Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.

1. An increase in government spending will raise interest rates.

2. Tax cuts are better for stimulating the economy than spending increases because they can be implemented more quickly.

3. As long as the study is not focused on the transactions role of money, putting money in the utility function is simply a convenient short-cut.

4. Inefficient levels of unemployment cannot occur in a competitive general equilibrium.

5. Hyperinflation is impossible in an economy populated by rational agents.

6. During the past few years, household savings rates have increased, even as the economy has grown very slowly. This is evidence against the permanent income hypothesis.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Optimal Growth with depreciation spillovers.

Time: Discrete, infinite horizon

Demography: A continuum, mass normalized to 1, of (representative) infinite lived consumer/worker households. There is a large number, mass $N$, of firms owned jointly and equally by the households.

Preferences: The instantaneous household utility function over, consumption, $c_t$ is $u(c_t)$. The function $u(.)$ is twice differentiable, strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology over capital and labor such that the output $y_t$ associated with the input of capital, $k_t$, and labor, $h_t$, is given by $y_t = F(k_t, h_t)$. The production function, $F(., .)$ is twice differentiable, strictly increasing in both arguments, concave and exhibits constant returns to scale. This means that aggregate output, $Y_t$ is given by $F(K_t, H_t)$ where $K_t$ is the aggregate capital stock and $H_t$ is the aggregate labor supply. Capital depreciates with usage. The extent of depreciation, however, depends on the aggregate amount of labor with which it is employed. Thus a firm that employs $k_t$ units of capital will have $(1 - \delta_t)k_t$ units remaining at the end of the period where $\delta_t = \delta H_t$ and $0 < \delta < 1$.

Endowments: Each households’ initial capital stock is $k_0$. Each household has 1 unit of time which it can devote in any proportions to labor or leisure. (This means that for any household, $h_t \in [0, 1]$ and, because the number of households is normalized to 1, $H_t \in [0, 1]$.)

(a) Write down the Planner’s problem for this economy and solve for the system of difference equations (in $C_t$, $K_t$ and $H_t$) that characterize its solution.

(b) Assuming the Planner is not at a corner solution, what (local) condition controls the relationship between $H_t$ and $K_t$ (i.e. what determines the sign of $\frac{dH_t}{dK_t}$)?

Now consider the decentralized economy.

(c) Write down and solve the representative household’s optimization problem (given $r_t$ and $w_t$, the interest rate and hourly wage, respectively).

(d) Write down and solve the representative firms’s optimization problem.

(e) Write down the market clearing conditions, define a competitive equilibrium and solve for the system of equations in $C_t$, $K_t$ and $H_t$ that specify equilibrium dynamics. ($C_t$ is aggregate consumption.)

(f) What is the relationship between $H_t$ and $K_t$ in the competitive equilibrium? Explain any difference from the Planner’s solution.
8. (Inspired by Shiller, 1981.) Consider the following variant of the Lucas tree model. The preferences of the representative consumer are

$$E_0 \left( \sum_{t=0}^\infty \beta^t u(c_t) \right), \quad 0 < \beta < 1,$$

where $c_t \geq 0$ denotes consumption. Assume that $u'(\cdot) \geq 0$ and $u''(\cdot) \leq 0$.

Output is produced by an infinite-lived tree: each period, the tree produces $d_t$ units of non-storable output. The dividend $d_t$ is a non-negative random variable governed by an i.i.d. stochastic process with mean $\bar{d}$ and variance $\sigma_d^2$. Let $p_t = p(d_t)$ be the price at time $t$ of a title to all future dividends from a tree, and let $R^{-1}_t = R^{-1}(d_t)$ be the price of a risk-free discount bond that pays one unit of consumption at time $t + 1$.

(a) Write down the consumer’s problem in recursive form and find the first order conditions.

(b) Find the equilibrium prices $p(d_t)$ and $R^{-1}(d_t)$.

(c) Suppose that the flow utility function is linear: $u(c_t) = \chi c_t$, $\chi > 0$. Find the risk-free interest rate $R^{-1}(d_t)$, and $\bar{R}^{-1} \equiv E(R^{-1}(d_t))$, the unconditional mean of the risk-free rate. Next, suppose the flow utility function is logarithmic: $u(c_t) = \ln(c_t)$. Find the risk-free interest rate, $R^{-1}(d_t)$ and its unconditional mean: $\bar{R}^{-1} = E(R^{-1}(d_t))$. Recalling that $\{d_t\}$ is i.i.d., show that $\bar{R}^{-1}$ is higher when preferences are logarithmic. Explain your findings.

(d) Find the price of a share of stock, $p(d_t)$, and its unconditional variance, for both the linear and logarithmic specifications.

(e) Let $p_t^* = \sum_{j=1}^\infty \bar{R}^{-j} d_{t+j}$ denote the realized discounted dividend stream. Assuming that flow utility is linear, what is the unconditional variance of $p_t^*$? Is the variance of the stock price $p_t$ greater than or less than the variance of the realized sum $p_t^*$? Explain.

(f) Repeat the analysis in part (e) under the assumption of logarithmic flow utility. To facilitate your analysis, let $\tilde{\beta} > \beta$ denote the average price of a discount bond under logarithmic utility.
9. Consider the following stochastic version of the Sidrauski Model. There are a fixed number of price-taking producers that solve

$$
\max_{\ell_t \geq 0} X_t = P_t y_t - W_t \ell_t,
$$

$$
y_t = \ell_t^{1-\alpha}, \quad 0 < \alpha < 1,
$$

(PRF)

where: $X_t$ is nominal profit; $\ell_t$ is labor; $W_t$ is the nominal wage; $y_t$ is real output; and $P_t$ is the nominal price level. The population and number of firms are normalized to 1.

The preferences of the representative household over consumption, $c_t$, real balances, $m_t = M_t / P_t$, and labor, $\ell_t$, are given by

$$
E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) + \eta \ln(m_t) - \chi \frac{1}{1+\gamma} \ell_t^{1+\gamma} \right] \right),
$$

$$
0 < \beta < 1, \quad \eta > 0, \quad \gamma > 0, \quad \chi > 0.
$$

Households receive labor income and profits from firms. They receive lump-sum transfers, $s_t = P_t s_{t-1}$, from the government; a larger value of $s_t$ implies an increase in income. They hold two assets: money, and capital, $K_t = P_t k_t$, which earns a real return of $(1+r)$, with $\beta (1+r) = 1$. Assume that capital held at the beginning of period $t+1$, $k_{t+1}$, is chosen in period $t$. Note that capital is used only as a storage device, and not as a factor of production. Households face the usual initial, non-negativity and No-Ponzi-Game conditions.

Households also pay consumption taxes. The consumption tax rate is $\tau_t$, so that the household’s total consumption-related expenditures are $(1 + \tau_t) c_t$. The gross tax rate $1 + \tau_t$ follows an AR(1) process around the log of its steady state value:

$$
\tilde{\tau}_t \equiv \ln \left( \frac{(1 + \tau_t)}{(1 + \tau_{SS})} \right) = \phi \tilde{\tau}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1,
$$

(TS)

where the exogenous shock $\varepsilon_t$ is i.i.d. and zero-mean.

The nominal money supply $M_t$ evolves according to

$$
M_t = (1 + \theta_t) M_{t-1},
$$

(MS)

with $\theta_t$ following an exogenous stochastic process. In equilibrium, lump-sum transfers equal seigniorage and consumption tax revenues:

$$
s_t = \frac{M_t - M_{t-1}}{P_t} + \tau_t c_t.
$$

(GBC)

(a) Solve the producer’s problem.

(b) Let $\pi_t = P_t / P_{t-1} - 1$ denote the rate of inflation, and let $w_t = W_t / P_t$ denote the real wage. Derive the consumer’s flow budget constraint in both nominal and real terms. Combining budget constraints, derive the aggregate (physical) resource constraint for the overall economy.

(c) Set up the household’s problem and find the optimality conditions.
(d) Find the equilibrium labor optimality condition. Is money neutral in this model? Why or why not? Conversely, does the consumption tax affect inflation? Why or why not?

(e) Let letters with carats “^” denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

\[
\begin{align*}
\hat{\ell}_t &= -\theta (\hat{\tau}_t + \hat{c}_t), \quad \theta > 0, \\
\hat{y}_t &= -\lambda (\hat{\tau}_t + \hat{c}_t), \quad 0 < \lambda < \theta.
\end{align*}
\]

(f) New York City Mayor Michael Bloomberg has proposed to prohibit the sale of large soda drinks in an effort to reduce the consumption of unhealthy, sugary beverages. Because smaller sodas have a higher per unit (per ounce) cost, this effectively a tax on a consumption good. According to the analysis above, would Mayor Bloomberg’s proposal work?

10. Mortensen-Pissarides with ex ante surplus sharing

**Time:** Discrete, infinite horizon

**Demography:** A mass of 1 of ex ante identical workers with infinite lives and a large mass of firms who create individual vacancies.

**Preferences:** Workers and firms are risk neutral (i.e. \(u(x) = x\)). The common discount rate is \(r\). The value of leisure for workers is \(b\) utils per period. The cost of holding a vacancy for firms is \(a\) utils per period.

**Productive Technology:** Matched firm/worker pairs produce \(p\) units of the consumption good per period. With probability \(\lambda\) each period, jobs (filled or vacant) experience a catastrophic productivity shock and the job is destroyed. Assume \(p > b\).

**Matching Technology:** Unemployed workers encounter vacancies at the rate \(m(\theta)\) where \(\theta = v/u, v\) is the mass of vacancies and \(u\) is the mass of unemployed workers. The function \(m(.)\) is increasing and concave, and \(m(\theta) < 1\) for all \(\theta\). Also \(\lim_{\theta \to 0} m'(\theta) = 1, \lim_{\theta \to \infty} m'(\theta) = 0\), and \(m(\theta) > \theta m'(\theta)\). The rate at which vacancies encounter unemployed workers is then \(m(\theta)/\theta\).

**Institutions:** The terms of trade are determined by symmetric division of the negotiable surplus which means that the wage \(w = (p + b)/2\).

(a) Write down the set of flow value equations or Bellman equations for workers and firms.

(b) Show that the outside option for the worker cannot bind (i.e. that \(w > rU_w\) where \(U_w\) is the value to unemployment for a worker).

(c) Define a steady state free-entry equilibrium and solve for a single equation in \(\theta\).

(d) Obtain an expression for the unemployment rate, \(u\) in terms of \(\theta\).

(e) How do changes in \(a\) and \(b\) affect the level of unemployment? Provide intuition for your results.