Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer.

1. The “equity premium puzzle” vanishes if one assumes that asset-holders are extremely risk-averse.

2. Explain why Ricardian Equivalence should not, in general, hold true in economies in which individuals have finite lives.

3. Economic analysts have long argued that comovement is an important feature of the business cycle, as output and employment rise and fall together across different industries. Furthermore, consistently some industries are cyclical, but others are not. Unfortunately neither general equilibrium theory nor Keynesian macroeconomic theory explains comovement.

4. Suppose that an econometric study shows that, after controlling for government spending, periods of high deficits are associated with rapid economic growth. This result should be taken as evidence against the Ricardian Equivalence Proposition.

5. Explain how money might be neutral (with respect to output) but not super-neutral.

6. General equilibrium models of business cycles always imply that recessions are efficient responses to bad events.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. The production function for this economy is given by
\[ Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha}, \quad 0 < \alpha < 1, \]  
(PRF)  
with productivity, \( Z_t \), following a random walk in logs:
\[ z_t \equiv \ln (Z_t) = z_{t-1} + \varepsilon_t, \]  
(TS)  
The preferences of the representative household are
\[ E_0 \left( \sum_{t=0}^{\infty} \beta^t [\ln (C_t) - \chi L_t] \right) \]  
The capital accumulation equation is
\[ K_{t+1} = Y_t - C_t. \]  
(CA)  

(a) In recursive form, the social planner’s problem is
\[ V(K_t, Z_t) = \max_{(C_t, L_t)} \ln (C_t) - \chi L_t + \beta E_t \left( V(K_{t+1}^\alpha (Z_{t+1} L_{t+1})^{1-\alpha} - C_t, Z_{t+1}) \right). \]  
The first order conditions are
\[ \frac{1}{C_t} = \beta E_t \left( \frac{\partial V(K_{t+1}, Z_{t+1})}{\partial K_{t+1}} \right), \]  
(FOC1)  
\[ \chi = \beta E_t \left( \frac{\partial V(K_{t+1}, Z_{t+1})}{\partial K_{t+1}} \right) (1 - \alpha) \frac{Y_t}{L_t}. \]  
(FOC2)  
Using Benveniste and Scheinkman’s results, we find that
\[ \frac{\partial V(K_t, Z_{t+1})}{\partial K_t} = \beta E_t \left( \frac{\partial V(K_{t+1}, Z_{t+1})}{\partial K_{t+1}} \right) \alpha \frac{Y_t}{K_t}. \]  
Inserting equation (FOC1), this reduces to
\[ \frac{\partial V(K_t, Z_{t+1})}{\partial K_t} = \left( \frac{1}{C_t} \right) \alpha \frac{Y_t}{K_t}, \]  
and (FOC1) becomes
\[ \frac{1}{C_t} = \beta E_t \left( \frac{1}{C_{t+1} \alpha} \frac{Y_{t+1}}{K_{t+1}} \right). \]  
(EE)  
Combining equations (FOC1) and (FOC2) yields
\[ \chi = \frac{1}{C_t} (1 - \alpha) \frac{Y_t}{L_t} \]  
which reduces to
\[ L_t = \left( \frac{1 - \alpha}{\chi} \right) \frac{Y_t}{C_t}. \]  
(LL)  
The capital accumulation equation (CA) was derived above when formulating the social planner’s problem.
(b) We will conjecture that labor hours are constant: \( L_t = L, \forall t \), and that aggregate expenditures on consumption and investment are constant shares of output, so that

\[
C_t = \Pi_1 Y_t, \quad \text{(CRULE)}
\]

\[
K_{t+1} = (1 - \Pi_1) Y_t. \quad \text{(KRULE)}
\]

Imposing these conjectures on equation (EE) implies that

\[
\frac{1}{\Pi_1 Y_t} = \beta E_t \left( \frac{1}{\Pi_1 Y_{t+1}} \alpha \frac{Y_{t+1}}{(1 - \Pi_1) Y_t} \right),
\]

which simplifies to

\[
1 - \Pi_1 = \beta \alpha \Rightarrow \Pi_1 = 1 - \beta \alpha.
\]

Imposing these conjectures on equation (LL) shows that

\[
L_t = \left( \frac{1 - \alpha}{\chi} \right) \frac{Y_t}{\Pi_1 Y_t} = \frac{(1 - \alpha)}{(1 - \beta \alpha) \chi} = L.
\]

(c) Working with equations (KRULE) and (PRF), we get

\[
\frac{Z_{t+1}}{Z_t} \left( \frac{K_{t+1}}{Z_t} \right) = (1 - \Pi_1) \frac{Y_t}{Z_t} = (1 - \Pi_1) \frac{1}{Z_t} K_t^\alpha (Z_t L_t)^{1 - \alpha}.
\]

This simplifies to

\[
\exp \left( \ln \left( \frac{Z_{t+1}}{Z_t} \right) \right) = (1 - \Pi_1) L^{1 - \alpha} \exp \left( \tilde{k}_{t+1} \right).
\]

Taking logs and imposing equation (TS), we get:

\[
\tilde{k}_{t+1} = \ln \left( (1 - \Pi_1) L^{1 - \alpha} \right) + \alpha \tilde{k}_t - \varepsilon_{t+1}
\]

\[
\equiv (1 - \alpha) \Omega + \alpha \tilde{k}_t - \varepsilon_{t+1}
\]

\[
\Rightarrow \tilde{k}_{t+1} - \Omega = \alpha \left( \tilde{k}_t - \Omega \right) - \varepsilon_{t+1}. \quad \text{(CA')}
\]

Returning to equation (PRF), we have

\[
\frac{Y_t}{Z_t} = \left( \frac{K_t}{Z_t} \right)^\alpha L^{1 - \alpha},
\]

which can be written as

\[
\tilde{y}_t = \alpha \left( \tilde{k}_t - \Omega \right) + \alpha \Omega + \ln \left( L^{1 - \alpha} \right)
\]

\[
\equiv \Lambda + \alpha \left( \tilde{k}_t - \Omega \right), \quad \text{(PRF')}
\]
(d) Let’s consider the effects of a one-unit increase in $\varepsilon_t$.

1. The effects of this shock are.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\varepsilon_t$</th>
<th>$z_t$</th>
<th>$\hat{k}_t - \Omega$</th>
<th>$\alpha(\hat{k}_t - \Omega) + z_t$</th>
</tr>
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<td>-1</td>
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<td>1.00</td>
<td>$-\alpha$</td>
<td>$1 - \alpha^2$</td>
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<tr>
<td>3</td>
<td>0.00</td>
<td>1.00</td>
<td>$-\alpha^2$</td>
<td>$1 - \alpha^3$</td>
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2. Noting that the log of total output is given by $\ln(Y_t) = \Lambda + \alpha(\hat{k}_t - \Omega) + z_t$, we see that it takes several periods for the shock to have its full effect. This is because it takes time to build up the capital stock. In the limit, output rises one for one with the permanent shock, consistent with the balanced growth properties of the model.

8. **One-sided search with long-term unemployment.**

**Time:** Discrete; infinite horizon.

**Demography:** A continuum, mass 1, of infinite-lived workers. Workers can either be employed, short-term unemployed or long-term unemployed.

**Preferences:** Workers are risk-neutral:

$$U(\{c_t\}_{t=s}) = \sum_{t=s}^{\infty} \beta^{t+1-s} c_t,$$

where $\{c_t\}_{t=s}$ is the sequence of consumption quantities at each time period $t$. $\beta < 1$ is the discount factor, and $r = (1 - \beta)/\beta$ is the discount rate.

**Endowments:**

- Employed workers all receive the single wage, $w$. Each period, with probability $\lambda$, they are laid-off.
- Short-term unemployed workers (those who just lost their jobs) receive $b$ units of the consumption good per period. Also, each period with probability $\alpha_0$ they get a job offer, with probability $\gamma$ they become long-term unemployed and with probability $1 - \alpha_0 - \gamma$ they remain in short-term unemployment.
- Long-term unemployed workers also receive $b$ units of the consumption good per period. Each period with probability $\alpha_1 < \alpha_0$ they get a job offer.

(a) Since the wage distribution is degenerate, the model will have a meaningful solution only if unemployed workers accept their job offers with probability 1. This leads to the following Bellman equations:

$$rV_u = b + \alpha_0(V_e - V_u) + \gamma(V_t - V_u)$$

$$rV_i = b + \alpha_1(V_e - V_i)$$

$$rV_e = w + \lambda(V_u - V_e)$$
where $V_i$ is the value to being in state $i = e, u, l$ (employment, short-term un-
employment, long-term unemployment respectively). Algebra reveals that the
difference between the short-term and long-term reservation wage is

$$r(V_u - V_l) = \frac{(\alpha_0 - \alpha_1)(w - b)}{(r + \alpha_1)(r + \alpha_0 + \gamma + \lambda) + \gamma \lambda} > 0.$$  

(There may be other ways to express the numerator.)

(b) The difference $V_u - V_l$ is decreasing in both $\gamma$ and $\alpha_1$. As $\gamma$ increases, workers in
short-term unemployment are more likely to transition to long-term unemploy-
ment, making the two states more similar. As $\alpha_1$ increases, the probability of
receiving a job offer in long-term unemployment moves closer to the probability
of receiving a job offer in short-term unemployment, making the two states more
similar.

(c) The flow rates are as follows

(d) Let $e, u, l$ denote the fraction of workers that are employed, short-term unem-
ployed, and long-term unemployed, respectively. It follows from the diagram
above that in a steady state, where flows in and out of each employment state
must balance,

$$\alpha_0 u + \gamma u = \lambda e,$$

$$\lambda e = \alpha_0 u + \alpha_1 l,$$

$$\gamma u = \alpha_1 l.$$ 

It must also be the case that

$$u + e + l = 1.$$ 

Combining, we get

$$l = \frac{\gamma}{\alpha_1} u,$$

$$e = 1 - u - l,$$

$$= 1 - \frac{\lambda}{\alpha_0 + \gamma} e - \left(\frac{\gamma}{\alpha_1}\right) \frac{\lambda}{\alpha_0 + \gamma} e,$$

$$= \frac{\alpha_1 (\alpha_0 + \gamma)}{\gamma (\alpha_1 + \lambda) + \alpha_1 (\alpha_0 + \lambda)}.$$


(e) Note that
\[
\frac{de}{d\gamma} = \frac{\alpha_1 \lambda (\alpha_1 - \alpha_0)}{[\gamma (\alpha_1 + \lambda) + \alpha_1 (\alpha_0 + \lambda)]^2} < 0
\]
so that the proportion of the population who are employed is decreasing with $\gamma$. As $\gamma$ increases, an increasing fraction of the short-term unemployed transition into long-term unemployment, where the odds of getting a job offer are lower. This means that once a worker loses his job, the probability of receiving a new job offer is lower. In equilibrium, this lowers the number of employed.

9. The preferences of the representative consumer are
\[
E_0 \left( \sum_{t=0}^{\infty} \beta^t \left( fc_t - \frac{g}{2} c_t^2 \right) \right), \quad 0 < \beta < 1, \quad f, g > 0,
\]
where $c_t$ denotes consumption. You can assume that $0 < c_t < f/g$.

(a) Writing the consumer’s problem as a Lagrangean, we get
\[
V(x_t, d_t) = \min_{\lambda_t \geq 0} \max_{c_t \geq 0, s_{t+1}, b_{t+1}} fc_t - \frac{g}{2} c_t^2 + \lambda_t \left( x_t - c_t - p_t s_{t+1} - R_t^{-1} b_{t+1} \right)
+ \beta E_t \left( V \left( [p_{t+1} (d_{t+1}) + d_{t+1}] s_{t+1} + b_{t+1}, d_{t+1} \right) \right)
\]
The FOC for an interior solution are:
\[
\begin{align*}
 f - gc_t &= \lambda_t, \\
 \lambda_t p_t &= \beta E_t \left( \frac{\partial V \left[ t + 1 \right]}{\partial x_{t+1}} [p_{t+1} + d_{t+1}] \right), \\
 \lambda_t R_t^{-1} &= \beta E_t \left( \frac{\partial V \left[ t + 1 \right]}{\partial x_{t+1}} \right).
\end{align*}
\]
Since (following Benveniste-Scheinkman),
\[
\frac{\partial V \left[ t \right]}{\partial x_t} = \lambda_t,
\]
the Euler equations are
\[
\begin{align*}
 p_t [f - gc_t] &= \beta E_t ([f - gc_{t+1}] [p_{t+1} + d_{t+1}]), \quad \text{(EE1)} \\
 R_t^{-1} &= \frac{1}{f - gc_t} \beta E_t (f - gc_{t+1}). \quad \text{(EE2)}
\end{align*}
\]

(b) Given the random variable $d_0$, the conditional distribution $F (d_{t+1} | d_t)$, and the initial endowments $s_0 = 1$ and $b_0 = 0$, a recursive rational expectations equilibrium consists of pricing functions $p (d)$ and $R^{-1} (d)$, a value function $V (x, d)$, and decision functions $c (x, d)$, $s (x, d)$, and $b (x, d)$ such that:

1. Given the pricing functions $p (d)$ and $R^{-1} (d)$, the value and policy functions $V (x, d)$, $c (x, d)$, $s (x, d)$, and $b (x, d)$ solve the consumer’s problem.
2. Markets clear: for $x = p(d) + d$, $c(x, d) = d$, $s(x, d) = 1$, and $b(x, d) = 0$.

(c) Imposing our answer to part (b), we have that equilibrium stock prices follow

$$p_t = \beta E_t \left( \frac{f - gd_{t+1}}{f - gd_t} (p_{t+1} + d_{t+1}) \right).$$  \hspace{1cm} (EE1')$$

Since $0 < \beta < 1$, it makes sense to solve equation (EE) forward:

$$E_t \left( (1 - \beta L^{-1}) (p_t [f - gd_t]) \right) = \beta E_t \left( [f - gd_{t+1}] d_{t+1} \right)$$

$$p_t [f - gd_t] = \frac{1}{1 - \beta L^{-1}} \beta E_t \left( [f - gd_{t+1}] d_{t+1} \right) + b_t$$

$$= E_t \left( \sum_{j=1}^{\infty} \beta^j [f d_{t+j} - gd_{t+j}^2] \right) + b_t,$$

with the “bubble term” $b_t$ obeying

$$E_t (b_{t+1}) = \beta^{-1} b_t.$$

We now require that

$$\lim_{j \to \infty} E_t \left( \beta^j p_{t+j} [f - gd_{t+j}] \right) = 0, \quad \forall t. \hspace{1cm} (TVC)$$

(TVC) will be satisfied only if $b_t = 0$ and the price of a stock is

$$p_t = p(d_t) = \frac{1}{f - gd_t} E_t \left( \sum_{j=1}^{\infty} \beta^j [f d_{t+j} - gd_{t+j}^2] \right).$$

(d) The expected rate of return on stocks is

$$E_t (R^S_t) = E_t \left( \frac{p_{t+1} + d_{t+1}}{p_t} \right).$$

Rewriting equation (EE1') yields:

$$1 = \beta E_t \left( \frac{f - gd_{t+1}}{f - gd_t} \left( \frac{p_{t+1} + d_{t+1}}{p_t} \right) \right)$$

$$= \beta E_t \left( \frac{f - gd_{t+1}}{f - gd_t} R^S_t \right)$$

$$= \beta E_t \left( \frac{f - gd_{t+1}}{f - gd_t} \right) E_t (R^S_t) + Cov_t \left( \beta \frac{f - gd_{t+1}}{f - gd_t}, R^S_t \right).$$

In equilibrium, equation (EE2) becomes:

$$R_t = \frac{f - gd_t}{\beta E_t (f - gd_{t+1})}.$$
Inserting this result and rearranging, we get:

\[ 1 = R_t^{-1} E_t \left( R_t^S \right) + \text{Cov}_t \left( \frac{\beta f - gd_{t+1}}{f - gd_t}, R_t^S \right), \]

\[ E_t \left( R_t^S \right) = R_t - R_t \text{Cov}_t \left( \frac{\beta f - gd_{t+1}}{f - gd_t}, R_t^S \right), \]

\[ = R_t - \frac{f - gd_t}{E_t (f - gd_{t+1})} \text{Cov}_t \left( \frac{\beta f - gd_{t+1}}{f - gd_t}, R_t^S \right), \]

\[ = R_t + g \frac{\text{Cov}_t (d_{t+1}, R_t^S)}{E_t (f - gd_{t+1})} \]

\[ \equiv R_t + e_t, \]

so that the expected return on stocks equals the expected return on the risk-free bond plus the risk-premium \( e_t \). As long as periods of high dividends are also periods of high realized stock returns, \( \text{Cov}_t \left( d_{t+1}, R_t^S \right) \) and thus \( e_t \) will be positive, and stocks will provide higher returns than bonds. Risk-aversion implies that the most desirable assets yield well when marginal utility is high. If \( \text{Cov}_t \left( d_{t+1}, R_t^S \right) > 0 \), however, stocks will yield their highest returns when marginal utility is low (\( d_t \) is high). This will lead investors to demand higher returns on stocks.

(e) Hall (1978) argued that under certain linear-quadratic preferences, consumption should follow a martingale process: \( c_t = E_t (c_{t+1}) \). Our results show, however, that unless the process for dividends is itself a martingale, \( c_t = d_t \) will exhibit serial correlation. The reason for the discrepancy is that Hall adopts a partial equilibrium perspective, where the rate of return is the constant \( R \), while we allow the rate of return \( R_t \) to vary in order to clear markets in general equilibrium. (Hall also assumes that \( \beta R = 1 \), but relaxing this assumption does not change his main message.)

10. Consider the following model:

**Time:** Discrete, infinite horizon.

**Demography:** \( N \) newborns in every period. Everyone lives for 2 periods except for the first generation of old people (no population growth).

**Preferences:** for the generations born in and after period 0:

\[ U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \beta u(c_{2,t+1}), \]

where \( c_{i,t} \) is consumption in period \( t \) and stage \( i \) of life, \( u(.) \) is increasing strictly concave and twice differentiable. For the initial old generation \( U(c_{2,0}) = u(c_{2,0}) \).

**Endowments:**

- \( \{e_t, e_t\} \), of the single perishable consumption good, where \( e_t = \gamma^t e, \gamma > 0, t \geq 0 \).
- \( H \) per original old person of “cash”.

We will consider two different institutional arrangements.
(a) We first consider an economy with inside money only (i.e., cash has no value).

**Definition 1** A competitive (inside money) equilibrium is a sequence of prices \( \{R_{t+1}\}_1^\infty \) and an allocation \( \{c_{1,t}, c_{2,t}\}_0^\infty \) such that given prices, the allocation solves the individual consumption problem and markets clear.

Turning to specifics, (all but the first generation of old) consumers solve

\[
\max_{c_{1,t}, c_{2,t+1}, s_t} u(c_{1,t}) + \beta u(c_{2,t+1}),
\]

s.t.

\[
c_{1,t} = e_t - s_t,
\]

\[
c_{2,t+1} = e_t + R_{t+1} s_t,
\]

or

\[
\max_{s_t} u(e_t - s_t) + \beta u(e_t + R_{t+1} s_t).
\]

The first-order condition is

\[-u'(c_{1,t}) + \beta u'(c_{2,t+1}) R_{t+1} = 0.\]

In the absence of money or any other long-lived asset (such as capital), there will be no trade across generations. Moreover, agents within a generation are identical. Market clearing thus requires: \( s_t = 0 \). So \( c_{1,t} = c_{2,t+1} = e_t \) and it follows from the Euler Equation that

\[R_{t+1} = \frac{1}{\beta} \]

(b) Next we consider an economy where outside money (cash) is used (ignore inside money).

**Definition 2** A competitive (inside money) equilibrium is a sequence of prices \( \{p_t\}_1^\infty \) and an allocation \( \{c_{1,t}, c_{2,t}\}_0^\infty \) such that given prices, the allocation solves the individual consumption problem and markets clear.

Turning to specifics, (all but the first generation of old) consumers solve

\[
\max_{c_{1,t}, c_{2,t+1}, s_t} u(c_{1,t}) + \beta u(c_{2,t+1}),
\]

s.t.

\[
c_{1,t} = e_t - \frac{M_{t+1}^d}{p_t},
\]

\[
c_{2,t+1} = e_t + \frac{M_{t+1}^d}{p_{t+1}},
\]

where \( M_{t+1}^d \) is the nominal value of cash held at the end of period \( t \) and \( p_t \) is the price of goods in period \( t \). The first-order condition is

\[-u'(c_{1,t}) + \beta u'(c_{2,t+1}) \frac{p_t}{p_{t+1}} = 0.\]
Market clearing requires

\[ c_{1,t} + c_{2,t} = (1 + \gamma) e_t, \]
\[ M_{t+1}^d = H. \]

We will focus on stationary equilibria. Stationarity requires that

\[ \frac{p_t}{p_{t+1}} = 1 + g, \text{ for all } t, \]

so that the rate of return on money holdings is constant. Imposing this condition on the Euler Equation, it follows that a stationary equilibrium is characterized by:

\[ u'(e_t - \frac{H}{p_t}) = (1 + g)\beta u'(e_t + \frac{H}{p_{t+1}}), \]

so that

\[ u'(e_t - \frac{H}{p_t}) = (1 + g)\beta u'(e_t + \frac{H(1+g)}{p_t}). \]

(c) Restricting attention to \( u(.) = \log(.) \), we wish to find an equilibrium in which \( c_{1,t} \) and \( c_{2,t+1} \) both grow at the (gross) rate \( \gamma \). Imposing the budget constraint and the definition of \( g \), this can be expressed as

\[ \frac{c_{1,t+1}}{c_{1,t}} = \frac{e_{t+1} - \frac{H}{p_{t+1}}}{e_t - \frac{H}{p_t}} = \frac{e_{t+1} - \frac{H(1+g)}{p_t}}{e_t - \frac{H}{p_t}} = \gamma. \]

As \( e_{t+1} = \gamma e_t \), the equality holds iff \( (1 + g) = \gamma \).

To find \( p_t \), we impose \( (1 + g) = \gamma \) on the final version of the Euler Equation. With log utility, this implies

\[ \left[ e_t - \frac{H}{p_t} \right]^{-1} = \beta \gamma \left[ e_t + \frac{H\gamma}{p_t} \right]^{-1}, \]

which can be rearranged to yield

\[ p_{t+1} = \frac{H(1 + \beta)}{(\beta \gamma - 1) e_t}. \]

This expression imposes the that \( \beta \gamma - 1 > 0 \). Otherwise the price level would be negative. This would imply transfers of goods from the old to the young, which would be sub-optimal for the first generation of old agents, who could never recoup their losses.