Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.

1. The U.S. will likely have to increase its military activities in the Middle East. This should cause output to rise.

2. Suppose that the data shows that times of high consumer and firm borrowing are also times of high economic growth. We could then conclude that financial constraints are an important determinant of economic activity.

3. What Macroeconomic models miss is that consumers do not want to borrow during recessions

4. Policies used to bring us out of one recession sow the seeds of the next.

5. During a liquidity trap, it is possible that increases in money supply can cause deflation.

6. We observe that during the Great Recession, housing prices drop substantially. Meanwhile, mortgage interest rate also drop. Therefore, a drop in interest rates causes declines in housing price.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Consider an economy populated by a large number of yeoman farmers. Each farmer’s preferences over consumption and leisure are:

\[ E_t \left( \sum_{j=0}^{\infty} \beta^j \left[ \ln (c_{t+j}) + \chi \ln (1 - \ell_{t+j}) \right] \right), \quad 0 < \beta < 1, \quad \chi > 0. \]

Let \( I_t \) denote the overall state of the economy, a random vector governed by a Markov process with the stationary transition density \( f(I_t, I) \). Let \( q(I_t, I_{t+1}) \) be the kernel used to price the one-step-ahead contingent claim that delivers one unit of the consumption good when \( I_{t+1} = I' \), and let \( z(I') \) denote the quantity of such claims held by the farmer. The farmer’s resources evolve according to

\[ \int z(I_{t+1}) q(I_{t+1}, I) dI_{t+1} + c_t = d_t \ell_t + z(I_t), \quad \text{(FBC)} \]

where \( d_t = d(I_t) > 0 \) is an exogenous shifter. Output is not storable—there is no capital.

(a) Write down the farmer’s problem in recursive form and find the first order conditions.

(b) Now suppose there are two types of farms, occurring in equal proportions. Half the farms have productivity level \( d_{1t} \), while the remainder have productivity level \( d_{2t} \).

i. Assuming that all farms have the same initial wealth, set up the social planner’s problem.

ii. Find the equations that characterize the optimal allocation.

(c) For any variable \( x \), let \( x_t = \frac{1}{2} (x_{1t} + x_{2t}) \) denote per capita averages taken across the two types of farms.

i. Using your answer to part (b), show that \( y_t = d_t \bar{\ell} \), where \( \bar{\ell} \) is constant.

ii. Is \( d_t \) the complete state vector for this economy, that is, does \( I_t = d_t \)? Explain.

(d) Consider the equilibrium allocation.

i. What is equilibrium consumption?

ii. Find the equilibrium value of \( q(I_{t+1}, I) \).

iii. In this model, is consumption insured against idiosyncratic productivity shocks? Is such insurance typically found in the real world? Why or why not?
8. Diamond OG model with immigration policy

We will investigate the possibility that the government has control over the rate of population growth, \( n \in [0, \bar{n}] \) through controlling the amount of immigration.

**Time:** discrete, infinite horizon, \( t = 1, 2, ... \)

**Demography:** A mass \( N_t \equiv N_0(1 + n)^t \) of newborns enter in period \( t \). Of these, \( N_{t-1} \) are native born and the rest are immigrants so that \( n \in [0, \bar{n}] \) is the rate of immigration. Everyone lives for 2 periods except for the first generation of old people who live for one.

**Preferences:** for the generations born in and after period 1;

\[
U(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1})
\]

where \( c_{i,t} \) is consumption in period \( t \) and stage \( i \) of life. For the initial old generation \( \check{U}(c_{2,1}) = \ln(c_{2,1}) \).

**Productive technology:** The production function available to firms is \( F(K, N) = K^\alpha N^{1-\alpha} \) where \( K \) is the capital stock and \( N \) is the number of workers employed. You may find it convenient to use the implied per young person production function, \( f(k) = k^\alpha \) where \( k \) is the capital stock per worker.

**Endowments:** Everyone has one unit of labor services when young. The initial old share an endowment, \( K_1 \) of capital so they have \((1 + n)k_1\) units each.

**Institutions:** There are competitive markets every period for labor and capital. (You can think of a single collectively owned firm which takes wages and interest rates as given.)

(a) Write down and solve the problem faced by the individuals born (or immigrating) in period \( t \).

(b) Write down and solve the representative firm’s problem in period \( t \).

(c) Write down the market clearing condition for capital and define a competitive equilibrium.

(d) Solve for an equation that characterizes the dynamics of the capital stock.

(e) Solve for the steady state(s) and obtain their dynamical properties.

(f) The government will use immigration policy to set the value for \( n \) (which is taken as given by the population). The government will maximize the utility of a representative household in a representative generation, \( U(c_{1,t}, c_{2,t+1}) \) taking as given the time \( t \) capital stock per worker, \( k_t \). Use the above results to write down an equation for this utility in terms of \( k_t \) and \( n \).

(g) Is there an optimal level of population growth? Explain your answer.
9. Consider the following version of the Sidrauski model. The economy is populated by a large number of identical yeoman farmers. Each farmer derives utility from consumption, $c_t$, real balances, $m_t = M_t/P_t$, and labor, $\ell_t$, according to

$$ E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln (c_t) + \eta \ln (m_t) - \frac{1}{1+\gamma} \ell_t^{1+\gamma} \right] \right), $$

$$ 0 < \beta < 1, \quad \eta > 0, \quad \gamma > 0, \quad \chi > 0. $$

The farmer has access to the production technology

$$ y_t = A_t \ell_t^{1-\alpha}, \quad 0 < \alpha < 1. $$

(Productivity, $A_t$, follows an exogenous AR(1) process in logs:

$$ \tilde{\alpha}_t \equiv \ln (A_t/A_{ss}) = \phi \tilde{\alpha}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1. $$

where $\{\varepsilon_t\}$ is an exogenous stationary martingale difference sequence. There is no population or productivity growth, and the population is normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

Farmers receive lump-sum transfers, $S_t = P_t s_t$, from the government; a larger value of $s_t$ implies an increase in farm income. They hold two assets: money, and capital, $k_t$, which earns a real return of $(1 + r)$, with $\beta (1 + r) = 1$. Note that capital is used only as a storage device, and not as a factor of production. The farmer also faces the usual initial, non-negativity and No-Ponzi-Game conditions.

The nominal money supply $M_t$ evolves according to

$$ M_t = (1 + \theta_t) M_{t-1}, \quad \theta > -1. $$

(MS)

In equilibrium, lump-sum transfers equal seigniorage:

$$ s_t = \frac{M_t - M_{t-1}}{P_t}. $$

(GBC)

The farmer, however, takes $s_t$ as given.

(a) Let $\pi_t = P_t/P_{t-1} - 1$ denote the rate of inflation. Set up the yeoman farmer’s problem in real terms, and find the first order conditions.

(b) Find: the labor allocation condition; the Euler equation for money; the Euler equation for capital; and the resource constraint.

(c) Let letters with carats “$^\wedge$” denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$ \tilde{\ell}_t = \zeta (\tilde{\alpha}_t - \tilde{\gamma}_t), \quad \zeta > 0, $$

$$ \tilde{y}_t = (1 + \lambda) \tilde{\alpha}_t - \lambda \tilde{\gamma}_t, \quad 0 < \lambda < \zeta. $$

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(d) Let $\hat{\pi}_t = \ln \left( \frac{1 + \pi_t}{1 + \pi_{s,t}} \right)$ denote inflation deviations. Log-linearize the Euler equations for money and capital to show that

$$\hat{m}_t = \hat{c}_t - \omega E_t (\hat{\pi}_{t+1})^\prime, \quad \omega > 0.$$ 

**Hint:** at low levels of variance,

$$\ln (E (X_t)) \approx E (\ln (X_t)).$$

(e) Interpret your answer to part (d).

10. **One-sided search with benefit expiry and re-entitlement**

   **Time:** Discrete, infinite horizon.

   **Demography:** A single infinite lived worker.

   **Preferences:** The worker is risk neutral (i.e. $u(x) = x$) and discounts the future at the rate $r$.

   **Endowments:** The endowments of the worker depend on his state within the labor market. There are 4 possible states determined by employment and eligibility status for benefits.

   - When *unemployed and eligible for benefits* he receives flow income $b$ per period, with probability $\alpha$ he receives a job offer and with probability $\gamma$ he becomes ineligible for benefits. (Note: getting a job and becoming ineligible are mutually exclusive events. Assume $\alpha + \gamma < 1$.)

   - When *unemployed and ineligible for benefits* he receives flow income $s < b$ per period ($s$ is called subsistence allowance). He continues to receive job offers with probability $\alpha$ per period.

   - All jobs pay the same wage, $w > b$. A worker who gets hired while eligible for benefits becomes *employed and eligible for benefits*. He receives flow income $w$ per period and with probability $\lambda$ he loses his job becoming unemployed and eligible for benefits.

   - A worker who gets hired while ineligible for benefits becomes *employed and ineligible for benefits*. He receives flow income $w$ per period, with probability $\lambda$ he loses his job becoming unemployed and ineligible for benefits, and with probability $\gamma$ he remains employed and becomes eligible for benefits. (Note: losing a job and becoming eligible for benefits are mutually exclusive events. Assume $\lambda + \gamma < 1$.)

   - The key is that you have to have a job to become eligible for benefits and you have to be unemployed to become ineligible.

a. Write down the flow asset value (flow Bellman) equations for the worker. (Notation: use $V^x_y$ for the value to being in eligibility status, $x = e, i$ and employment status $y = u, e.$)
b. Show that if the capital gain associated with gaining eligibility when employed exceeds the capital loss associated with losing eligibility when unemployed, then the gain from employment over unemployment is larger for the ineligible than the eligible.

c. Draw a flow diagram showing the population movements between states when there is a continuum (mass 1) of similar workers.

d. Write down a system of equations that can be solved for the steady state populations (you don’t need to solve them).

e. Without solving for all of the numbers, what is the unemployment rate? (Hint: look at the diagram)