Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.

1. As long as utility is separable in real balances and consumption
leisure, money is neutral.

2. Changes in measured total factor productivity not accompanied by changes in technology are evidence of labor hoarding.

3. Search models of unemployment give insight into why employment has recovered much more slowly after the financial crisis than after previous recessions.

4. No macroeconomic model argues that lack of financing can bring about recession. If prospective investment is profitable, then saving rises to finance it.

5. There is no reason to suppose that tax increases are any worse for the economy than the same amount of spending cuts.

6. The principle objective of monetary policy should be to avoid deflation.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Asset Pricing with Habit Formation. Consider the following application of the Lucas tree model. The preferences of the representative consumer over current and lagged consumption are

\[ E_0 \left( \sum_{t=0}^{\infty} \beta^t \ln(c_t - \sigma c_{t-1}) \right), \quad 0 < \beta < 1, \quad 0 \leq \sigma < 1. \]

Output is produced by an infinite-lived tree: each period, the tree produces \( d_t \) units of non-storable output. The growth rate of dividends follows a stationary process, with

\[ G_t \equiv d_t/d_{t-1} = \exp(\varepsilon_t), \]

where \( \varepsilon_t \) is an i.i.d. continuous non-negative random variable with \( E(\exp(\varepsilon_t)) = \bar{G} > 1 \).

Each consumer begins life owning one tree.

Let \( p_t = p(G_t, d_t) \) be the price at time \( t \) of a title to all future dividends from a tree. Let \( R_t^{-1} = R^{-1}(G_t) \) be the time-\( t \) price of a risk-free discount bond that pays one unit of consumption at time \( t+1 \) under any future state. Finally, let \( x_t \) denote the consumer’s financial resources, which she allocates between stocks, bonds and consumption.

(a) Write down the Bellman equation for the consumer’s problem. What are the state variables in this expression? Why are they included?

(b) Find the first order conditions for the consumer’s problem.

(c) Find the equilibrium bond price. Using \( d_{t+1} = G_{t+1}d_t \), etc., express your answer completely in terms of growth rates. Next, suppose for the moment that \( G_t = \bar{G} \), \( \forall t \). How does the bond price depend on the average rate of output growth, \( \bar{G} \)? How does it depend on the habit persistence parameter \( \sigma \)?

(d) Assuming that \( p(G_t, d_t) = p(G_t)d_t \), find \( p(G_t) \) and verify the assumption. (Hint: When solving the difference equation for stock prices, you will have some voluminous terms to manipulate. You may find it useful to define a variable that collects a lot of these terms under a single, simple heading.)
8. Diamond OG model with inherited capital

**Time:** discrete, infinite horizon

**Demography:** A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in period $t$. Everyone lives for 2 periods except for the first generation of old people who live for one.

**Preferences:** for the generations born in and after period 1;

$$U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \beta u(c_{2,t+1})$$

where $c_{i,t}$ is consumption in period $t$ and stage $i$ of life. For the initial old generation $\bar{U}(c_{2,1}) = u(c_{2,1})$.

**Productive technology:** The production function available to firms is $F(K, N)$ where $K$ is the capital stock and $N$ is the number of workers employed. $F(.,.)$ has constant returns to scale, is twice differentiable, strictly increasing in both arguments, concave and satisfies the Inada conditions. You may find it convenient to use the implied per young person production function, $f(k)$, where $k$ is the capital stock per worker. After production occurs, a proportion $\phi$ of period $t$ capital stock is left over. This is distributed lump-sum to the next generation of old people.

**Endowments:** Everyone has one unit of labor services when young. Old people cannot work but do receive a share of the unworn out capital stock from the previous period. The initial old share an endowment, $K_1$, of capital.

**Institutions:** There are competitive markets every period for labor and capital. (You can think of a single collectively owned firm which takes wages and interest rates as given.)

(a) Write down and solve the problem faced by the individuals born in period $t$. (Ignore the possibility of negative savings.)

(b) Write down and solve the representative firm’s problem in period $t$.

(c) Write down the market clearing condition for capital and define a competitive equilibrium.

(d) Solve for an implicit equation that characterizes the dynamics of the capital stock.

(e) Write down the Planner’s problem and obtain the relevant first order conditions. (Assume the planner treats each generation equally.)

(f) Under what conditions is the steady state competitive equilibrium Pareto optimal?

(g) Explain why this condition is affected by the size of $\phi$. 
9. Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t;$$

$$Y_t = G_t^\alpha (Z_t L_t)^{1-\alpha}, \quad 0 < \alpha < 1,$$

(PRF)

where: $\Pi_t$ is profit; $L_t$ is labor; $W_t$ is the real wage; $Y_t$ is output; and $G_t$ is government spending. Productivity, $Z_t$, follows an AR(1) process in logs:

$$\hat{z}_t \equiv \ln (Z_t/Z_{ss}) = \phi \hat{z}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1,$$

(TS)

where $\{\varepsilon_t\}$ is an exogenous stationary martingale difference sequence. There is no population or productivity growth, and the population is normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption, $C_t$, and labor are given by

$$E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln (C_t) - \frac{1}{1+\gamma} L_t^{1+\gamma} \right] \right),$$

$$0 < \beta < 1, \quad \gamma > 0, \quad \chi > 0.$$

Households receive labor income and profits from firms. They pay lump-sum taxes, $H_t$, to the government. Households earn a gross return of $(1 + r) K_t$ on their assets, $K_t$, with $\beta (1 + r) = 1$. As usual, assume that assets held at the beginning of period $t + 1$, $K_{t+1}$, are chosen in period $t$. Note that capital is used only as a storage device, and not as a factor of production. Households face the usual initial, non-negativity and No-Ponzi-Game conditions.

The government follows a balanced budget rule:

$$H_t = G_t.$$  \hspace{1cm} (BB)

(a) Write down the social planner’s problem for this economy as a dynamic programming problem. Find all the first order conditions, including the one for government spending, $G_t$.

(b) Express government spending as a function of productivity and labor. Using these substitutions, find the equations that characterize the equilibrium allocation: the labor leisure condition, the Euler equation, and the aggregate resource constraint. (Hint: You may find it useful to define $A = \alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)} = (1-\alpha)\alpha^{\alpha/(1-\alpha)}$.)

(c) How might you calibrate $\alpha$ and $\gamma$?

(d) Let lower-case letters with carats “$\hat{\cdot}$” denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$\hat{t}_t = \theta_1 \hat{z}_t - \theta_2 \hat{c}_t,$$

$$\hat{y}_t = \lambda_1 \hat{z}_t - \lambda_2 \hat{c}_t,$$

$$\hat{g}_t = \zeta_1 \hat{z}_t - \zeta_2 \hat{c}_t$$

where all coefficients are positive. (Hint: You should be able to express your answers with much simpler notation.)
(e) Suppose that the steady state consumption-to-capital ratio, \( \frac{C_{ss}}{K_{ss}} \), is \( \psi > r \). Show that the steady state private output-to-capital ratio, \( \frac{(Y_{ss} - G_{ss})}{K_{ss}} \), is \( (\psi - r) \). Using this result, log-linearize the capital accumulation equation to show that
\[
\hat{k}_{t+1} = (1 + r) \hat{k}_t + \omega_1 \hat{z}_t - \omega_2 \hat{z}_t, \quad \omega_1, \omega_2 > 0.
\]

(f) It is often argued that government spending should be countercyclical. Is such a claim consistent with the model? Briefly explain.

10. Mortensen-Pissarides with out-of-laborforce workers

Time: Discrete, infinite horizon

Demography: A mass of 1 of ex ante identical workers with infinite lives and a large mass of firms who create individual vacancies.

Preferences: Workers and firms are risk neutral (i.e. \( u(x) = x \)). The common discount rate is \( r \). The value of leisure for workers is \( b \) utils per period. The cost of holding a vacancy for firms is \( a \) utils per period.

Productive Technology: Matched firm/worker pairs produce \( p \) units of the consumption good per period. With probability \( \lambda \) each period, jobs (filled or vacant) experience a catastrophic productivity shock and the job is destroyed. Assume \( p > b \).

Matching Technology: With probability \( m(\theta) \) each period unemployed workers encounter vacancies. Here \( \theta = v/u \), \( v \) is the mass of vacancies and \( u \) is the mass of unemployed workers. The function \( m(.) \) is increasing concave and \( m(\theta) < 1 \) for all \( \theta \). Also \( \lim_{\theta \to 0} m'(\theta) = 1 \), \( \lim_{\theta \to \infty} m'(\theta) = 0 \), and \( m(\theta) > \theta m'(\theta) \). The rate at which vacancies encounter unemployed workers is then \( m(\theta)/\theta \).

Out-of-laborforce transitions: With probability \( \delta \) each period unemployed workers drop out of the labor force. (Dropping out and encountering a vacancy are mutually exclusive events; assume that \( \delta + m(\theta) < 1 \)). Being out of the laborforce means they continue to receive the value of leisure, \( b \), but they no longer encounter firms. With probability \( \mu \), workers who are out of the laborforce re-enter as unemployed workers.

Institutions: The terms of trade are determined by generalized Nash bargaining with firms having bargaining power \( \phi \).

(a) Using \( X_w \) as the value to being out the labor force, write down the set of flow value equations for workers and firms for given values of the wage, \( w \), and labor market tightness, \( \theta \).

(b) Define a steady state free-entry equilibrium and specify a system of equations that characterize the equilibrium. (Do not solve the system at this point.)

(c) Draw a diagram showing how workers flow between each of the possible states. Write down a system of equations that can be used to solve for the steady state population in each state. Solve for the population, \( u \), who are unemployed.
(d) Here we will consider a measure of wage dispersion introduced by Hornstein et al (2011). They define the mean-min ratio, $Mm$, as the ratio of the mean to the lowest observed wage in a labor market. Here only one wage, $w$, is observed. Clearly $w$ is the mean wage. Rather than the lowest wage observed we will use the reservation wage. (It is the lowest wage any worker would work for.) So

$$Mm = \frac{w}{rU_w}$$

where $U_w$ is the value to unemployment. Let $b = \rho w$ ($\rho$ is called the replacement ratio and is simply a new parameter that helps with the algebra).

1. Solve for $Mm$ using the equations you wrote down in part a. (Hint: You only need the workers’ equations.)
2. Show that $Mm$ is increasing in $\delta$ and explain your answer.