Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.

1. The fact that consumption is procyclical is evidence that government spending shocks cannot be an important source of business cycles.

2. Utility cannot be negative.

3. Unemployment cannot be too low.

4. U.S. citizens will soon be receiving a tax rebate from the Federal government. If this tax rebate is followed by an increase in personal consumption expenditures, we will have evidence that the Ricardian Equivalence Proposition is false.

5. Steady-state and equilibrium are the same concept.

6. Researchers have shown that when a person retires, his or her consumption experiences a discrete drop. This drop is inconsistent with rational economic behavior.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Consider a model of search with high- and low-destruction-rate jobs:

**Time:** Discrete, infinite horizon.

**Demography:** One worker who lives forever.

**Preferences:** The worker is risk-neutral (i.e. \( u(x) = x \)). He discounts the future at the rate \( r \).

**Endowments:** While unemployed the worker gets \( b \) units of the consumption good as benefits every period. With probability \( \alpha \) he also gets a wage offer. A proportion \( \gamma \) of the offers are for a low-destruction-rate job, \( \lambda = \lambda_L \) and the proportion \((1 - \gamma)\) are for a high-destruction-rate job, \( \lambda = \lambda_H > \lambda_L \). The wages in both types of job are the same, \( w > b \). (I.e. the wage distribution is degenerate; no need for a distribution function.)

(a) Write down the value functions for the worker under the proviso (assumption) that he accepts both types of job. (So you do not need the \( \max \{..,.\} \) operator.)

(b) What is the reservation wage in terms of the parameters of the model \( (b, r, w, \alpha, \gamma, \lambda_L, \lambda_H) \)?

(c) Are there parameter values under which the worker turns down the high-destruction-rate job? Explain your answer.

(d) Now, assuming a population mass 1 of workers, draw the steady-state flow diagram and solve for the steady-state unemployment rate.
8. Consider the following version of the Lucas tree model. The economy is populated by two representative consumers, Hans and Frans. Hans’ preferences are logarithmic:

\[ E_0 \left( \sum_{t=0}^{\infty} \beta^t \ln (c_t) \right), \quad 0 < \beta < 1, \]

where \( c_t \) denotes consumption. Frans’ preferences are linear:

\[ E_0 \left( \sum_{t=0}^{\infty} \beta^t c_t \right), \quad 0 < \beta < 1. \]

Output is produced by an infinite-lived tree: each period, the tree produces \( d_t \) units of non-storable output, following the stationary one-step transition density \( f(d_{t+1}, d_t) \). The economy starts off with Hans and Frans each owning one tree.

Let \( p_t = p(d_t) \) be the price at time \( t \) of all future dividends from a tree, and let \( q(d', d) \) be the price of a one-step-ahead contingent claim that delivers one unit of fruit when \( d_t = d \) and \( d_{t+1} = d' \).

(a) Let \( s_t \) denote the number of trees that a consumer owns at the beginning of time \( t \), let \( y(d') \) denote the “purchasing kernel” for contingent claims, and let \( x_t \) denote the consumer’s financial resources.

1. Using \( u(c) \) to denote a “generic” utility function, write down Bellman’s functional equation for the consumer’s problem, and derive the “generic” Euler equations associated with stocks and contingent claims.

2. Derive exact Euler equations for Hans and Frans.

(b) Find the equilibrium values of \( q(d_{t+1}, d_t) \) and \( p(d_t) \). (The stock price should not include any expected future prices.)

(c) Using a recursive approach, define an equilibrium in this economy. Describe the market clearing conditions in as much detail as possible. (Hint: You can assume that Hans’ and Frans’ initial endowments are consistent with a recursive equilibrium, but you don’t need to be precise.)

(d) Let \( R_t^{-1} = R^{-1}(d_t) \) be the time-\( t \) price of a risk-free discount bond that pays one unit of consumption at time \( t+1 \) under any future state. Use the pricing kernel to find \( R_t^{-1} \).

(e) Define the return on stocks, \( R_t^S \), by

\[ R_t^S = \frac{p_{t+1} + d_{t+1}}{p_t}, \]

and define the equity premium, \( e_t \), by \( e_t = E_t \left( R_t^S \right) - R_t \). Calculate the equity premium, and briefly explain your findings.
9. Consider the following model

**Time:** Discrete, infinite horizon

**Demography:** \( N \) newborns in every period (no population growth). Everyone lives for 2 periods except for the first generation of old people.

**Preferences:** For the generations born in and after period 0,

\[
U_t(c_{1t}, c_{2t+1}) = \log(c_{1t}) + \beta \log(c_{2t+1}),
\]

where \( c_{it} \) is consumption in period \( t \) and stage \( i \) of life. For the initial old generation

\[
\bar{U}(c_{20}) = \log(c_{20}).
\]

**Endowments:** of the single, perishable consumption good are \( \{e_{1t}, e_{2t+1}\} = \{e, e\} \) for all \( t \). (That is, everyone gets the same endowment in youth and old age and every generation gets the same endowment.)

(a) Define, characterize and solve for the competitive equilibrium with inside money.

(b) Now suppose the country is comprised of 2 states, Franklin and Hamilton. Half of the country’s population lives in each state. The Governor of Franklin wants to introduce a pay-as-you-go social security scheme. For a given value of \( d \), the constant per capita transfer from young to old (which could be negative), define, characterize and solve for the competitive equilibrium with inside money. Assume all endogenous variables are constant over time. Note: there are no restrictions to trade within the country and contracts are enforceable across state borders.

(c) Characterize (but do not solve) the problem the Governor of Franklin must solve to obtain the optimal transfer, \( d^* \), if the first generation of old are ignored.

(d) Assuming parameters are such that the transfer \( d^* \) is Pareto improving (i.e., \( \beta > 1 \)), how will this policy affect the people of Hamilton (not Franklin)?
10. (Inspired by Edge, Laubach and Williams, 2007.) Consider the following simplified version of a stochastic growth model, where technology is imperfectly observed. The representative price-taking firm solves
\[
\max_{L_t \geq 0} E_t (\Pi_t) = E (Y_t| I_t) - W_t L_t,
\]
where \(\Pi_t\) is profit; \(L_t\) is labor; \(Z_t\) is the exogenous technology level; and \(W_t\) is the real wage; and \(E_t (\cdot) = E (\cdot| I_t)\) denotes the conditional expectation operator. The preferences of the representative household over consumption, \(C_t\), and labor, \(L_t\), are given by:
\[
E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln (C_t) - \chi L_t \right] \right),
\]
\(0 < \beta < 1, \quad \chi > 0\).
Households receive labor income and profits from firms. Households earn a gross return of \((1 + r) K_t\) on their assets, \(K_t\), with \(\beta (1 + r) = 1\). As usual, assume that assets held at the beginning of period \(t + 1\), \(K_{t+1}\), are chosen in period \(t\). Note that capital is used only as a storage device, and not as a factor of production. Households spend their income on consumption and investment in capital. Households also face the usual initial, non-negativity and No-Ponzi-Game conditions.
The log of technology follows an AR(1) process
\[
z_t \equiv \ln (Z_t) = \phi z_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1,
\]
where \(\varepsilon_t\) is i.i.d., zero-mean and normally-distributed, with variance \(\sigma_\varepsilon^2 > 0\). However, at time \(t\) households and firms only observe \(A_t = Z_t U_t\), where \(u_t \equiv \ln (U_t)\) is i.i.d., zero-mean and normally-distributed, with variance \(\sigma_u^2 > 0\). \(U_t\) and \(Z_t\) are independent at all leads and lags. As a result
\[
a_t \equiv \ln (A_t) = z_t + u_t,
\]
\(z_t \sim N(0, \sigma_z^2); \quad u_t \sim N(0, \sigma_u^2); \quad \text{cov} (z_t, u_s) = 0, \quad \forall t, s\).
Agents do observe lagged values of \(Z\), however, so that
\[
I_t = \{A_t, Z_{t-1}, K_t\}.
\]
While hours, real wages and consumption are determined in advance of observing \(Z_t\) (and thus depend on \(A_t\)) profits and future assets \((K_{t+1})\) are not known until \(Z_t\) is realized.

(a) Define \(\tilde{z}_t = E (z_t| I_t)\). Show that \(\tilde{z}_t\) can be written as
\[
\tilde{z}_t = \beta_1 a_t + \beta_2 z_{t-1} = \beta_1 (\varepsilon_t + u_t) + \phi z_{t-1},
\]
\(\beta_1 = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_u^2}\).

Hint: Recall that \(\sigma_z^2 = \sigma_\varepsilon^2 / (1 - \phi^2)\).
(b) Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation. **Hint:** although the changes in the information structure will affect your derivation, the first order conditions will otherwise be standard.

(c) Let lower-case letters with carats “^” denote deviations of logged variables around their steady state values. Show that log-linear approximations for labor and output are

\[
\hat{\ell}_t = \theta [\hat{z}_t - \hat{c}_t], \quad (LM)
\]
\[
\hat{y}_t = (1 + \lambda) \hat{z}_t - \lambda \hat{c}_t + \eta_t, \quad (PRF')
\]
\[
\eta_t \equiv z_t - \hat{z}_t.
\]

**Hint:** at low levels of variance,

\[
\ln (E_t (Z_t)) \approx E_t (\ln (Z_t)).
\]

(d) It is often argued that an important feature of the U.S. economy is that output **growth** is positively autocorrelated: high output growth today is likely to lead to high output growth in the near future. Based on equations (LM), (PRJ) and (PRF'), do you think this model can generate a positive correlation in output growth? Why or why not?