similar workers.

There are many possible equations basically flows in and of any blob will have to be equal in steady state. Also flows in and out of say employment or ineligibility have to be equal too. There are many such equations but some are colinear with others.
If $n_u^e$ is the share of the population who are unemployed and eligible for benefits etc.

d. Write down a system of equations that can be solved for the steady state populations (you don’t need to solve them).

\[
\begin{align*}
    n_u^e + n_e^e + n_e^i + n_u^i & = 1 \\
    \lambda n_e^e & = (\alpha + \gamma) n_u^e \\
    \alpha n_u^i & = (\lambda + \gamma) n_e^i \\
    \gamma n_e^e & = \gamma n_e^i
\end{align*}
\]

e. Without solving for all of the numbers, what is the unemployment rate? (**Hint:** use the diagram)

Because the flow rates between unemployment and employment are $\alpha$ and $\lambda$. The standard result applies so that

\[
\alpha n_u = \lambda (1 - n_u)
\]

where $n_u$ is the mass of unemployed workers. Then

\[
n_u = \frac{\lambda}{\alpha + \lambda}.
\]